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LETTER TO THE EDITOR

# A constrained variational calculation for beta-stable matter

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**Abstract.** A method of lowest-order constrained variation previously applied by us to asymmetric nuclear matter is extended to include electrons and muons making the nucleon fluid electrically neutral and stable against beta decay. The equilibrium composition of a nucleon fluid is calculated as a function of baryon number density and an equation of state for beta-stable matter is deduced for the Reid soft-core interaction.

In a series of publications Owen *et al* (1976a, b, 1977) have developed a lowest-order constrained variational (LOCV) method for calculating the equations of state for nuclear and neutron matter. The basis of the method is to take a Jastrow-style trial wavefunction, make a cluster expansion of the expectation value of a Hamiltonian composed of a one-body kinetic energy term and a two-body interaction term, then to terminate the cluster expansion at the two-body term and carry out a functional variation with respect to the two-body correlation function subject to a constraint. In the case of a fermion fluid, the constraint adopted has been one which requires the correlated two-body wavefunction to be normalised, or equivalently that the one-body density $n^{(1)}(\boldsymbol{r}_1)$ should be related to the two-body density $n^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2)$ by the familiar relation

$$n^{(1)}(\boldsymbol{r}_1) = \frac{1}{N-1} \int n^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2)\, \mathrm{d}\boldsymbol{r}_2. \tag{1}$$

This work was extended by Howes *et al* (1978) to derive the equation of state of a nucleon fluid with an arbitrary proton to neutron ratio and the symmetry parameter for the semi-empirical mass formula was deduced for the case of the Reid (1968) soft-core interaction.

In the present Letter we extend the work of Howes *et al* to include electrons and muons and we investigate the composition and equation of state of an equilibrium mixture of neutrons, protons, electrons and muons which is electrically neutral and at absolute zero temperature. The nucleons are assumed to interact through the Reid soft-core potential; the requirement of charge neutrality means that we can ignore the electromagnetic interaction and the weak interactions are neglected.

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The total baryon number density $n_B$ is the sum of the proton and neutron number densities

$$n_B = n_p + n_n \tag{2}$$

and the condition of electrical neutrality requires

$$n_p = n_e + n_\mu . \tag{3}$$

The leptons form two highly relativistic, degenerate Fermi seas. The contribution to the energy per baryon arising from these Fermi seas is

$$E_L = \sum_{i=e,\mu} \frac{m_i^4 c^5}{8\pi^2 \hbar^3 n_B} [x_i(1 + x_i^2)^{1/2}(2x_i^2 + 1) - \sinh^{-1} x_i] \tag{4}$$

where

$$x_i = \hbar k_i / m_i c \tag{5}$$

and $k_e$ and $k_\mu$ are the electron and muon Fermi momenta respectively. These Fermi momenta are not independent, but are related through the condition of beta stability, i.e.

$$\mu_n - \mu_q = \mu_e = \mu_\mu \tag{6}$$

where $\mu_i$ is the chemical potential of the $i$th species of particle. This implies that

$$m_e c^2 (1 + x_e^2)^{1/2} = m_\mu c^2 (1 + x_\mu^2)^{1/2}, \tag{7}$$

which determines the muon to electron abundance.

Our approximation to the free energy per nucleon for beta-stable matter is thus

$$F = \sum_{i=n,p} \frac{n_i}{n_B} \left( \frac{3}{5} \frac{\hbar^2}{2m_i} k_i^2 + m_i c^2 \right) + E_L + E_2 \tag{8}$$

where $k_n$ and $k_p$ are the neutron and proton Fermi momenta and $E_2$ is the two-body cluster energy calculated by our LOCV method (Owen *et al* 1977, Howes *et al* 1978).

The equilibrium configuration of beta-stable matter is obtained at each total baryon number density $n_B$ by minimising $F$ with respect to the two-body correlation functions and the proton abundance $n_p/n_B$ subject to the constraints of equations (1), (2), (3) and (6).

In figure 1 we plot our calculated number densities as a function of the total baryon number density. We see that the proton abundance exhibits a peak at $n_B \sim 0{\cdot}45$ N fm$^{-3}$ where it reaches 7% of the baryon abundance. This is due to the system taking advantage of the strong $T = 0$ tensor components in the Reid interaction as observed by Pandharipande and Garde (1972).

