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A covariant Lagrangian for stable nonsingular bounce

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Abstract: The nonsingular bounce models usually suffer from the ghost or gradient instabilities, as has been proved recently. In this paper, we propose a covariant effective theory for stable nonsingular bounce, which has the quadratic order of the second order derivative of the field ϕ but the background set only by $P(\phi, X)$. With it, we explicitly construct a fully stable nonsingular bounce model for the ekpyrotic scenario.

Keywords: Classical Theories of Gravity, Spacetime Singularities

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Contents

1 Introduction

General relativity (GR) suffers the singularity problem [1], which indicates the incompleteness of our understanding about the gravity theory as well as the origin of the Universe [2, 3]. Instead of looking for a UV(ultraviolet)-complete theory to describe what happens at the "singularity", investigating the possibility of a nonsingular origin of the Universe with the effective theory, which captures low energy behaviors of the complete theory, is a significant direction.

It seems that since [4], the perturbations of the Friedmann-Roberson-Walker background usually suffer from the ghost or gradient instabilities in nonsingular cosmological models, see [5] for a review. Recently, this observation has been proved, up to the cubic Galileon theory [6] and the Horndeski theory [7]. Based on the effective field theory (EFT) of nonsingular cosmologies [8–10], this No-go result has been more clearly illustrated. It is found that the stable nonsingular cosmological models can be implemented only in the theories beyond cubic Galileon, (see also [11, 12]).

Recent progresses have inspired a wave of looking for stable nonsingular bounce [13–15] (see also [16, 17]), along the road beyond the cubic Galileon (even the Horndeski theory [18–20]). Moreover, the developments of scalar-tensor theory (the GLPV [21] and DHOST theory [22–24], the mimetic gravity [25, 26]) might also be able to provide us with some chances to implement stable nonsingular cosmologies. However, due to the complexity of relevant theories, which component is required for a stable bounce is not clear. Thus so far building a realistic and stable model is still difficult.

In refs. [8, 9], with the EFT of nonsingular cosmologies, it has been found that the operator $R^{(3)}\delta g^{00}$ is significant for the stability of nonsingular bounce. Actually, in unitary gauge, without getting involved in the specific theories,

$$L_{\rm add-oper} \sim \frac{M_2^4(t)}{2}(\delta g^{00})^2 + \frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00} \tag{1.1}$$

might be the least set of operators added to GR to cure the instabilities, since $(\delta g^{00})^2 \sim \dot{\zeta}^2$ while $R^{(3)}\delta g^{00} \sim (\partial\zeta)^2$ at quadratic order.

In this paper, based on the covariant description of the $R^{(3)}\delta g^{00}$ operator, we propose a covariant theory for stable nonsingular bounce, which has the quadratic order of the second order derivative of the field ϕ but the background set only by $P(\phi, X)$. We illuminate its application by constructing a fully stable nonsingular bounce model for the ekpyrotic scenario [27, 28].

Note added. Several days after our paper appeared in arXiv, the preprint [29] appeared, in which somewhat similar analysis is done in beyond Horndeski model with sort of similar result.

2 Covariant description of $R^{(3)}\delta g^{00}$

In unitary gauge, $\phi = \phi(t)$. We have

$$\delta g^{00} = \frac{X}{\dot{\phi}^2(t)} + 1 = \frac{X}{f_2(t(\phi))} + 1, \tag{2.1}$$

where $X = \phi_\mu \phi^\mu$, $\phi_\mu = \nabla_\mu \phi$ and $\phi^\mu = \nabla^\mu \phi$.

