Asian Journal of Probability and Statistics

8(2): 1-16, 2020; Article no.AJPAS.58405
ISSN: 2582-0230

A Critique on the Foundational Response Surface Methodology for Exploring Optimal Regions

John E. Usen[1*], Essien J. Okoi[1], Eric M. Egomo[1], Ekeng N. Henshaw[1] and Edet B. Hogan[1]

[1]Department of Statistics, Cross River University of Technology, P.O.Box 1123, Calabar, Nigeria.

Authors' contributions

This work was done in collaboration among all the five authors. Author JEU designed the study, performed the analysis and wrote the first draft of the manuscript. Authors EJO, EME, ENH and EBH supervised the study and analysis of the data. All the authors managed the literature search, as well as wrote the final manuscript. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJPAS/2020/v8i230201
Editor(s):
(1) Dr. Belkacem Chaouchi, Professor, Khemis Miliana University, Algeria.
Reviewers:
(1) Zheng Mingliang, University of Wuxi, China.
(2) Sanjay Jain, SPC Government College Ajmer (Affiliated to MDS University), India.
Complete Peer review History: http://www.sdiarticle4.com/review-history/58405

Review Article

Received: 24 April 2020
Accepted: 29 June 2020
Published: 28 July 2020

Abstract

The interest of most process engineers in industries is usually to optimize the yield of their processes. Not until 1951, imprecise methodologies were used in industries for this purpose. However, in 1951, G. E. P. Box and K. B. Wilson invented the technique of Response Surface Methodology (RSM) as one used for the optimization of the yield of processes. Being an initial idea, this paper has considered RSM as a foundational idea. In particular, it criticizes this foundational idea from the angle of its intuitive approach to searching for near-optimal settings of industrial processes, should such processes fail to run at optimal settings. RSM uses the tools of canonical transformation and analysis (a trial-and-error routine) for this search. Regardless, the foundational response surface methodology is acknowledged to be primarily efficient for determining the optimum response.

Keywords: Foundational response surface methodology; near-optimal settings; canonical transformation; canonical analysis.

**Corresponding author: E-mail: usenjohn@crutech.edu.ng;*

1 Introduction

Response surface methodology is a collection of mathematical and statistical techniques useful for analyzing problems where several factors influence a response variable and the goal is to optimize (maximize or minimize) this response [1]. Response surface methodology is an efficient experimental design technique, invented by [2] for this purpose. The foundational response surface methodology is executed in two broad stages. The first stage locates the optimum (maximum or minimum) operating setting which yields the optimum response; whereas the second stage explores the region around the optimum operating setting for near-optimal operating conditions.

The stage of locating the optimum (maximum or minimum) operating setting involves implementing a sequence of routines. This sequence of routines starts with planning and running a factorial or fractional factorial design around the current operating condition [3,4,5,6]. Next, a linear model (with no interaction or quadratic terms) is fit to the data; and, thereafter, the path of steepest ascent (or descent) is determined [7,8,9]. Following this, several tests are run on the path of steepest ascent until response no longer improves [5,10,11]. The operating setting at which response no longer improves indicates the presence of large curvature in the system; and this implies that the process is in the optimal region [11,12]. In this region, design, run and fit a quadratic model using least squares technique. Based on this quadratic model, locate the optimal setting of the factors using classical derivative technique [11,12].

However, should the tests on the path of steepest ascent (or descent) indicate little or no curvature in the system, the entire process must be repeated from the beginning since such a result indicates that the experiment is remote from the optimum [5,13,14]. But, according to [8,11], if this is not the case, experience shows that the located optimal setting usually enables process engineers in industries to determine the levels at which sensitive factors of their processes can be positioned in other to maximize yield (e.g. volume or quality of a product) or minimize yield (e.g. cost of production or waste from production).

Indeed, obtaining the optimum operating condition indicates an end to the first stage of response surface methodology [15,16,5,17]. But often times, circumstances such as drop in efficiency of machines overtime and sporadic fluctuations in voltage usually initiate momentary drifts away from the optimum operating condition that, in turn, affects the nature of the yield especially within durations prior to the fixing of such machines or adjustments of such fluctuations. In view of such set-back, it becomes imperative for process engineers to manage the situation using carefully selected factors settings that are near-optimal in order to maintain near-optimality of the process yield. According to [18,19,20], making such selections involves exploring the optimal region for near-optimal settings; hence, the second stage of response surface methodology. The foundational procedure of response surface methodology explores the optimal region using the technique of canonical analysis. This paper is concerned with critically reviewing the technique of canonical analysis.

