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ORIGINAL ARTICLE

A constitutive model for mechanical response characterization of pumpkin peel and flesh tissues under tensile and compressive loadings

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Abstract Enhancing quality of food products and reducing volume of waste during mechanical operations of food industry requires a comprehensive knowledge of material response under loadings. While research has focused on mechanical response of food material, the volume of waste after harvesting and during processing stages is still considerably high in both developing and developed countries. This research aims to develop and evaluate a constitutive model of mechanical response of tough skinned vegetables under postharvest and processing operations. The model focuses on both tensile and compressive properties of pumpkin flesh and peel tissues where the behaviours of these tissues vary depending on various factors such as rheological response and cellular structure. Both elastic and plastic response of tissue were considered in the modelling process and finite elasticity combined with pseudo elasticity theory was applied to generate the model. The outcomes were then validated using the published results of experimental work on pumpkin flesh and peel under uniaxial tensile and compression. The constitutive coefficients for peel under tensile test was $\alpha=25.66$ and $\beta=-18.48$ Mpa and for flesh $\alpha=-5.29$ and $\beta=5.27$ Mpa. under compression the constitutive coefficients were $\alpha=4.74$ and $\beta=-1.71$ Mpa for peel and $\alpha=0.76$ and $\beta=-1.86$ Mpa for flesh samples. Constitutive curves predicted the values of force precisely and close to the experimental values. The curves were fit for whole stress versus strain curve as well as a section of curve up to bio yield point. The modelling outputs had presented good agreement with the empirical values and the constructive curves exhibited a very similar pattern to the experimental curves. The presented constitutive model can be applied next to other agricultural materials under loading in future.

Keywords Pseudo elasticity · Tensile loading · Rheological response · Compressive loading · Finite elasticity · Uniaxial loading · Constitutive modelling · Post harvesting · Food processing

Introduction

Quality of food product varies with changes in physical and mechanical properties of food tissues. Parameters such as colour, shape, texture, hardness, tenderness and firmness can be changed after ripening due to the harvesting, post harvesting and processing operations on food tissues. Analysing the cause of these changes and their influence on food materials provides essential knowledge of food tissue damage and material loss during industrial processes. There have been previous studies on mechanical and physical properties of food materials, applying experimental, mathematical and computational approaches. Their main focus has been on linear elastic and viscoelastic behaviours (Lu and Puri 1992; Lu and Chen 1998) which is limited to the small deformation of tissue. Due to the soft nature of food material, large deformation and plastic changes need to be addressed and studied as a common reason to cause damage and loss on these materials. Although the rate of deformation directly changes regarding both type of process and properties of food materials, the response of tissues are complex. Constitutive models usually apply mathematical equations and theories to illustrate the behaviours of materials under different loading process. The aim in applying constitutive laws in studying the behaviours of food materials is to develop equations which display compatible results in comparison with the experimental response of materials.

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There are different mechanical loading types that agricultural tissues undergo during and after harvesting operations. Tensile, compression, vibration, impact and cutting are some of these processes. Post harvesting and processing stages are usually a combination of these loading types, in analysing material behaviours it is difficult to consider all different loading operations at once.

The responses of agricultural and biomaterials under large deformation conditions however, is described as nonlinear and inelastic (Qiong et al. 1989; Lu and Puri 1991, 1992; Gao et al. 1993; Lu and Chen 1998). The other important characteristic of food material is their time-dependent behaviours under loading conditions, in fact both current and past loading conditions play an important role in the behaviours of these materials under loading. Due to the complex and unique behaviours of these materials under loadings, it is not possible to describe stress versus strain relationships of all these materials with only single constitutive law. It is required to analyse the response of each group or class of these materials to the loading (Lu and Chen 1998). Additionally, in investigating response of food material under loading, there are specific concepts and assumptions that need to be considered.

There are exponential and polynomial equations that are used to describe the behaviour of bio material under loading conditions (Lu and Puri 1991, 1992; Fung 1993; Gao et al. 1993; Tang et al. 1997; Lu and Chen 1998). As Lu and Chen (Lu and Chen 1998) reported, the polynomial equations are used to model plant material behaviours while the exponential functions are used to describe response of living biological materials. One of the common theories, which has been used for plant material, is the Mooney-Rivlin equation; this theory considers the material behaviour as a viscoelastic response under small deformation rates. The results of stress relaxation tests have been used to develop and validate the Mooney-Rivlin relationship. Further, use of finite elasticity theory and the concept of pseud elasticity have been introduced to develop a new constitutive relationship for biological materials under large deformation (Lu and Chen 1998). It is essential to consider the plastic deformation happening on agricultural crop tissue regarding the actual behaviours of these materials under loading. In this study a constitutive equation that has been used previously for food materials under large deformation was selected and used for further data analysis to estimate coefficients of equations for pumpkin peel and flesh tissues. The core goal of this study was to determine a mathematical relationship between the deformation ratio and stress value for uniaxial loading of peels and flesh samples. To simplify the process of constitutive modelling, the following assumptions were considered and applied; behaviour of material is nonlinear, isotropic and homogenous under uniaxial loading. A series of experimental tests was performed in work completed by our research team including uniaxial tensile and compression tests on peel and flesh samples. The results of loading tests were used to fit a curve and determine the two coefficients of equation as described earlier.

This article presents the results of developing a nonlinear constitutive model for the stress versus strain relationship of pumpkin flesh and peel tissue as a tough skinned vegetable. The core goal of this study was to determine a mathematical equation which demonstrates a reasonable similar response to the uniaxial stress versus strain results obtained from experimental studies. The developed model was based on finite elasticity theory and illustrated a good agreement with the experimental curves. The authors of this article have recommended the same theory and formulation to be examined and established for other varieties of pumpkin as well as other members of tough skinned vegetables. The outcomes of this study in combination with experimental results were also applied as input data in the FE model of mechanical loading of pumpkin tissue.

Theory and calculation

Agricultural crops undergo different types and rates of loading during and after harvesting and processing stages. There are prior studies with a focus on properties of tissues under small and large deformation processes. The majority of these studies considered the behaviours of food material as linear elastic or viscoelastic under small deformation conditions (Lu and Chen 1998). In this study, the actual response of material considered and constitutive model developed based on it. For this reason a series of assumption were considered which were described in the following parts of this section. Regarding the high rate of moisture content in food materials, they are considered as incompressible materials (Gao et al. 1993; Hamann et al. 2006; Coburn and Pandit 2007; Miller et al. 2007; Daubert and Foegeding 2010). Previous studies on apple and potato tissues under external source of load indicated an incompressible behaviour (Mitsuhashi–Gonzalez et al. 1995, Scanlon and Long 1995). The Jap variety of pumpkin flesh and peel has a high rate of moisture content (87 and 83 % respectively) (Shirmohammadi 2013) similar to the fore-mentioned crops (potato and apple) (Mitsuhashi–Gonzalez et al. 1995, Scanlon and Long 1995). As a result, in constitutive modelling of pumpkin peel and flesh samples, it was assumed they are incompressible under uniaxial tensile and compression. This assumption has been made to simplify the mathematical study of these materials under loading. Also, the three dimensional equations were simplified as the compression and tensile experimentations were performed in axial direction of samples.