$R^{(3)}$ is the Ricci scalar on the 3-dimensional spacelike hypersurface. Using the Gauss-Codazzi relation, it is straightforward (though tedious) to find

$$\begin{aligned} R^{(3)} &= R - \frac{\phi_{\mu\nu}\phi^{\mu\nu} - (\Box\phi)^2}{X} + \frac{2\phi^\mu \phi_{\mu\nu}\phi^{\nu\sigma}\phi_\sigma}{X^2} - \frac{2\phi^\mu \phi_{\mu\nu}\phi^\nu \Box\phi}{X^2} \\ &\quad + \frac{2({\phi^\nu}_{\nu\mu}\phi^\mu - {\phi_\nu}^\mu{}_\mu \phi^\nu)}{X}, \end{aligned} \tag{2.2}$$

with $\phi_{\mu\nu} = \nabla_\nu \nabla_\mu \phi$ and ${\phi^\nu}_{\nu\mu} = \nabla_\mu \nabla_\nu \nabla^\nu \phi$. It is simple to check that the right hand side of eq. (2.2) is 0 at the background level.

We define $S_{\delta g^{00}R^{(3)}} = \int d^4x \sqrt{-g} L_{\delta g^{00}R^{(3)}}$, and have

$$\begin{aligned} L_{\delta g^{00}R^{(3)}} &= \frac{f_1(\phi)}{2}\delta g^{00} R^{(3)} \\ &= \frac{f}{2}R - \frac{X}{2}\int f_{\phi\phi} d\ln X - \left(f_\phi + \int \frac{f_\phi}{2} d\ln X\right)\Box\phi \\ &\quad + \frac{f}{2X}\left[\phi_{\mu\nu}\phi^{\mu\nu} - (\Box\phi)^2\right] - \frac{f - 2Xf_X}{X^2}\left[\phi^\mu \phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu - (\Box\phi)\phi^\mu \phi_{\mu\nu}\phi^\nu\right] \end{aligned} \tag{2.3}$$

after integration by parts, where $f(\phi, X) = f_1\left(1 + \frac{X}{f_2}\right)$ has the dimension of mass squared, $f_2(\phi)$ is defined in (2.1), and the total derivative terms have been discarded. One useful formula for obtaining eq. (2.3) is

$$2\mathcal{B}(\phi, X)\phi^\mu \phi_{\mu\nu}\phi^\nu = \nabla_\mu\left(\phi^\mu \int \mathcal{B} dX\right) - X\int \frac{\partial \mathcal{B}}{\partial \phi} dX - \Box\phi \int \mathcal{B} dX\,. \tag{2.4}$$

3 Stable nonsingular bounce

3.1 The covariant theory

Here, the EFT proposed is

$$S=\int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R+P(\phi,X)\right)+S_{\delta g^{00}R^{(3)}}\,,\tag{3.1}$$

which is a covariant theory equivalent to GR plus the set of operators in (1.1), since $M_2^4(t)=\dot\phi^4P_{XX}$ and $\tilde m_4^2(t)=f_1(\phi)$.

The covariant action (3.1) actually belongs to a subclass of the DHOST theory [22, 23] (see appendix A for details), which could avoid the Ostrogradski instability, up to quadratic order of the second order derivative of ϕ. Ijjas and Steinhardt used the quartic Horndeski action in [13]. In (2.3), though the nonminimal coupling $f(\phi,X)R$ is similar to that in [13], terms $\sim\Box\phi$, $\phi_{\mu\nu}\phi^{\mu\nu}$, $(\Box\phi)^2$, $(\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu$ and $\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu$ also appear simultaneously with the coefficients set by $\delta g^{00}R^{(3)}$, so that the effect of $S_{\delta g^{00}R^{(3)}}$ on background is canceled accurately. Here, the background is set only by $P(\phi,X)$. In [14], $(\Box\phi)^2$ is used, which shows itself the Ostrogradski ghost, see also earlier [30], how to remove it requires argumentation.

