A Critical Look at the Standard Cosmological Picture[*]

Daryl Janzen[†]

Abstract

The discovery that the Universe is accelerating in its expansion has brought the basic concept of cosmic expansion into question. An analysis of the evolution of this concept suggests that the paradigm that was finally settled into prior to that discovery was not the best option, as the observed acceleration lends empirical support to an alternative which could incidentally explain expansion in general. I suggest, then, that incomplete reasoning regarding the nature of cosmic time in the derivation of the standard model is the reason why the theory cannot coincide with this alternative concept. Therefore, through an investigation of the theoretical and empirical facts surrounding the nature of cosmic time, I argue that an enduring three-dimensional cosmic present must necessarily be assumed in relativistic cosmology—and in a stricter sense than it has been. Finally, I point to a related result which could offer a better explanation of the empirically constrained expansion rate.

1 Introduction

Many of our basic conceptions about the nature of physical reality inevitably turn out to have been false, as novel empirical evidence is obtained, or paradoxical implications stemming from those concepts are eventually realised. This was expressed well by Einstein, who wrote [1]

> What is essential, which is based solely on accidents of development?. . . Concepts that have proven useful in ordering things, easily attain such an authority over us that we forget their Earthly origins and accept them as unalterable facts. . . The path of scientific advance is often made impassable for a long time through such errors. It is therefore by no means an idle trifling, if we become practiced in analysing the long-familiar concepts, and show upon which circumstances their justification and applicability depend, as they have grown up, individually, from the facts of experience.

Or, as he put it some years later [2],

> The belief in an external world independent of the percipient subject is the foundation of all science. But since our sense-perceptions inform us only indirectly of this external world, or Physical Reality, it is only by speculation that it can become comprehensible to us. From this it follows that our conceptions of Physical Reality can never be definitive; we must always be ready to alter them, to alter, that is, the axiomatic basis of physics, in order to take account of the facts of perception with the greatest possible logical completeness.

And so it is in the same spirit, and with the greatest respect, that I shall argue against a number of concepts in the standard cosmological picture that have changed little in the past century, by making note of original justifications upon which they were based, and weighing those against empirical data and theoretical developments that have been realised through the intervening years.

The essay will concentrate initially on the nature of cosmic expansion, which lacks an explanation in the standard cosmological model. Through a discussion of the early developments in cosmology, a familiarity with the pioneering conception of expansion, as being always driven by a cosmological constant Λ, will be developed, upon which basis it will be argued that the standard model—which cannot reconcile with this view—affords only a very limited description. Then, the nature of time in relativistic cosmology will be addressed, particularly with regard to the formulation of Weyl's postulate of a cosmic rest-frame. The aim will therefore be towards a better explanation of cosmic expansion in general, along with the present acceleration that has become evident, by reconceiving the description of time in standard cosmology, as an approach to resolving this significant shortcoming of the big bang Friedman-Lemaître-Robertson-Walker (FLRW) models, and particularly the flat ΛCDM model that describes the data so well.

2   On Cosmic Expansion

The expansion of our Universe was first evidenced by redshift measurements of spiral nebulae, after the task of measuring their radial velocities was initiated in 1912 by Slipher; and shortly thereafter, de Sitter attempted the first relativistic interpretation of the observed shifts, noting that 'the frequency of light-vibrations diminishes with increasing distance from the origin of co-ordinates' due to the coefficient of the time-coordinate in his solution [3]. But the concept of an expanding Universe, filled with island galaxies that would all appear to be receding from any given location at rates increasing with distance, was yet to fully form.

For one thing, when de Sitter published his paper, he was able to quote only three reliable radial velocity measurements, which gave merely $2:1$ odds in favour of his prediction. However, in 1923 Eddington produced an updated analysis of de Sitter space, and showed that the redshift de Sitter had predicted as a phenomenon of his statical geometry was in fact due to a cosmical repulsion brought in by the Λ-term, which would cause inertial particles to all recede exponentially from any one [4]. He used this result to support an argument for a truly expanding Universe, which would expand everywhere and at all times due to Λ. This, he supported with an updated list of redshifts from Slipher, which now gave $36:5$ odds in favour of the expansion scenario.

