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A continuous time Markov model for the length of stay of elderly people in institutional long-term care

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Summary. The paper develops a Markov model in continuous time for the length of stay of elderly people moving within and between residential home care and nursing home care. A procedure to determine the structure of the model and to estimate parameters by maximum likelihood is presented. The modelling approach was applied to 4 years' placement data from the social services department of a London borough. The results in this London borough suggest that, for residential home care, a single-exponential distribution with mean 923 days is adequate to provide a good description of the pattern of the length of stay, whereas, for nursing home care, a mixed exponential distribution with means 59 days (short stay) and 784 days (long stay) is required, and that 64% of admissions to nursing home care will become long-stay residents. The implications of these findings and the advantages of the proposed modelling approach in the general context of long-term care are discussed.

Keywords: Length-of-stay modelling; Long-term care; Markov model; Survival

1. Introduction

In the UK, the National Audit Office has recently reported that the most common causes of delay in discharges from hospital are patients awaiting placement in a nursing or residential home and awaiting assessment of their needs (National Audit Office, 2003). Under the 1990 National Health Service and Community Care Act and the Care Standard Act 2000, local authorities in Great Britain are responsible for the placement and finance of adults in publicly funded residential and nursing home care that conforms to national standards. Discharge to long-term care is a central component of plans for acute hospital care and the demand for long-term care is expected to increase substantially as the population ages (Wittenberg *et al.*, 2001). In England, already 1 in 5 people aged 85 years or over live in a long-term care institution (Laiho, 2001). In addition, the UK Government is planning to fine local authorities for failing to provide vacancies in residential and nursing home care for hospital discharges. Therefore, it is important for both health authorities and local authorities to have a sound understanding of the patterns of the length of stay (LOS) and movements of residents in long-term care.

A recent survey showed that nearly 70% of the residents in residential and nursing homes were publicly funded and were there permanently (Netten *et al.*, 2001). In earlier research, we found that older people who are placed in nursing homes are more likely to have complex problems. Factors such as being male, immobile, dependent in feeding, urine incontinent, having open wounds and taking multiple drugs are associated with nursing home care placements, whereas older people who are admitted to residential home care are likely to be more independent (Xie

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et al., 2002). Therefore, we would expect differences in the pattern of LOS in residential and nursing home care.

Research in the UK shows that the mortality rate for residents in nursing home care is particularly high in the first few months and then gradually levels out (Smith and Lowther, 1976; Bebbington *et al.*, 2001; Rothera *et al.*, 2002). This observation supports the notion of phases in residents' stay in care homes. In the context of hospital geriatric departments, Harrison and Millard (1991) and Taylor *et al.* (1998, 2000) have shown that, despite the great heterogeneity between individuals (Millard, 1988), compartmental and Markov models, which divide patients' LOSs into short-stay and long-stay phases, capture successfully the behaviour of patients' LOSs. Similar results for residential and nursing home care can be expected.

We model the flow of elderly residents within and between residential and nursing home care by using a continuous time Markov model, in which residents' stay in care homes is modelled as a two-phase process: short stay and long stay. First, we describe the model that we propose and present a procedure for determining the model structure and estimating parameters by the method of maximum likelihood. We also show and discuss results that are obtained from fitting the model to a real data set.

2. A model for movement of elderly people in residential and nursing home care

The proposed conceptual model for the movement of elderly people in residential and nursing care facilities is depicted in Fig. 1. In this model, elderly people can be admitted into residential home care or nursing home care directly, either from the community or following discharge from hospital. In each type of care, residents start their stay in the short-stay phase and either leave care after a short period of time or continue their stay to become long-stay residents. People in residential home care can move to nursing home care if their conditions deteriorate to such an extent that residential home care is no longer adequate. In this paper, we consider only those residents who require local authority funding, and we exclude residents whose admissions are meant to provide short respites for their carers. This restriction is imposed because most local authorities have means of determining suitable care placements for applicants requiring public funds; therefore, these admissions will better reflect residents' physical conditions and needs. Movements from nursing home care to residential home care rarely occur among residents who are supported by local authority funds (Bebbington *et al.*, 2001) and are not modelled.

