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A
Crack Growth
Retardation Model
Using an
Effective Stress Concept

J. Willenborg
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FOREWORD

This work was conducted by the authors under the supervision of Mr. R.M. Bader, Technical Manager, Analysis Group at the Air Force Flight Dynamics Laboratory, under project 1467, "Structural Analysis Methods," Task 146704, "Structural Fatigue and Fracture Analysis Methods for Aerospace Vehicles."

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ABSTRACT

This report describes a crack growth retardation model which utilizes an "effective stress" concept to reduce the applied stresses and hence the crack tip stress intensity factor. The derivation of the model is presented as well as comparisons with existing experimental and analytical spectrum crack growth data for D6ac steel and 7075-T6 aluminum.

CONTENTS

I INTRODUCTION

Current predictive analysis techniques for crack propagation under cyclic loading rely on the integration of basic constant amplitude growth rate data derived from laboratory tests on simple coupons. Such an automated procedure is contained in Reference 1.

Variations between predicted and actual growth lives have been noted for cases of variable amplitude spectrum loading due to the interaction of the stress applications (References 3 and 4). The occurrence of a tensile overload will retard growth below that normally expected. To neglect these interaction effects results in grossly conservative prediction of crack growth life.

Several attempts at developing mathematical models for growth retardation have been made. (References 2 and 5). In Reference 2, Wheeler calculates a retardation factor C_p which operates directly by reducing growth rate da/dn. The procedure requires previous spectrum growth data to derive a retardation exponent "m". Moderate success has been achieved by the author in fitting existing spectrum data.

In the current study, retardation is accounted for by operating directly on the crack growth driving function ΔK. An effective value of the stress intensity factor range is computed by assuming a form of the residual crack tip stress present after the application of the overload. Once obtained, the modified ΔK is used in conjunction with ordinary constant growth rate data and the CRACKS computer routine (Reference 1) to calculate life. No other empirical data or factors are required.

The model development is contained in Section II. Application of the procedure to existing D6ac and 7075-T6 spectrum data is contained in Section III.

SECTION II

DESCRIPTION OF THE MODEL

To best describe the operation of the retardation model, consider the simple spectrum as shown in Figure 1. The step-by-step solution of this problem is outlined below:

1. Load layer 1 is applied. Using the maximum stress, σ_1, the plastic zone radius, a_p, is calculated and saved for reference.

$$a_p = \frac{K_1^2}{2\pi\sigma_y^2} + a_1 \qquad 1$$

2. The first load cycle in layer 2 is applied. The maximum stress, σ_2, is compared to σ_1. Since σ_2 is less than σ_1, the retardation model is applied.

3. The first step is to determine the applied stress, σ_{ap} required to reach a_p. This stress is determined as follows:

The yield zone radius for σ_{ap} is given by

$$R_y = \frac{K_{ap}^2}{2\pi\sigma_y^2} = \frac{\left(\sigma_{ap}\sqrt{\pi a_c}\,\beta_T\right)^2}{2\pi\sigma_y^2} = a_p - a_c$$

Solving for σ_{ap} we obtain

$$\sigma_{ap} = \frac{\sigma_y}{\beta_T}\sqrt{\frac{2(a_p - a_c)}{a_c}} \qquad 2$$

where a_c is the crack length at the beginning of the load cycle or load layer. Hence, for the first cycle of layer 2, equation (2) becomes

$$\sigma_{ap} = \frac{\sigma_y}{\beta_T}\sqrt{\frac{2(a_p - a_1)}{a_1}}$$

4. Next, we obtain the reduction in the applied stress, σ_{red} due to the progress through the plastic zone for a given layer.

$$\sigma_{red} = \sigma_{ap} - \sigma_{max} \qquad 3$$

For layer 2 , equation 3 becomes

$$\sigma_{red} = \sigma_{ap} - \sigma_2$$

When the crack has propagated through the plastic zone, σ_{red} is set equal to zero since the crack propagation is no longer being retarded.

5. Effective values of the maximum and minimum applied stresses are then calculated as follows:

$$(\sigma_{max})_{eff} = \sigma_{max} - \sigma_{red} \qquad 4$$

$$(\sigma_{min})_{eff} = \sigma_{min} - \sigma_{red} \qquad 5$$

If either of the effective stresses is less than zero, it is set equal to zero.