The next assumption was based on the pseudo elasticity concept in analysing the response of food materials under loading. Pseudo-elasticity is applied to describe behaviours of materials where it is not possible to analyse their behaviours with elasticity theory. Elastic materials in loading and unloading processes present a unique stress versus strain relationship, while in the case of pseudo-elastic materials there are different loading and unloading paths (Fung 1990; Lu and Chen 1998). The elastic materials return back to their original state after loads are removed and no plastic changes happens on the materials. Agricultural materials do not return to the original position after the unloading process, which is due to the cracks and discontinuities occurring on the tissue (Sitkei 1987). Although the volume of food damages on tissue varies with different parameters such as maturity and moisture content, there always will be an amount of hysteresis on tissue. Researchers have used the pseudo-elasticity concept for biological and food materials (Lu and Chen 1998; Sun and Sacks 2005; Peña and Doblaré 2009), this is a new attempt to apply this theory on tough skinned vegetable pumpkin peel and flesh tissues.

The other assumption was the isotropic response of tissue under loading, which was made to simplify the constitutive modelling process.

Nonlinear constitutive equations

Different constitutive laws have been applied to study and describe mechanical response of biological and food materials under mechanical loading. The Mooney-Rivlin equation was one of them that was applied to describe properties of food materials under loading, as Lu (Lu and Chen 1998) stated, this equation is not suitable for the tensile behaviours of biological materials. In addition, determination of two parameters of this equation is difficult as these parameters have inconsistent values (Lu and Puri 1992; Fung 1993). The current study focuses on the plastic deformation of tissue under compression, tensile and peeling process. The other applied approach in analysing behaviours of materials is to use finite elasticity theory (Lu and Chen 1998). Finite elasticity theory has been applied in constitutive modelling of biomaterials previously (Veronda and Westmann 1970; Lu and Chen 1998) where the assumption is based on strain energy function for elastic materials under large deformation. Finite elasticity theory is usually applied to the soft materials when they undergo large deformation due to the external loading. In this theory the final position of any particle "**X**" in a domain such as "φ" is determined as:

$$\varphi(X) = X + u(X) \tag{1}$$

Although the material is considered as elastic due to the large deformation of it under external loading, nonlinear phenomena are considered to be happened on the material body. In this theory the physical state of material is determined regarding the present deformation of material and not the history of it (Drozdov 1996).

Considering finite elasticity theory and the stress–strain relation of elastic materials (Lu and Puri 1992; Fung 1993):

$$\sigma_{ij} = \frac{\partial x_i}{\partial X_R}\frac{\partial x_j}{\partial X_s}\frac{\partial W}{\partial E_{RS}} - p\delta_{ij} \tag{2}$$

where in Eq. 2, σ_{ij}, δ_{ij}, E_{RS}, p, W, x_i, X_s are Cauchy stress, the Kronecker delta (equal 1 for $i=j$ and zero when $i\neq j$), the Lagrangian strain, hydrostatic pressure, strain energy function, spatial coordinates in Cartesian coordinate system with respect to the current configuration, spatial coordinates in Cartesian coordinate system with respect to the initial configuration, and i,j,R and S are indexes (values of 1,2,3 for x, y, or z) (Lu and Puri 1992).

The strain energy function for isotropic elastic materials with a three strain invariants, I_1, I_2 and I_3 for the right Cauchy-Green deformation tensor C (Spencer 2004):

$$W = W(I_1, I_2,\ I_3) \tag{3}$$

$$I_1 = {\lambda_1}^2 + {\lambda_2}^2 + {\lambda_3}^3 \tag{4}$$

$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 \tag{5}$$

$$I_3 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \tag{6}$$

In Eq. 3 to Eq. 6, I_1,I_2, *and* I_3 are three strain invariants and ${}_1,{}_2, and_3$ are three principal compression or stretches; the compression or stretch ratio is defined as the ratio of the deformed length over the original length of the line in three dimensional deformations. Since in this study the uniaxial loadings are investigated, the compression/extension ratio is described as the length after loading over the initial length of sample in the axial direction of samples:

$$\lambda = \frac{L}{L_0} = 1 \pm \frac{\Delta L}{L_0} \tag{7}$$

In Eq. 7, the plus is used for extension ratio under tensile loadings and minus for the compression ratio under compression loading.

Exponential and polynomial equations have been used to describe material properties of biological materials mathematically (Fung 1993). There are two types of function applied for constitutive modelling of food materials, including polynomial and exponential functions. In this study the exponential function has been used; which applies the concept of strain energy function (Lu and Chen 1998) for food products. The polynomial function which has been utilized for other food and biological materials (Lu and Puri 1991; Gao et al. 1993; Tang et al. 1997) was not considered in this study due to the complexity of determining strain energy constants. The emphasis in this study was on tensile and compressive properties of tissue and the failure phenomenon under large deformation, which happens in mechanical loading such as peeling process. The outcomes of this study was used in a study of FE modelling and simulation of mechanical peeling process and the mentioned focus was chosen in order to apply the outcomes of constitutive modelling in FE modelling and validation (Shirmohammadi 2013). For this reason the following strain energy function was used.

$$W = \frac{1}{2}\frac{\beta}{\alpha}\left(e^{\alpha trE^2}-1\right) \tag{8}$$

Eq. 8, α and β are the material parameters and they have been determined using results of uniaxial loading experiments. The trace of square matrix is:

$$trE^2 = E_{ij}E_{ij} \tag{9}$$

Substituting this function in the Cauchy stress formula:

$$\sigma_{ij} = -p\delta_{ij} + \beta\frac{\partial x_i}{\partial X_R}\frac{\partial x_j}{\partial X_s}E_{RS}e^{\alpha trE^2} \tag{10}$$

Considering the stress definition as load divided by the cross sectional area:

$$T_{zz} = \frac{P}{A_0} = \frac{\sigma_{zz}}{\lambda_z} = \frac{1}{2}\beta\left(1-\frac{1}{\lambda_z{}^3}\right)\left(\lambda_z{}^3-\lambda_z+1\right)e^{\frac{1}{4}\alpha\left[\left(\lambda_z{}^2-1\right)^2+2\left(\frac{1}{\lambda_z}-1\right)^2\right]} \tag{11}$$

In Eq. 11, P, A_0 and λ_z are total applied force (N), cross sectional area and ratio of deformed height over initial height of sample, α (dimensionless) and β (same unit as axial stress (T_{zz})) are the material parameters.