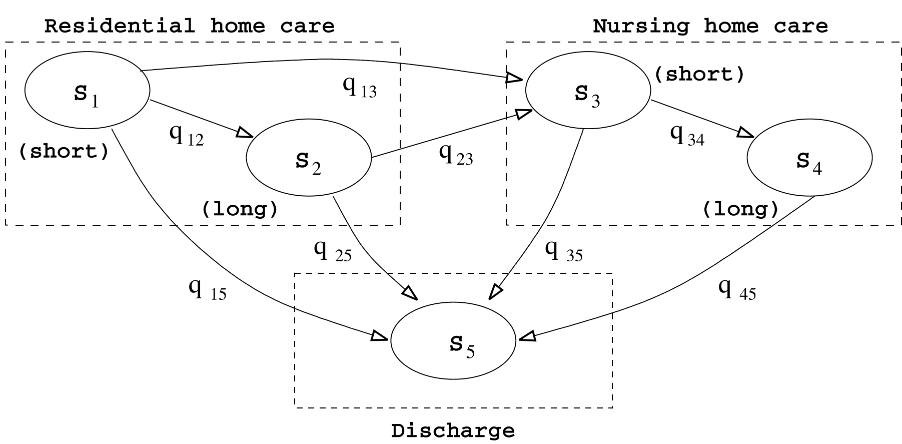

Fig. 1. Markov model for movements of elderly people in residential and nursing home care

Discharges from institutional long-term care are considered permanent. They occur predominately by death and, although a small number of residents are discharged to the community or hospital, they are not expected to return to institutional long-term care. Discharges to the community are rare for local-authority-funded residents, and those to hospital usually mean terminal care (Bebbington *et al.*, 2001).

We construct a continuous time Markov model of the flow of elderly people within and between residential and nursing home care. The phases in each type of care and the discharge state form the system states. Given the Markov model that is described in Fig. 1, the generator matrix **Q** is written as

$$\mathbf{Q}=\left(\begin{array}{cc:cc:c} q_{11} & q_{12} & q_{13} & 0 & q_{15} \\ 0 & q_{22} & q_{23} & 0 & q_{25} \\ \hdashline 0 & 0 & q_{33} & q_{34} & q_{35} \\ 0 & 0 & 0 & q_{44} & q_{45} \\ \hdashline 0 & 0 & 0 & 0 & 0 \end{array}\right), \tag{1}$$

where q_{ij} is the instantaneous transition rate between state i and state j $(i \neq j)$, and the elements in the main diagonal are defined such that row sums are 0, i.e. $q_{ii} = -\Sigma_{j \neq i}\, q_{ij}$.

3. Maximum likelihood estimation of model parameters

The actual states of the Markov model are not observable. We can only observe which type of care a person is in. For example, at any time, we observe that a person is in residential home care but we do not know whether she or he is in a short-stay (S_1) or long-stay (S_2) state. This is an aggregated Markov process, i.e. a Markov process in which system states are aggregated into a number of classes (Fredkin and Rice, 1986). There are three classes in the model that is outlined in Fig. 1, namely residential home care, nursing home care and discharge (denoted by $\mathcal{R}$, $\mathcal{N}$ and $\mathcal{D}$ respectively). We partition the matrix **Q** according to the class structure of the model, i.e.

$$\mathbf{Q}=\begin{pmatrix} \mathbf{Q}_{\mathcal{R}\mathcal{R}} & \mathbf{Q}_{\mathcal{R}\mathcal{N}} & \mathbf{Q}_{\mathcal{R}\mathcal{D}} \\ \mathbf{0} & \mathbf{Q}_{\mathcal{N}\mathcal{N}} & \mathbf{Q}_{\mathcal{N}\mathcal{D}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}, \tag{2}$$

where the submatrices correspond to those delimited by broken lines in equation (1) and the subscripts represent system classes. For instance, $\mathbf{Q}_{\mathcal{R}\mathcal{N}}$ is the submatrix of transition rates from states in $\mathcal{R}$ to states in $\mathcal{N}$, and $\mathbf{Q}_{\mathcal{R}\mathcal{R}}$ that of transition rates between states within $\mathcal{R}$.

The theory of aggregated Markov processes has been motivated by and applied to the modelling of ion channels in neurophysiological applications (Colquhoun and Hawkes, 1981, 1982; Fredkin *et al.*, 1985). Generalization and parameter estimation have been investigated by various researchers, including Ball and Sansom (1989), Fredkin and Rice (1986) and Qin *et al.* (1997). We adapt and modify the approach that was taken by these researchers to suit our modelling needs and to deal with the existence of an absorbing state and censored observations.