6. Effective values of R and ΔK are now calculated using equations

4 and 5. The crack growth law is then applied directly, using the effective R and ΔK, to obtain the growth during the interval. At the end of the first cycle of layer 2 we obtain $a_{2,1}$.

7. Compare the current value $a_{2,1}$ with a_p. Since $a_{2,1}$ is less than a_p, the growth is still retarded. We now return to step 3. Now we obtain

$$\sigma_{ap} = \frac{\sigma_y}{\beta_T}\sqrt{\frac{2(a_p - a_{2,1})}{a_{2,1}}}$$

We see that σ_{ap} diminishes as a_c approaches a_p. When σ_{ap} equals σ_{max}, σ_{red} is zero and retardation is no longer present.

SECTION III

CORRELATION

In order to test the validity of the model, a few problems were solved involving two different materials subjected to different types of spectra. The model was incorporated into the CRACKS computer program (Reference 1) to provide rapid solution capability.

The first problem is taken from Dr. Wheeler's report (Reference 2, figure 8). The spectrum is shown in figure 2 and the correlation with test data as well as Dr. Wheeler's retardation model is given in figure 3. The material was D6ac steel. The Paris form of the crack growth law was

$$\frac{da}{dN} = 0.0012 \times 10^{-6} (\Delta K)^{2.55}$$

In this example, the Paris form of the growth law was assumed valid for all values of load ratio, $R = \frac{\sigma min}{\sigma max}$.

A second problem, also a surface flaw in a D6ac steel specimen, was run using Forman's form of the crack growth equation.

$$\frac{da}{dN} = \frac{0.9998 \times 10^{-9} (\Delta K)^{2.74}}{(1-R)125 - \Delta K}$$

Both the 5g and 7.33g versions of the spectra were used.

The spectra from Reference 3 is given in figure 4 and the correlation is shown in figures 5a and 5b. For the 5.0g spectrum, four laboratory tests were conducted. Specimen P3F3 and P1MJ5 indicate the scatter of the test data. Also indicated on Figure 5a and 5b are predictions based on the Wheeler model using the same basic growth rate data.

Another material, 7075-T6 aluminum was also examined. A twelve inch wide center-cracked panel was subjected to various simple spectrum loadings (Reference 4.) The spectra and correlation data are presented in figure 6. For this example, the plane stress form of the yield zone was considered.

IV DISCUSSION

Although of preliminary nature, the data presented in this report have demonstrated the ability of the proposed crack retardation model to account for the growth delay due to the application of tensile overloads in a complex spectrum.

The most significant feature of the model is its ability to predict growth retardation without the assistance of empirical factors or test data. This is clearly demonstrated in examples 1 and 2 of Section III, where widely different values of the Wheeler parameter "m" are required to fit the spectrum data. This would indicate that "m" is sensitive to factors other than material difference.

Work is continuing to further validate the model with additional simple and complex spectra including the occurrence of single spike overloads. A test program currently underway at AFFDL will provide additional data for D6ac steel, 7075-T6 aluminum and 6al-4V titanium.

The successful use of the analysis scheme, of course, requires valid and adequate basic growth rate data. Current efforts are investigating the sensitivity of prediction, to the normal variation in reported da/dn vs ΔK data.

