

# A Constraint on the Standard Beaming Model for Superluminal Sources

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**Abstract:** We have used two subsamples of superluminal quasars to test the relativistic beaming model, and to place useful constraints on the radio source orientation hypothesis and cosmology. Based on the variation of the observed ratio  $R$  of the core-to-lobe radio luminosities with proper motion  $\mu$  for the subsample of lobe-selected quasars, we show that the observed  $R$ - $\mu$  data can be explained in terms of a bulk relativistic motion with Lorentz factor  $\gamma \approx 4$ . Also, from the observed proper motion versus redshift ( $\mu$ - $z$ ) plot for this subsample, we show that  $\gamma \approx 4$  implies a high density universe with deceleration parameter  $q_0 = 0.5$ . Furthermore, from the observed ( $\mu$ - $z$ ) plot for the two subsamples taken separately, we show that both  $\gamma$  and  $\mu$  for the core-selected subsample exceed those of the lobe-selected subsample by a factor of  $\sqrt{2}$  for the  $q_0 = 0.5$  world model. This result is demonstrated to be consistent with an orientation-based unified scheme in which lobe-selected quasars lie, on the average, at an angle which is a factor of  $\sim 2$ - $3$  larger than that of their core-selected counterparts.

**Keywords:** quasars : general — cosmology: theory, relativity

## 1 Introduction

In the relativistic beaming model for extragalactic radio sources (Scheuer & Readhead 1979; Blandford & Königl 1979), an object moving relativistically at close angles to the line of sight will appear to have a transverse velocity much greater than the speed of light. This superluminal motion has been detected in a number of sources using the VLBI and quite a significant fraction of them have a measured proper motion  $\mu$  and redshift  $z$ . If there is no evolution of proper motion with redshift, the observed upper envelope ( $\mu$ - $z$ ) function can be used to derive useful constraints for relativistic beaming and radio source orientation hypotheses as well as for cosmology (see e.g. Hough & Readhead 1987; Cohen et al. 1988; Vermeulen & Cohen 1994).

In the present paper, we derive the value of the Lorentz factor  $\gamma$ , which is a crucial parameter of the relativistic beaming scenario, using the observed variation of the ratio  $R$  of core-to-lobe radio luminosities as a function of proper motion for a sample of superluminal sources. We also show that the derived Lorentz factor  $\gamma$  can be used, at least in principle, to constrain the values of the deceleration parameter  $q_0$ , which is a powerful discriminant for possible cosmological models.

## 2 Some Basic Relationships

In the simple relativistic beaming model, superluminal sources are believed to consist of one or more radiating components moving at relativistic velocity

$v$  relative to a stationary core, so that we can write the apparent velocity of separation in units of the speed of light  $c$  as

$$\beta_{\text{app}} = \frac{\beta \sin \theta}{1 - \beta \cos \theta}, \quad (1)$$

where  $\theta$  is the angle to the line of sight and  $\beta = v/c$  is related to the bulk Lorentz factor  $\gamma$  through

$$\gamma = (1 - \beta^2)^{-1/2}. \quad (2)$$

In the Friedmann–Robertson–Walker (FRW) universe parametrised by the Hubble constant  $H_0$  and deceleration parameter  $q_0$ , the apparent velocity is also given in terms of the observed proper motion  $\mu$  as (see e.g. Pearson & Zensus 1987)

$$\beta_{\text{app}} = \frac{47 \cdot 4 \mu z}{h(1+z)} \left[ \frac{1 + (1 + 2q_0 z)^{1/2} + z}{1 + (1 + 2q_0 z)^{1/2} + q_0 z} \right], \quad (3)$$

where  $h$  is a dimensionless parameter which is the Hubble constant in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Also, if we assume a simple continuous jet model, the ratio of the beamed flux density of the core to that of the lobe (which is usually considered isotropic) is given by (see e.g. Hough & Readhead 1989)

$$R = \frac{R_T}{2} [(1 + \beta \cos \theta)^{-2} + (1 - \beta \cos \theta)^{-2}], \quad (4)$$

where  $R_T = R(\theta = 90^\circ)$ . In the absence of any large spread in the intrinsic distributions of  $R_T$  and  $\gamma$ ,  $R$  is usually used as an orientation indicator, and this predicts a direct correlation between  $R$  and  $\beta_{app}$  and hence  $\mu$  (see Vermeulen et al. 1993). This implies that sources with high values of  $R$  are expected to have large values of  $R$ . Considering equations (1) and (4), we can show that a linear regression of  $R$  against  $\beta_{app}$  (or  $\mu$  using equation 3) should yield an intercept given by (see Ubachukwu 1998)

$$R_0 = R_T \gamma^2 (2\gamma^2 - 1), \tag{5}$$

which can be used to test specific beaming models.

For the present analysis, we have used the sample of superluminal sources compiled by Vermeulen & Cohen (1994). This heterogeneous sample consists of 66 sources out of which 25 were selected on the basis of their core emission, 13 were lobe-selected and the rest were unclassified. Note that two of the sources 0906+430 and 1040+123 were classified as both core- and lobe-selected, but were treated as lobe-selected in the present analysis. Since we are only interested in the classified sources, the final sample consists of 13 lobe-selected and 23 core-selected quasars. The lobe-selected quasars are usually used to test specific predictions of the beaming models since their radio axes are believed to have a random orientation in the sky and their extended emissions to be isotropic.