3.1. *Distribution of sojourn time in a class*

Calculating the first-passage time (Cox and Miller, 1965) leads to the probability density function (PDF) of the sojourn time in a class, say class $\mathcal{R}$ (Colquhoun and Hawkes, 1981)

$$f_{\mathcal{R}}(t) = -\phi_{\mathcal{R}}^{\mathrm{T}} \exp(\mathbf{Q}_{\mathcal{R}\mathcal{R}} t)\mathbf{Q}_{\mathcal{R}\mathcal{R}}\mathbf{1}_{\mathcal{R}}, \tag{3}$$

where $\phi_{\mathcal{R}}$ is a column vector of the probabilities of entering class $\mathcal{R}$ via each of its member states (in the case of the model depicted in Fig. 1, $\phi_{\mathcal{R}}=(1,0)^{\mathrm{T}}$) and $\mathbf{1}_{\mathcal{R}}$ is a column vector of 1s whose dimension is $|\mathcal{R}|$, i.e. the cardinality of $\mathcal{R}$. The PDF of sojourn time in a class, in equation (3), is a weighted sum of exponential distributions and the number of terms is equal to the number of states in the class (Colquhoun and Hawkes, 1981). This characteristic enables us to determine a lower bound on the number of states in a class from observed data, i.e. we can fit a series of mixtures of exponentials with increasing number of components and determine the smallest number of components that are needed to provide a good fit to observed data. Throughout, we use $\mathbf{M}^{-1}$ to denote the matrix inverse and $\mathbf{M}^{\mathrm{T}}$ the matrix transpose. The matrix exponential is defined in the usual manner; see for example Schott (1997).

From equation (3), the probability distribution function of the sojourn time in $\mathcal{R}$ becomes

$$F_{\mathcal{R}}(t)=\int_0^t f_{\mathcal{R}}(u)\,\mathrm{d}u=1-\phi_{\mathcal{R}}^{\mathrm{T}}\exp(\mathbf{Q}_{\mathcal{R}\mathcal{R}}t)\,\mathbf{1}_{\mathcal{R}}, \tag{4}$$

and, in the terminology of survival analysis (see for example Cox and Oakes (1984)), the survivor function is

$$S_{\mathcal{R}}(t)=1-F_{\mathcal{R}}(t)=\phi_{\mathcal{R}}^{\mathrm{T}}\exp(\mathbf{Q}_{\mathcal{R}\mathcal{R}}t)\,\mathbf{1}_{\mathcal{R}}. \tag{5}$$

For class $\mathcal{N}$, the PDF and distribution function of the sojourn time and the survivor function are obtained by substituting $\mathcal{N}$ for $\mathcal{R}$ in equations (3), (4) and (5) respectively.

3.2. Likelihood function

For each individual in the system, we observe the sequence of types (classes) of care that is received, and their duration until a fixed time point T, i.e. the end of the data collection period. In this model, there are six possible sequences: $\mathcal{R}\to\mathcal{D}$, $\mathcal{N}\to\mathcal{D}$ and $\mathcal{R}\to\mathcal{N}\to\mathcal{D}$, together with the three equivalent sequences where the individual is still in care at time T, i.e. right censored. We define $\mathbf{t}=(t_{\mathcal{R}},t_{\mathcal{N}})^{\mathrm{T}}$, i.e. the duration in each type of care. An element of $\mathbf{t}$ is equal to 0 if the corresponding class is not visited. Derived from the joint PDF of an observed sequence of classes and durations (Fredkin and Rice, 1986), the likelihood function of observing each of such sequences given the model is (subscripts represent the sequence of classes visited)

$$l_1(\boldsymbol{\theta}|\mathbf{t})=l_{\mathcal{R}\mathcal{D}}(\boldsymbol{\theta}|\mathbf{t})=\phi_{\mathcal{R}}^{\mathrm{T}}\exp(\mathbf{Q}_{\mathcal{R}\mathcal{R}}t_{\mathcal{R}})\,\mathbf{Q}_{\mathcal{R}\mathcal{D}}\mathbf{1},$$

