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A CONTROL THEORY APPROACH ON THE DESIGN OF A MARX GENERATOR NETWORK

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Abstract

A Marx generator is a well-known type of electrical circuit first described by Erwin Otto Marx in 1924. It has been utilized in numerous applications in pulsed power with resistive or capacitive loads. To-date the vast majority of research on Marx generators designed to drive capacitive loads relied on experimentation and circuit-level modeling to guide their designs. In this paper we describe how the problem of designing a Marx generator to drive a capacitive load is reduced to that of choosing a diagonal gain matrix F that places the eigenvalues of the closed-loop matrix $A + BF$ at specific locations. Here A is the identity matrix and B characterizes the elements of the Marx generator and depends on the number of stages N . Due to the special structure of matrix F , this formulation is a well-known problem in the area of feedback control and is referred to as the *structured static state feedback problem*. While the problem is difficult to solve in general, due to the specific structures of matrices A and B , various efficient numerical algorithms exist to find solutions in specific cases. In a companion paper by Buchenauer [1] it is shown that if certain conditions hold, then setting the natural frequencies of the circuit to be harmonically related guarantees that all the energy is delivered to the load capacitor after a suitable delay. A theorem

formalizing this result is presented. Earlier aspects of this research have been published in two theses [2,3].

I. State Space Realization

The circuit model for the Marx generator described in this work is shown in Fig. 1. The circuit components are intended of be lossless. An external voltage source simultaneously charges capacitors $C_1, C_2, \dots, C_{n-1}, C_n$. After charging these capacitors to the desired voltage (and hence accumulating the desired charge), the discharging process starts into the corresponding parasitic capacitors through their respective inductances and resistances.

We assume that the resistances are identical and that the energy storage capacitors are identical as well. By assuming that the current through the resistors is small compared to the current through the inductors, the discharging process for each stage can be approximated as shown in Fig. 2. Antoun [2] proceeds to use circuit theory to redraw the Marx circuit (Fig. 1) as a connection of branches, where tree branches represent voltage sources (capacitors in our case) and links represent current sources (inductors in our case) and resistors (Fig. 3). The direction of the arrows is along the voltage drop in the case of a voltage source, or along the current in the case of a current source.

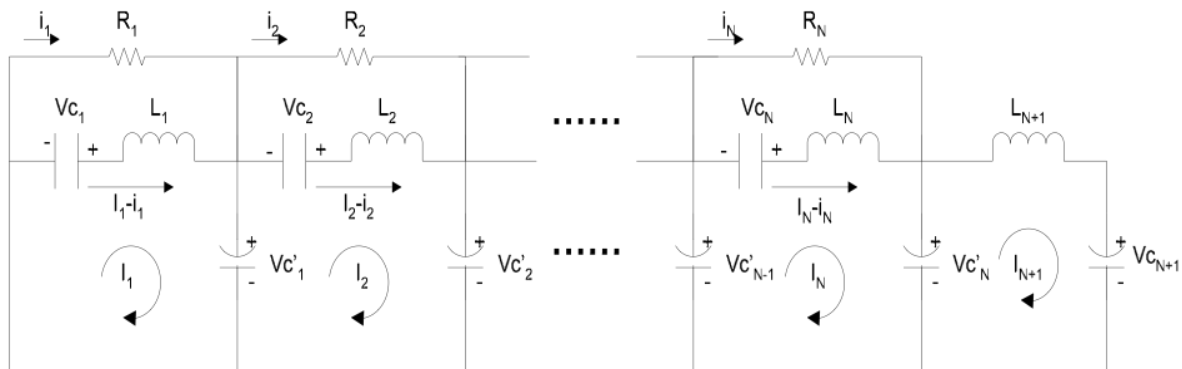


Figure 1. Network model of an N -stage Marx generator driving a capacitive load.