That same year, Weyl published a third appendix to *Raum, Zeit, Materie*, and an accompanying paper [5], where he calculated the redshift for the 'de Sitter cosmology',

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + e^{2\sqrt{\frac{\Lambda}{3}}t}(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2), \tag{1}$$

the explicit form of which would only be found later, independently by Lemaître [6] and Robertson [7]. Weyl was as interested in the potential relevance of de Sitter's solution for an expanding cosmology as Eddington [5], and had indeed been confused when he received a postcard from Einstein later that year (Einstein Archives: [24-81.00]), stating,

> With reference to the cosmological problem, I am not of your opinion. Following de Sitter, we know that two sufficiently separate material points are accelerated from one another. If there is no quasi-static world, then away with the cosmological term.

Eight days after this was posted, Einstein's famous second note [8] on Friedman's paper, which he now referred to as 'correct and clarifying', arrived at *Zeitschrift für Physik*. Einstein evidently had in mind that
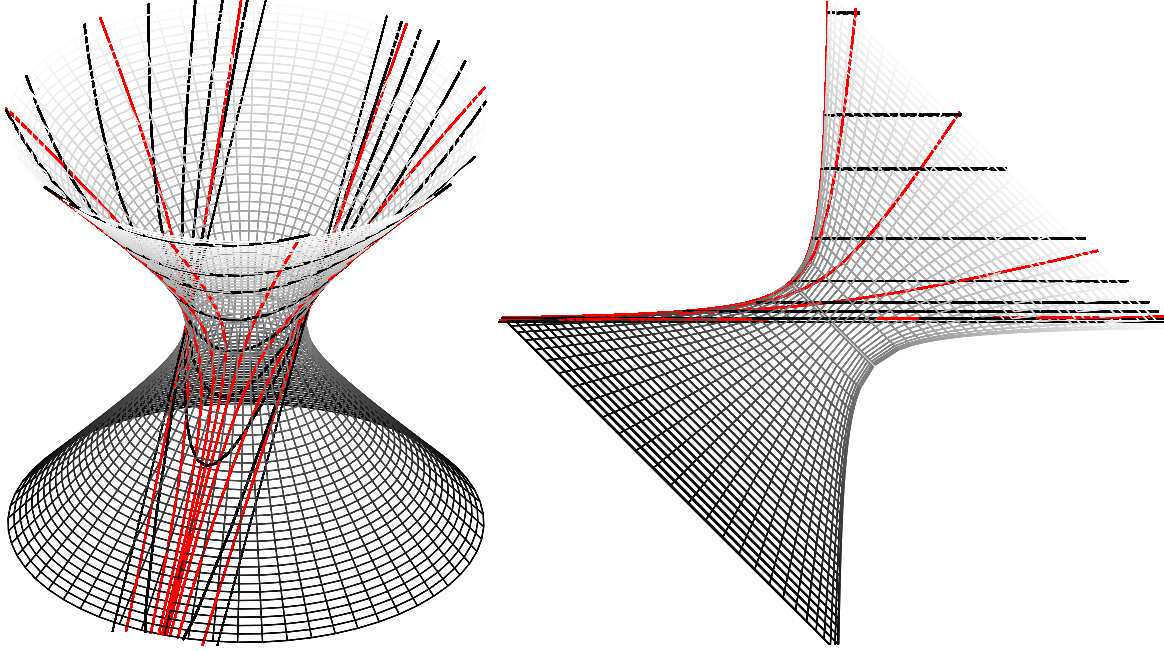
Figure 1: Slices of constant time in the Lemaître-Robertson coordination of de Sitter space (black lines), along with comoving world-lines (red lines), drawn on a two-dimensional slice of de Sitter space in three-dimensional Minkowski space.

the cosmic expansion can be described with Λ set to zero in Friedman's solution, and he might have thought Weyl would notice [8] and make the connection—but the latter evidently did not, as he wrote a dialogue the following year [9] in which the proponent of orthodox relativity [1] eventually states, 'If the cosmological term fails to help with leading through to Mach's principle, then I consider it to be generally useless, and am for the return to the elementary cosmology'—that being a particular foliation of Minkowski space, which, of the three cosmological models known to Weyl, was the only one with vanishing Λ.