2 The Foundational Response Surface Methodology

Once the quadratic model for the response surface is obtained from the first stage discussed previously, the two-staged process of the foundational response surface methodology reduces to performing the following routines:

2.1 Locating the optimal setting of the factors using classical derivative technique

In matrix notation we write the quadratic model of the response surface as:

$$\hat{y} = \hat{\beta}_0 + x^T b + x^T B x \tag{1}$$

where:

$$x = \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{pmatrix}}_{k\times1} \qquad b = \underbrace{\begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_k \end{pmatrix}}_{k\times1} \qquad B = \underbrace{\begin{pmatrix} \hat{\beta}_{11} & \hat{\beta}_{12}/2 & \hat{\beta}_{13}/2 & \cdots & \hat{\beta}_{1k}/2 \\ & \hat{\beta}_{22} & \hat{\beta}_{23}/2 & \cdots & \hat{\beta}_{2k}/2 \\ & & \hat{\beta}_{33} & \cdots & \hat{\beta}_{3k}/2 \\ & & & \ddots & \vdots \\ \text{sym.} & & & & \hat{\beta}_{kk} \end{pmatrix}}_{k\times k}$$

That is, b is a vector of the first-order regression coefficients and B is a symmetric matrix whose main diagonal elements are the pure quadratic coefficients and whose off-diagonal elements are one-half of the mixed quadratic coefficients. Differentiating equation (1), and equating the derivate to zero gives:

$$\frac{\partial \hat{y}}{\partial x} = b + 2Bx = 0 \qquad (2)$$

The optimal setting is the solution to equation (2); that is:

$$x_0 = -\frac{1}{2}B^{-1}b \qquad (3)$$

On substituting for equation (3) in equation (1), the expected response at the optimal setting is obtained as:

$$\hat{y}_0 = \hat{\beta}_0 + \frac{1}{2}x_0^T b \qquad (4)$$

2.2 Exploring the optimal region around the optimal setting

In order to explore the optimal region around the optimal setting for near-optimal settings, the second stage of the foundational response surface methodology usually starts with characterizing the obtained optimal setting. To characterize the obtained optimal setting it transforms the fitted model to a new coordinate system with the origin at the optimal setting, and then rotates the axes of this system until they are parallel to the principal axes of the fitted response surface [15,8,1]. Fig. 1 illustrates this transformation.

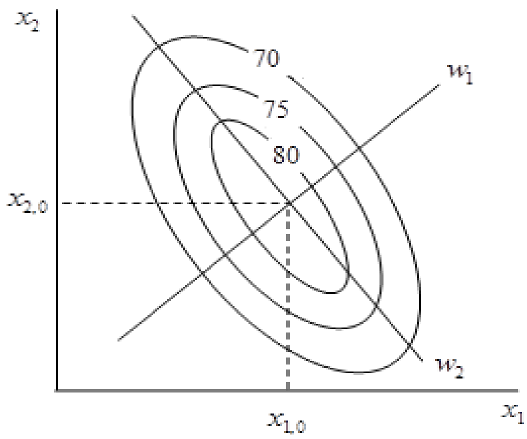

Fig. 1. Canonical form of an SOM

This results in the fitted model

$$\hat{y} = \hat{y}_0 + \lambda_1 w_1^2 + \lambda_2 w_2^2 + \cdots + \lambda_k w_k^2 \qquad (5)$$

where the w_i's are the transformed independent variables and λ_i's are just the eigen-values or characteristic roots of the matrix B. We call equation (5) the canonical form of the model.

The nature of the response surface can be determined from the optimal setting and the sign and magnitude of the λ_i's [5,6]. First, suppose that the stationary point is within the region of exploration for fitting the SOM. If the λ_i's are all positive then x_0 is a point of minimum response [6,11,1]. If the λ_i's are all negative then x_0 is a point of maximum response. If the λ_i's have different signs then x_0 is a saddle point [5,6,1]. Furthermore the surface is steepest in the w_i direction for which the absolute values of the λ_i's are the greatest [7]. If one or more of the λ_i's are very small, then the system is insensitive to the variable w_i. This type of surface is called a stationary ridge [11,1]. Fig. 2 illustrates it.