$$l_2(\boldsymbol{\theta}|\mathbf{t})=l_{\mathcal{N}\mathcal{D}}(\boldsymbol{\theta}|\mathbf{t})=\phi_{\mathcal{N}}^{\mathrm{T}}\exp(\mathbf{Q}_{\mathcal{N}\mathcal{N}}t_{\mathcal{N}})\,\mathbf{Q}_{\mathcal{N}\mathcal{D}}\mathbf{1},$$

$$l_3(\boldsymbol{\theta}|\mathbf{t})=l_{\mathcal{R}\mathcal{N}\mathcal{D}}(\boldsymbol{\theta}|\mathbf{t})=\phi_{\mathcal{R}}^{\mathrm{T}}\exp(\mathbf{Q}_{\mathcal{R}\mathcal{R}}t_{\mathcal{R}})\,\mathbf{Q}_{\mathcal{R}\mathcal{N}}\exp(\mathbf{Q}_{\mathcal{N}\mathcal{N}}t_{\mathcal{N}})\,\mathbf{Q}_{\mathcal{N}\mathcal{D}}\mathbf{1},$$

$$l_4(\boldsymbol{\theta}|\mathbf{t})=l_{\mathcal{R}}(\boldsymbol{\theta}|\mathbf{t})=S_{\mathcal{R}}(t_{\mathcal{R}}),$$

$$l_5(\boldsymbol{\theta}|\mathbf{t})=l_{\mathcal{N}}(\boldsymbol{\theta}|\mathbf{t})=S_{\mathcal{N}}(t_{\mathcal{N}}),$$

$$l_6(\boldsymbol{\theta}|\mathbf{t})=l_{\mathcal{R}\mathcal{N}}(\boldsymbol{\theta}|\mathbf{t})=\phi_{\mathcal{R}}^{\mathrm{T}}\exp(\mathbf{Q}_{\mathcal{R}\mathcal{R}}t_{\mathcal{R}})\,\mathbf{Q}_{\mathcal{R}\mathcal{N}}\exp(\mathbf{Q}_{\mathcal{N}\mathcal{N}}t_{\mathcal{N}})\,\mathbf{1},$$

where $S_{\mathcal{R}}(t_{\mathcal{R}})$ is as defined in equation (5), and $\boldsymbol{\theta}$ is the parameters of the model, i.e. elements of the matrix $\mathbf{Q}$. The first three equations correspond to complete observations, and the last three to right-censored observations.

Define $\mathbf{\Delta} = (\delta_1, \ldots, \delta_6)^{\mathrm{T}}$ for an individual where, for $j = 1, \ldots, 6$,

$$\delta_j = \begin{cases} 1 & \text{the individual has taken the } j\text{th sequence,} \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Hence, the likelihood of an observed sequence of classes visited and duration spent for an individual can be written as

$$l(\boldsymbol{\theta}|\mathbf{\Delta}, \mathbf{t}) = \prod_{j=1}^{6} l_j(\boldsymbol{\theta}|\mathbf{t})^{\delta_j}, \tag{7}$$

or the log-likelihood function is

$$L(\boldsymbol{\theta}|\mathbf{\Delta}, \mathbf{t}) = \sum_{j=1}^{6} \delta_j \log\{l_j(\boldsymbol{\theta}|\mathbf{t})\}. \tag{8}$$

Therefore, the log-likelihood function for observing a group of n individuals is

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{i=1}^{n} L_i(\boldsymbol{\theta}|\mathbf{t}), \tag{9}$$

where $L_i(\boldsymbol{\theta}|\mathbf{t}) = L(\boldsymbol{\theta}|\mathbf{\Delta}_i, \mathbf{t}_i)$ is the log-likelihood of the ith individual given the observed sequences indicator $\mathbf{\Delta}_i$ and $\mathbf{t}_i$, i.e. the ith realization of $\mathbf{\Delta}$ and $\mathbf{t}$. Individuals are assumed to flow through the system independently.

The goal here is to maximize the log-likelihood function (9) with respect to $\boldsymbol{\theta}$ given the observed $\mathbf{\Delta}$s and $\mathbf{t}$s. Analytical expressions for the first derivatives of the likelihood function with respect to $\boldsymbol{\theta}$ can then be derived and used in the maximization process (for details, see Xie (2004)).