The quadratic action of scalar perturbation for (3.1) is

$$S_\zeta^{(2)}=\int a^3Q_s\left(\dot\zeta^2-c_s^2\frac{(\partial\zeta)^2}{a^2}\right)d^4x\,,\tag{3.2}$$

in which

$$Q_s=\frac{2\dot\phi^4P_{XX}-M_p^2\dot H}{H^2},\qquad c_s^2Q_s=M_p^2\left(\frac{\dot c_3}{a}-1\right)\tag{3.3}$$

and $c_3=a(1+\frac{2f_1}{M_p^2})/H$. We can see that the sound speed of scalar perturbation can be directly modified by $f_1(\phi)$, namely, the function before $\delta g^{00}R^{(3)}$ operator. Therefore, the gradient instability of scalar perturbation could be cured by proper choice of $f_1(\phi)$, while that of tensor perturbation is unaffected by $S_{\delta g^{00}R^{(3)}}$, hence is same with that of GR.

A fully stable nonsingular bounce ($Q_s>0$ and $c_s^2=1$) can be designed with (3.1). In the bounce phase, $\dot H>0$. However, $Q_s>0$ can be obtained, since $P(\phi,X)$ contributes $\dot\phi^4P_{XX}$ in Q_s. While around the bounce point $H\simeq0$,

$$c_s^2\sim-\dot H\left(1+\frac{2f_1}{M_p^2}\right).\tag{3.4}$$

Thus we will have $c_s^2>0$ for $2f_1<-M_p^2$, as has been clarified in refs. [8, 10]. It should be mentioned that if $f_1=0$, we have $c_s^2\sim-\dot H<0$ around the bounce point, thus $S_{\delta g^{00}R^{(3)}}$ is needed to contribute f_1. Here, we always could set $c_s^2\sim\mathcal{O}(1)$ with a suitable $f_1(\phi)$ (see also [10]) which satisfies

$$2f_1(\phi)=\frac{H}{a}\int a\left(Q_sc_s^2+M_p^2\right)dt-M_p^2.\tag{3.5}$$

3.2 A stable nonsingular bounce model

With (3.1), building a nonsingular bounce model is simple. The ghost-free nonsingular bounce is set by $P(\phi, X)$, while $c_s^2 \simeq 1$ is set by using suitable f_1 and f_2 in (2.1).

As a specific model, we set $P(\phi, X)$ in (3.1) as

$$P(\phi, X) = \left[\frac{k_0}{(1+\kappa_1\phi^2)^2} - 1\right] X/2 + \frac{q_0}{(1+\kappa_2\phi^2)^2} X^2 - V(\phi)\,, \tag{3.6}$$

where the potential is ekpyrotic-like

$$V(\phi) = -\frac{V_0}{2} e^{\phi/\mathcal{M}_1} \left[1 - \tanh\left(\frac{\phi}{\mathcal{M}_2}\right)\right], \tag{3.7}$$

with constant $\mathcal{M}_1, \mathcal{M}_2, V_0$, and k_0, κ_1 responsible for the switching of the sign before $X/2$ around $\phi \simeq 0$, and q_0, κ_2 for the appearance of X^2 around $\phi \simeq 0$, see [31] for a similar $P(\phi, X)$, which might allow for a supersymmetric counterpart [32].

The background equations are

$$3M_p^2 H^2 = -2\dot{\phi}^2 P_X - P\,, \tag{3.8}$$

$$M_p^2 \dot{H} = \dot{\phi}^2 P_X\,. \tag{3.9}$$

Initially $\phi \ll -\mathcal{M}_2, -1/\sqrt{\kappa_1}, -1/\sqrt{\kappa_2}$, we have $P(\phi, X) = -X/2 + V_0 e^{\phi/\mathcal{M}_1}$, the Universe is in the ekpyrotic phase with the equation of state parameter

$$\omega_{ekpy} = \frac{M_p^2}{3\mathcal{M}_1^2} - 1 > 1. \tag{3.10}$$

Around $\phi \simeq 0$, we have

$$\dot{H} \simeq \left(\frac{k_0 - 1}{2} - 2q_0\dot{\phi}^2\right)\dot{\phi}^2 > 0. \tag{3.11}$$

Thus the bounce could occur. However, after the bounce the field ϕ will be canonical again but with $V(\phi) = 0$. It is possible that the phase after the bounce might be the inflation [33–36], we will consider it elsewhere.