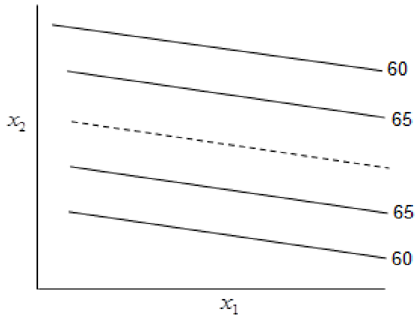

Fig. 2. A Stationary ridge system

If the stationary point is far outside the region of exploration for fitting the SOM, and one or more λ_i is near zero, then the surface may be a rising ridge. Fig. 3 illustrates a rising ridge for $k = 2$ variables with λ_1 near zero and λ_2 negative. In this type of ridge system, we cannot draw inferences about the true surface or the stationary point since it is outside the region where we have fit the model. However, further exploration in the w_1 direction is warranted. If λ_2 had been positive we would call this system a falling ridge.

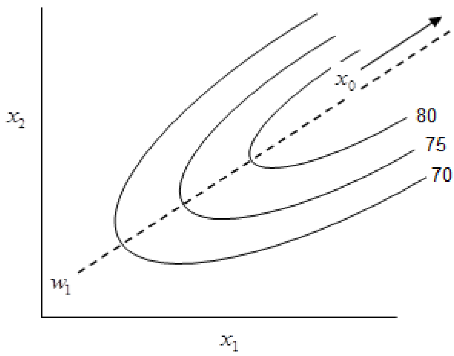

Fig. 3. A rising ridge system

However, in some response surface problems it may be necessary to find the relationship between the canonical variables and the design variables [5,21]. This is particularly true if it is impossible to operate the process at the stationary point. According to [1,17], exploration in the canonical form requires converting points in the canonical variable space to points in the design variable space. In general both types of variables are related by

$$w = M^T(x - x_0) \tag{6}$$

where M is a $k \times k$ orthogonal matrix. The columns of M are the normalized eigen-vectors associated with the λ_i. That is if m_i is the ith column of M then m_i is the solution to

$$(B - \lambda_i I) m_i = 0 \tag{7}$$

for which

$$\sum_{j=1}^{k} m_{ji}^2 = 1$$

2.3 Illustrating the deficiency of foundational RSM approach

The contact process is a method of producing sulfuric acid in the high concentrations needed for industrial processes [22]. A case study in [23] shows that the yield of a chemical process (contact process) is studied. The chemical engineer had chosen three controllable variables (temperature, pressure and time) that influenced the yield of his process each at a high and low level. He was interested in determining the operating conditions that maximize the yield of his process. He was operating the process with reaction temperature of 450 degree Celsius, pressure of 1.0 atmosphere and time of 30 minutes resulting in yield around 97 percent. Since it was unlikely that this region contained the optimum, an FOM was fit and the MSA applied. We decided that the region of exploration for fitting the FOM should be (445, 455) degree Celsius of temperature, (0.9, 1.1) Atmosphere of pressure and (29, 31) minutes of time. To simplify the calculations, we coded the independent variables to a (-1, 1) interval. Thus, if ξ_1 denotes the natural variable temperature, ξ_2 denotes the natural variable pressure and ξ_3 denotes the natural variable time, then the coded variables are

$$x_1 = \frac{\xi_1 - 450}{5} \qquad x_2 = \frac{\xi_2 - 1.0}{0.1} \qquad x_3 = \frac{\xi_3 - 30}{1.0}$$

The quadratic model this problem is given as:

$$y = 97.6 + 0.447x_1 + 0.314x_2 + 0.357x_3 - 0.150x_1^2 - 0.450x_2^2 - 0.203x_3^2 + 0.025x_1x_2 - 0.075x_1x_3 + 0.225x_2x_3$$

We now perform the canonical analysis. Note that:

$$b = \begin{pmatrix} 0.447 \\ 0.314 \\ 0.357 \end{pmatrix} \qquad B = \begin{pmatrix} -0.150 & 0.0125 & -0.0375 \\ 0.0125 & -0.450 & 0.1125 \\ -0.0375 & 0.1125 & -0.203 \end{pmatrix}$$