At this point in the dialogue, the protagonist Paulus perseveres, citing the evidence for an expanding Universe, and therefore the de Sitter cosmology as the most likely of the three known alternatives. Weyl's excitement over its description is evident in Paulus' final statement: 'If I think about how, on the de Sitter hyperboloid the world lines of a star system with a common asymptote rise up from the infinite past [see Fig. 1], then I would like to say: the World is born from the eternal repose of 'Father Æther'; but once disturbed by the 'Spirit of Unrest' (*Hölderlin*), which is at home in the Agent of Matter, 'in the breast of the Earth and Man', it will never come again to rest.' Indeed, as Eq. (1) indicates, and as illustrated in Fig. 1, the universe emerges from a single point at $t = -\infty$, even though slices of constant cosmic time are infinitely extended thereafter—and comoving geodesics *naturally* disperse throughout the course of cosmic time.

Thus, we have a sense of the concept of cosmic expansion that was common amongst the main thinkers in cosmology in the 1920s, who were considering the possibility of expansion driven by the cosmical repulsion in de Sitter space. Indeed, Hubble was aware of this concept, as he wrote of the 'de Sitter effect' when he published his confirmation of cosmic expansion in 1929 [10]; and de Sitter himself, in 1930, wrote of Λ as 'a measure of the inherent expanding force of the universe' [11]. Thus, along with the evidence that our Universe actually *does* expand, one had in-hand the description of a well-defined force to drive that

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[1] The dialogue is situated between Saints Peter and Paul, with the latter presenting Weyl's 'apostatical' and 'heretical' views against the 'Relativity Church'. The following statement, which seems to be loosely quoted from the postcard sent by Einstein, was made by Peter.

expansion.

It was therefore a huge blow to Eddington, e.g., when in 1932 Einstein and de Sitter finally rejected that in favour of a model that could afford no prior explanation for why the Universe *should* expand. As he put it [12],

> the theory recently suggested by Einstein and de Sitter, that in the beginning all the matter created was projected with a radial motion so as to disperse even faster than the present rate of dispersal of galaxies,[*] leaves me cold. One cannot deny the possibility, but it is difficult to see what mental satisfaction such a theory is supposed to afford.

To see why the big bang FLRW models with matter provide no explanation of expansion, for the reason stated by Eddington, we need only look at Friedman's equation,

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3} - \frac{\kappa}{2}\left(p + \frac{\rho}{3}\right), \tag{2}$$

which describes the dependence of the scale-factor, a, on Λ and the density, ρ, and pressure, p, of matter. Since $p + \rho/3$ goes like $1/a^4$ for radiation or $1/a^3$ for dust, these models must be decelerating at the outset, with the 'inherent expanding force of the universe' only contributing to the expansion of space later on. Therefore, aside from Weyl's vacuous de Sitter cosmology, with its big bang singularity at $t = -\infty$, the big bang FLRW models can never explain the cosmic expansion they describe, which must be caused by the big bang singularity itself—i.e., where the theory blows up.

But since the cosmic microwave background (CMB) indicates that the Universe *did* begin in a hot dense state at a finite time in the past, the model Eddington had favoured instead—in which an Einstein universe that existed since eternity would inevitably begin expanding purely due to Λ [13]—also can't be accepted.

The explanatory deficit of standard cosmology lies in the fact that although the non-vacuous big bang FLRW models do *describe* expanding universes, they afford no justification for why those universes *should* expand, since that could only be due to the initial singularity; i.e., as we follow the models back in time, looking for a possible cause of expansion, we eventually reach a point where the theory becomes undefined, and call that the cause of it all. In contrast, I've discussed two FLRW models, neither of which is empirically supported, which would otherwise better *explain* the expansion they describe, as the result of a force that is well-defined in theory.

The basic cause and nature of cosmic expansion, along with its recently-observed acceleration, are significant problems of the standard model; so, condisering the evidence that the acceleration is best described by pure Λ [14], there is strong motivation to search for an alternative big bang model that would respect the pioneering concept of expansion, as a direct consequence of the 'de Sitter effect' in the modified Einstein field equations. It is therefore worth investigating the axiomatic basis of the Robertson-Walker (RW) line-element. As I'll eventually argue that the problem has to do with the way that time enters into the description, I'll begin by discussing some issues related to the problem of accounting for a cosmic present.