A linear regression of  $R$  against  $\mu$  using the lobe-selected quasars in the current sample yields

$$R = 13.1 \pm 1.3 + (3.5 + (3.5 \pm 0.8)\mu), \tag{6}$$

with correlation coefficient  $r \sim 0.8$ . We have considered only the largest observed proper motion in each case. Using  $R_0 = 13.1$  obtained above in equation (5) and  $R_T = 0.024$  (see e.g. Orr and Browne 1982) shows that the observed  $R-\mu$  data are consistent with  $\gamma \approx 4$ , which is in close agreement with  $\gamma = 5$  found by Orr & Browne (1982) for a sample of flat- and steep-spectrum quasars.

### 3 Implications for Cosmology

Following Pearson & Zensus (1988) we write the upper envelope proper motion-redshift ( $\mu-z$ ) function in any FRW model as

$$\mu = \frac{2.11 \times 10^{-2} \gamma h (1+z)}{z} \left[ \frac{1 + (1 + 2q_0 z)^{1/2} + q_0 z}{1 + (1 + 2q_0 z)^{1/2} + z} \right] \tag{7}$$

for  $\gamma^2 \gg 1$ . This equation shows that superluminal motion depends on redshift and in the absence of any large intrinsic spread in  $\gamma$ , the observed ( $\mu-z$ )

data for any well-defined sample of superluminal sources can be used to test cosmological models. Thus, for any assumed value of  $h$ , we can deduce the value of  $q_0$  required to provide a reasonable fit to the observed ( $\mu-z$ ) data using the value of  $\gamma$  obtained in the preceding section.

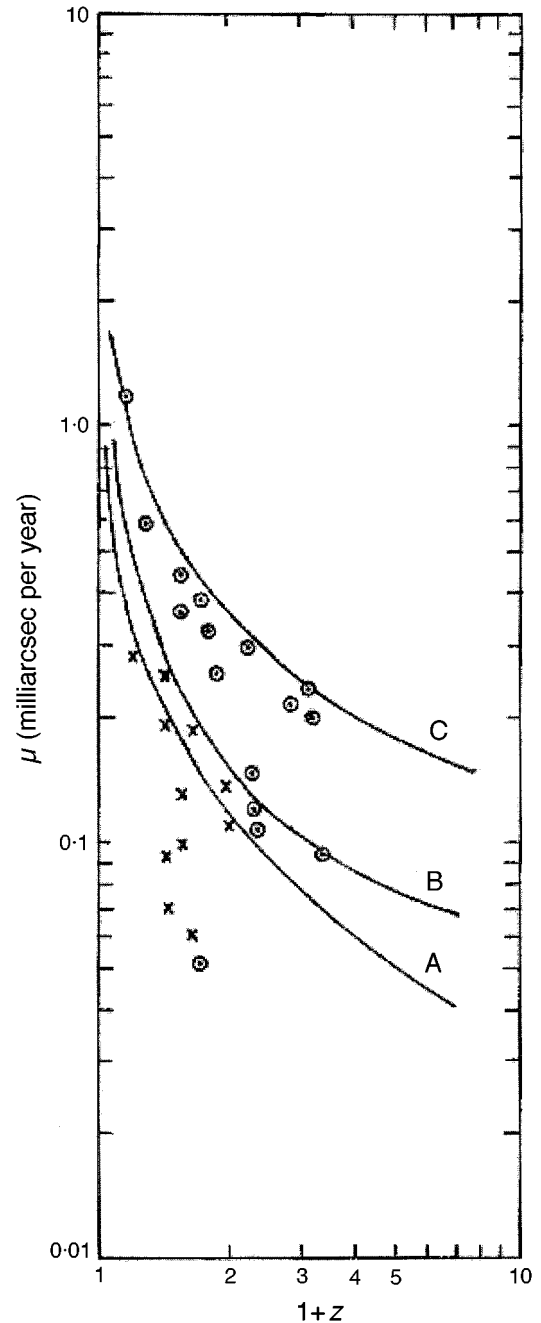


Figure 1—Plot of ( $\mu-z$ ) data for the core-selected quasars (circles) and lobe-selected quasars (crosses). All sources with  $\mu \leq 0$  have been omitted.

Figure 1 shows the ( $\mu-z$ ) plot for the present core-selected and lobe-selected quasars. Two curves (A and B) representing two Friedmann world models have been superimposed: one with  $q_0 = 0.1$  (curve A), which appears to be consistent with the density

of the universe derived from its luminous matter (see e.g. Peebles 1988), and one with  $q_0 = 0.5$  (curve B) chosen on the supposition of an inflationary world model (Guth 1981). We adopted  $h = 1$  and  $\gamma = 4$  (obtained in the preceding section). Figure 1 shows that the  $(\mu-z)$  data for the lobe-selected quasars used in the present analysis are consistent with a Friedmann universe with  $q_0 = 0.5$ . The present result thus favours a closed universe over an open universe.