We have limited our calculations to the range 0·05 N fm$^{-3}$ to 0·85 N fm$^{-3}$. At lower densities our LOCV equations become numerically unstable. At higher densities, a non-relativistic treatment of the nucleons become extremely questionable. At extremely low densities the two-body energy $E_2$ becomes insignificant and we are left with three degenerate Fermi gases. At very low densities, $\sim 5 \times 10^{-4}$ N fm$^{-3}$, the proton abundance starts to grow again and by $\sim 5 \times 10^{-8}$ N fm$^{-3}$ the protons are dominant. Of course this is unrealistic because before this happens the nucleons

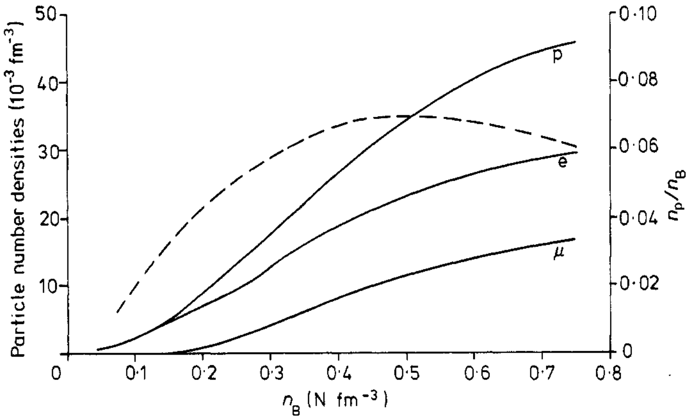


**Figure 1.** Calculated number densities for protons, electrons and muons as a function of total baryon number density in beta-stable matter. The broken curve represents the relative proton abundance.

will have taken advantage of the attractive long-range components of the N–N interaction to cluster into nuclei which at the same time will minimise their Coulomb energy by forming a lattice, e.g. a neutron star crust (Irvine 1978).

In figure 2 we present our calculated equation of state and compare it with that for pure neutron matter. We see that the extra degree of freedom offered by beta decay leads to a slight softening of the equation of state, as we would expect.

A very early estimate of the proton abundance in beta-stable matter was given by Salpeter (1960); this gave a maximum of 6% proton abundance but badly overestimated the symmetry energy. Later calculations were based on the Bethe–Brueckner–Goldstone approach. Amongst the earliest of such calculations are the works of Sprung and collaborators. Originally Nemeth *et al* (1967) obtained a maximum proton concentration of 0·8% near to the nuclear saturation density $n_B \simeq 0{\cdot}17\ \mathrm{N\,fm^{-3}}$.

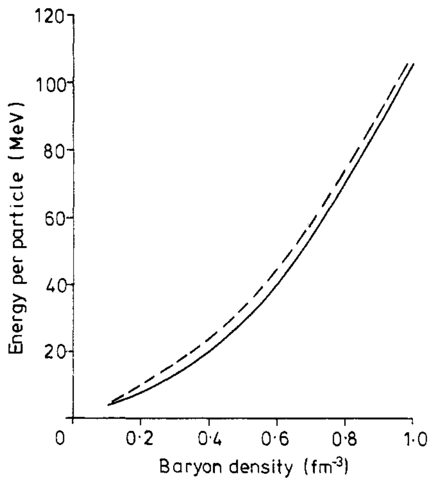


**Figure 2.** The full curve is our calculated equation of state for beta-stable matter. The broken curve represents our results for pure neutron matter.

These results, however, are extremely sensitive to the dependence of their $G$-matrix elements on the relative proton to neutron abundance which they took from a simple linear interpolation between the $G$-matrix elements for neutron matter and nuclear matter. Ellis and Sprung (1972), in an extension of the work of Nemeth and Sprung (1968), attempted to examine the sensitivity of their results to the $G$-matrix elements at higher densities. They used the forces G-0 and G-1 of Sprung and Banerjee (1971), which they adjusted by two different methods to yield nuclear matter saturation at a binding energy of 16·5 MeV and density 0·17 N fm$^{-3}$. Ellis and Sprung then considered the spread in their results arising from the two different interactions and the different renormalisation procedures to be a measure of the uncertainty arising from the Brueckner approach. At densities greater than 0·1 N fm$^{-3}$ we are in good agreement with their results, especially those obtained with the force G-1. The Sprung calculations were all based upon the Reid potential. Buchler and Ingber (1971) have also carried out Brueckner calculations based on Ingber's potential (Ingber 1968, Ingber and Potensa 1970). They obtain a 10·8% proton abundance at $n_B = 0{\cdot}35$ N fm$^{-3}$. However this is not a peak value, but is the highest density they consider. The proton abundance is still rising at this point.

We would conclude by pointing out that we have carried out a calculation of the equation of state and composition of beta-stable matter using techniques which we believe to be more reliable than low-order Brueckner calculations, especially at higher densities $n_B \gtrsim 0{\cdot}2$ N fm$^{-3}$, and which is free from the uncertainties associated with effective interactions obtained by interpolating between nuclear matter and neutron matter results.

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