3   The Cosmic Present

The problem of recognising a cosmic present is that, according to relativity theory, it should not be possible to assign one time-coordinate to the four-dimensional continuum of events that could be used to describe objective simultaneity, since two events that are described as simultaneous in one frame will not be described as such by an observer in relative motion. However, as noted by Bondi [15],

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[*]They do not state this in words, but it is the meaning of their mathematical formulae. [Eddington's footnote.]

The Newtonian concept of the uniform omnipresent even-flowing time was shown by special relativity to be devoid of physical meaning, but in 1923 H. Weyl suggested that the observed motions of the nebulae showed a regularity which could be interpreted as implying a certain geometrical property of the substratum .... This in turn implies that it is possible to introduce an omnipresent *cosmic time* which has the property of measuring *proper time* for every observer moving with the substratum. In other words, whereas special relativity shows that a set of arbitrarily moving observers could not find a common 'time', the substratum observers move in such a specialized way that such a public or cosmic time exists.

Although the existence of such a time concept seems in some ways to be opposed to the generality, which forms the very basis of the general theory of relativity, the development of relativistic cosmology is impossible without such an assumption.

In fact, as Einstein himself noted in 1917 [16],

The most important fact that we draw from experience as to the distribution of matter is that the relative velocities of the stars are very small as compared with the velocity of light. So I think that for the present we may base our reasoning upon the following approximative assumption. There is a system of reference relatively to which matter may be looked upon as being permanently at rest.

Thus, he justified the assumption of a cosmic rest-frame—and a corresponding cosmic time—in deriving his 'cylindrical' model.

So, we have two opposing descriptions of relativistic time, and what I'll now argue is that developments both in cosmology and in our understanding of relativity theory which have taken place in the past century demand the latter—that there is one absolute cosmic time relative to which every observer's proper time will measure, as space-time will be perceived differently due to their absolute motion through the cosmic present that must be uniquely and objectively defined—rather than the former implication of Einstein's 1905 theory of relativity.

In the case of special relativity, a description in which space-time emerges as a clearly defined absolute cosmic present endures, can be realised by considering four-dimensional Minkowski space, as a background structure, and a three-dimensional universe that actually flows equably though it—with the past space-time continuum emerging as a purely ideal set of previous occurrences in the universe. Then, if we begin in the cosmic rest-frame, in which fundamental observers' world-lines will be traced out orthogonal to the cosmic hyperplane, photons can be described as particles that move through that surface at the same rate as cosmic time, thus tracing out invariant null-lines in space-time. In this way, the evolution of separate bodies, all existing in one three-dimensional space, forms a graduating four-dimensional map.

The causal and inertial structures of special relativity are thus reconciled by describing the world-lines of all observers in uniform motion through the cosmic present as their proper time axes, and rotating their proper spatial axes accordingly, so that light will be described as moving at the same rate in either direction of proper 'space'. And then, so that the speed of photons along invariant null-lines will actually be the same magnitude in all inertial frames, both the proper space and time axes in these local frames must also be scaled hyperbolically.