3.3. Implementation procedure

The states in each class of the Markov model are not observable. Therefore we must determine the number of states in each class before fitting the model. This is done by fitting mixtures of exponential distributions to observed LOS data of a cohort in each class separately, exploiting the fact that the functional form of the PDF of the LOS in a class, i.e. equation (3), is a weighted sum of exponential distributions with as many terms as there are states in that class. We select the mixture representing the best compromise between model complexity and goodness of fit according to both Akaike's information criterion (AIC) (Akaike, 1974) and the Bayesian information criterion (BIC) (Schwarz, 1978), along with a visual inspection of the fit (McLachlan and Peel, 2000).

We then fit the Markov model to the overall LOS data by using a general purpose optimizer to maximize the log-likelihood function. The departure rate for each state in a class is estimated from the parameters that are obtained by fitting the mixture of exponential distributions (see equation (3)) in the first stage. Then a starting value of the transition rate for all paths out of a state can be obtained by dividing the departure rate by the number of paths out of that state. Standard errors of the estimated parameters are approximated from the curvature of the log-likelihood surface at its maximum, i.e. from the Hessian matrix at the point of convergence (Cox and Hinkley, 1974).

In this paper, all computations were carried out in MATLAB® (MathWorks, 1992). These included maximization of the log-likelihood function by using quasi-Newton methods and numerical calculation of matrix exponentials.

Table 1. Estimated parameters for simulated data

Parameter	*True value*	*Estimate*	*Standard error*	*95% confidence interval*
q_{12}	0.070	0.0641	0.0066	(0.0512, 0.0770)
q_{13}	0.050	0.0497	0.0033	(0.0433, 0.0560)
q_{23}	0.005	0.0053	0.0003	(0.0048, 0.0059)
q_{34}	0.040	0.0381	0.0022	(0.0338, 0.0425)
q_{15}	0.010	0.0096	0.0012	(0.0073, 0.0118)
q_{25}	0.002	0.0022	0.0002	(0.0019, 0.0026)
q_{35}	0.080	0.0817	0.0027	(0.0764, 0.0870)
q_{45}	0.005	0.0052	0.0002	(0.0047, 0.0055)

3.4. Testing the procedure

We simulated an instance of the proposed model depicted in Fig. 1 by using the computer package for discrete event simulation MICRO SAINT® (Micro Analysis and Design, 1998). Pathways and LOSs for 2500 entities were simulated and recorded in a data set, to which the fitting procedure was applied.

Table 1 shows that the estimated parameters are close to the true values that were used in the simulation and the estimated standard errors are small, about 10% of the estimated parameter value.

4. Application of the model

4.1. Data

Data were provided by the Housing and Social Services Department of the London Borough of Merton in the form of annual spreadsheet records of payments that had been made for placements in residential and nursing homes. An anonymized cohort data set containing the date of admission, place of admission, date of discharge and destination at discharge was created. The data set contained information for all publicly funded admissions to residential home care ($\mathcal{R}$) and nursing home care ($\mathcal{N}$) between April 1997 and April 2001. It consisted of 935 records of 889 distinctive individuals. In all, 438 individuals were admitted to $\mathcal{R}$; at the end of observation, 217 of them were still living in $\mathcal{R}$, two had left the borough (considered as censored), 46 had been transferred to $\mathcal{N}$, seven had been discharged to hospital (not returning) and 166 had died. 451 people were directly admitted to $\mathcal{N}$; together with the 46 people who had been transferred from $\mathcal{R}$, 155 of them were still alive in $\mathcal{N}$, one had left the borough, one had been discharged to hospital and 340 had died; of the 46 who had been transferred from $\mathcal{R}$, 25 were still alive in $\mathcal{N}$ and 21 had died. Residents who die in the community shortly after leaving the care home are considered as having died in the care home, and residents with acute hospital episodes while their beds are kept open for their return are treated as if there were no interruption in their stay in a care home.

