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A constitutive model for polyurethane foam with strain rate sensitivity †

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Abstract

The present work investigates the strain rate dependent behavior of polyurethane foams and formulates a new constitutive model in order to improve the fit of the experimental data at various strain rates. The model has seven parameters that are decided by quasi-static compression tests at two strain rates. Two models for low and high density polyurethane foams are shown to give stress strain relation at various strain rates. Dynamic compression tests were carried out to give stress strain data at high strain rate and the results are compared with those of the constitutive model.

Keywords: Polyurethane foam; Constitutive model; Dynamic loading; Strain rate

1. Introduction

The improvement of the crashworthiness of automobiles cannot be overestimated. US Department of Traffic estimated that there were over 400,000 fatalities and 20 million injuries requiring hospitalization from 1999 to 2009 [1, 2]. This, together with a range of environmental concerns and social pressures backed by legislation has led, and will continue to lead, to highly innovative designs involving advanced materials such as nonferrous alloys, smart structures, composites and foams. Of particular interest to this study is the use of polymeric foams in crashworthiness structures. Polymeric foams are currently being used as a filler material in bumpers and as reinforcement in roof and door beams and their applications will be continuously extended. They can reinforce weak parts of structures so that they respond effectively to impact loads, i.e., enhancing the crashworthiness. The energy absorbing capacity of foams is derived from their ability to undergo large deformation while maintaining a nearly constant stress value. Among other polymers, polyurethane foam is widely used industrially.

Foams have been the subjects of numerous experimental, numerical and theoretical investigations. Shim et al. [3] performed the normal impact testing at velocities ranging from 2 to 4 m/s for rigid polyurethane foams. Avalle et al. [4] examined the energy absorption characteristics of polymeric foam using the energy absorption diagram method. Rusch [5] proposed the relationship between compressive stress and strain as initial compressive modulus of the foam and strain function. Meinecke and Schwaber [6] described the initial modulus as a function of strain and changed the form of strain function as a power series of strain. Sherwood and Frost [7] modified the shape function of the constitutive model for the compressive behavior of the polyurethane foam. Langseth et al. [8] performed impact tests with extruded aluminum under the constant impact mass with varying velocities.

The stress-strain curve for polyurethane foams exhibits a curve consisting of three regions: linear elastic, plateau, and densification regions [9, 10]. Several models to describe the curve have been proposed where most of them are phenomenological or empirical based on experimental data [11, 12]. Although these models well describe the three-stage stress-strain relationship, they only work at a specific strain rate. So it is desirable to have a constitutive equation describing the stress strain relation at various strain rates. The aim of the present work is to investigate the strain rate dependent behaviors of polyurethane foams and to formulate a new constitutive model in order to improve the fit of the experimental data at various strain rates. Quasi-static compression tests were performed with cylindrical flexible polyurethane foam specimens to find the model parameters of the proposed constitutive model. Dynamic compression tests were done and its results compared with simulation results of the constitutive model.

2. Constitutive equation

2.1 Quasi-static models

The stress-strain curve of polymeric foams, in general, ex-

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hibits three regions: a linear elastic regime followed by a plasticity-like plateau region, and a densification region in which the stress rises steeply. The Gibson model was formulated by three equations describing each region as follows [9]:

$$\sigma(\varepsilon) = E\varepsilon \quad \text{when} \quad \sigma \le \sigma_y, \tag{1}$$

$$\sigma(\varepsilon) = \sigma_y \quad \text{when} \quad \varepsilon_y \le \varepsilon \le \varepsilon_D(1 - D^{-1}) + \varepsilon_y, \tag{2}$$

$$\sigma(\varepsilon) = \sigma_y \frac{1}{D}\left(\frac{\varepsilon_D}{\varepsilon_D - \varepsilon}\right)^m \quad \text{when} \quad \varepsilon > \varepsilon_D(1 - D^{-1}) + \varepsilon_y \tag{3}$$

where σ and ε are engineering stress and engineering strain, respectively. The model has five parameters: E Young's modulus for the elastics part, σ_y the yield stress, ε_D the densification strain, and two constants D and m. This model has limits that the stress has a constant value at the plateau region and that the stress strain curve is not smooth at the boundaries of two regions.