This description of the emergence of space-time in a special relativistic universe can be illustrated in the following way. Consider a barograph, consisting of a pen, attached to a barometer, and a sheet of paper that scrolls under the pen by clockwork. The apparatus may be oriented so that the paper scrolls downwards, with changes in barometric pressure causing the pen to move purely horizontally. We restrict the speed of the pen's horizontal motion only so that it must always be less than the rate at which the paper scrolls underneath it. The trace of the barometric pressure therefore represents the worldline of an arbitrarily moving observer in special relativistic space-time, with instantaneous velocity described in this frame by the ratio of its speed
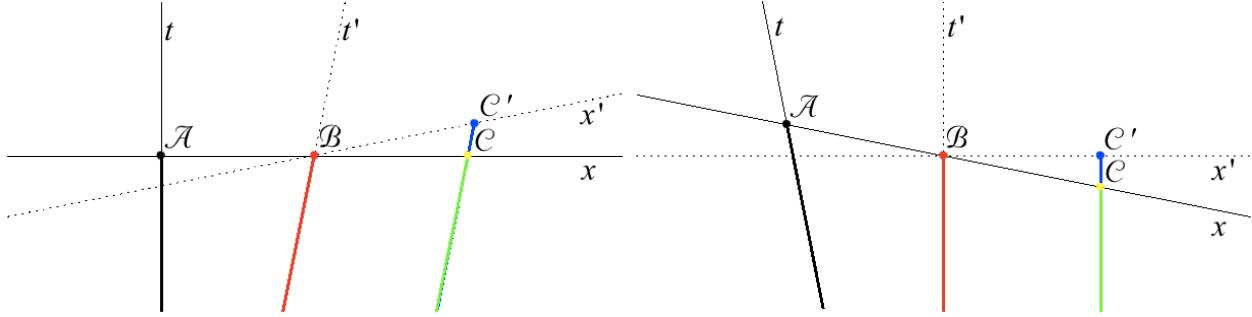
Figure 2: Snapshots, in two proper reference frames, of an emergent space-time. Although the proper times of $\mathscr{C}'$ of $\mathscr{B}$ appear to coincide, $\mathscr{C}'$ is disconnected from the causally coherent set, $\{\mathscr{A}, \mathscr{B}, \mathscr{C}\}$.

through the horizontal cosmic present and the graph paper's vertical speed, with 'speed' measured in either case relative to the ticking of the clockwork mechanism, which therefore cancels in the ratio.

Now, in order to illustrate the relativity of simultaneity, we detach the pen (call it $\mathscr{A}$) from the barometer so that it remains at rest absolutely, and add another pen, $\mathscr{B}$, to the apparatus, at the exact same height, which moves horizontally at a constant rate less than clockwork time—therefore, with *absolute velocity* less than the absolute speed limit. Furthermore, we make $\mathscr{A}$ and $\mathscr{B}$ 'observers', by enabling them to send and receive signals that transmit horizontally at the same rate (in clockwork time) as absolute time rolls on (in clockwork time), thus tracing out lines on the graph with unit speed.

As this system evolves, the two 'timelike observers' can send these 'photons' back and forth while a special relativistic space-time diagram is traced out. If we'd rather plot the map of events in coordinates that give the relevant description from $\mathscr{B}$'s perspective, we use the Lorentz transformation equations corresponding to the description of the map as Minkowski space-time: a line is drawn, tilted towards $\mathscr{B}$'s world-line by the appropriate angle, and the events along that surface are described as synchronous in that frame, even though they take place sequentially in reality. In particular, at the evolving present, $\mathscr{B}$'s proper spatial axis extends, in one direction, onto the empty sheet of graph paper in which events have not yet occurred, and, in the other direction, into the past space-time continuum of events that have already been traced onto the paper—while the real present hyperplane is tilted with respect to that axis of relative synchronicity.

This can be understood more clearly by adding two more 'observers', $\mathscr{C}$ and $\mathscr{C}'$, which remain at rest relative to $\mathscr{B}$, with $\mathscr{C}$ positioned along the same hyperplane as $\mathscr{A}$ and $\mathscr{B}$, and $\mathscr{C}'$ positioned precisely at the intersection of $\mathscr{C}$'s worldline (so that the worldlines of $\mathscr{C}$ and $\mathscr{C}'$ exactly coincide, as they are traced out on the space-time graph) and $\mathscr{B}$'s proper spatial axis (therefore, on a different hyperplane than $\mathscr{A}$, $\mathscr{B}$, and $\mathscr{C}$); thus, $\mathscr{C}'$ shall not be causally connected to $\mathscr{A}$, $\mathscr{B}$, and $\mathscr{C}$, since *by definition* information can only transmit along the cosmic hyperplane; see Fig. 2.

The significant point that is clearly illustrated through the addition of $\mathscr{C}$ and $\mathscr{C}'$, is that although in the proper coordinate system of $\mathscr{B}$ (or $\mathscr{C}$ or $\mathscr{C}'$), $\mathscr{C}'$ apparently exists synchronously and at rest relative to $\mathscr{B}$, $\mathscr{C}$—which in contrast appears to exist in $\mathscr{B}$'s (spacelike separated) past or future (depending on the direction of absolute motion)—is really the causally connected neighbour that remains relatively at rest, with which it should be able to synchronise its clock in the usual way. Thus, simultaneous noumena will not be perceived as synchronous phenomena in any but the cosmic rest-frame.