Hence, from the above quadratic model, the optimal setting is obtained as:

$$x_0 = -\frac{1}{2}B^{-1}b$$

$$|B| = -0.011245$$

$$\text{Adj}\,B = B_c^T$$

But

$$B_c = \begin{pmatrix} 0.07869375 & -0.00168125 & -0.01546875 \\ -0.00168125 & 0.02904375 & 0.01640625 \\ -0.01546875 & 0.01640625 & 0.06734375 \end{pmatrix}$$

$$\text{Adj}\,B = B_c^T = \begin{pmatrix} 0.07869375 & -0.00168125 & -0.01546875 \\ -0.00168125 & 0.02904375 & 0.01640625 \\ -0.01546875 & 0.01640625 & 0.06734375 \end{pmatrix}$$

$$B^{-1} = \frac{\text{Adj}\ B}{|B|} = \begin{pmatrix} -6.998110271 & 0.149510893 & 1.375611383 \\ 0.149510893 & -2.582814584 & -1.45898177 \\ 1.375611383 & -1.45898177 & -5.988772788 \end{pmatrix}$$

$$\Rightarrow x_0 = -\frac{1}{2}\begin{pmatrix} -6.998110271 & 0.149510893 & 1.375611383 \\ 0.149510893 & -2.582814584 & -1.45898177 \\ 1.375611383 & -1.45898177 & -5.988772788 \end{pmatrix}\begin{pmatrix} 0.447 \\ 0.314 \\ 0.357 \end{pmatrix}$$

$$= -\frac{1}{2}\begin{pmatrix} -2.590115607 \\ -1.265028902 \\ -1.981213873 \end{pmatrix}$$

$$= \begin{pmatrix} 1.295057804 \\ 0.632514451 \\ 0.990606936 \end{pmatrix}$$

That is $x_{1,0} = 1.295057804$, $x_{2,0} = 0.632514451$ and $x_{3,0} = 0.990606936$. In terms of the natural variables the stationary point is

$$1.295057804 = \frac{\xi_1 - 450}{5}, \quad 0.632514451 = \frac{\xi_2 - 1.0}{0.1}, \quad 0.990606936 = \frac{\xi_3 - 30}{1.0}$$

which yields $\xi_1 = 456.475289$, $\xi_2 = 1.063251445$ and $\xi_3 = 30.99060694$.

Hence, the predicted response at the optimal setting becomes $\hat{y}_0 = 98.16557353$.

$$\hat{y}_0 = 97.6 + \frac{1}{2}(1.29505780\ 4,\ 0.63251445\ 1,\ 0.99060936\ )\begin{pmatrix}0.447\\0.314\\0.357\end{pmatrix}$$

$$= 97.6 + \frac{1}{2}(1.13147052\ )$$

$$= 97.6 + 0.56557352\ 6$$

$$= 98.1655735\ 3$$

To further characterize the stationary point we obtain the canonical form the response equation above. The Eigen values λ_1, λ_2 and λ_3 are the roots of the determinant equation

$$|B - \lambda I| = 0$$

$$\begin{vmatrix}-0.150-\lambda & 0.0125 & -0.035\\ 0.0125 & -0.450-\lambda & 0.1125\\ -0.0375 & 0.1125 & -0.203-\lambda\end{vmatrix} = 0$$

which reduces to

$$\Rightarrow \lambda^3 + 0.803\lambda^2 + 0.17508125\lambda + 0.011244999 = 0$$

$$\Rightarrow \lambda_1 = -0.122900982,\quad \lambda_2 = -0.495411202,\quad \lambda_3 = -0.184687815$$

Thus the canonical form of the fitted model is:

$$\hat{y} = 98.1655735\ 3 - 0.12290098\ 2w_1^2 - 0.49541120\ 2w_2^2 - 0.18468715\ w_3^2$$

Since λ_1, λ_2 and λ_3 are negative and the stationary point is within the region of exploration, we conclude that the stationary point is a maximum.

However, as an illustration of the deficiency of the foundational response surface methodology discussed previously, suppose that a process engineer could not operate the process at $\xi_1 = 456.475289$, $\xi_2 = 1.063251445$ and $\xi_3 = 30.99060694$ owing to machine deterioration overtime. If we now wish to slightly drift from the optimal setting to a point of lower cost, but without large losses in process yield, then the canonical form of the model indicates that the surface is least sensitive to yield loss in the w_1 direction.