The Rusch model can be described by the sum of two powers:

$$\sigma(\varepsilon) = a\varepsilon^{-p} + b\varepsilon^q \tag{4}$$

where the four parameters, a, b, p and q, can be empirically determined. Although this model describes the stress strain relation by one equation, it has a drawback of inaccuracy in describing the densification phase [11].

To fit the typical form of experimental stress-strain relation of the foam material better, Liu and Subhash [13] suggested the model shown in Eq. (5). This model has six parameters. By setting B to be a unit, it becomes a model of five parameters.

$$\sigma(\varepsilon) = A\frac{e^{\alpha\varepsilon} - 1}{B + e^{\beta\varepsilon}} + e^C(e^{\gamma\varepsilon} - 1). \tag{5}$$

Recently, a model was presented by Avalle et al. [11] as follows:

$$\sigma(\varepsilon) = A\left(1 - e^{-(E/A)\varepsilon(1-\varepsilon)^m}\right) + B\left(\frac{\varepsilon}{1-\varepsilon}\right)^n. \tag{6}$$

This model also has five parameters. The parameters E, A and B are density dependent, while m and n are not. The first term fits the elastic and plateau region, while the second one fits the densification region. Although this model describes the stress-strain curve of polymeric foams well, it does not include the effect of strain rate due to dynamic loading.

2.2 Models including strain rate effect

A constitutive model including the strain rate effect was proposed by Nage et al. [14].

$$\sigma_d = f(\varepsilon)M(\varepsilon,\dot{\varepsilon}) \tag{7}$$

where σ_d is the engineering compressive stress, $f(\varepsilon)$ is the shape function of strain representing the stress-strain relationship at the reference strain rate $\dot{\varepsilon}_0$, and $M(\varepsilon,\dot{\varepsilon})$, the strain rate function, is a function of strain and strain rate and has a unit value at the reference strain rate. Sherwood and Frost [7] described the shape function by a tenth order polynomial function of strain whose coefficients are determined from compression tests for the reference strain rate. The strain rate function based on the assumption that log-log plot of stress vs. strain rate exhibits the linear function of strain can be represented as follows:

$$f(\varepsilon) = \sum_{n=0}^{10} A_n\varepsilon^n, \tag{8}$$

$$M(\varepsilon,\dot{\varepsilon}) = \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{a+b\varepsilon}. \tag{9}$$

Two parameters, a and b, can be found by linearly approximating the experimental stress-strain data for several strain rates.

Since the shape function described in Eqs. (7) and (8) is nothing but a stress-strain equation, it is much more efficient to replace it by the five-parameter model of Eq. (6) than to use a tenth order polynomial.

The strain rate function of Eq. (9) is based on the assumption that log-log plot of stress-strain data for a certain strain exhibits a linear function of strain. Careful examination of experimental data [14] shows that this is not true especially for the semi-flexible urethane. A new strain rate function is introduced to describe the log-log plot of stress vs. strain rate, which is concave up, at constant strain as follows:

$$M(\varepsilon,\dot{\varepsilon}) = 1 + (a + b\varepsilon)\ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right). \tag{10}$$

By replacing the shape function and the strain rate function of Eq. (7) by Eq. (6) and Eq. (10), respectively, a stress-strain equation including strain rate effect is derived as:

$$\begin{aligned}\sigma_d &= \sigma(\varepsilon)\left\{1 + (a + b\varepsilon)\ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right\} \\ &= \left\{A\left(1 - e^{-(E/A)\varepsilon(1-\varepsilon)^m}\right) + B\left(\frac{\varepsilon}{1-\varepsilon}\right)^n\right\}\left\{1 + (a + b\varepsilon)\ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right\}.\end{aligned} \tag{11}$$

The proposed model of Eq. (11) has seven parameters. The evaluation of these parameters is divided into two steps. First, five parameters, E, A, B, m, and n, are determined by fitting the experimental stress-strain data at the reference strain rate. Next, the remaining two parameters, a and b, are found by linearly approximating the experimental stress-strain data for another strain rate. Most of the parameters are considered to be dependent on the material, density, and temperature. Thus, two-step experiments should be done on the specimens with

Table 1. Five parameters calculated from curve fitting.