It is usually difficult to determine a mathematical equation which illustrates the nonlinear behaviours of biological material under loadings. In the previous equation, T_{zz} is function of the material parameters (α and β) and the compression or extension ratio. In this analysis, compression and extension ratios were determined from experimental test results, while the other material parameters were obtained by fitting a curve to the experimental results.

Uniaxial compression and tensile testing was performed on cylindrical and bone-shaped samples of peel and flesh, and the deformation ratio was calculated (Shirmohammadi 2013). In the constitutive equation obtained, an overall pattern for two coefficients (α and β) was determined using the experimental results. The stress (T_{zz}) versus compression ratio ($\Delta L/L_0$) was sketched using an experimental compression ratio where $\lambda_z = 1-\Delta L/L_0$.

In (a) the value for β is constant (and equal 10) and α values changes, and in (b) the value for α is constant (and equal one) and β values varies.

From the outputs of compression testing on flesh samples (Fig. 1) under smaller deformation (compression ratio <0.05) the stress changes linearly with the increase in compression ratio.

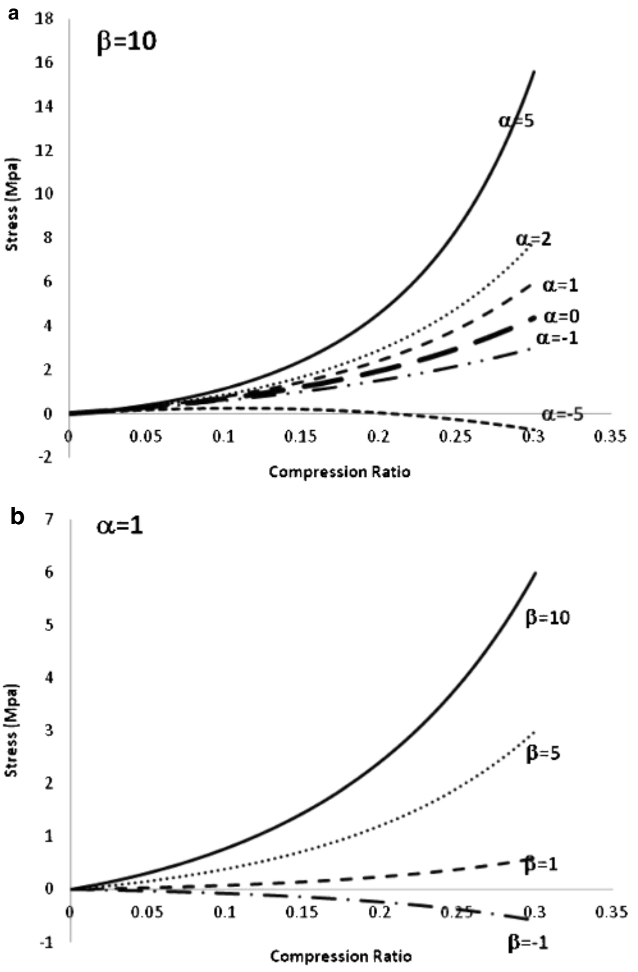


Fig. 1 Stress versus deformation ratio graphs for flesh samples compressive loading test obtained from constitutive modelling

In (a) the value for β is constant (and equal 10) and α values changes, and in (b) the value for α is constant (and equal 1) and β values varies.

In the first part of Fig. 1a, for the constant value of β, the values of α did not have significant effects on the slope of the curve in smaller deformation ratios while for the higher values of compression ratio it is clearly influencing the curve slope. For negative values of α, the curve shape changes to convex, similar to the previous study by Lu (Lu and Chen 1998). Change in β values affects the slope of the curve clearly and the negative values epitomized the curve in the negative section of the stress axis.

For the tensile results the same approach was used to determine stress versus deformation ratio curve (Fig. 2). A similar pattern was observed for fixed β value and changes in values of α, where there was a linear relationship between stress and deformation ratio for small deformation range. With an increase in deformation ratio the slope of the curve changes completely with a change in the values of α. After determining the overall pattern of the equation and the effects of two parameter on the stress versus deformation ratio curve, a series of analyses was applied for both tensile and compression results of peel and flesh samples to estimate values of α and β using MATLAB (1994–2013). The estimated values will be presented in the following section.

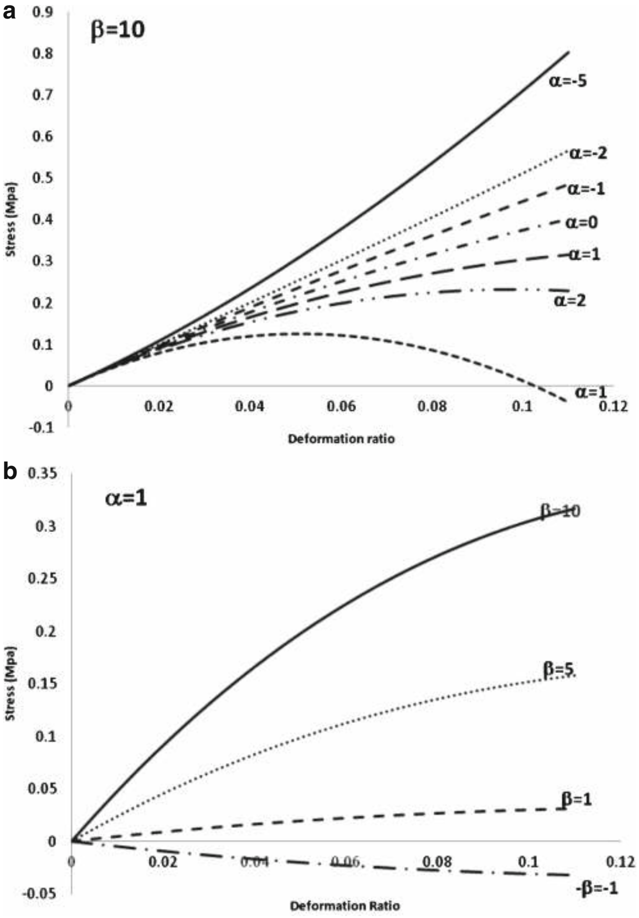


Fig. 2 Stress versus deformation ratio graphs for flesh samples tensile loading test obtained from constitutive modelling