Exploration of the canonical form requires converting points in the (w_1, w_2, w_3) space to points in the (x_1, x_2, x_3) space. Using the quadratic model in the design variables above, we make the following illustration. For $\lambda_1 = -0.122900982$, we have:

$$\begin{bmatrix}-0.027099018 & 0.0125 & -0.0375\\ 0.0125 & -0.372099018 & 0.1125\\ -0.0375 & 0.1125 & -0.080099018\end{bmatrix}\begin{bmatrix}m_{11}\\m_{21}\\m_{31}\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

We wish to obtain the normalized solution to these equations, that is, the one for which $m_{11}^2 + m_{21}^2 + m_{31}^2 = 1$. There is no unique solution to these equations and it is most convenient to assign an arbitrary value of one of the unknowns, solve the system and then normalize the solution. Letting $m_{21}^* = 1$.

$$\begin{array}{rcrcrcl} -0.027099018m_{11} & + & 0.0125m_{21} & - & 0.0375m_{31} & = & 0 \\ 0.0125m_{11} & - & 0.372099018m_{21} & + & 0.1125m_{31} & = & 0 \\ -0.0375m_{11} & + & 0.1125m_{21} & - & 0.080099018m_{31} & = & 0 \end{array}$$

Taking the first two equations,

$$\begin{array}{rcrcrcl} -0.027099018m_{11}^* & + & 0.0125 & - & 0.0375m_{31}^* & = & 0 \\ 0..0125m_{11}^* & - & 0.372099018 & + & 0.1125m_{31}^* & = & 0 \end{array}$$

$$\begin{array}{rcrcl} -0.027099018m_{11}^* & - & 0.0375m_{31}^* & = & -0.0125 \\ 0.0125m_{11}^* & + & 0.1125m_{31}^* & = & 0.372099018 \end{array}$$

$$\begin{array}{rcrcl} 0.027099018m_{11}^* & + & 0.0375m_{31}^* & = & 0.0125 \\ 0.0125m_{11}^* & + & 0.1125m_{31}^* & = & 0.372099018 \end{array}$$

$$\begin{pmatrix} 0.027099018 & 0.0375 \\ 0.0125 & 0.1125 \end{pmatrix} \begin{pmatrix} m_{11}^* \\ m_{31}^* \end{pmatrix} = \begin{pmatrix} 0.0125 \\ 0.372099018 \end{pmatrix}$$

$$m_{11}^* = \frac{\Delta_1}{\Delta}, \quad m_{21}^* = 1, \quad m_{31}^* = \frac{\Delta_2}{\Delta}$$

But

$$\begin{aligned} \Delta &= \begin{vmatrix} 0.027099018 & 0.0375 \\ 0.0125 & 0.1125 \end{vmatrix} \\ &= 0.003048639525 - 0.00046875 \\ &= 0.002579889525 \end{aligned}$$

$$\begin{aligned} \Delta_1 &= \begin{vmatrix} 0.0125 & 0.0375 \\ 0.372099018 & 0.1125 \end{vmatrix} \\ &= 0.00140625 - 0.013953713 \\ &= -0.012547463 \end{aligned}$$

$$\Delta_2 = \begin{vmatrix} 0.027099018 & 0.0125 \\ 0.0125 & 0.372099018 \end{vmatrix}$$
$$= 0.010083517 - 0.00015625$$
$$= 0.009927267$$

$$m^*_{11} = \frac{\Delta_1}{\Delta} = -\frac{0.012547463}{0.002579889525} = -4.863566009$$

$$m^*_{21} = 1$$

$$m^*_{31} = \frac{\Delta_2}{\Delta} = \frac{0.009927267}{0.002579889525} = 3.847942675$$

To normalize this solution divide m^*_{11}, m^*_{21} and m^*_{31} by:

$$\left((m_{11})^2 + (m_{21})^2 + (m_{31})^2\right)^{\frac{1}{2}} = \left((4.863566009)^2 + (1)^2 + (3.847942675)^2\right)^{\frac{1}{2}}$$
$$= 6.281794103$$

This yields the normalized solutions

$$m_{11} = \frac{m^*_{11}}{6.281794103}$$
$$= -\frac{4.863566009}{6.281794103}$$
$$= -0.774231999$$

$$m_{21} = \frac{m^*_{21}}{6.281794103}$$
$$= \frac{1}{6.281794103}$$
$$= 0.15919019$$

$$m_{31} = \frac{m^*_{31}}{6.281794103}$$
$$= -\frac{3.847942675}{6.281794103}$$
$$= 0.612554727$$

which is the first column of the M matrix.

