A Control Lyapunov Function Approach to Multiagent Coordination

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Abstract—In this paper, the multiagent coordination problem is studied. This problem is addressed for a class of robots for which control Lyapunov functions can be found. The main result is a suite of theorems about formation maintenance, task completion time, and formation velocity. It is also shown how to moderate the requirement that, for each individual robot, there exists a control Lyapunov function. An example is provided that illustrates the soundness of the method.

Index Terms—Coordinated control, Lyapunov methods, mobile robots, multirobot system, robot formation control.

I. INTRODUCTION

I N THIS PAPER, we investigate the problem of how to coordinate a collection of robots in such a way that they maintain a given formation relative to each other. The main assumption about the dynamics of the individual robots that we initially make in this paper is that they have control Lyapunov functions (CLFs). Based on this assumption, an abstract and theoretically sound coordination strategy can be developed.

Multiagent formation control problems have been extensively studied in the literature, and our main contribution is that we use CLFs to define the formation. By doing this, we convert the formation control problem, typically a constrained motion control problem of multiple systems, into a stabilization problem for one single system. By this approach, we neither cast the problem without real dynamics [2], nor with an explicit nonlinear robot model [5]. Instead, we believe that by requiring the existence of CLFs, we can capture at least some aspects of the platform dynamics, while not having to spend our main effort on nonlinear robot control. Thus, we can focus on the coordination problem at a higher level.

In addition to the CLF approach, we use the idea of virtual vehicles discussed in [6]. Concepts of similar flavor are the "action reference" suggested by Kang *et al.* [7] and the "dynamic coordination variable" proposed by Beard *et al.* [3]. Furthermore, in the terminology of Beard *et al.* in [9], our approach would fall

into the category of "virtual structures." The formation function we introduce has similarities with the task function of [13], but there is no connection with Lyapunov theory in that approach.

The motivation for studying this type of multiagent coordination problem mainly stems from the observation that there is robustness and strength in numbers. If more than one agent is asked to carry out a given task, e.g., search a disaster area, the likelihood of success increases as more agents are included in the mission. In other situations, cost and energy efficiency indicates that using many small robots might be more beneficial than using one big robot.

The outline of this paper is as follows. In Section II, we define what we mean by a formation, and show how this can be formalized in terms of a formation function. We define a subclass called Lyapunov formation functions, and show how these can be constructed. Next, in Section III, we add error feedback to the time evolution of the formation. In Section IV, we then prove a suite of theorems about bounded formation errors, task completion times, and group velocities. We conclude, in Section V, with an example, illustrating the usefulness of our proposed method.

II. FORMATION FUNCTIONS

Our primary object of study is a collection of m robots, whose dynamics can be described by the following set of controlled differential equations:

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i, \quad i = 1, \ldots, m \tag{1}$$

where $f_i, g_i \in C^\infty, x_i \in \mathbb{R}^n$, and $u_i \in \mathbb{R}^{p_i}$.

Now, a desired formation in $\mathbb{R}^{nm}$ is simply a set $\{x_{10}, \ldots x_{m0}\} \in \mathbb{R}^{nm}$, and we define this set implicitly through the null set of a so-called formation function.

Definition II.1—Formation Function: Given a formation $\{x_{10}(s), \ldots, x_{m0}(s)\} \in \mathbb{R}^{nm}$ parameterized by a scalar s. We say that a positive definite, continuously differentiable map $F : \mathbb{R} \times \mathbb{R}^{nm} \to \mathbb{R}$ is a formation function to the set $\{x_{10}(s), \ldots, x_{m0}(s)\}$ if $\{s, x_1, \ldots, x_m\} \in F^{-1}(0)$ gives a unique $\{x_1, \ldots, x_m\}$ for each choice of $s \in \mathbb{R}$.

Example II.1: One choice is $F(s, x_1, \ldots, x_m) = \Sigma_i \|x_i - x_{i0}(s)\|^2$. Clearly, $F = 0$ implies a unique x_i for each s, as demanded in the definition. But as we will see, robot dynamics (*Definition II.2*) can be taken into account when choosing $F(s, x)$.

In order to focus on the high-level coordination issues, we connect the formation function with the concept of CLFs.

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Definition II.2—Lyapunov Formation Function: A formation function $F(s, x)$ is a Lyapunov formation function if there exists a class $\mathcal{K}$ function σ and a control $u(s, x)$, such that

$$\frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\sigma(F(s, x))} \geq 1 \quad \forall s \in \mathbb{R}, \quad x \in \mathbb{R}^{nm} \,|\, 0 < F(s, x) \leq F_U.$$

Furthermore, the left-hand side of this expression goes to infinity as F approaches zero.