Material and methods

Experimental studies

A series of uniaxial compression and tensile tests were conducted on samples of the Jap variety of pumpkin. The details of tests and results have been presented thoroughly in previous papers published by our research team (Shirmohammadi et al. 2011a, b, 2013); the results of stress versus strain curves were used to determine parameters of constitutive equations. Figure 3 presents the force versus time curves for tensile and compression tests of pumpkin peel and flesh samples. In all the cases there was an increasing trend in force deformation curve up to the peak value, which is defined as bio-yield point, and after that rupturing happened in the tissues which was similar to what other researchers have stated previously (Shirmohammadi et al. 2011a, b, a, b, 2012, Mohsenin 1986a, b; Lu and Chen 1998, Shirmohammadi and Yarlagadda 2012). Agricultural materials undergo large deformations during different stages of mechanical processing operations which lead to the permanent changes on tissue even before the rupturing phenomenon happens; this is due to the soft nature and structure of these tissue (Lu and Chen 1998).

The stress versus strain curves were obtained and two coefficients, α and β in Eq.12 of the selected constitutive equation were determined by fitting a curve to the experimental results. According to the determined equation for axial stress, two parameters α and β needed to be estimated for each sample and loading type results:

$$T_{zz} = \frac{P}{A_0} = \frac{\sigma_{zz}}{\lambda_z} = \frac{1}{2}\beta\left(1-\frac{1}{{\lambda_z}^3}\right)\left({\lambda_z}^3-\lambda_z+1\right)e^{\frac{1}{4}\alpha\left[\left({\lambda_z}^2-1\right)^2+2\left(\frac{1}{\lambda_z}-1\right)^2\right]} \quad (12)$$

MATLAB (1994–2013) software was used to fit the experimental results to T_{zz} (exponential function) equation and values for α and β were obtained for both tensile and compression results. The process of data analysis with MATLAB included, fitting an exponential

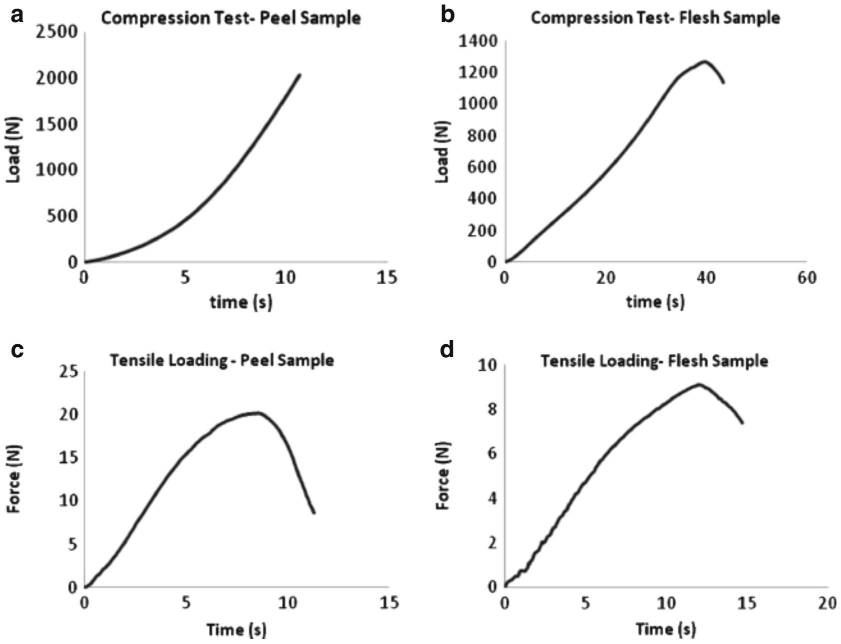


Fig. 3 Typical force versus deformation curve for peel and flesh samples under compression (**a & b**) and tensile (**c & d**) testing under loading speed of 20 mm/min

curve to the experimental data and estimation the values of coefficients, as well as determining the residual values for each set of data and calculate Root Mean Square Error (RMSE) for each curve fitting process.

$$Stress = \frac{1}{2}\beta\, F(x).e^{\frac{1}{4}\alpha.G(x)} \tag{13}$$

In Eq. 13, F(x) and G(x) are functions of deformation ratio and α (dimensionless) and β (same unit as stress) are the material parameters. A non-linear least square method was used to estimate the best values for α and β. In this method, sum of squares are computed and the minimum of them will be presented as the optimal value for the coefficients (1994–2013):

$$\min_{x}\left\|f(x)\right\|_2^2 = \min_{\mathrm{x}}\left(f_1(x)^2 + f_2(x)^2 + \ldots + f_n(x)^2\right) \tag{14}$$

Where in Eq. 14, x is a vector or matrix and f(x) is a function which returns the values a vector or matrix value.

The residual value illustrates the difference between the observed value from experiment and the predicted value in curve fitting process (1994–2013):

$$R_i = y_i - \widehat{y}_i \tag{15}$$

$$RMSE = \sqrt{\sum_{i=1}^{n}\frac{\left(y_i - \widehat{y}_i\right)^2}{n}} \tag{16}$$

In Eqs. 15 and 16, R_i, y_i, $\widehat{y}_i$ and n are residual value, observed valued from experiment, predicted value in curve fitting, and the number of data respectively.