We now use $\lambda_2 = 0.495411202$. Thus, equation (7) becomes

$$\begin{bmatrix} 0.345411202 & 0.0125 & -0.0375 \\ 0.0125 & 0.045411202 & 0.1125 \\ -0.0375 & 0.1125 & 0.495411202 \end{bmatrix} \begin{bmatrix} m_{12} \\ m_{22} \\ m_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Letting $m_{22}^* = 1$,

$$\begin{array}{ccccccc} 0.345411202m_{12} & + & 0.0125m_{22} & - & 0.0375m_{32} & = & 0 \\ 0.0125m_{12} & + & 0.045411202m_{22} & + & 0.1125m_{32} & = & 0 \\ -0.0375m_{12} & + & 0.1125m_{22} & + & 0.495411202m_{32} & = & 0 \end{array}$$

Taking the first two equations,

$$\begin{array}{ccccccc} 0.345411202m_{12}^* & + & 0.0125 & - & 0.0375m_{32}^* & = & 0 \\ 0..0125m_{12}^* & + & 0.045411202 & + & 0.1125m_{32}^* & = & 0 \end{array}$$

$$\begin{array}{ccccc} 0.345411202m_{12}^* & - & 0.0375m_{32}^* & = & -0.0125 \\ 0.0125m_{12}^* & + & 0.1125m_{32}^* & = & -0.045411202 \end{array}$$

$$\begin{pmatrix} 0.345411202 & -0.0375 \\ 0.0125 & 0.1125 \end{pmatrix} \begin{pmatrix} m_{12}^* \\ m_{32}^* \end{pmatrix} = \begin{pmatrix} -0.0125 \\ -0.045411202 \end{pmatrix}$$

$$m_{12}^* = \frac{\Delta_1}{\Delta}, \quad m_{22}^* = 1, \quad m_{32}^* = \frac{\Delta_2}{\Delta}$$

But

$$\Delta = \begin{vmatrix} 0.345411202 & -0.0375 \\ 0.0125 & 0.1125 \end{vmatrix}$$
$$= 0.03885876 + 0.00046875$$
$$= 0.03932751$$

$$\Delta_1 = \begin{vmatrix} -0.0125 & -0.0375 \\ -0.045411202 & 0.1125 \end{vmatrix}$$
$$= -0.00140625 - 0.001702920075$$
$$= -0.003109170075$$

$$\Delta_2 = \begin{vmatrix} 0.345411202 & -0.0125 \\ 0.0125 & -0.045411202 \end{vmatrix}$$
$$= -0.015685537 + 0.00015625$$
$$= -0.015529287$$

$$m_{12}^* = \frac{\Delta_1}{\Delta} = -\frac{-0.003109170075}{0.03932751} = -0.079058401$$

$$m_{22}^* = 1$$

$$m_{32}^* = \frac{\Delta_2}{\Delta} = -\frac{0.015529287}{0.03932751} = -0.394870842$$

To normalize this solution divide m_{12}^*, m_{22}^* and m_{32}^* by

$$\left((m_{12})^2 + (m_{22})^2 + (m_{32})^2\right)^{\frac{1}{2}} = \left((0.079058401)^2 + (1)^2 + (0.394870842)^2\right)^{\frac{1}{2}}$$
$$= 1.078041378$$

This yields the normalized solutions

$$m_{12} = \frac{m_{12}^*}{1.078041378}$$
$$= -\frac{0.079058401}{1.078041378}$$
$$= -0.073335219$$

$$m_{22} = \frac{m_{22}^*}{1.078041378}$$
$$= \frac{1}{1.078041378}$$
$$= 0.927608179$$

$$m_{32} = \frac{m_{32}^*}{1.078041378}$$
$$= -\frac{0.394870842}{1.078041378}$$
$$= 0.366285422$$

which is the second column of the M matrix.