A	*B*	*E*	*m*	*n*
0.3863	0.3151	13.9996	3.4162	1.3680

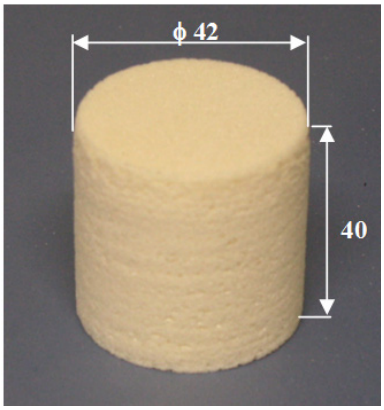

Fig. 1. Specimen of the polyurethane foam.

the same density at the same temperature.

3. Experiments and parameter identification

3.1 Low density polyurethane foam models

The quasi-static tests were performed on a MTS 810 machine having 100 kN maximum load capacity at room temperature, where the displacement measuring device and force transducers were equipped. The open-cell type cylindrical polyurethane foam (Fig. 1, supplied by Lacomtech, Co. Ltd., Republic of Korea) with densities of 67 kg/m^3 was tested at room temperature. The diameter and length of the specimen are 42 mm and 40 mm, respectively.

A quasi-static test at 0.001 s^{-1} strain rate was done and the experimental stress-strain data were obtained. The five parameters of Eq. (6) were found by fitting the data using the least square method. The calculated parameters are affected by the number of the points of the fitting curve. Table 1 shows the parameters obtained from the least square method by fitting 33 points of the experimental stress-strain data. The experimental and fitted stress-strain curves are shown in Fig. 2. The fitted curve well approximates the experimental curve.

To determine the two parameters, *a* and *b* in Eq. (11), the experimental data at another strain rate are needed. The experiment was done for the polyurethane foam of the same density of 67 kg/m^3 at the strain rate of 0.1 s^{-1}. Since the five parameters were obtained from the least square method at the strain rate of 0.001 s^{-1}, the two parameters are to be determined by fitting the experimental data with the constitutive equation of Eq. (11). Table 2 shows the parameters calculated by fitting the experimental data by least square method. As shown in Fig. 3, the fitted stress-strain curve well approximates the experimental data.

Having obtained all parameters by approximating the proposed constitutive model with two quasi-static experiments, it is possible to obtain the stress-strain curve for any strain rate. Six stress-strain curves are obtained as shown in Fig. 4 by substituting the values of strain rate, which are 0.001, 0.01, 0.1, 1, 10, and 100 s^{-1}, into Eq. (11). The lowest curve corresponds to the lowest strain rate of 0.001 s^{-1}.

For the same strain, more energy will be stored in the foam when the foam is compressed at higher strain rate than at lower strain rate. The seven parameters to describe the stress strain relation of polyurethane foam of 67 kg/m^3 density under dynamic loading were obtained by fitting two experimental data with the constitutive equation. Verification of the consti-

Table 2. Two parameters calculated from curve fitting.

a	*b*
0.0401	0.1222

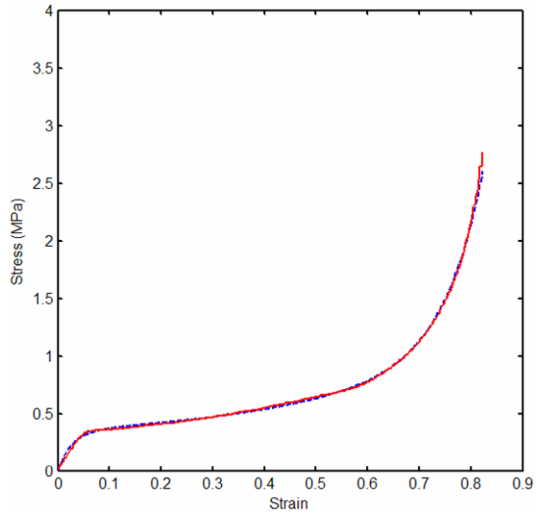

Fig. 2. Stress-strain relation (ρ = 67 kg/m^3, $\dot{\varepsilon}_0$ = 0.001 s^{-1}).