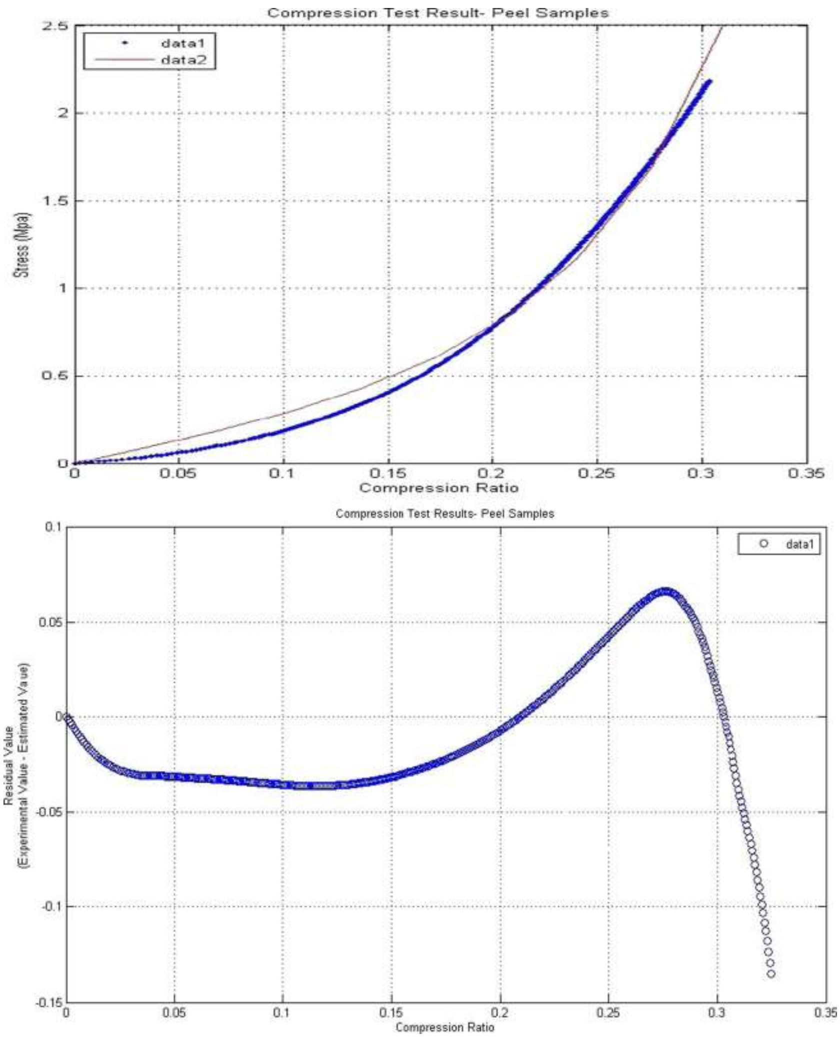
Results

The mentioned constitutive relationship was fitted for the experimental values

Figure 3 and the values of constitutive coefficients (αand β) were determined for compression and tensile tests. Figure 4 shows the fitted curve for the compression test results on peel samples (top) and the residual value for the predicted value (bottom). The determined values for α and β were 4.74 and −1.71 respectively and the RMSE calculated for the fitted curve was 0.061 Mpa (Fig. 4).

(top) Experimental and constitutive modelling curve fitting for peel samples under compression loading (data 1 is experimental results and data 2 results of constitutive equation),(bottom) residual value versus deformation ratio where “data 1” is experimental results an “data 2” is result of curve fitting.

Fig. 4 Experimental and constitutive modelling curve fitting for flesh samples under tensile loading where "data 1" is experimental results an "data 2" is result of curve fitting



There was a good agreement between the fitted values and the experimental values although there was a small gap between the two curves in deformation under 0.2, the RMSE value was 0.061 Mpa which shows a good agreement between fitted and observed results. In the case of flesh samples due to the complexity of curve, fitting was done in two steps including, for whole length of compression rate (up to 0.30) and also for compression rate under 0.30 similar value in peel samples (see Fig. 3 for the experimental curves).

The selected constitutive equation was chosen regarding the pattern of the curve as the first section is semi-linear and after reaching the peak of the curve there was a descending trend, which makes it difficult to fit an exponential equation for it.

As is shown in Fig. 5a, residual value for whole stress-deformation ratio curve was double the value for the part of curve with residual values of R=0.1 and R<0.05, which was expected. Consequently, the RMSE values for whole flesh curve were 0.077 Mpa while the value for the partial curve was 0.039 Mpa, which indicates the fitted curve for a part of curve was a better estimation.

Comparing the curve fitting values of flesh and peel samples, RMSE was lower in the curve fitted for the partial stress versus compression ratio curve for flesh samples. The stress-deformation ratio curve in peel samples showed a convex curve for a lower rate of deformation while the curve for flesh results was more linear shaped. This could be related to the thickness and curved form of peel samples. The values calculated for α, β and RMSE of each test have been listed in Table 1.

The results of curve fitting process for tensile test on peel samples has been shown in Figs. 6 and 7, the curve fitting was completed for whole stress-deformation ratio curve as well as part of the curve.

The results of curve fitting for flesh under tensile testing has been shown in Fig. 7, the fitted curve for both whole and part of stress-deformation ratio. The