Here, in the quadratic action (3.2) of scalar perturbation,

$$Q_s = -\frac{M_p^2 \dot{H}}{H^2} + \frac{4q_0}{(1+\kappa_2\phi^2)^2 H^2}\dot{\phi}^4 > 0 \tag{3.12}$$

can be obtained, while $c_s^2 = 1$ can be obtained by setting suitable $f_1(\phi)$ in (2.3), which is given by (3.5), and $f_2(\phi) = \dot{\phi}(t(\phi))$.

The background evolution is numerically plotted in figure 1. We show the behaviors of $f_1(\phi)$ and $f_2(\phi)$ with respect to ϕ in figure 2 while we require $c_s^2 = 1$ throughout. In both figures 1 and 2, we set $k_0 = 1.2$, $\kappa_1 = 30$, $q_0 = 1.25$, $\kappa_2 = 20$, $V_0 = 2 \times 10^{-7}$, $\mathcal{M}_1 = 0.22$ and $\mathcal{M}_2 = 0.1$. We set the initial condition of ϕ as $\phi_{ini} = -0.54$ and $\dot{\phi}_{ini} = 2.24 \times 10^{-4}$, while the initial value of t is $t_{ini} = -2000$. We see that with f_1 and f_2 plotted in figure 2, the Lagrangian (3.1) with $P(\phi, X)$ in (3.6) will bring a fully stable nonsingular bounce ($Q_s > 0$ and $c_s^2 = 1$).

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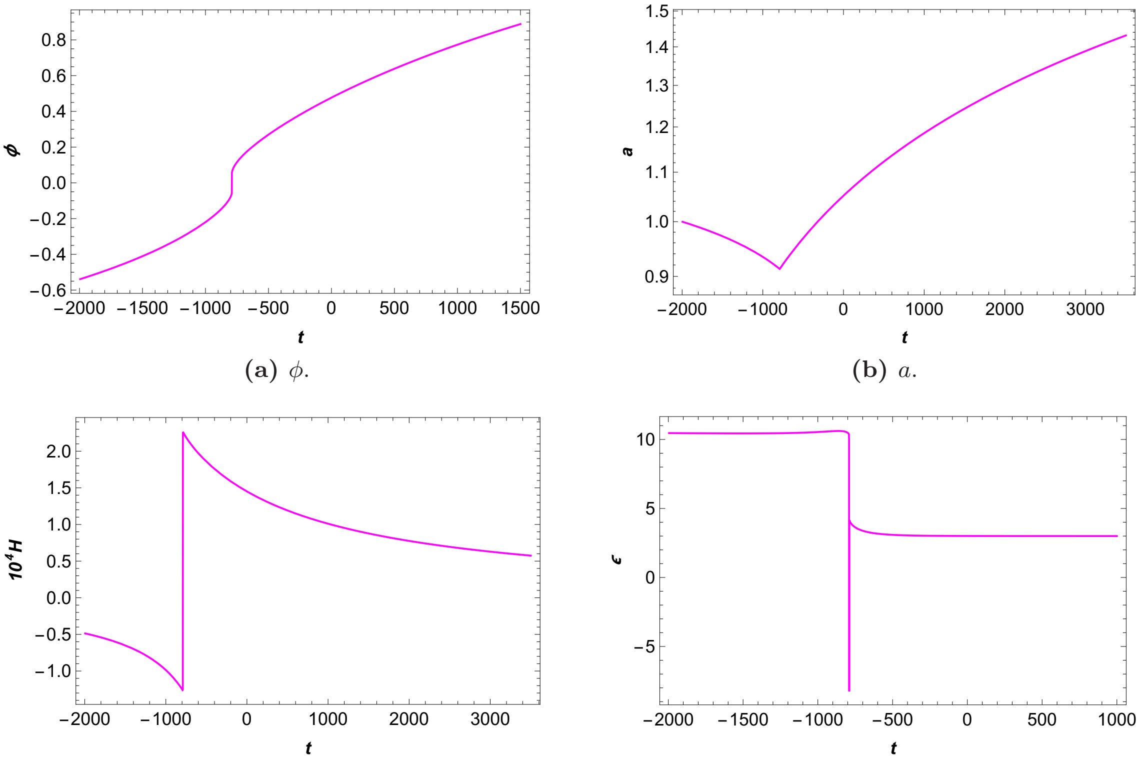
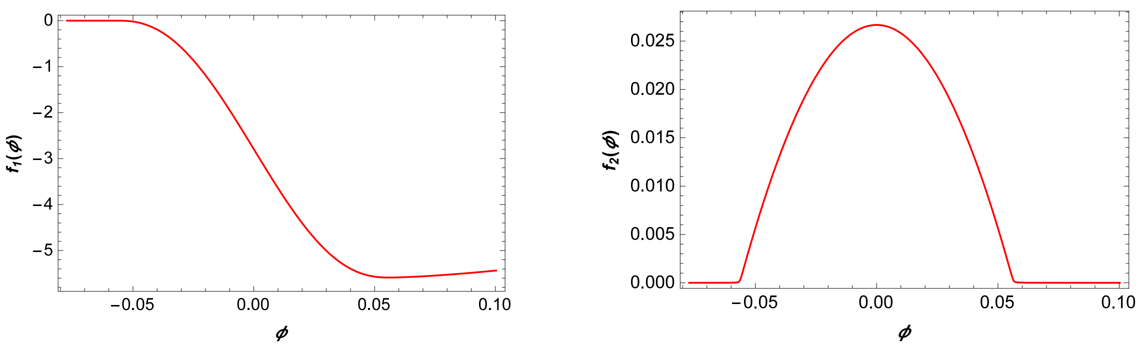