According to this description, we should have to relinquish the concept that there can be no priviliged observers, as well as Einstein's light-postulate in its original form. With regard to the latter, consider that photons will still be perceived as travelling at a constant speed in all directions of all reference frames, due to the invariance of null-lines. But this is indeed a matter of perception, since an observer moving through the universe will keep pace better with a photon in their direction of motion, and will remain closer to

that photon at all later times, on the cosmic hyperplane. Therefore, although light actually won't recede as quickly through the universe in the direction of absolute motion, it can always be described as such in the proper coordinate frame because it travels along invariant null-lines.

And with regard to the former concept, it is useful to note Galileo's argument that, to a person riding in the cabin of a moving ship, everything inside the cabin should occur just as if the ship were at rest. It was crucial for Galileo to make this point by *isolating* the inertial system from its relatively moving surroundings—as the point would have been less clear, e.g., if he had argued that when riding in the back of a wagon one can toss a ball straight in the air and have it fall back to the same point within the wagon. However, if one should argue that there *really* can't be privileged observers in the Universe, due to the relativity of inertia, one must go beyond this local-inertial effect and consider the frame with respect to its cosmic surroundings—in which case the argument can't be justified.

For consider a neutrino, created in a star shortly after the Big Bang: in the neutrino's proper frame, only minutes may have elapsed since it left the star, throughout which time the galaxies would have formed, etc., all moving past it in roughly the same direction, at nearly the speed of light. Clearly the most reasonable interpretation, however, is that the neutrino has *really* been travelling through the Universe for the past 13.7 billion years—and this description may be given, with the cosmic present uniquely defined, in all frames including the neutrino's.

Furthermore, if we'd rather assume that there are no privileged observers, it should be noted that the consequence of describing simultaneity and synchronicity as one and the same thing in all frames is a block universe [17]—a temporally singular 'absolute world' [18] in which 'the distinction between past, present, and future has only the significance of a stubborn illusion' [19]; i.e., 'The objective world simply *is*, it does not *happen*. Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time' [20]; 'There is no dynamics within space-time itself: nothing ever moves therein; nothing happens; nothing changes. …one does not think of particles "moving through" space-time, or as 'following along" their world-lines. Rather, particles are just "in" space-time, once and for all, and the world-line represents, all at once, the complete life history of the particle' [21].

And so I've argued against the simultaneity of synchronicity,—a reasonably intuitive concept held in common between the theories of both Newton and Einstein. But is there any *sensible* justification for the concept that the space in which events *really* take place simultaneously *must* be orthogonal to the proper time-axis of an inertial observer? When our theories are interpreted in this way, is that because one can, e.g., sit down on the floor with legs out in front, raise their right arm out to the side and their left arm up in the air, and then stick out their tongue in the direction in which time is flowing, for them as much as it is for their entire surroundings? Of course not. This is no more justified for someone who thus defines a right-handed coordinate system while sitting on solid ground, than it is for a person in the cabin of a ship—whether that is floating on water or flying through space. Therefore, intuition justifies only existence in space that endures with the ticking of everyone's watch—and relativity theory demands that this cannot be both coherently defined and synchronous with every inertial observer.

Now, although it may been argued that the alternative assumption of cosmic time is unobservable meta-physics, and therefore unsceintific, that is simply not true—for cosmology does provide strong empirical evidence of an absolute rest-frame in our Universe, as follows. As Einstein noted already in 1917 [16], there appears to be a frame relative to which the bodies of our Universe are at rest, on average. Now, Einstein had no idea of the scope of the Universe at that time, but already by 1923 Weyl realised the significance of this point, which has indeed stood the test of time, when he wrote that [5] 'Both the papers by de Sitter [3] and Eddington [4] lack this assumption on the "state of rest" of stars—by the way the only possible one compatible with the homogeneity of space and time. Without such an assumption nothing can be known about the redshift, of course.' For it is true, even in de Sitter space, that a cosmic time must be assumed in order to calculate redshifts; e.g., for particles in the comoving Lemaître-Robertson frame illustrated in

