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A CONSIDERATION OF THE INERTIA OF THE ROTATING PARTS OF A TRAIN.

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The problem of calculating the motor capacity and the amount of power necessary to maintain a certain train service involves a consideration of where all the power which is developed by the motors is expended; of just how much is used in overcoming train resistance; how much in overcoming the force of gravity, and how much in overcoming the inertia. The train resistance is a variable quantity depending on the track, bearings, wind and speed. A considerable number of formulæ have been produced to assist in calculating the train resistance under different speeds, but at the best the amount to allow for train resistance is only approximate. The energy required to overcome the force of gravity and the inertia of the train, however, is susceptible of the most exact calculation, but the latter is seldom estimated correctly. There is one element in the inertia factor which has been almost entirely neglected, either because it has not been recognized at all or because its importance has not been appreciated. This feature is the inertia of the rotating parts of the train.

It has been generally understood that armatures of small diameter and light weight are desirable because their small fly-wheel capacity makes easier braking, but it has seldom been considered that this means also less power developed by the motor. A recent investigation of this subject has led to some very interesting results. It is found that the wheels, with their low speed, and the armatures, which usually revolve at a much higher rate than the wheels, together constitute an important element in the determination of the power required for operating the train. A specific instance will show this most clearly. Take the case of a double-truck car weighing 30 tons loaded. It has eight 33-inch wheels weighing about 700 lbs. each, and two motors rated, we will say, at 150 h.p. each. The radius of gyration of the wheels is about 77 per cent. of the radius of the wheel. The center of gyration of the wheel, therefore, moves at a rate 77 per cent. of that of the train. The flywheel effect of each wheel then is equal to a weight of $700 \times 77^2 = 415$ lbs., when reduced to the speed of the car. Eight wheels will, therefore, add 3,320 lbs. to the inertia weight of the car.

The armatures have each a flywheel effect of 1,400 lbs. at a radius of 6 inches. With a gear ratio of 18: 53 the center of gyration of the armature will move a distance of 925 feet for every revolution of the axle, or for a corresponding movement of the car of 8.6 feet. Its relative speed, is, therefore, $\frac{9.25}{8.6} = 1.08$ times the car speed, and the flywheel effect is therefore $1,400 \times 1.08^2 = 1,640$ lbs. reduced to the car speed. The two armatures thus add an equivalent weight of 3,280 lbs. to the inertia weight of the car.

The wheels and armatures together add an equivalent of $3,320 + 3,280 = 6,600$ lbs., or about 11 per cent., to the inertia weight of the car.

The following paragraph will show the effect of a change in the gear ratios:

With a gear ratio of 20 : 51 the flywheel effect of the armatures would be equivalent to the addition of 2,400 lbs. to the inertia weight of the car. This, together with that of the wheels, adds a total of 5,720 lbs., or about 9.5 per cent., to the inertia weight of the car.

From a considerable number of instances that have been taken, the flywheel effect of the rotating parts of an electric car is found to average about 10 per cent. of the inertia of the entire weight of the train. This means that 10 per cent. more energy is stored in every train than is accounted for by the dead weight; 10 per cent. more power is required for accelerating; 10 per cent. more energy is lost in braking, and the train resistance measured by the retardation in coasting is 10 per cent. below the true resistance.

The actual increase in energy supplied to a train on account of the flywheel effect of the rotating parts is the energy in these parts which is lost in braking. The relation this bears to the total power developed by the motors is dependent on the number of stops, the speed at the time the brakes are applied and on the energy absorbed by the train resistance. Where the stops are frequent, the energy lost in brakes may be from 50 per cent. to 75 per cent. of the entire power developed by the motors, in which case the energy required by the rotating parts will be from 5 to 7½ per cent. of the total.

There are two simple methods for including this item in the calculations. The first is by basing the calculations on a weight of car 10 per cent. heavier than the actual in determining the acceleration and drifting. The second is by assuming that the force required to produce a certain rate of acceleration is 10 per cent. higher than would be necessary if it were simply a dead weight. This is probably the simpler method, for it gives round numbers for calculations, as 10 per cent. added to 91.3 gives practically 100 lbs. per ton as the force required for accelerating at the rate of one mile per hour per second. All that is necessary then in correcting calculations for accelerations is to use this figure of 100 lbs. per ton instead of 91.3. It will give a good average correction, although if great accuracy is desired it will be preferable to calculate the flywheel effect of each of the rotating parts of the train separately. It will usually be found that for slower speed service, where the gear reduction is considerable, more than 10 per cent. will be required, while for high speed interurban work where it would really amount to very little anyway on account of the small number of accelerations, the amount to be added for the correction will be less than 10 per cent.

As will be readily recognized, this factor will also enter into the determination of the train resistance from the coasting line. Just how much of a correction will have to be made on this account depends somewhat on what is considered to be the train resistance. If this includes the friction in the motor then the train resistance obtained from the coasting line will be 10 per cent. lower than the actual train resistance. On this assumption, if the retardation in coasting is .2 miles per hour per second, the train resistance will be 20 lbs. per ton instead of 18.2 as calculated by the ordinary method. If, however, the train resistance does not include the friction of the motor, the correction necessary to be made for inertia of rotating parts will be small, because it will be nearly balanced by the motor friction. When the train is coasting, the inertia of the rotating parts is added to the inertia of the dead weight of the train in tending to keep up the speed. When the train is accelerating, or moving with power on the motors, the motor friction is taken into account in the efficiency curve of the motor, so that the train resistance to be used in calculating the acceleration should properly not include the motor friction. It will thus be seen that the error due to motor friction and the inertia of the rotating parts will tend to counterbalance each other in the determination of the train resistance from the coasting line. For accurate determinations, however, the train resistance should include a consideration of both the inertia of the rotating parts and the motor friction. The most accurate way to obtain this is to plot the friction curve of the motor and to obtain the inertia of the armature and wheels either from tests or from calculations based on the drawings.

It may be considered that this is an undue refinement, but if the matter is carefully investigated it will be found that the motor friction is a considerable portion of the total train resistance, in the same way as the inertia of the rotating parts is a considerable portion of the inertia of the train.

It is understood that general solutions for the railway problem may be offered which will give fair approximations of the motor capacity and the amount of power required. It is understood that any good engineer with a fair amount of experience can give a pretty good estimate of the power required for a given service even when considering the capacity of the motor according to the old horse power rating. But where accuracy is required, every known element should be considered at its proper value and there will still be at best enough variable quantities in the railway problem. The most reliable estimate for train resistance should be used; the inertia of the rotating parts should be obtained and considered; the weight of the entire train should be known; the acceleration and power curves should be carefully plotted and the average heating effect in the motors should be accurately determined before any large equipment is finally decided upon.