(a) ϕ. (b) a.

(c) H. (d) $\epsilon = -\dot{H}/H^2$.

Figure 1. The background evolution of ekpyrotic Universe.



(a) $f_1(\phi)$ for $c_s^2 \equiv 1$. (b) $f_2(\phi)$ for $c_s^2 \equiv 1$.

Figure 2. The expressions of $f_1(\phi)$ and $f_2(\phi)$ with respect to ϕ.

4 Discussion

The exploration of stable nonsingular bounce has been still a significant issue. Recently, it has been found in refs. [8, 9] that the operator $R^{(3)}\delta g^{00}$ in EFT of nonsingular cosmologies is significant for the stability of bounce. Here, based on the covariant description of the $R^{(3)}\delta g^{00}$ operator, we propose a covariant theory (3.1) for stable nonsingular bounce.

Our (3.1) is actually a subclass of the DHOST theory [22, 23], but the cosmological background is set only by $P(\phi, X)$. The $P(\phi, X)$ nonsingular bounce model could be ghost-free [31, 37], but suffers the problem of $c_s^2 < 0$, which can not be dispelled by using the Galileon interaction $\sim \Box\phi$ [6–9]. Actually, in [10, 38], it is observed that the Galileon interaction only moves the period of $c_s^2 < 0$ to the outside of the bounce phase, but can not remove it, see also earlier [39]. Thus it could be imagined that the quadratic order of the second order derivative of ϕ, i.e., $\phi_{\mu\nu}\phi^{\mu\nu}$, $(\Box\phi)^2$, $\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu$ and $(\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu$, might play crucial roles in stable nonsingular bounce model. However, due to the complexity of relevant theories, what kind of combination of these components is required for a stable cosmological bounce is unclear. Here, the corresponding combination (2.3) is just what told by the covariant description of the $R^{(3)}\delta g^{00}$ operator.