Fig. 1 and described by Eq. (1), the redshift will be different from that in the frame of comoving particles in the three-sphere which contracts to a finite radius and subsequently expands (as illustrated by the gridlines of the de Sitter hyperboloid in Fig. 1) according to

$$\mathrm{d}s^2 = -\mathrm{d}T^2 + \frac{3}{\Lambda}\cosh^2\left(\sqrt{\frac{\Lambda}{3}}T\right)\mathrm{d}\Omega_3{}^2, \tag{3}$$

where $\mathrm{d}\Omega_3$ describes the three-sphere. The existence of more than one formally distinct RW cosmological model in one and the same space-time thus illustrates the importance of defining a cosmic time.

Since 1923, a number of novel observations have strengthened the evidence for a cosmic present, such as Hubble's confirmation of cosmic expansion, the detailed measurement of the expansion rate that has lately been afforded through type Ia supernovae observations, and the discovery of the CMB, which gives a detailed signature of the cosmic rest-frame relative to which we are in fact moving, according to the common interpretation of its dipole anisotropy. Thus, the assumption of a cosmic present is now justified.

4 Implications for Cosmology

Although many points should be considered in connection to the description of an absolute cosmic present, such as concepts of time travel, free will, and a causally coherent local description of gravitational collapse in the Universe—notwithstanding space-time curvature in general,—the one consequence that I will note pertains to cosmology, and a better explanation for cosmic expansion.

To start, note that in deriving the general line-element for the background geometry of FLRW cosmology, Robertson required four basic assumptions [22]: i. a congruence of geodesics, ii. hypersurface orthogonality, iii. homogeneity, and iv. isotropy. i. and ii. are required to satisfy Weyl's postulate of a causal coherence amongst world-lines in the entire Universe, by which every single event in the bundle of fundamental world-lines is associated with a well-defined three-dimensional set of others with which it 'really' occurs simultaneously. However, it seems that ii. is therefore mostly required to satisfy the concept that synchronous events in a given inertial frame should have occurred simultaneously, against which I've argued above.

In special relativity, if we allow the fundamental world-lines to *set* the cosmic rest-frame, then the cosmic hyperplane should be orthogonal—but that shouldn't be the case in general. Indeed, as I've shown in my PhD thesis [23], in the cosmological Schwarzschild-de Sitter (SdS) solution,

$$\mathrm{d}s^2 = -\frac{r}{\frac{\Lambda}{3}r^3 + 2M - r}\mathrm{d}r^2 + \frac{\frac{\Lambda}{3}r^3 + 2M - r}{r}\mathrm{d}t^2 + r^2\mathrm{d}\Omega^2, \tag{4}$$

for which $\Lambda M^2 > 1/9$ and $r > 0$ is timelike, the r-coordinate should well describe the cosmic time *and* factor of expansion in a universe in which, in the coordinates carried by fundamental observers, the cosmic present would not be synchronous, and r would evolve in proper time τ as

$$r(\tau) \propto \sinh^{2/3}[(\sqrt{3\Lambda}/2)\tau], \tag{5}$$

which is *incidentally* also the flat ΛCDM scale-factor of the standard model that has been empirically constrained this past decade [14]. This is the rate of expansion that *all* observers would measure, if distant galaxies were themselves all roughly at rest with respect to fundamental world-lines. But in contrast to FLRW theory, this universe actually has to expand—at all $r > 0$—as a result of the 'de Sitter effect'; i.e., if such a universe did come to exist at any infinitesimal time, it would *necessarily* expand—and in exactly the manner that we observe—which may be the closest to an explanation of that as we can achieve.

It is, of course, important to stress that this intriguing result is utterly meaningless if simultaneity should rather be defined as synchronicity in a given frame of reference. In that case, as Lemaître noted [24], the solution describes flat spatial slices extending from $r = 0$ to ∞, with particles continuously ejected from the origin. It is therefore only by reconceiving the relativistic concepts of time and simultaneity that SdS can be legitimated as a coherent cosmological model with a common origin—and one with the very factor of expansion that we've measured—which really *should* expand, according to the view of expansion as being always driven by Λ.

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