4.2. Model fitting

The model fitting procedure that was described previously (Section 3.3) was applied to the data set. In the first stage of the procedure, the best compromise between model complexity and goodness of fit was obtained with a single state in $\mathcal{R}$ and two states in $\mathcal{N}$, which show the smallest AIC and BIC values in Table 2. The structure of the Markov model was modified accordingly

Table 2. Determination of the number of states in $\mathcal{R}$ and $\mathcal{N}$

Number of states	*Results for residential home care*		*Results for nursing home care*	
	AIC	*BIC*	*AIC*	*BIC*
1	3430.651	3434.733	4879.295	4883.504
2	3433.142	3445.388	4774.788	4787.414
3	3437.142	3457.553	4778.792	4799.835

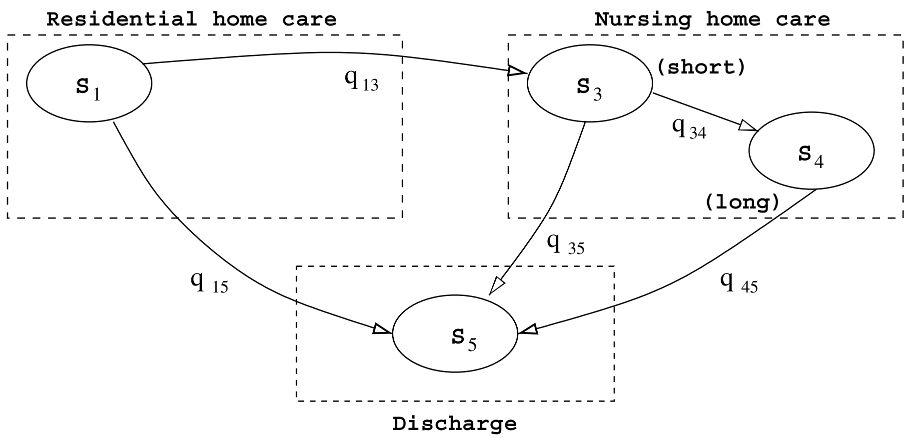

Fig. 2. Structure of the Markov model for the Merton data set

(Fig. 2). The second-stage Markov model fitting procedure converged quickly with the starting-point proposed in Section 3.3. One-dimensional views of the log-likelihood surface along all parameter axes suggested that the maximum was well defined and that the log-likelihood surface was relatively quadratic near the maximum. For each type of care, the close agreement between the survivor curve that was derived from the estimated matrix **Q** (see equation (5)) and the survivor curve that was estimated by the Kaplan–Meier estimator (Kaplan and Meier, 1958) indicates that the Markov model provides a good fit to the data (Fig. 3). This is confirmed by the probability plots (Fig. 4).

4.3. Results

The estimated parameters for the Markov model are summarized in Table 3. These results give interesting insights into the survival patterns of elderly people in institutional long-term care in the London Borough of Merton. A single state provides a good fit to the LOS pattern in residential home care ($\mathcal{R}$), thus indicating a constant rate of departure from $\mathcal{R}$. The average LOS for $\mathcal{R}$ is estimated by $1/(q_{13}+q_{15})$, i.e. 923 days (about 2.5 years). On leaving $\mathcal{R}$, about 79% of the residents will be discharged (permanently) and 21% of them will transfer to nursing home care ($\mathcal{N}$). Two distinctive states are observed in $\mathcal{N}$: a short-stay state with an average LOS of 59 days and a long-stay state with an average LOS of 784 days (about 2.1 years). The rate of discharge from the short-stay state is about five times that from the long-stay state. This agrees with empirical observations that initial mortality is higher for the first few months following admission to nursing care (Smith and Lowther, 1976; Bebbington *et al.*, 2001; Rothera *et al.*,