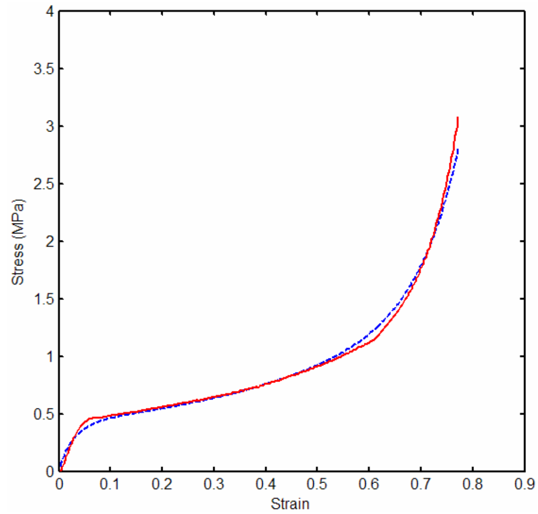

Fig. 3. Stress-strain relation (ρ = 67 kg/m^3, $\dot{\varepsilon}_0$ = 0.1 s^{-1}).

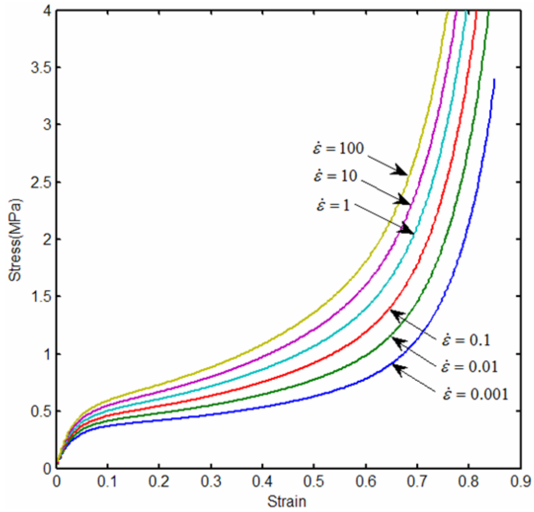

Fig. 4. Stress-strain relations at various strain rates (ρ = 67 kg/m^3).

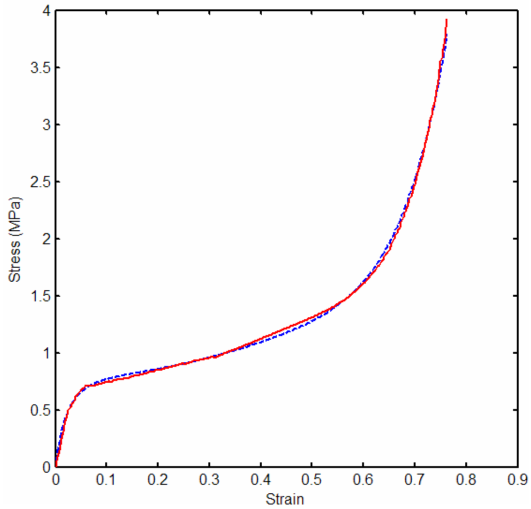

Fig. 6. Stress-strain relation (ρ = 89 kg/m^3, $\dot{\varepsilon}_0$ = 0.001 s^{-1}).

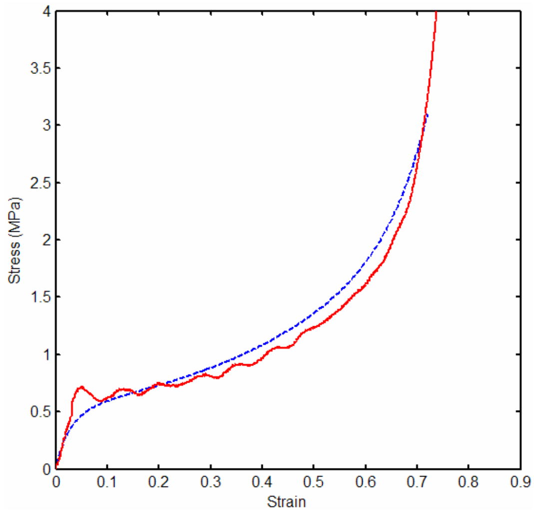

Fig. 5. Stress-strain relation (ρ = 67 kg/m^3, $\dot{\varepsilon}_0$ = 97 s^{-1}).