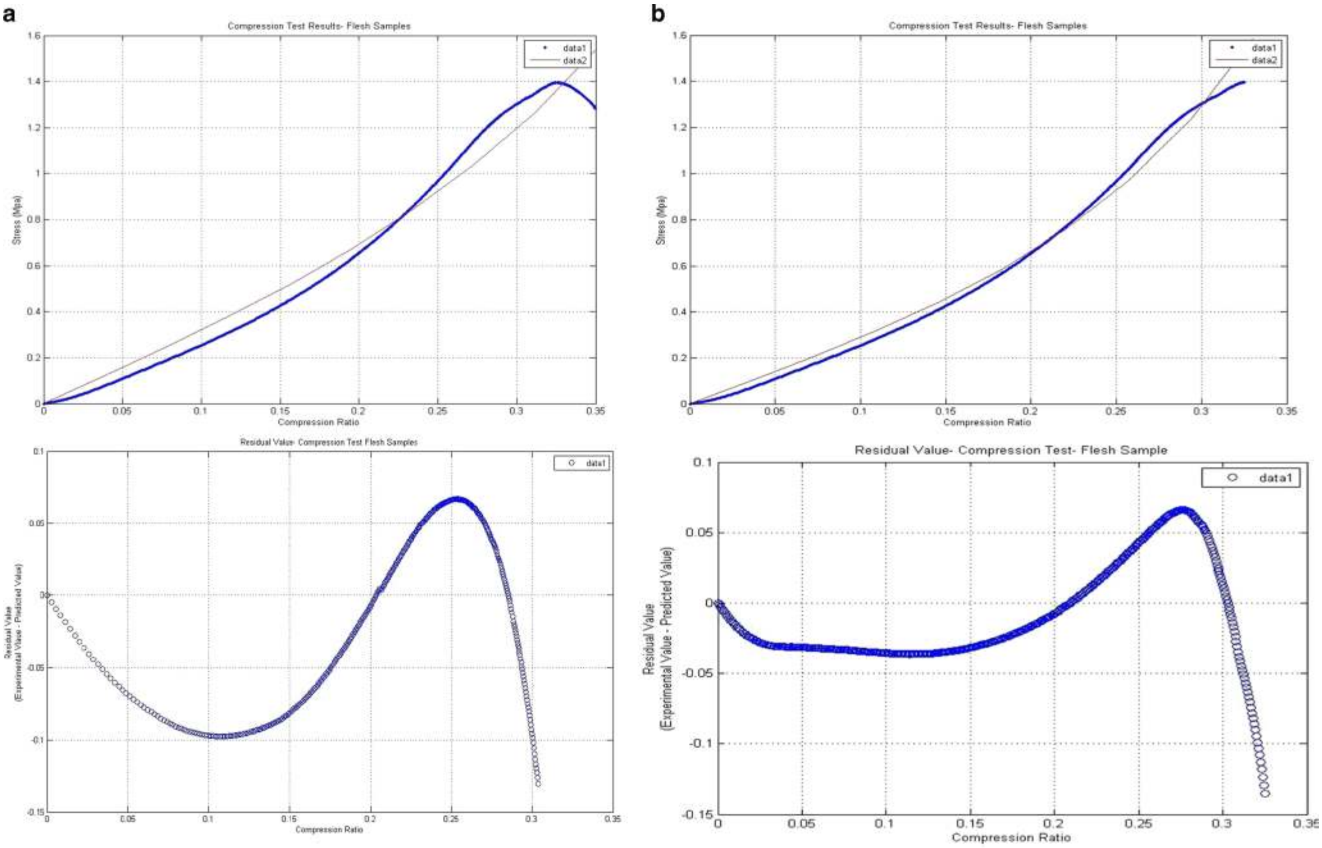


Fig. 5 Experimental and constitutive modelling curve fitting for flesh samples under compression loading where “data 1” is experimental results an “data 2” is result of curve fitting (**a**) whole curve, (**b**) part of curve

residual value for whole stress-deformation curve (Fig. 7a) was higher than the residual values for part of curve (Fig. 7b). The root mean squared error value for partial fitted curve in the tensile test of peel samples is 0.042. The residual values for fitted curve of whole peel results shows a high difference between fitted and experimental values, the fitted curve (Fig. 6a) illustrated a similar pattern to the actual experimental curve, which can be one of the few constitutive results for plant materials which include both curve zones before and after yielding point. In the case that high accuracy of the constitutive model is essential, the curve fitted for a part of stress versus deformation ratio curve (Fig. 7b) is more desirable which limits the deformation ratio to the values under 0.07. Comparing the compression test results for peel samples and the tensile test results, the first section of the curve before yielding happens had more linear shape in tensile than the convex shape in compression results. This can be related to the curved shape of the peel in comparison with flesh samples as well as low thickness of peel tissue.

The maximum residual value for the whole curve was over 0.04 Mpa (see Fig. 7a) while the value for a part of curve (Fig. 7b) was 0.015. The fitted curve for the whole stress–strain curve had higher agreement with the experimental results in comparison with the results for peel samples, after reaching the peak point of the curve, it did not follow the experimental pattern in flesh samples (Figs. 6a and 7a). Both second fitted curves (Figs. 6b and 7b) had a good agreement with the experimental curve however the residual value was much lower for the flesh samples results than the peel samples.

Table 1 Constitutive equation’s coefficients for peel and flesh samples under tensile and compression loadings[a]

	Sample type	α	β (Mpa)	RMSE (Mpa)
Tensile	Peel	−25.66	−18.48	0.042
	Flesh	−5.29	5.27	0.008
Compression	Peel	4.74	−1.71	0.061
	Flesh	0.7618	−1.86	0.039

[a] The parameter α is dimensionless and the parameter β and RMSE (root mean squared error) have same unit as stress (Mpa). The RMSE value is the average value for the data sets

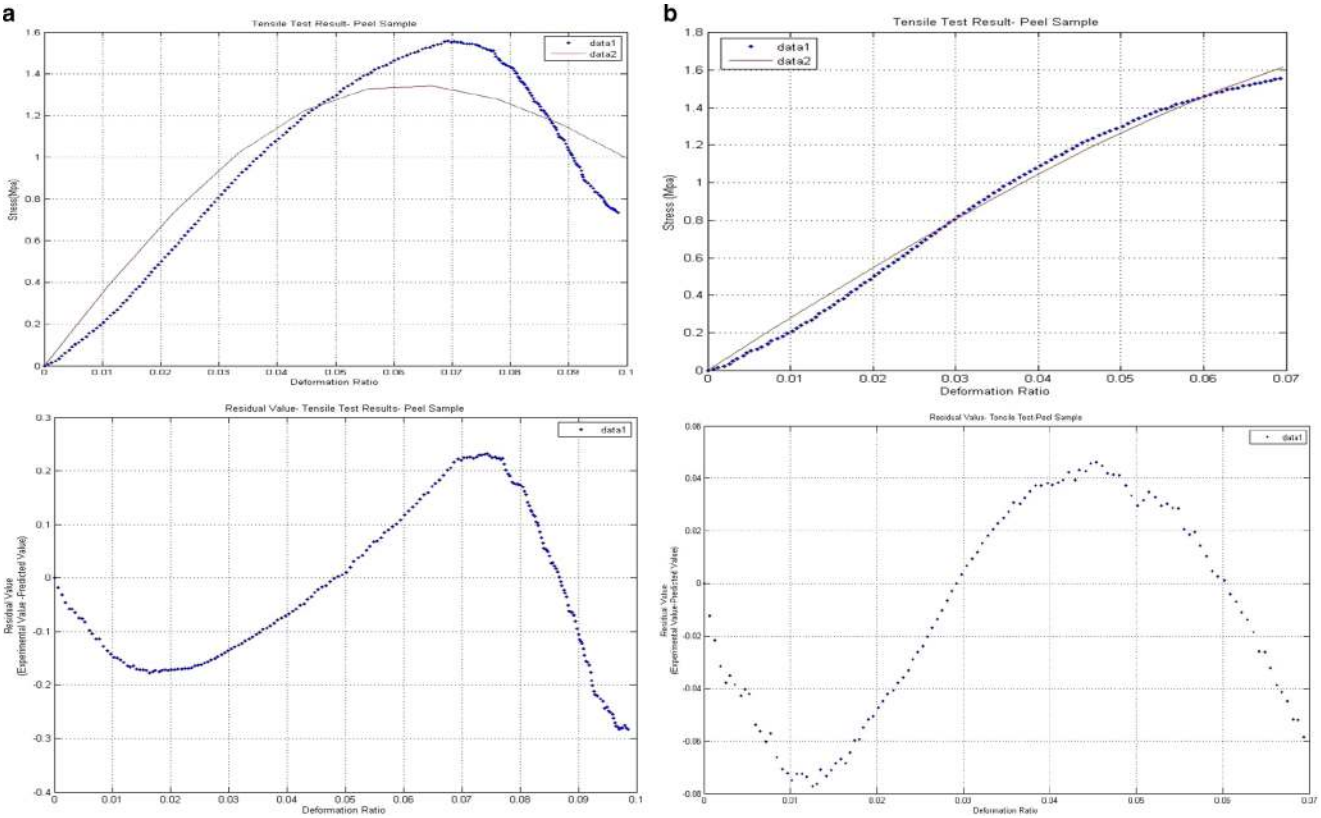


Fig. 6 Experimental and constitutive modelling curve fitting for peel samples under tensile loading where "data 1" is experimental results an "data 2" is result of curve fitting (**a**) whole stress-deformation curve and (**b**) part of the curve

Discussion

In this study, a constitutive model of tensile and compressive response of pumpkin peel and flesh tissue as a part of study on FE modelling and simulation of mechanical peeling process of tough skinned vegetables (Shirmohammadi 2013) was developed.