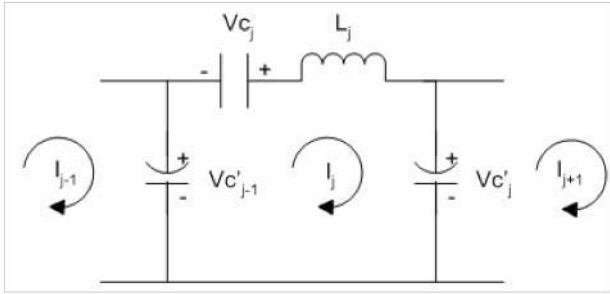


Figure 2. The discharging process of a Marx generator's j^{th} stage.

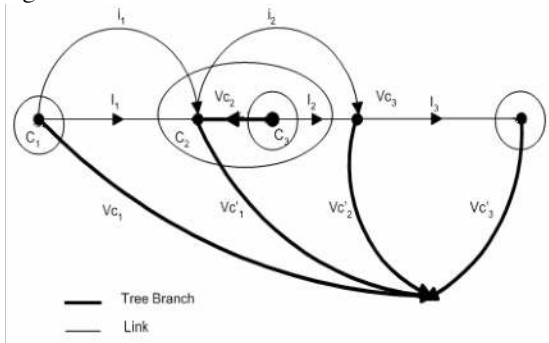


Figure 3. Graph of an $N=2$ stage Marx generator.

Having chosen the states to be voltages across capacitors and currents across inductors we follow two simple rules stated in [4]:

1. Write KCL for every fundamental cut set (*i.e.*, one tree branch and a number of links) in the network formed by each capacitor in the tree.
2. Write KVL for every fundamental loop (*i.e.*, one link and a number of tree branches) in the network formed by each inductor in the co-tree (complement of a tree).

The details of this analysis may be found in [2].

II. Currents and Voltages for 1 and 2 Stage Networks

Francaviglia [3] followed up on the work of Antoun [2] and considered the simplified Marx generator circuit shown in Figs. 4,5. In his analysis, for values of the circuit elements in Fig. 5 given by $C_1 = C = 1\text{F}$, $L_1 = L = 1\text{H}$, $C_2 = C/N = C$, $L_2 = NL = L$, and $V(0) = V_0 = 1\text{V}$, the curves shown in Fig. 6 are obtained on the circuit corresponding to a one stage network, namely $N = 1$.

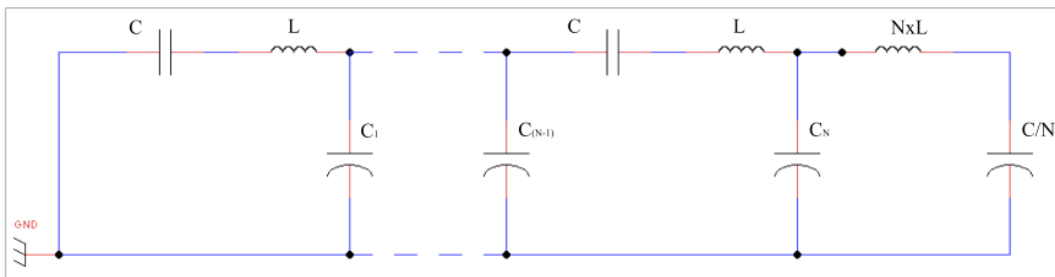


Figure 4. A simplified Marx generator circuit under consideration.

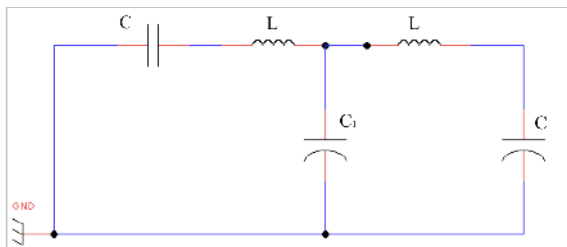


Figure 5. One stage of the Marx generator circuit shown in Fig. 4.