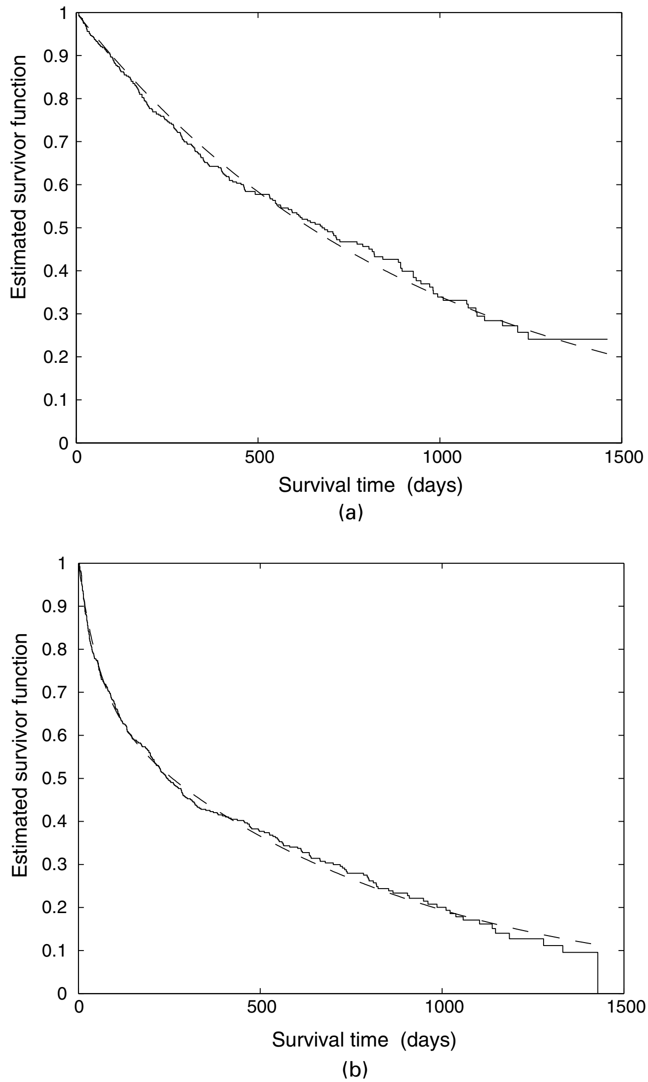

Fig. 3. Kaplan–Meier estimated (———) and Markov model fitted (– – – –) survivor curves for (a) residential home care and (b) nursing home care for the Merton data set

2002). About 64% of elderly people entering $\mathcal{N}$ will become long-stay residents. The average LOS in the long-stay state of $\mathcal{N}$ is similar to that of $\mathcal{R}$, which suggests that, once residents stay through the short-stay state, their LOS pattern will be similar to those in $\mathcal{R}$.

Based on the estimated parameters of the Markov model, the fitted survivor function (see equation (5)) for $\mathcal{R}$ is $S_{\mathcal{R}}(t) = \exp(-0.0011t)$, and for $\mathcal{N}$ it is

$$S_{\mathcal{N}}(t) = 0.309 \exp(-0.0173t) + 0.691 \exp(-0.0013t),$$

i.e. a mixture of two exponentials. The average LOSs in $\mathcal{R}$ and $\mathcal{N}$ are 2.5 years and 1.5 years respectively. A property of the exponential distribution is that a proportion of people will live substantially longer than average, and in the case of the mixture of exponentials the longer their stay the longer their expected further stay will be, as noted in Harrison (2001). Among the

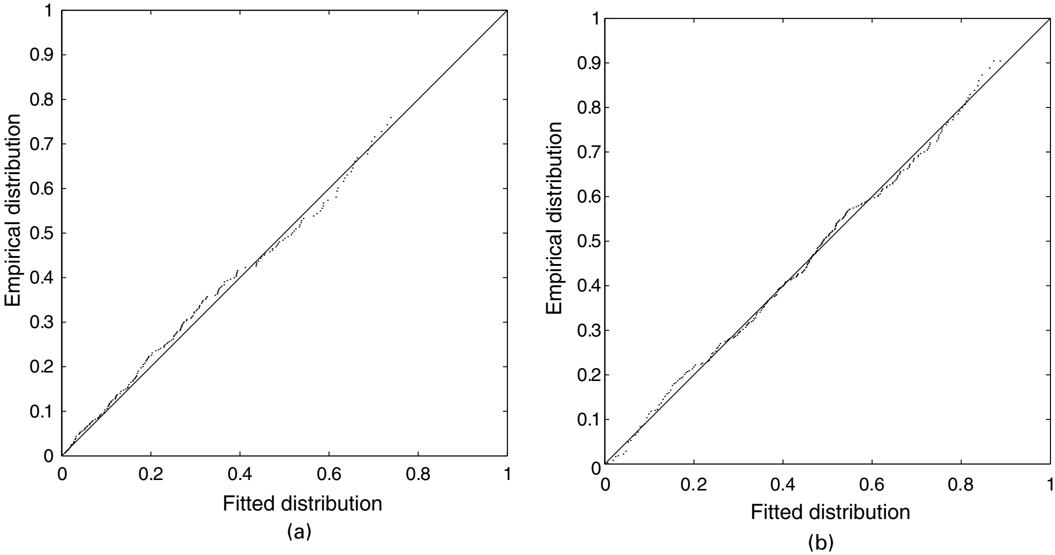

Fig. 4. Probability (*P*–*P*-) plot of the Markov model fitted survivor curves for (a) residential home care and (b) nursing home care for the Merton data set