Exponential function was used to establish the relationship between stress and deformation ratio happening during mechanical loading process. Applying exponential function provided the possibility of establishing a close pattern of stress versus strain curve for both peel and flesh tissue where the experimental curve presented a clear curvature for both samples. The curvature of experimental diagram has been reported in literature for similar type of tissues such as pumpkin and watermelon. Additionally the two parameters of selected constitutive model were computable with performing a series of uniaxial mechanical loading. Considering the difficulty of defining coefficients for common polynomial function based constitutive models such as Mooney-Rivlin equation (Lu and Puri 1991). The applied force versus time curve was obtained from experimental work, showed a steady increase in force and deformation value up to a certain level, which usually is defined as bio-yielding point (Mohsenin 1986a, b). After yielding, the curves had descending trend as rupturing happened in the structure. The failure was considered to happen at the defined bio-yield point. The constitutive equation (Lu and Puri 1991) was applied for compression ratio versus stress two other parameters needed to be defined including α (dimensionless) and β (same unit as stress). The investigation on the effects of each coefficient on the shape of stress versus deformation ratio curve illustrated that the slope and curvature of the curve for each case of mechanical loading vary with the values of coefficients. The changes in the values of coefficients to the positive and negative values showed the high capability of the equation in transforming the data to a desirable shape of curve in comparison with the empirical curve. Best values of constitutive coefficients (α and β) were determined for each loading style and the sample using MATLAB Values of α and β for tensile of peel samples were −25.66 and −18.48 Mpa respectively, while α and β for flesh samples under tensile loading were −5.29 and 5.27 Mpa. For the compressive loading also the values of α and β of peel sample were 4.74 and −1.71 Mpa, and for flesh samples were 0.76 and −1.86 Mpa. After defining values for parameters of constitutive model, the stress versus strain curves resulted from constitutive modelling were compared with the actual experimental outcomes. The comparison was made by comparing

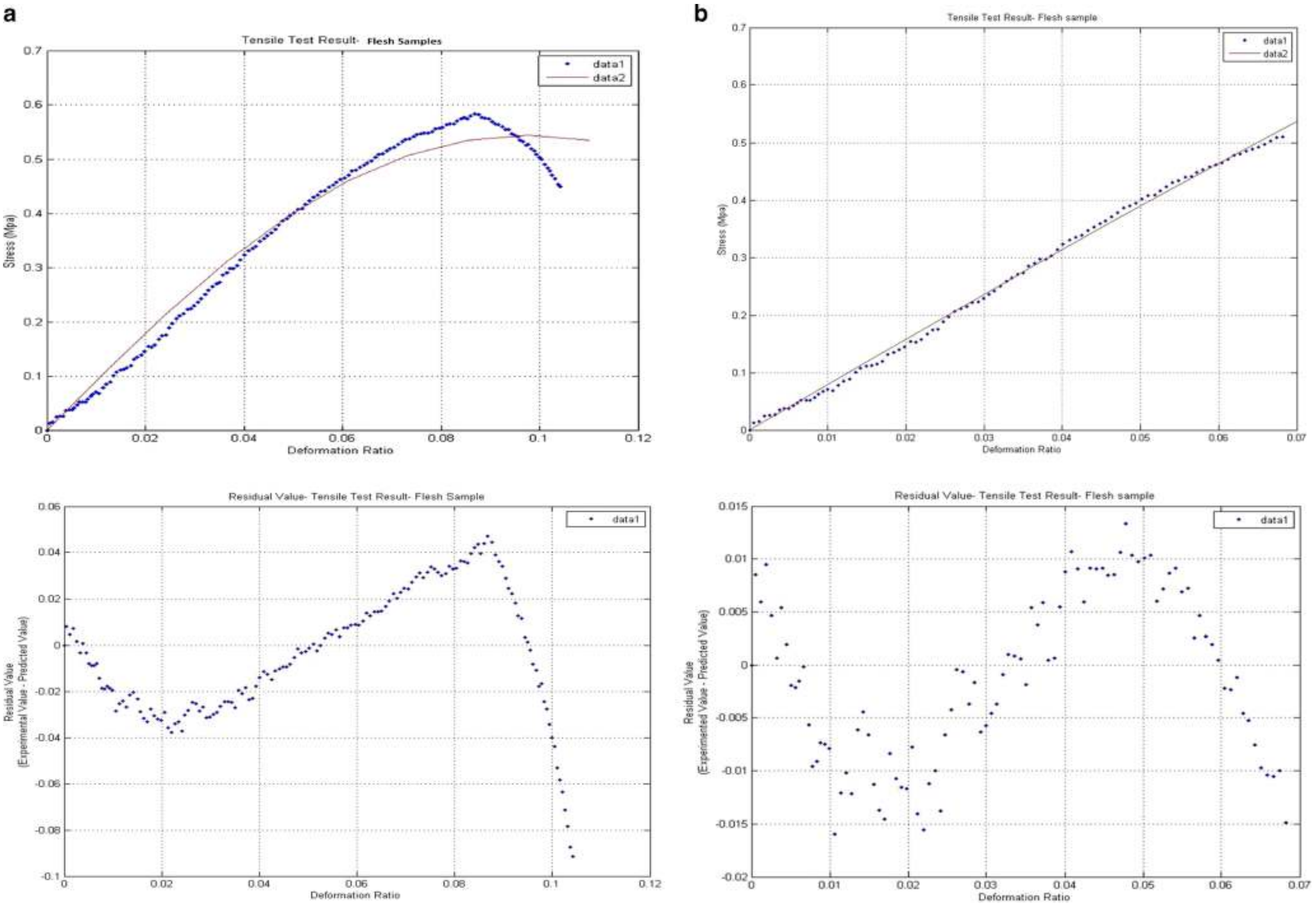


Fig. 7 Experimental and constitutive modelling curve fitting for flesh samples under tensile loading where "data 1" is experimental results an "data 2" is result of curve fitting

the values of residuals of each individual value predicted by constitutive model and experimental result in combination with the Root Mean Square Error values. The tensile loading curve for flesh samples had the lowset value of RMSE (0.008 Mpa) and the compressive loading of peel samples had the highest value (0.061 Mpa). For accuracy of the predicted values and obtained constitutive stress versus compression ratio curves, the constitutive equation was also fitted for a portion of the curve where the difference between predicted and experimental values showed a lower deference.