SECTION V

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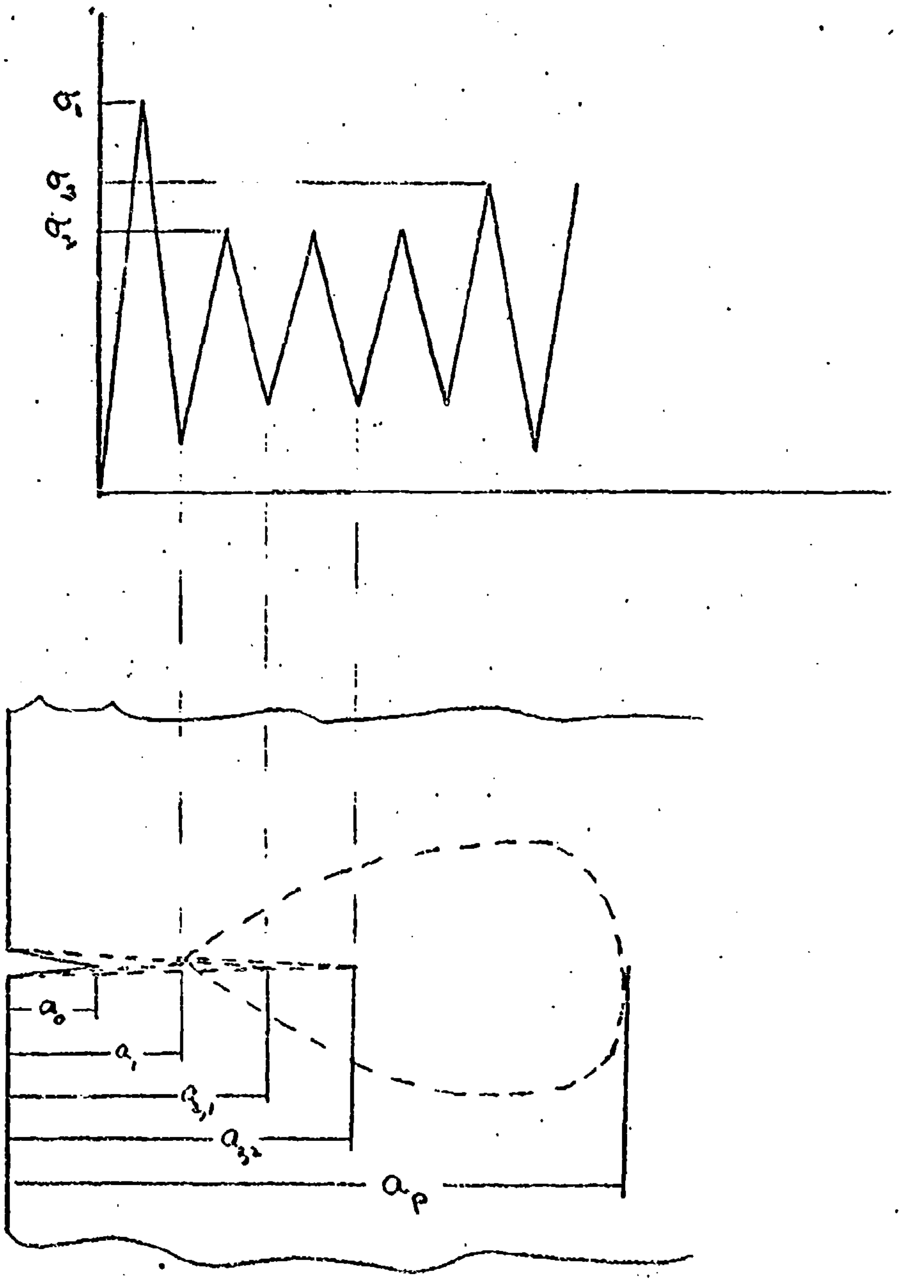
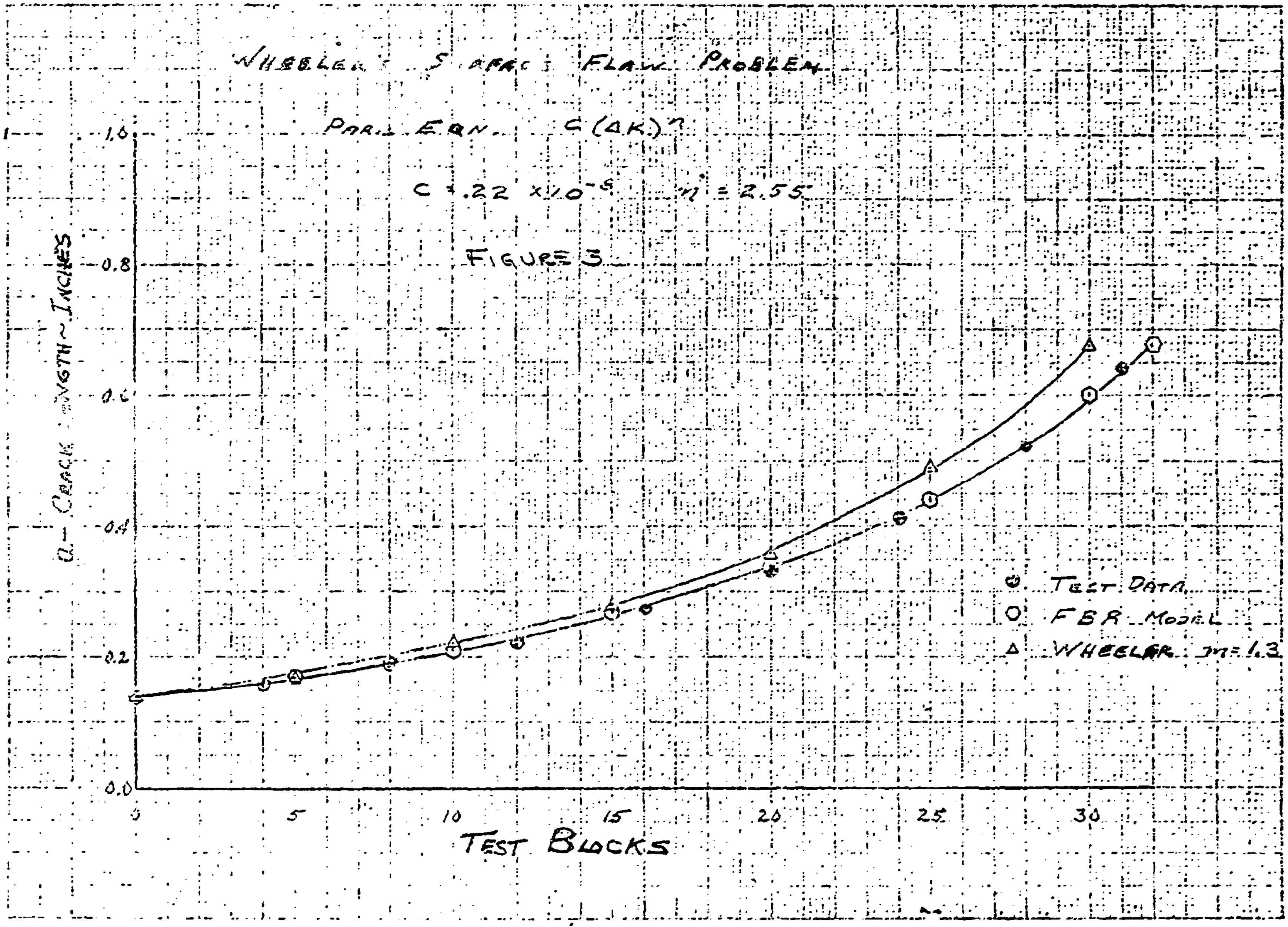