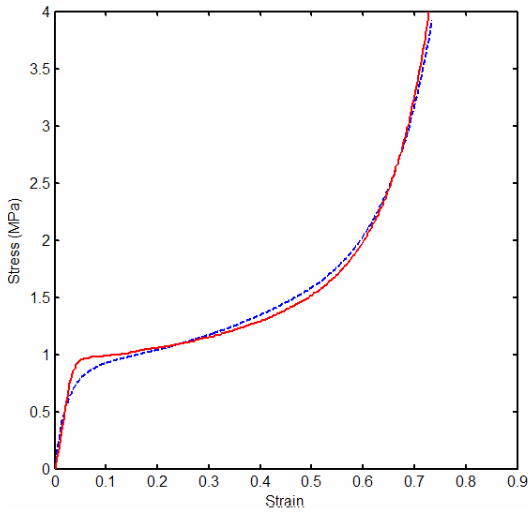

Fig. 7. Stress-strain relation (ρ = 89 kg/m^3, $\dot{\varepsilon}_0$ = 0.1 s^{-1}).

tutive equation can be done by using the dynamic compression test.

A drop tower type impact testing machine (Instron Dynatup 9250 HV) was used for the dynamic test. The machine can raise a discretely changeable weight to a specific height and drop it to the specimen either using gravitation or accelerating springs. When the striker of 26.5 kg hit the specimen, the impact velocity was measured to be 3.88 m/s, and the strain rate was calculated to be 97.0 s^{-1}. The solid line in Fig. 5 shows the experimental stress-strain relation, and the dotted line describes the relation using the proposed constitutive model. The model well approximates the experimental results, although it does not follow the oscillation due to the dynamic effect.

3.2 High density polyurethane foam

The quasi-static tests were carried out for high density polyurethane foam of 89 kg/m^3 density. First, experimental stress-strain data at the strain rate of 0.001 s^{-1} was obtained and the five parameters of Eq. (6) found by fitting the data using the least square method. The experimental and fitted stress-strain curves are shown in Fig. 6.

The fitted curve well approximates the experimental curve. The experimental data of another quasi-static strain rate are needed to find the two parameters *a* and *b* in Eq. (11). A quasi-static test at strain rate of 0.1 s^{-1} was done and two parameters obtained by curve fitting. As shown in Fig. 7, the fitted stress-strain curve well approximates the experimental data.

Since all parameters of the constitutive model have been found, the stress-strain curves can be drawn for any strain rate. Six stress-strain curves are drawn in Fig. 8 by substituting the values of strain rate, which are 0.001, 0.01, 0.1, 1, 10, and 100 s^{-1}, into Eq. (11).

To determine if the proposed model was working at high strain rate, dynamic compression tests were done. When the striker hit the specimen, the impact velocity was 3.88 m/s and

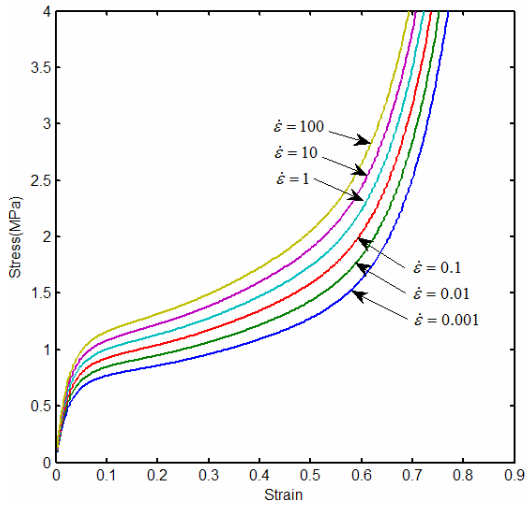

Fig. 8. Stress-strain relations at various strain rates (ρ = 89 kg/m^3).

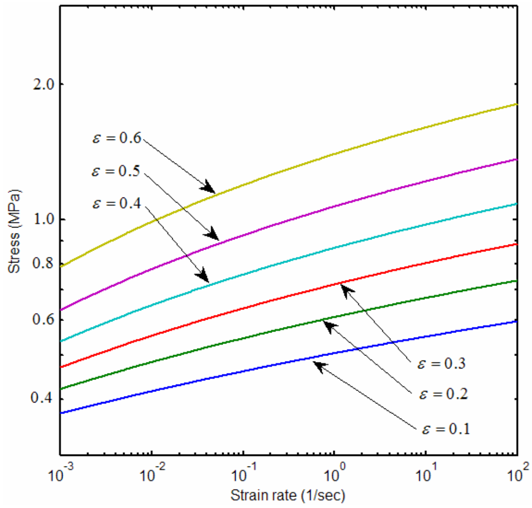

Fig. 10. Stress vs. strain rate at constant strains (ρ = 67 kg/m^3).

