

A CORRECTION FACTOR FOR ROCH'S STABILITY INDEX OF SLAB AVALANCHE RELEASE

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ABSTRACT. Roch (1966) proposed that the ratio of the strength of a bed layer to the load on it could be used to predict slab avalanche release. However, both he and Perla (unpublished) found that the mean strength was approximately twice the mean stress for snow layers known to have failed. A strength-size relationship is proposed to explain this discrepancy.

RÉSUMÉ. *Un facteur de correction pour l'indice de stabilité de Roch pour le déclenchement des avalanches de plaque.* Roch (1966) avait proposé que le rapport entre la résistance d'une couche de neige et la charge à laquelle elle était soumise serve pour la prévision des déclenchements d'avalanches. Cependant lui-même et Perla (unpublished) ont trouvé que la résistance moyenne était approximativement le double de la contrainte moyenne subie par les couches de neige que l'on sait avoir cassé. Une relation entre la résistance et la taille des grains est proposée pour expliquer cette discordance.

ZUSAMMENFASSUNG. *Ein Korrekturfaktor für Rochs Stabilitätsindex der Auslösung von Schneebrettlawinen.* Roch (1966) hat vorgeschlagen das Verhältnis zwischen der Festigkeit einer Lagerschicht und ihrer Auflast zur Vorhersage der Auslösung von Schneebrettlawinen heranzuziehen. Sowohl er selbst und Perla (unpublished) stellten jedoch fest, dass die mittlere Festigkeit etwa doppelt so gross wie die mittlere Spannung für Schneeschichten, die als Lawinen abgingen, war. Zur Erklärung dieser Unstimmigkeit wird eine Beziehung zwischen der Festigkeit und der Grösse herangezogen.

RECENT work by Perla (in press) and Smith and Curtis (in press) indicates that some event occurring in the region of the bed surface is necessary to produce the crown-region tensile stress involved in snow-slope instability. Their work supports the class of release mechanisms proposed by Roch (1966) and Bradley and Bowles (1967), as opposed to those of Haefeli (1963) and Sommerfeld (1969). Roch (1966) attempted a quantitative evaluation of slab stability with a Coulomb-Mohr failure criterion. He evaluated 35 slabs that had avalanched and thus were known to be unstable. His data showed a very high scatter, and his mean shear strength at the bed surface was 2.05 times his mean shear stress. Perla (unpublished) performed similar measurements on 23 avalanches, and found a mean Roch stability index of 2.45.

The large scatter is not surprising, but the Roch index would be expected to average close to one. Clearly some problem exists with the ratio of strength to load, since it indicates a safety factor of more than two in slabs known to have failed.

Sommerfeld (1973) pointed out that the properties of snow are inherently variable, and that this variation must be taken into account in any evaluation of properties. The effects of significant, random variations of strength within a material body on the observed strength of the body have been analyzed by several workers. The results are reviewed by Epstein (1948). A feature common to these analyses is that, when there is significant variation of strength within the body of a material, the mean strength of several specimens of the material depends on the specimen size. Such an effect is also likely in snow. Sommerfeld's (1974) prediction of a volume effect on tensile strength has received support from recent measurements by McClung (unpublished, p. 14).

The shear strength of a snow layer varies widely throughout the layer. As the stress increases the weakest parts will fail first, but when an extensive layer of snow is under shear stress it is unlikely that the failure of a small part will lead to catastrophic failure along the whole plane. Rather, the failure of one part would throw an extra load on the rest of the plane.

This is in contrast to failure under tension, where the failure of a small part may produce instability leading to catastrophic failure of the whole body. In such a case, extreme value statistics (e.g. Weibull, 1939[a], [b]) are probably applicable. Daniels (1945) considered a problem analogous to the plane-failure problem in the breaking of a bundle of fibers. If the fibers in a bundle have a significant distribution of strengths, the breaking of a few of the weakest fibers will not result in complete failure; the

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load will be carried by the remaining fibers. There is a critical load, however, depending on the strength distribution, above which the whole bundle will fail. Daniels found that failure stress of a bundle of fibers is given by

$$\frac{d(S/n)}{dS} - \frac{d}{dS} \left\{ s \int_s^{\infty} \theta(\sigma) d\sigma \right\} = 0 \quad (1)$$

where $\theta(\sigma)$ is the probability density of the breaking stress σ of n threads, S is the total load, and s is the load on each surviving thread. The theory is fairly general, since it only assumes a distribution of strengths in the sample and the lack of a Griffith-type instability in the failure mechanism.

If, in the case of a plane under shear stress, we take n to be the number of unit areas, then S/n becomes the failure stress of the plane.

To test the hypothesis that Daniels's theory applies to snow layers under shear stress, it would be necessary to have a large number of shear-strength measurements taken on the sliding surfaces of several avalanches. From such data $\theta(\sigma)$ could be determined for each case and the calculated failure stress compared with the shear stress. No such data exist at the present time.

TABLE I. RESULT OF FITTING NORMAL DISTRIBUTIONS TO PERLA'S DATA

Density range	Mean shear strength N m ⁻²	Standard deviation	Probability of a larger χ^2	Daniels's (1945) failure stress N m ⁻²	Daniels's stress: mean strength
0-100	193	88	0.73	103	0.53
100-150	487	345	0.56	244	0.50
150-200	976	782	0.27	488	0.50
200-250	2 079	864	0.56	1 139	0.55
250-300	2 988	1 476	0.80	1 574	0.53
300-350	3 983	1 953	0.52	2 101	0.53
350-400	3 633	1 995	0.84	1 877	0.52

R. I. Perla (personal communication) measured shear strengths of many avalanche sliding layers, but only took a few samples in each case. Although use of such mixed data is risky, such an analysis might show if the hypothesis has any merit at all. We therefore fitted normal distributions to his data, breaking it into seven density ranges. The results are shown in Table I. As indicated in column 4, the fits are very good except for the range 150-200, which is still acceptable. Apparently the populations sampled were similar enough to fit the same distribution. Column 5 gives the stress values (S/n) for which Equation (1) is satisfied, and the last column gives the ratio of this stress value to the mean stress. Daniels's theory predicts that, in each density range, snow layers with strength distributions like those found by Perla would fail at 0.50 to 0.55 times the mean strength. The average ratio is 0.52. Applying this ratio as a correction factor to Roch's mean stability index, we obtain 1.1; with Perla's, we obtain 1.3.

A possible explanation for the failure of Roch's (1966) stability index as applied by Roch and by Perla (unpublished) is that the mean shear strength of a layer is a function of its size. Evaluation of limited existing data provides some support for such a conclusion, and also indicates that Daniels's (1945) strength theory may be applicable to slab shear failure. To provide an adequate test of the hypothesis it would be necessary to collect enough tests from several known bed surfaces (perhaps 50 to 100 samples each) to determine the distribution of strengths in each layer that failed.

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