A class $\mathcal{K}$ function is a function $\sigma : [0, k) \rightarrow \mathbb{R}_+$, continuous, strictly increasing, and satisfying $\sigma(0) = 0, \sigma(x) > 0, \forall x > 0$ [11].

To set the stage for our main existence theorem on Lyapunov formation functions, we now state two lemmas.

Lemma II.1—Locally Positive Definite (LPD) Implications: If the Lyapunov function V is LPD and decrescent and $-\dot{V}$ is LPD, then the bound of *Definition II.2* is fulfilled.

Proof: The conditions on V implies [11] that there exist class $\mathcal{K}$ functions α, β, and γ, such that

$$\alpha(\|x\|) \leq V(x) \leq \beta(\|x\|)$$
$$\gamma(\|x\|) \leq -\dot{V}.$$

Since the functions are of class $\mathcal{K}$, they are all invertible and positive, and we have $\|x\| \geq \beta^{-1}(V(x))$ and

$$-\dot{V} \geq \gamma(\|x\|) \geq \gamma(\beta^{-1}(V(x))) = \sigma(V(x)).$$

Note that this makes σ a class $\mathcal{K}$ function. Now we have

$$\frac{-\dot{V}}{\sigma(V)} \geq 1$$

and we can choose $\sigma(V) = \gamma(\beta^{-1}(V(x)))$. Furthermore, if the property only holds locally, i.e., when $\|x\| \leq \epsilon$, then this condition can be replaced by $V(x) \leq \alpha(\epsilon) = V_U$, since $\|x\| \leq \alpha^{-1}(V(x))$. ∎

Remark II.1: These are the standard Lyapunov assumptions for showing that a system is asymptotically stable [11]. The case of a semidefinite Lyapunov function for the whole formation is discussed in [12].

Lemma II.2—Limit Property: Given a σ such that

$$\frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\sigma(F(s, x))} \geq 1 \quad \forall s \in \mathbb{R}, \quad x \in \mathbb{R}^{nm} \,|\, 0 < F(s, x) \leq F_U.$$

The limit property of *Definition II.2* will be fulfilled if we choose a new $\sigma' = (\sigma^2)/(1 + \sigma)$.

Proof: Since $\sigma' \leq \sigma$ the bound still holds. Furthermore

$$\frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\sigma'(F(s, x))} = \frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\sigma(F(s, x))} \frac{1 + \sigma(F(s, x))}{\sigma(F(s, x))}$$
$$\geq \frac{1 + \sigma(F(s, x))}{\sigma(F(s, x))}$$

which clearly approaches infinity as $F \rightarrow 0$. ∎

Remark II.2: The local stability property of *Definition II.2* in combination with the feedback of (3) in Section III will be shown to guarantee successful traversal of the whole trajectory.

Remark II.3: In general, finding a CLF is an open problem, however, it is known that CLFs exist for a large class of prac-

tically important systems [8], including feedback linearizable systems (as will be seen in Section V). Note also the Artstein Sonntag theorem on existence of CLFs [1].

We now go on to state and prove the main existence theorem of Lyapunov formation functions.

Theorem II.1—Lyapunov Formation Function: If the n vehicles have translationally invariant (in position coordinates) dynamics and LPD CLFs V_i, with locally negative definite time derivatives, then we can form a Lyapunov formation function by a weighted sum of the parameterized CLFs

$$F(s, x) = \sum_{i=1}^m \beta_i V_i(s, x_i). \tag{2}$$

Proof: By *Lemma II.1*, the bound of *Definition II.2* holds, pointwise. The hypothesis that the vehicle dynamics are invariant with respect to position translations lets us parameterize them with respect to s, $V_i(s, x)$, and we have that

$$\frac{-\left(\frac{\partial V_i}{\partial x_i}\right)^T \dot{x}_i}{\sigma_i(V_i(s, x_i))} \geq 1 \quad \forall s \in \mathbb{R}, \quad x_i \in \mathbb{R}^n \,|\, V_i(s, x_i) \leq V_{Ui}$$

where the V_{Ui} are given by $\alpha(\epsilon)$ as in *Lemma II.1*.