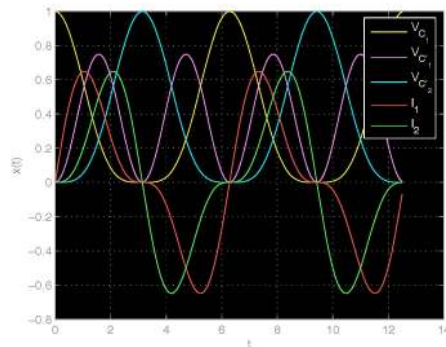


Figure 6. Voltages and currents through the elements of the circuit shown in Fig. 5.

When extended to a 2-stage network the curves shown in Fig. 7 are obtained.

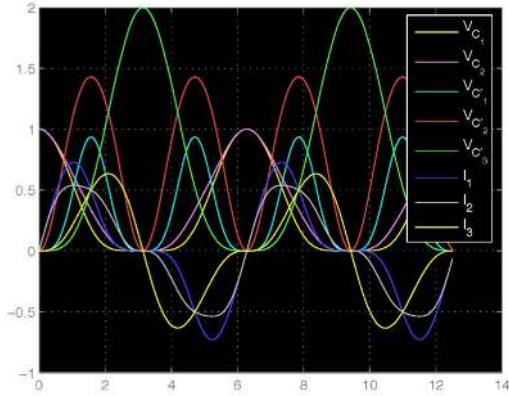


Figure 7. Voltages and currents through the elements of the 2-stage network.

The results plotted in Figs. 6 and 7 are qualitatively identical to the optimized results obtained by Buchenauer through PSPICE simulations [1].

III. Similarity to Feedback Control

Consider the following dynamic state-space equation:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

where $x(t)$ denotes the state of a certain physical system to be controlled by the input $u(t)$. The two matrices A and B are time invariant so that equation (1) models a linear, time-invariant (LTI) system. One classical problem in control theory is to select the input $u(t)$ so that the dynamics of the resulting $x(t)$ is driven into a certain desired state.

Depending on how the input $u(t)$ is calculated, there are generally two types of controls, both of which have been extensively studied and documented in the literature. In state feedback control $u(t)$ is selected as a linear function of state $x(t)$ such that $u(t) = Fx(t)$. In this way the original system is changed to a closed-loop dynamical system

$$\dot{\tilde{x}}(t) = (A + BF)x(t) \quad (2)$$

The general goal in such a control scheme is to choose the gain matrix F so as to achieve stability or to speed up the response. However, it turns out for the Marx network, the particular form of F (diagonal

matrix) renders this a structured static state feedback control problem that is an open problem.

III.A. Theorem

For the circuit shown in Fig. 4, define the following matrices A , B , and F of suitable dimensions:

$$A = -Id_N$$

$$B = \begin{bmatrix} -2 & 1 & 0 & \cdots & 0 \\ 1 & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & -2 & 1 \\ 0 & \cdots & 0 & 1 & -\frac{(N+1)}{N} \end{bmatrix}$$

(3)

$$F = \begin{bmatrix} \frac{C}{C_1} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{C}{C_N} \end{bmatrix}$$

where Id_N denotes the identity matrix having dimensions $N \times N$.

Given any selection of $N + 1$ constants $1 = \alpha_0 < \alpha_1 < \dots < \alpha_N$, if the eigenvalues of $A + BF$ are placed in $\alpha_1^2, \alpha_2^2, \dots, \alpha_N^2$, then the transfer function of the circuit from the initial conditions $V_i(0) = V_0, i = 1, \dots, N$ to the output V_N has its poles in $\pm j \frac{\alpha_i}{\sqrt{LC}}, i = 0, \dots, N$.

III.B. Remarks

Normally based on $\alpha_0 = 1$, which is fixed, one would choose $\alpha_i = 2i\alpha_0$ for $i = 1, 2, \dots, N$, namely all the natural frequencies are multiples of each other (following Buchenauer [1], where it is shown that under certain conditions, setting the natural frequencies of the circuit to be harmonically related guarantees that all the energy is delivered to the load capacitor after a suitable delay); however, alternative solutions would involve selecting e.g. $\alpha_i = 2\alpha_{i-1}$ for $i = 1, 2, \dots, N$.