With (3.1), the design of stable nonsingular bounce model is simple, as illuminated for the ekpyrotic scenario. Our work actually offers a concise way to the fully stable nonsingular cosmologies. See also [40–48] for other interesting studies.

Here, the importance of the EFT of nonsingular cosmologies is obvious. Actually, the role of $R^{(3)}\delta K$ in EFT [8] is similar to that of $R^{(3)}\delta g^{00}$, where $K_{\mu\nu}$ is the extrinsic curvature on the 3-dimensional spacelike hypersurfaces. The covariant description of $R^{(3)}\delta K$ involves the term $\sim (\Box\phi)R$, which might have the Ostrogradski ghost unless certain constraint is imposed. This issue will be revisited. In mimetic gravity [25, 26] (see e.g. [49] for review), since the mimetic constraint suggests $\delta g^{00} = 0$ (which is the source of instabilities [50–53]), one might apply the operator $R^{(3)}\delta K$ to make the (possibly-built) nonsingular bounce stable,[1] instead of $R^{(3)}\delta g^{00}$. The mimetic gravity with the couple $(\Box\phi)R$ has been proposed in ref. [54]. We will back to the relevant issues.

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A Correspondence with a subclass of DHOST theory

Up to cubic order of $\phi_{\mu\nu}$, the covariant action of DHOST can be written as (see e.g., [24])

$$S_{DHOST} = \int d^4x\sqrt{-g}\Big[p(\phi,X) + q(\phi,X)\Box\phi + g_2(\phi,X)R + C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma} + g_3(\phi,X)G_{\mu\nu}\phi^{\mu\nu} + C_{(3)}^{\mu\nu\rho\sigma\alpha\beta}\phi_{\mu\nu}\phi_{\rho\sigma}\phi_{\alpha\beta}\Big], \tag{A.1}$$

[1] Communication with Mingzhe Li.

where R and $G_{\mu\nu}$ denote the usual 4-dimensional Ricci scalar and Einstein tensor associated with the metric $g_{\mu\nu}$, respectively;

$$C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma} = \sum_{A=1}^{5} a_A(\phi, X)\, L_A^{(2)}\,, \tag{A.2}$$

with

$$\begin{aligned} &L_1^{(2)} = \phi_{\mu\nu}\phi^{\mu\nu}\,, \qquad L_2^{(2)} = (\Box\phi)^2\,, \qquad L_3^{(2)} = (\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu\,,\\ &L_4^{(2)} = \phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu\,, \quad L_5^{(2)} = (\phi^\mu\phi_{\mu\nu}\phi^\nu)^2\,, \end{aligned} \tag{A.3}$$

and

$$C_{(3)}^{\mu\nu\rho\sigma\alpha\beta}\phi_{\mu\nu}\phi_{\rho\sigma}\phi_{\alpha\beta} = \sum_{A=1}^{10} b_A(\phi, X) L_A^{(3)}\,, \tag{A.4}$$

with

$$\begin{aligned} &L_1^{(3)} = (\Box\phi)^3\,, && L_2^{(3)} = (\Box\phi)\phi_{\mu\nu}\phi^{\mu\nu}\,, && L_3^{(3)} = \phi_{\mu\nu}\phi^{\nu\rho}\phi_\rho^\mu\,,\\ &L_4^{(3)} = (\Box\phi)^2\,\phi_\mu\phi^{\mu\nu}\phi_\nu\,, && L_5^{(3)} = \Box\phi\,\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\,, && L_6^{(3)} = \phi_{\mu\nu}\phi^{\mu\nu}\phi_\rho\phi^{\rho\sigma}\phi_\sigma\,,\\ &L_7^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^{\rho\sigma}\phi_\sigma\,, && L_8^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\,\phi_\sigma\phi^{\sigma\lambda}\phi_\lambda\,, &&\\ &L_9^{(3)} = \Box\phi\,(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2\,, && L_{10}^{(3)} = (\phi_\mu\phi^{\mu\nu}\phi_\nu)^3\,; && \end{aligned} \tag{A.5}$$

extra conditions on the functions a_A and b_A need to be satisfied so that there is no extra propagating degree of freedom, see [24] and references therein for further discussions.