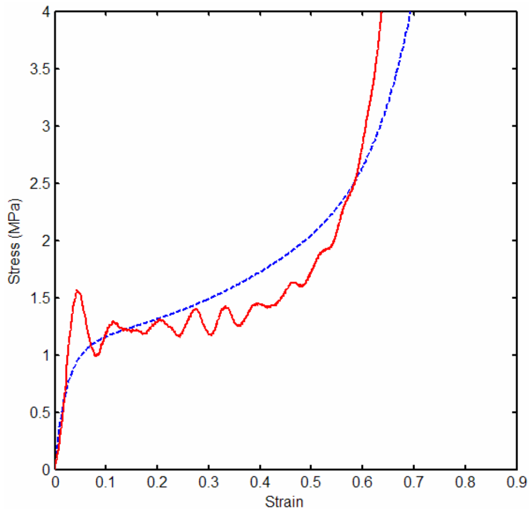

Fig. 9. Stress-strain relation (ρ = 89 kg/m^3, $\dot{\varepsilon}_0$ = 97 s^{-1}).

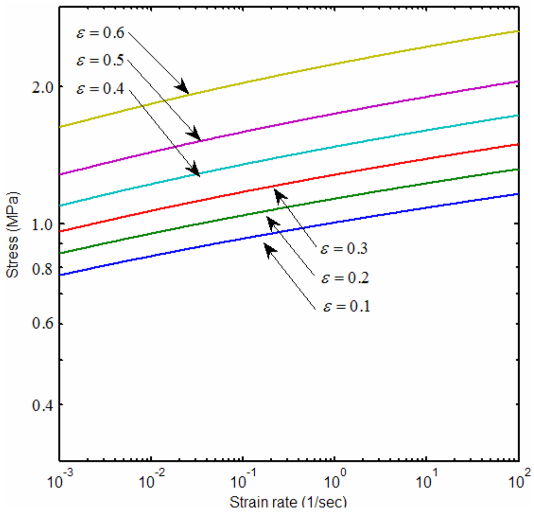

Fig. 11. Stress vs. strain rate at constant strains (ρ = 89 kg/m^3).

the strain rate was calculated to be 97 s^{-1}. The experimental stress strain curve was plotted and compared to that of the proposed model. The solid line in Fig. 9 shows the experimental stress-strain relation, and the dotted line describes the relation using the proposed constitutive model.

Since the parameters of the proposed model are decided by two quasi-static experiments at very low strain rates, this model could not follow the oscillation of the experiment due to the dynamic effect. Experimental results of Figs. 9 and 5 show that the higher density specimen underwent the large oscillation than the lower one.

3.3 Plot of stress vs. strain rate

After all parameters of the proposed model are obtained, the stress can be plotted against the strain rate on log-log scale for different scale levels. For the low density polyurethane foam of 67 kg/m^3, the log-log plots of stress vs. strain rate are shown in Fig. 10; for the high density foam of 89 kg/m^3, in Fig. 11. Each curve seems almost linear but a little convex up. Nage et al. [14] proposed a constitutive model as shown in Eq. (7) by assuming that log-log plot of stress vs. strain rate exhibits a linear function of strain. The log-log plots of those of their experiments generally follow straight lines but a little convex up for most foam materials, which tends to follow characteristics of the proposed model.

4. Conclusions

When designing energy absorbing structural foam, mathematical description of mechanical behavior of the foam is needed. Several models well describing the stress strain relation of the foam are work only at a specific strain rate. In this paper a constitutive model for polyurethane foam has been proposed to describe the stress-strain relation at a wide range of strain rates.

The proposed model has seven parameters. The parameters are evaluated through two-step optimization procedures. (1) The first step is conducted at the reference strain rate and five parameters are found in this step. (2) Remaining two parameters affected by the magnitude of the strain rate are found in the next step by fitting the curves between the presented model and the experimental measured value at another strain rate.

Dynamic compression tests were carried out to give stress strain data at initial strain rate of 97 s^{-1}. These data are plotted with those of the proposed model. The stress strain curve of the proposed model well follows that of the experimental data, although it does not accurately approximate the oscillation due to the ringing of the system caused by the impact.

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