Table 3. Estimated parameters for the Merton data set

Parameter	*Estimate*	*Standard error*	*95% confidence interval*
q_{13}	0.000228	0.000034	(0.000162, 0.000293)
q_{15}	0.000855	0.000065	(0.000728, 0.000983)
q_{34}	0.010874	0.002961	(0.005071, 0.016677)
q_{35}	0.006138	0.000793	(0.004584, 0.007692)
q_{45}	0.001275	0.000135	(0.001010, 0.001540)

older people who have been placed in $\mathcal{R}$ by the local authority, 50% will stay more than 21 months, 25% will live longer than 3.5 years and 10% will be there after 5.7 years. Of those who have been placed in $\mathcal{N}$, 50% will stay for more than 8 months, 25% will live longer than 2.1 years and 10% will still be there 4.1 years after they have been admitted.

5. Discussion

We have built a continuous time Markov model which captures the flow of elderly people within and between residential and nursing home care. Using the framework of aggregated Markov processes, we derived a procedure for fitting the model to observed data. By modelling the system of long-term care as a whole, we captured the movements between facilities and estimated parameters by using the overall joint likelihood function. Using a real data set we showed that the LOS in residential home care can be approximated by a single-exponential distribution with mean 923 days, whereas in nursing home care a mixed exponential distribution with short-stay mean 59 days and long-stay mean 784 days is needed to provide a good fit. About 21% of residential home care vacancies were created by transfers to nursing home care and 64% of all admissions to nursing home care will become long-stay residents. In nursing home care, the mortality rate in the short-stay state is about five times that in the long-stay state. Thus, the model quantifies the large heterogeneity in mortality rates that is widely observed in nursing home care.

Extensive research in the UK has been conducted to identify the characteristics that are associated with differences in survival patterns in long-term care. This research has mainly focused on identifying risk factors that are associated with mortality, e.g. Bebbington *et al.* (2001), Dale *et al.* (2001) and Rothera *et al.* (2002). From the point of view of individual elderly people, their doctors and social workers, the identification of risk factors that are associated with transfer, early death and long-term survival is of considerable importance. But, for planning, care managers and budget holders need to know the overall pattern of LOS in long-term care. Our model complements other research in providing a full picture of the overall behaviour of LOS in residential and nursing home care.

Methods that explicitly model the survival time (or the LOS in care) of elderly people have consistently shown that a mixture of exponentials gives a good fit to observed LOS data (Harrison and Millard, 1991; McClean and Millard, 1993; Taylor *et al.*, 1998, 2000). Struthers (1963) first reported that LOS in a hospital geriatric department in Southampton followed a combination of two exponential curves: one had a 'half-life' of 2 months and the other had a half-life of 2 years. A mixed exponential distribution implies that a proportion of elderly people in residential and nursing home care will live substantially longer than the mean and the longer their stay the longer their expected further stay will be. A large proportion of older people who have been placed by the Merton Social Service Department in residential and nursing home care will stay substantially longer than their expected LOS, 2.5 years and 1.5 years respectively. In residential home care, 25% will live longer than 3.5 years and 10% will live longer than 5.7 years; in nursing home care, 25% will live longer than 2.1 years and 10% will live longer than 4.1 years. This means that short-term decisions to increase the number of permanent admissions to residential and nursing home care will have serious long-term financial and organizational consequences. Such action will result in, as time passes, a reduction in the places that are available for new admissions since the number of beds occupied by residents admitted in earlier years increases.

The model that we have developed in this paper could help planning authorities to understand the overall pattern of usage of resources for elderly people in their catchment area. Our model can be extended to cope with possible differences in survival pattern between nursing care residents who are admitted directly and those who are transferred from residential care, although we did not find significant evidence to suggest that such differences existed in the data set that we used. Further work is needed to confirm our findings and to extend the model to take into account the attributes of elderly people, e.g. their age, gender and physical and mental conditions.

Given the importance of having vacancies in long-term care to run acute hospitals efficiently and the significant costs that are associated with maintaining elderly people in care homes, the findings of this paper should be of great interest to Government departments, insurance companies, health and social services planners, and purchasers and providers of residential and nursing home care.

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