We have that

$$-\left(\frac{\partial F}{\partial x}\right)^T \dot{x} = \sum_{i=1}^m \beta_i \left(-\frac{\partial V_i}{\partial x}\right)^T \dot{x}$$
$$\geq \sum_{i=1}^m \beta_i \sigma_i(V_i(s, x))$$
$$\geq \min_i \beta_i \sigma_i \left(\frac{F}{\beta_{\max} m}\right)$$
$$= \sigma(F(s, x))$$

where

$$\sigma(F(s, x)) = \min_i \beta_i \sigma_i \left(\frac{F}{\beta_{\max} m}\right).$$

The second inequality above follows, since there exists a j such that $V_j \geq (F/\beta_{\max} m)$, and thus

$$\sigma_j(V_j(s, x)) \geq \min_i \sigma_i \left(\frac{F}{\beta_{\max} m}\right).$$

The limit property follows from *Lemma II.2*. ∎

Remark II.4: Note that the different vehicles in the formation can have completely different dynamics [and therefore Lyapunov functions $V_i(s, x_i)$ in (2)]. Furthermore, the choice of coefficients in (2) reflects how large deviations from zero are allowed for each V_i.

III. COORDINATED CONTROL

By establishing these observations about the Lyapunov formation functions derived from the individual CLFs, we can now shift our attention to actually controlling the evolution of the formation. The one parameter that we can control is the s parameter, i.e., the parameterization of the time evolution of the desired positions. We do this by specifying the trajectory that we want the so-called *virtual leader*, $x_0(s(t))$, to follow.

This nonphysical leader is a reference point in the state space with respect to which we can define the rest of the formation. We denote the trajectory executed by the virtual leader by $x_0(s(t)) = p(s(t))$. Intuitively, one might want to set $s(t) = t$. But, due to robustness considerations, we incorporate error feedback into the time evolution of s (see, for example, [6]) and let $\dot{s}$ be given by

$$\dot{s} = \min\left\{ \frac{v_0}{\delta + \left\|\frac{\partial p(s)}{\partial s}\right\|}, \frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\delta + \left|\frac{\partial F}{\partial s}\right|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))}\right) \right\}. \quad (3)$$

Here, $\delta > 0$ is a small positive constant that prevents $\dot{s}$ from becoming singular, and F_U is the bound of *Definition II.2* or something smaller chosen by the user. It will be shown to be an upper bound on the Lyapunov formation function $F(s,x)$. The idea is to say that the formation is being respected as long as $F(s,x) \leq F_U$. σ is the class $\mathcal{K}$ function of *Definition II.2*. Furthermore, v_0 is the nominal velocity that we want the formation to move with, and as we will see later, it holds that $\|\dot{x}_0(s(t))\| \approx v_0$ when F is small.

IV. THEORETICAL PROPERTIES

In the following paragraphs we will investigate what theoretical properties the evolution of the multi-agent formation exhibits when letting $\dot{s}$ be given by (3). We will show that if $F(s(t_0), x(t_0)) \leq F_U$, then F remains bounded by F_U along trajectories for all times greater than t_0. We will also show that if $s \in [s_{\text{start}}, s_{\text{final}}]$ and if $\dot{s}$ is governed by (3), then s reaches s_{final} in finite time. We will conclude our theoretical investigations by showing that $\|\dot{x}_0(s(t))\| \approx v_0$ if the formation function is small enough.

Theorem IV.1—Error Bound: If $F(s(t_0), x(t_0)) \leq F_U$, then

$$F(s, x) \leq F_U, \quad \forall t \geq t_0$$

i.e., the Lyapunov formation function will never exceed F_U.

Proof: We directly have

$$\dot{F} = \left(\frac{\partial F}{\partial x}\right)^T \dot{x} + \frac{\partial F}{\partial s} \dot{s}(s,x)$$

$$\leq \left(\frac{\partial F}{\partial x}\right)^T \dot{x} + \frac{\partial F}{\partial s} \frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\delta + \left|\frac{\partial F}{\partial s}\right|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))}\right)$$

$$= \left(\frac{\partial F}{\partial x}\right)^T \dot{x} \left(1 - \frac{\frac{\partial F}{\partial s}}{\delta + \left|\frac{\partial F}{\partial s}\right|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))}\right)\right).$$

To show that the set $\{(s,x) \,|\, F(s(t), x(t)) \leq F_U\}$ is invariant, we will note that for any $(s(t_0), x(t_0))$ such that $F(s(t_0), x(t_0)) \geq F_U$, we have

$$\dot{F}(s(t_0), \quad x(t_0)) \leq 0.$$

For such a $(s(t_0), x(t_0))$, it holds that

$$\frac{\sigma(F_U)}{\sigma(F(s(t_0), x(t_0)))} \leq 1$$

$$\frac{\frac{\partial F(s(t_0), x(t_0))}{\partial s}}{\left(\delta + \left|\frac{\partial F(s(t_0), x(t_0))}{\partial s}\right|\right)} \leq 1$$

and

$$\left(\frac{\partial F(s(t_0), x(t_0))}{\partial x}\right)^T \dot{x}(t_0) \leq 0.$$