#### 4 Implications for Unified Theories

In the Orr & Browne (1982) unified scheme, core-dominated quasars are believed to be lobe-dominated quasars seen at smaller angles to the line of sight. This immediately predicts that the superluminal velocity will be higher and more frequent in core-dominated than in lobe-dominated quasars. This implies that a larger value of the Lorentz factor would be required for core-selected quasars to provide a good fit to the observed  $(\mu-z)$  data than for their lobe-dominated counterparts.

We can recast equation (7) in the form (see Pearson & Zensus 1987)

$$\mu = 2.11 \times 10^{-2} h \gamma g(q_0, z), \quad (8)$$

where

$$g(q_0, z) = \frac{1+z}{z} \left[ \frac{1 + (1 + 2q_0z)^{1/2} + q_0z}{1 + (1 + 2q_0z)^{1/2} + z} \right]. \quad (9)$$

It follows immediately from equation (8) that for given values of  $q_0$  and  $h$ , we can write

$$\frac{\mu_c(z)}{\mu_l(z)} = \frac{\gamma_c}{\gamma_l}, \quad (10)$$

where the subscripts c and l stand for core- and lobe-selected quasars respectively. This equation thus suggests that the apparent values of the Lorentz factor derived from superluminal data will be different for the two classes of object because of orientation effects (see Padovani & Urry 1992). A good fit to the  $(\mu-z)$  data for the core-selected quasars is also indicated in Figure 1 (curve C) for the  $q_0 = 0.5$  world model. This curve corresponds to a Lorentz factor  $\gamma_c = 9$  obtained by Cohen et al. (1988). It therefore follows that  $\gamma_c/\gamma_l \approx 2.3$  for the present sample. Curves B and C also show that within the region of overlap in redshift ( $0.2 \leq z \leq 2$ ), we have  $\mu_c/\mu_l \approx 2.4$ . These results are consistent with equation (10) and suggest that the Lorentz factor and proper motion are a factor 2 larger for core-selected quasars than for lobe-selected quasars.

Furthermore, in a simple standard beaming model, the boosting factor  $\delta$  is given by (see e.g. Vermeulen & Cohen 1994)

$$\delta(\theta, \gamma) = [\gamma(1 - \beta \cos \theta)]^{-1}. \quad (11)$$

Vermeulen & Cohen (1994) have shown that the optimum value of  $\delta$  can be obtained by setting  $d\delta/d\theta = 0$  in equation (11) and this corresponds to

$$\beta = \cos \theta \quad \text{and} \quad \gamma = \frac{1}{\sin \theta}. \quad (12)$$

We can therefore write

$$\frac{\mu_c(z)}{\mu_l(z)} = \frac{\gamma_c}{\gamma_l} = \frac{\sin \theta_l}{\sin \theta_c}. \quad (13)$$

This clearly demonstrates that equation (10) is consistent with a pure-orientation-based unified scheme in which the core-dominated quasars are their lobe-dominated counterparts seen end-on. This can be quantitatively tested by estimating the values of  $\theta_c$  and  $\theta_l$  from the observed  $R$  distributions for the core- and lobe-selected quasars respectively. These distributions give median values of  $R_{m,c} = 12.6$  and  $R_{m,l} = 0.22$ . Using  $\gamma_c = 9$  and  $\gamma_l = 4$  (obtained earlier) in equation (4) and for a typical value of  $R_T = 0.024$  (e.g. Orr & Browne 1982), we obtain  $\theta_{m,c} \approx 13^\circ$  and  $\theta_{m,l} \approx 38^\circ$  respectively. This yields  $\sin \theta_l / \sin \theta_c \approx 2.7$  which agrees closely with  $\mu_c/\mu_l \approx 2.4$  obtained above.

#### 5 Summary

We have examined a simple statistical consequence of the standard model of superluminal sources and its implications for cosmology and unification schemes. Although the sample statistics are necessarily small, the overall results are, however, very intriguing. The present analysis shows a strong correlation between the core-to-lobe flux ratio  $R$  and proper motion  $\mu$  for a subsample of lobe-selected quasars used. This is consistent with the scenario in which  $R$  and  $\mu$  are good beaming indicators. The observed  $R-z$  correlation is shown to be consistent with a Lorentz factor of  $\gamma \approx 4$ . Also, from a plot of the proper motion–redshift  $(\mu-z)$  function, it is shown that a value of  $\gamma_l = 4$  implies that the data could be understood in terms of a high density universe with deceleration parameter  $q_0 = 0.5$ .

Furthermore, from a comparison of the observed  $(\mu-z)$  plot for core- and lobe-selected quasars at similar redshifts, it is shown that  $\mu$  and  $\gamma$  for core-selected quasars exceed those of lobe-selected quasars by a factor of  $\sim 2$ . This is also shown to be consistent with a pure orientation-based unified scheme in which the core-selected quasars are, on the average, at an angle of  $\approx 13^\circ$  to the line of sight, while their lobe-dominated counterparts are at  $\approx 38^\circ$ .

to the line of sight. We need more data, however, especially at high redshifts to confirm these results.

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