The “transfer function of the circuit” considered in the theorem is in fact the transfer function characterizing

the response from an initial condition (corresponding to the initial voltage configuration of the Marx generator) to an output (the voltage across the last capacitor) under zero input; this actually corresponds to considering the impulse response of the circuit where the input distribution matrix has exactly the same pattern of the considered initial condition (that is, all the elements acting on the stage capacitors equal to 1, and all the other elements equal to 0). Since this transfer matrix has exactly $2N + 2$ poles, but the whole state space model of the circuit is characterized by $3N + 2$ eigenvalues, it follows that some natural modes are either unobservable or unreachable from the considered input and output. Also, it is interesting to notice that the matrices A , B , and F in (3) are not the matrices characterizing the state space realization of the whole circuit (in fact, the matrices in (3) correspond to a state of dimension N), but just a compact representation of the “useful” dynamics of the Marx generator (the part that the designer wants to assign). In fact, $(A + BF)$ only accounts for $2N$ complex conjugate imaginary eigenvalues of the circuit (by keeping track of the modulus of N pairs of eigenvalues), with the understanding that the above cited transfer function also contains two additional poles in 0; as mentioned above, the additional N eigenvalues belong to an unobservable/unreachable part which is of no interest when the only concern is guaranteeing that the whole energy in the system is stored on the last capacitor at a given time.

From the above formulae, it is evident that the solution F is independent of L and linear in C .

The theorem also holds for generalizations of the circuit, as example, if in the last branch the values L and C are replaced by NL and C/N ; it is currently under study to see whether similar transformations can also be performed in other branches of the circuit.

III.C. Similarity to Other Problems

The matrix $A + BF$ may be thought of as a decentralized state feedback problem (i.e. an output feedback problem). One can also simply concentrate on BF , which corresponds to adding 1 to each eigenvalue. Finding F via numerical algorithms is computationally attractive (the resulting optimization problem is quasi-convex locally).

The problem may be transformed via the change of coordinate $T = \sqrt{F}$ into the problem of finding a diagonal Λ such that $\Lambda B \Lambda$ has the eigenvalues $\alpha_i^2 + 1$ where $T = \Lambda = \sqrt{F}$. Since the matrix B is

symmetric but the last element on its diagonal is not equal to 2, this is a nonstandard Jacobi inverse eigenvalue problem (JIEP), which appears in the “bead on a string” problem [5] (see Fig. 8). The Jacobi matrix J_0 describing Fig. 8 is given by Eq. (4) and is identical in form to the matrix describing the Marx network.

$$J_0 = \begin{bmatrix} 2 & -1 & 0 & & & \\ -1 & 2 & -1 & & & \\ 0 & -1 & 2 & \cdot & \cdot & 0 \\ \cdot & & & \cdot & & \\ \cdot & & & & 2 & -1 \\ 0 & & & & -1 & 2 \end{bmatrix} \quad (4)$$

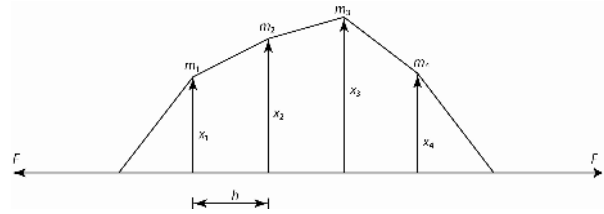


Figure 8. Similarity of the analysis of the circuit in Fig. 4 with the “bead on a string” problem in mechanics [5].

IV. Conclusions and Future Work

The synthesis of an N –stage Marx generator was modeled mathematically and related to a standard (but difficult) structured state feedback problem in control theory. Solutions to this problem are for now found numerically. The formalism allows for more general solutions to the problem to be found. Many aspects of the solutions identified to-date have been found to be in agreement with the solutions identified by Buchenauer [1] through PSPICE simulations.

Future work will focus on how to mathematically prove the existence of, then finding the parasitic capacitors to achieve perfect energy transfer for an N –stage Marx generator with a capacitive load.

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