This directly gives that if $F(s(t_0), x(t_0)) \geq F_U$, then $\dot{F}(s,x) \leq 0$ along trajectories. Thus, the set $\{(s,x) \,|\, F(s(t), x(t)) \leq F_U\}$ is invariant, i.e., if $F(s(t_0), x(t_0)) \leq F_U$, then $F(s(t), x(t)) \leq F_U$ for all $t \geq t_0$. ∎

Remark IV.1: In most cases, a careful choice of F_U can guarantee collision avoidance.

Theorem IV.2—Finite Completion Time: If the trajectory is given as an interval, $s \in [s_{\text{start}}, s_{\text{final}}]$, then by using the controller in (3) we can find an upper bound $M < \infty$, such that

$$T_{\text{final}} \leq M.$$

In other words, there is an upper bound on the completion time.

Proof: Let the completion time be defined in such a way that

$$\int_{T_{\text{start}}}^{T_{\text{final}}} \dot{s} = s_{\text{final}} - s_{\text{start}}.$$

If there exists a constant $v_s > 0$ such that $\dot{s} \geq v_s$, then we obviously have

$$T_{\text{final}} \leq \frac{s_{\text{final}} - s_{\text{start}}}{v_s} + T_{\text{start}}.$$

The proof thus consists of finding such a lower bound on $\dot{s}$. Let

$$K_1 = \max_{s \in [s_{\text{start}}, s_{\text{final}}]} \left\{ \delta + \left\|\frac{\partial p(s)}{\partial s}\right\| \right\}$$

$$K_2 = \max_{s \in [s_s, s_f], x \in \mathbb{R}^{nm} \,|\, F(s,x) \leq F_U} \left\{ \delta + \left|\frac{\partial F(s,x)}{\partial s}\right| \right\}.$$

We have that

$$\dot{s}(s,x) = \min\left\{ \frac{v_0}{\delta + \left\|\frac{\partial p(s)}{\partial s}\right\|}, \frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\delta + \left|\frac{\partial F}{\partial s}\right|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))}\right) \right\}$$

$$\geq \min\left\{ \frac{v_0}{K_1}, \frac{L\sigma(F_U)}{K_2} \right\} = v_s > 0$$

since $(F_U)/(F(s,x)) \geq 1$. Thus

$$T_{\text{final}} \leq \frac{s_{\text{final}} - s_{\text{start}}}{v_s} + T_{\text{start}} = M$$

which concludes the proof. ∎

Theorem IV.3—Formation Velocity: If the formation error is small, $F(s,x) \ll F_U$ and $\delta v_0 \ll \|\partial p(s)/\partial s\|$, then

$$\left\|\frac{dx_0}{dt}\right\| \approx v_0$$

i.e., the formation velocity is v_0.

Proof: We will start by showing that the right-hand term in the brackets of (3) grows to infinity as F approaches zero

$$\frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\delta + \left|\frac{\partial F}{\partial s}\right|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))}\right) \geq \frac{\sigma(F_U)}{K_2} \frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\sigma(F(s,x))} \to \infty$$

$$\text{as } F(s,x) \to 0$$

by *Assumption II.2*. Thus, the left-hand term governs $\dot{s}$ when F is small. In that case, we have

$$\left\| \frac{dx_0}{dt} \right\| = \left\| \frac{\partial p(s)}{\partial s} \right\| \dot{s}$$

$$= \left\| \frac{\partial p(s)}{\partial s} \right\| \frac{v_0}{\delta + \left\| \frac{\partial p(s)}{\partial s} \right\|}$$

$$= v_0 - \frac{\delta v_0}{\delta + \left\| \frac{\partial p(s)}{\partial s} \right\|} \approx v_0.$$

∎

V. SIMULATION EXAMPLES

We will now go on to illustrate *Theorem IV.1* and *Theorem IV.3*, as well as the effect of measurement noise on our proposed approach. In the following example, we model the robots using the standard unicycle model (see, for example, [4] and [5]). Such a model is applicable to the Nomadic Scout, which is the robot we work with at the Royal Institute of Technology in Stockholm. It is, furthermore, adequate for most all-terrain caterpillar vehicles as well.