Figure 1 SPECTRUM & YIELD ZONES

LAYER	σ_{max} (ksi)	σ_{min} (ksi)	CYCLES PER LAYER
1	50.0	11.3	867
2	55.4	11.3	239
3	64.6	11.3	53
4	72.8	11.3	15
5	80.6	11.3	11
6	41.5	9.6	632
7	48.1	9.6	176
8	54.5	9.6	66
9	60.0	9.6	48
10	66.6	9.6	7
11	27.2	2.7	25
12	35.5	2.7	10
13	37.8	2.7	5

LOAD SPECTRUM FOR WHEELER SURFACE FLAW

FIGURE 2


WHEELER'S SURFACE FLAW PROBLEM

PARIS EQN. $C(\Delta K)^n$

$C = .22 \times 10^{-8}$ $n = 2.55$

FIGURE 3

CONDENSED MAC SPECTRUM WRE

5.0 g* & 7.33 G NO COMPRESSION

Layer No.	σmin		σmax		n		Layer No.	σmin	σmax	n
1	0.2		48.0		63		34	22.8	106.6	1
2	20.3		77.9		76		35	4.7	18.3	265
3	1.3		39.5		371		36	2.3	59.9	34
4	17.0		76.0		37		37	22.5	58.1	318
5	2.3		50.5		111		38	10.6	34.2	6
6	30.6		73.2		2		39	0	32.7	21
7	2.2		40.8		363		40	20.7	51.7	374
8	11.6		82.6		5		41	5.8	40.0	478
9	10.5	23.2	30.7	80.4	1280	1	42	4.6	25.4	46
10	19.5		65.9		62		43	0.2	34.2	300
11	10.5		47.9		1		44	4.6	32.6	10
12	17.5	20.4	50.5	90.2	89	1	45	22.8	91.4	4
13	24.9		63.0		41		46	0	47.2	4
14	27.4		55.2		57		47	21.8	41.9	306
15	10.9	10.2	40.4	51.6	491	7	48	23.8	71.8	15
16	0		40.2		6		49	23.0	75.2	5
17	11.0		50.4		74		50	23.6	37.3	230
18	22.7	20.1	38.7	67.5	682	4	51	23.0	31.0	1338
19	2.1	20.4	29.9	82.2	1376	5	52	0.2	57.2	19
20	27.0		46.1		66		53	11.1	29.9	1546
21	1.5		49.7		34		54	0	18.4	238
22	19.5	20.4	24.8	84.2	1621	5	55	1.4	46.4	114
23	23.0	20.4	33.9	85.6	1589	2	56	20.4	43.1	370
24	1.3	22.9	30.7	108.3	1374	1	57	11.1	59.9	7
25	0		25.4		67		58	5.8	40.0	478
26	20.4		82.0		1					
27	21.3	11.5	65.7	93.7	250	2				
28	0.2		63.8		8					
29	4.7		40.1		2					
30	22.9		100.7		2					
31	10.5		46.3		37					
32	21.8	11.5	48.3	102.3	367	1				
33	20.6		73.9		109					