Presented work was one of the first efforts in applying constitutive relationships in analysis of plastic response of tough skinned vegetable tissues under mechanical loading. As a result, the outcomes were presented but there was a very limited literature in order to develop a detail comparison. However the results in the current work were compared with the experimental outcomes of previous studies on pumpkin. After constitutive modelling and validation, fitted curves and results were used as input material properties for the stress versus strain curve in FE modelling (Shirmohammadi 2013).

Conclusion

A constitutive model was established for two uniaxial compressive and tensile test of pumpkin peel and flesh tissues. The constitutive equation was selected based on a previous work on food products (Lu and Puri 1991) and the results of experimentation were applied to develop and validate the model. The outcomes of modelling and the constitutive parameters were determined for each case. The main focus in selecting the constitutive relationship was to get a constitutive response as close as possible to the experimental curve. Number of assumption was made and an exponential equation was selected and tested for the tissues. The outcomes of constitutive modelling were used later for FE modelling of mechanical peeling process of tough skinned vegetables (Shirmohammadi 2013). There were some differences between predicted value with constitutive model and the actual experimental data, the curves followed a similar pattern with the experimental curve. This model illustrated a good agreement with the experimental curves in both cases for peel and flesh tissues. The constitutive coefficients for peel under tensile test was α=25.66 and β=−18.48 Mpa and for flesh α=

−5.29 and β=5.27 Mpa. under compression the constitutive coefficients were α=4.74 and β=−1.71 Mpa for peel and α=0.76 and β=−1.86 Mpa for flesh samples. Constitutive curves predicted the values of force precisely and close to the experimental values. The curves were fit for whole stress versus strain curve as well as a section of curve up to bio yield point. The model was tested for peel and flesh tissues of Jap variety of pumpkin and in future can be applied for other member of tough skinned vegetables as well as other agricultural crops. The predicted curve for stress versus strain curve resulted from this constitutive model can be substituted as experimental results in analysing mechanical response of food particles under loadings.

References

(1994–2013) MATLAB R2012b. The MathWorks, Inc., Natick

Coburn J, Pandit A (2007) Development of naturally-derived biomaterials and optimization of their biomechanical properties. Top Tissue Eng 3:1–14

Daubert, CR and Foegeding EA (2010) Rheological principles for food analysis. Food analysis, Springer: 541–554

Drozdov AD (1996) Finite elasticity and viscoelasticity: a course in the nonlinear mechanics of solids. World Scientific, Singapore

Fung YC (1990) Biomechanics: motion, flow, stress, and growth. Springer, New York

Fung, YC (1993) "Biomechanics:{M} echanical {P} roperties of {L} iving {T} issues"

Gao Y, Lelievre J, Tang J (1993) A constitutive relationship for gels under large compressive deformation. J Texture Stud 24(3):239–251

Hamann DD, Zhang J, Daubert CR, Foegeding EA, Diehl KC (2006) Analysis of compression, tension and torsion for testing food gell fracture properties. J Texture Stud 37(6):620–639

Lu R, Chen YR (1998) Characterization of nonlinear elastic properties of beef products under large deformation. Trans ASAE 41(1):163–171

Lu R, Puri V (1991) Characterization of nonlinear creep behavior of two food products. J Rheol 35:1209

Lu R, Puri V (1992) Characterization of nonlinear behavior of apple flesh under stress relaxation. J Rheol 36:303

Miller K, Joldes G, Lance D, Wittek A (2007) Total Lagrangian explicit dynamics finite element algorithm for computing soft tissue deformation. Commun Numer Methods Eng 23(2):121–134

Mitsuhashi–Gonzalez K, Pitts M, Fellman J, Curry E, Clary C (1995) "Bruising profile of fresh apples associated with tissue and structure"

Mohsenin NN (1986a) Physical properties of plant and animal materials: structure, physical characteristics, and mechanical properties. Routledge, New York

Mohsenin NN (1986) Physical properties of plant and animal materials

Peña E, Doblaré M (2009) An anisotropic pseudo-elastic approach for modelling Mullins effect in fibrous biological materials. Mech Res Commun 36(7):784–790

Qiong G, Pitt R, Bartsch J (1989) Elastic–plastic constitutive relations of the cell walls of apple and potato parenchyma. J Rheol 33:233

Scanlon MG, Long AE (1995) Fracture strengths of potato tissue under compression and tension at two rates of loading. Food Res Int 28(4): 397–402

Shirmohammadi M (2013) Process modelling and simulation of tissue damage during mechanical peeling of pumpkin as tough skinned vegetable. Queensland University of Technology, Brisbane, p 301

Shirmohammadi M, Yarlagadda PKDV (2012) Properties of tough skinned vegetable-pumpkin tissue. 11th global congress on manufacturing and management GCMM2012. AUT University Auckland New Zealand, Auckland

Shirmohammadi M, Yarlagadda PKDV, Gudimetla P, Kosse V (2011a) Mechanical behaviours of pumpkin peel under compression test. J Adv Mater Res 337:3–9

Shirmohammadi M, Yarlagadda PKDV, Kosse V, Gu Y (2011b) Study of tissue damage during mechanical peeling of tough skinned vegetables during Mechanical Peeling Process (A Review). Global Science and Technology Forum, Singapore

Shirmohammadi, M, Yarlagadda PKDV and Gudimetla P (2012). Properties of tough skinned vegetable-pumpkin tissue. Global congress of manufacturing and management. Auckland New Zealand

Shirmohammadi M, Yarlagadda PKDV, Gu Y, Gudimetla P, Kosse V (2013) Tensile properties of pumpkin peel and flesh tissue, a review of current testing methods. Trans ASAE 56(4):1521–1527

Sitkei G (1987) Mechanics of agricultural materials. Access Online via Elsevier, New York

Spencer AJM (2004) Continuum mechanics. Courier Dover Publications

Sun W, Sacks MS (2005) Finite element implementation of a generalized Fung-elastic constitutive model for planar soft tissues. Biomech Model Mechanobiol 4(2–3):190–199

Tang J, Tung MA, Lelievre J, Zeng Y (1997) Stress–strain relationships for gellan gels in tension, compression and torsion. J Food Eng 31(4):511–529

Veronda D, Westmann R (1970) Mechanical characterization of skin—finite deformations. J Biomech 3(1):111–124