The equations of motion are

$$\dot{z}_1 = v \cos \theta, \quad \dot{z}_2 = v \sin \theta, \quad \dot{\theta} = \omega, \quad m \dot{v} = F, \quad J \dot{\omega} = \tau$$

where (z_1, z_2) is the center of the wheel axis, θ is the directional angle, and v, ω are forward and angular velocities. The controls F, τ are the applied force and torque. We choose the output to be the position of an off-axis point, $x_1 = z_1 + l \cos(\theta), x_2 = z_2 + l \sin(\theta)$, perhaps the center of gravity.

It was shown in [9] that this model can be feedback linearized to a two-dimensional double integrator $\ddot{x}_1 = u_1, \ddot{x}_2 = u_2$. (This property was also used implicitly in [5]). A parameterized CLF and feedback control of a one-dimensional double integrator is

$$V_i(s, x_i) = (x_i - p_i(s))^2 + ((x_i - p(s)) + \dot{x}_i)^2$$
$$u_i = 0.5(-(x_i - p_i(s)) - 3\dot{x}_i)$$

yielding $\dot{V}_i = -(x_i - p_i(s))^2 - \dot{x}_i^2$. We note that there is some flexibility in choosing the second term in V_i, since the only hard formation constraint is $x_i = p_i(s)$.

Now we can choose $F(s, x) = \sum_{i=1}^{m} V_i(s, x_i)$, as in (2), and it is straightforward to check that $\sigma(z) = ((0.3z)^2)/(0.3z + 1)$ fulfills the bound and limit property of *Definition II.2*.

A formation of three feedback linearized dynamic unicycles thus gives a 12–dimensional system (Fig. 1). In the first part of the simulation, the three desired trajectories meet to form a side-by-side formation. When the robots are close to horizontal coordinate 6 m, we increase $v_0 = 0.1$ to $v_0 = 0.3$. This will drive the formation function close to, but not above, the upper limit of $F_U = 1$. When this happens, the velocity is decreased to a value below $v_0 = 0.3$. Finally, in the last part, when passing the 12–m mark, we introduce a stochastic measurement error in the control of the topmost robot. Since we are already close to the upper bound $F_U = 1$, the disturbance makes the whole formation slow down when needed (as seen in the lowest plot) to respect the bound. This is not guaranteed by *Theorem IV.1*, since the proof is only valid in a deterministic setting. However,
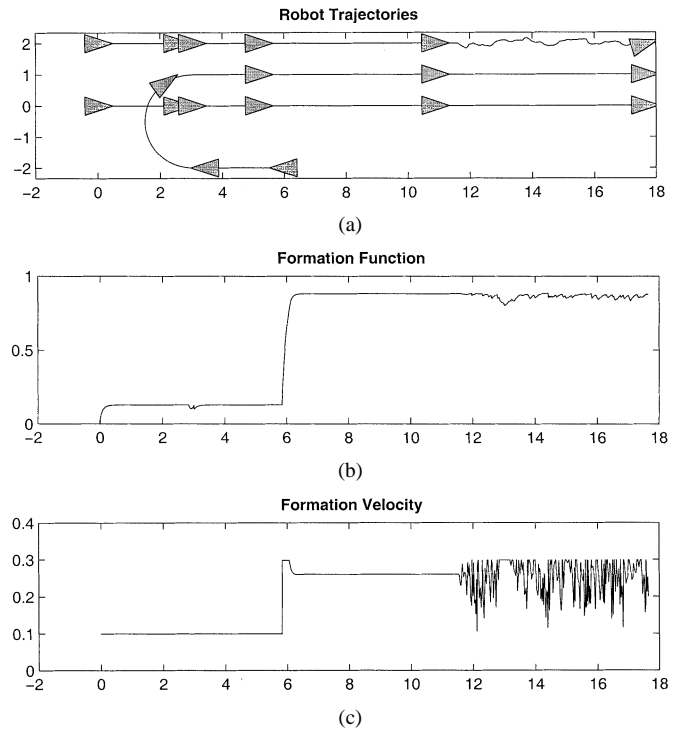

Fig. 1. (a) Top view of the robot trajectories. (b) The formation function. (c) The formation velocity. The horizontal axis of the two lower plots refer to the horizontal position of the topmost robot.

the successful simulation does indicate some robustness of the approach.

VI. CONCLUSION

In this paper, we propose a stable coordination strategy for a team of formation constrained autonomous agents. A Lyapunov formation function defined under standard assumptions can be constructed from individual-CLFs. The Lyapunov formation function is used to prove properties such as formation maintenance, task completion time, and formation velocity. Finally, we present an example that illustrates the soundness of our method.

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