* 5.0 g SPECTRUM - TYPED VALUES ONLY.

FIGURE 4

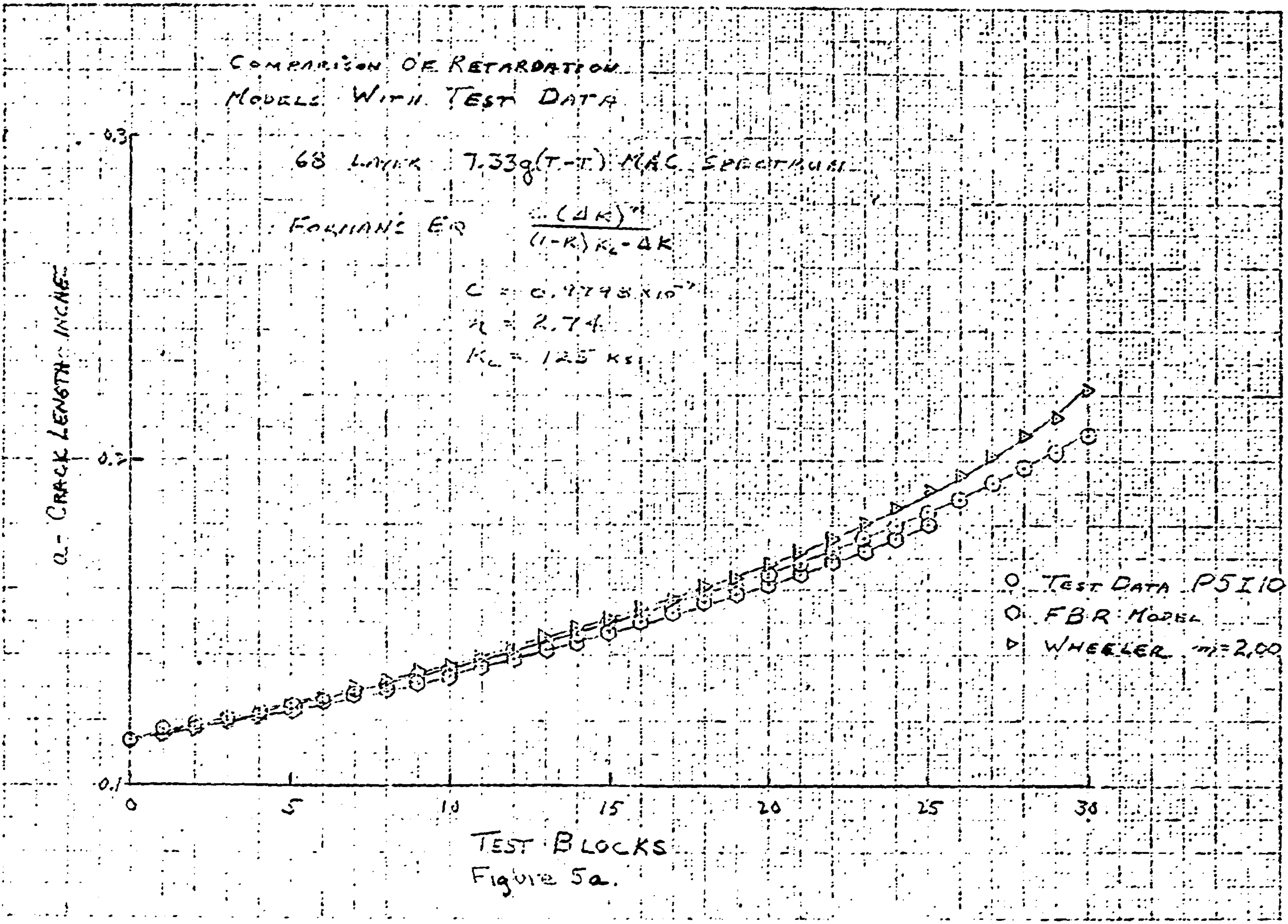


Figure 5a.