Comparing with (A.1), we find our model (3.1) corresponds to the covariant form of DHOST theory with

$$\begin{aligned} &p(\phi,X) = P(\phi,X) - \frac{X}{2}\int f_{\phi\phi} d\ln X\,, && q(\phi,X) = -f_\phi - \int \frac{f_\phi}{2} d\ln X\,,\\ &g_2(\phi,X) = \frac{M_p^2+f}{2}\,, && g_3(\phi,X) = 0\,,\\ &a_1 = -a_2 = \frac{f}{2X}\,, && a_3 = -a_4 = \frac{f-2Xf_X}{X^2}\,, \qquad a_5 = 0\,, \end{aligned} \tag{A.6}$$

and $b_A = 0$.

In the EFT formalism, the quadratic action for DHOST theory can be written as

$$\begin{aligned} S_{DHOST}^{(2)} = \int d^3x dt a^3 \frac{M^2}{2}\Big\{&\delta K_{\mu\nu}\delta K^{\mu\nu} - \left(1+\frac{2}{3}\alpha_L\right)\delta K^2 + (1+\alpha_T)\left(R^{(3)}\frac{\delta\sqrt{h}}{a^3} + \delta_2 R^{(3)}\right)\\ &+ H^2\alpha_K\delta N^2 + 4H\alpha_B\delta K\delta N + (1+\alpha_H)R^{(3)}\delta N + 4\beta_1\delta K\delta\dot{N} + \beta_2\delta\dot{N}^2 + \frac{\beta_3}{a^2}(\partial_i\delta N)^2\Big\}, \end{aligned} \tag{A.7}$$

where $\delta N = \delta g^{00}/2$, $\delta_2 R^{(3)}$ stands for the second order term in the perturbative expansion of $R^{(3)}$, the dimensionless time-dependent functions α_L, α_T, α_K, α_B, α_H, β_1, β_2 and β_3 satisfy certain conditions so that there is no extra propagating degree of freedom, see [24] for details.

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Comparing with (A.7), we find our model (3.1) corresponds to

$$\begin{aligned} &M = M_p\,, && \alpha_L = \alpha_T = \alpha_B = 0\,, && \beta_1 = \beta_2 = \beta_3 = 0\,, \\ &\alpha_K = \frac{4M_2^4}{M_p^2 H^2} = \frac{4X^2 P_{XX}}{M_p^2 H^2}\,, && \alpha_H = \frac{2\tilde{m}_4^2}{M_p^2} = \frac{2f_1(\phi)}{M_p^2}\,. \end{aligned} \tag{A.8}$$

Note that the results in eqs. (A.8) should be evaluated at background level in the quadratic action if we derive them from eqs. (A.6) by using formulae given in eqs. (2.14) of [24].

According to the above results, our model (3.1) belongs to a subclass of the DHOST theory with $\alpha_L = 0$ and $\alpha_T = 0$. As has been pointed out in ref. [24], in such a DHOST theory, in the linear regime for a Minkowski background (namely, at the limit $a = 1$ and $H = 0$) the Newton's constant is $G_N = \frac{1}{8\pi M_p^2}\frac{1}{(1+\alpha_H)^2}$. In our specific numerical example, the Universe is nearly slowly expanding Friedmann Universe at large positive times, which is nearly Minkowskian. However, in that limit, the contribution from $L_{\delta g^{00}R^{(3)}}$ (or the value of $f_1(\phi)$) is already vanishing, i.e., $\alpha_H = 0$, hence GR is retrieved and $G_N = \frac{1}{8\pi M_p^2}$ at large positive times.

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