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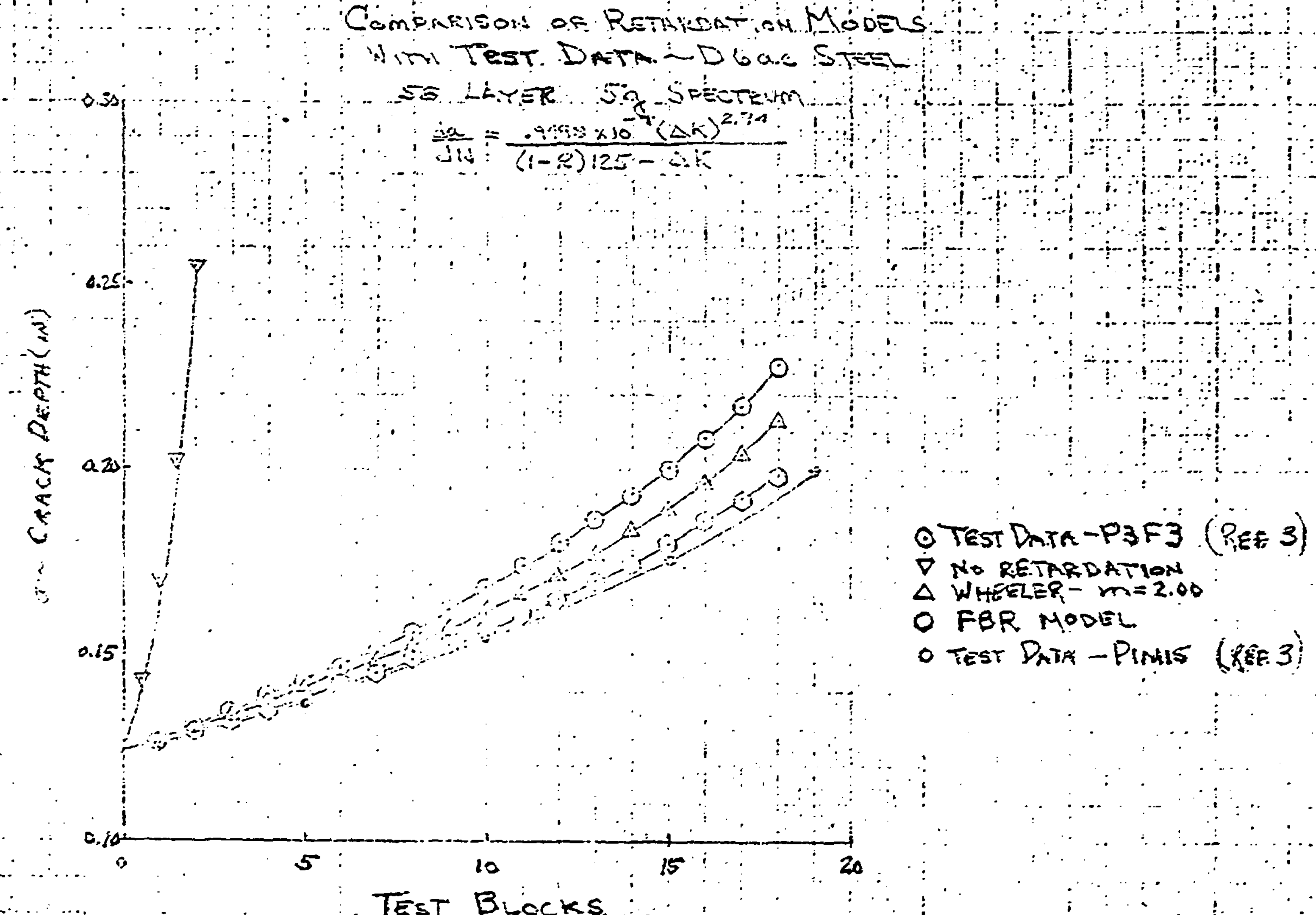
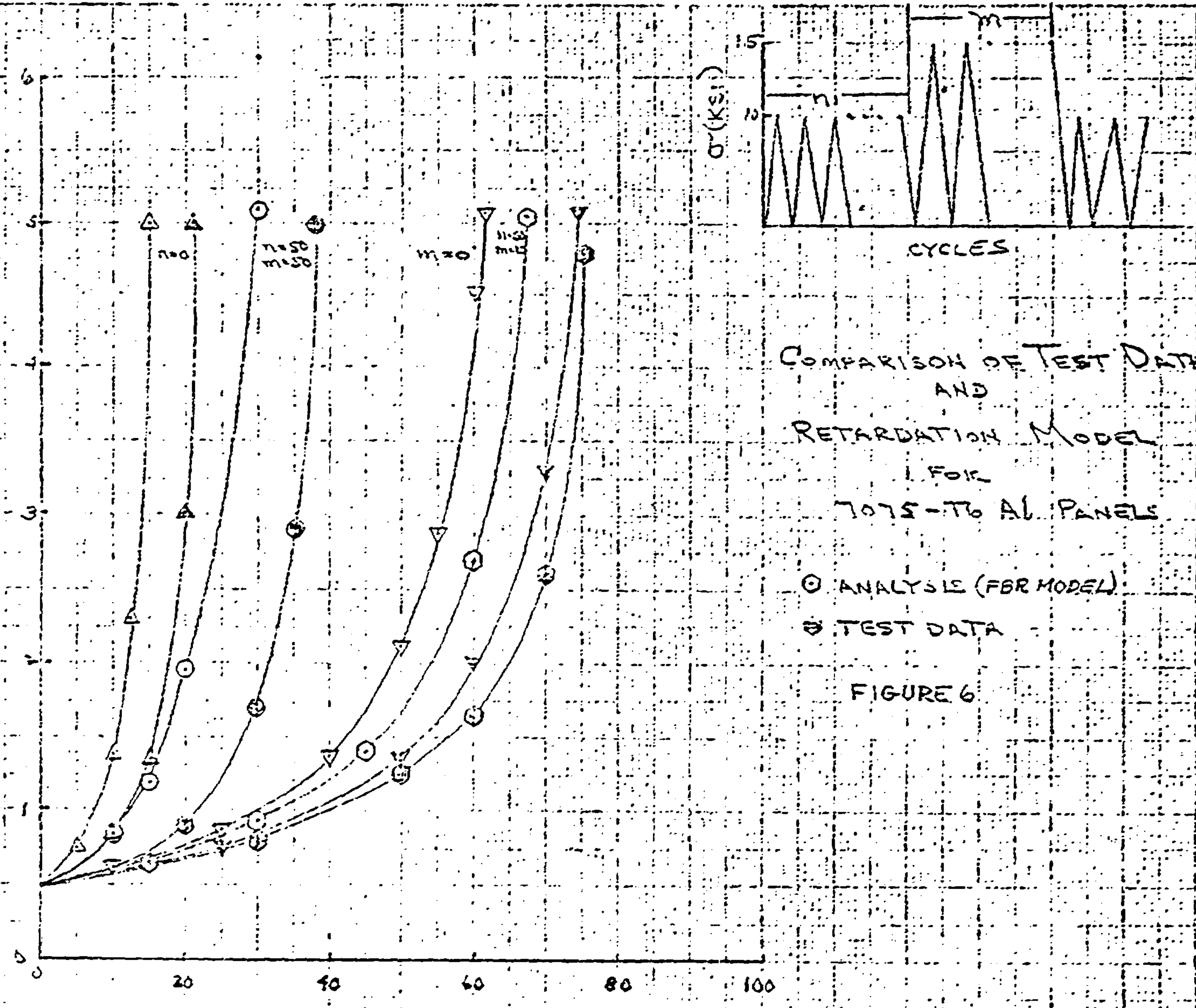

Comparison of Retardation Models With Test Data ~ D6ac Steel

55 Layer 5a Spectrum

$$\frac{da}{dN} = \frac{.9993 \times 10^{-4} (\Delta K)^{2.74}}{(1-R)125 - \Delta K}$$

Figure 5b



COMPARISON OF TEST DATA
AND
RETARDATION MODEL
FOR
7075-T6 AL PANELS

FIGURE 6