We now use $\lambda_3 = 0.184687815$. This gives:

$$\begin{bmatrix} 0.034687815 & 0.0125 & -0.0375 \\ 0.0125 & -0.265312185 & 0.1125 \\ -0.0375 & 0.1125 & -0.01832185 \end{bmatrix} \begin{bmatrix} m_{13} \\ m_{23} \\ m_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Letting $m^*_{23} = 1$

$$\begin{array}{rcccccl} 0.034687815m_{13} & + & 0.0125m_{23} & - & 0.0375m_{33} & = & 0 \\ 0.0125m_{13} & - & 0.265312185m_{23} & + & 0.1125m_{33} & = & 0 \\ -0.0375m_{13} & + & 0.1125m_{23} & - & 0.01832185m_{33} & = & 0 \end{array}$$

Taking the first two equations,

$$\begin{array}{rcccccl} 0.034687815m^*_{13} & + & 0.0125 & - & 0.0375m^*_{33} & = & 0 \\ 0.0125m^*_{13} & - & 0.265312185 & + & 0.1125m^*_{33} & = & 0 \end{array}$$

$$\begin{array}{rcccl} 0.034687815m^*_{13} & - & 0.0375m^*_{32} & = & -0.0125 \\ 0.0125m^*_{13} & + & 0.1125m^*_{32} & = & 0.265312185 \end{array}$$

$$\begin{pmatrix} 0.034687815 & -0.0375 \\ 0.0125 & 0.1125 \end{pmatrix} \begin{pmatrix} m^*_{13} \\ m^*_{33} \end{pmatrix} = \begin{pmatrix} -0.0125 \\ 0.265312185 \end{pmatrix}$$

$$m^*_{13} = \frac{\Delta_1}{\Delta}, \quad m^*_{23} = 1, \quad m^*_{33} = \frac{\Delta_2}{\Delta}$$

But

$$\Delta = \begin{vmatrix} 0.034687815 & -0.0375 \\ 0.0125 & 0.1125 \end{vmatrix}$$
$$= 0.003902379188 + 0.00046875$$
$$= 0.004371129188$$

$$\Delta_1 = \begin{vmatrix} -0.0125 & -0.0375 \\ 0.265312185 & 0.1125 \end{vmatrix}$$
$$= -0.00140625 - 0.009949206938$$
$$= 0.008542956938$$

$$\Delta_2 = \begin{vmatrix} 0.034687815 & -0.0125 \\ 0.0125 & 0.265312185 \end{vmatrix}$$
$$= 0.009203099991 + 0.00015625$$
$$= 0.009359349991$$

$$m_{13}^* = \frac{\Delta_1}{\Delta} = -\frac{0.008542956938}{0.004371129188} = 1.954405045$$

$$m_{23}^* = 1$$

$$m_{33}^* = \frac{\Delta_2}{\Delta} = -\frac{0.009359349991}{0.004371129188} = 2.141174417$$

To normalize this solution divide m_{13}^*, m_{23}^* and m_{33}^* by

$$\left((m_{13})^2 + (m_{23})^2 + (m_{33})^2\right)^{\frac{1}{2}} = \left((1.954405045)^2 + (1)^2 + (2.141174417)^2\right)^{\frac{1}{2}}$$
$$= 3.066647512$$

This yields the normalized solutions

$$m_{13} = \frac{m_{13}^*}{3.066647512} = \frac{1.954405045}{3.066647512} = 0.637309973$$

$$m_{23} = \frac{m_{23}^*}{3.066647512} = \frac{1}{3.066647512} = 0.326088993$$

$$m_{33} = \frac{m_{33}^*}{3.066647512} = \frac{2.141174417}{3.066647512} = 0.69821341$$

which is the third column of the M matrix. Thus, we have

$$\mathbf{M} = \begin{bmatrix} -0.774231999 & -0.073335219 & 0.637309973 \\ 0.15919019 & 0.927608179 & 0.326088993 \\ 0.612554727 & 0.366285422 & 0.69821341 \end{bmatrix}$$

The relationship between the $\mathbf{w}$ and $\mathbf{x}$ variables is

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} -0.774231999 & 0.15919019 & 0.612554727 \\ -0.073335219 & 0.927608179 & 0.366285422 \\ 0.637309973 & 0.326088993 & 0.69821341 \end{bmatrix} \begin{bmatrix} x_1 - 1.295057804 \\ x_2 - 0.632514451 \\ x_3 - 0.990606936 \end{bmatrix}$$

If the process engineer wished to explore the response surface in the vicinity of the optimal setting, he could determine appropriate points to take observations in the (w_1, w_2, w_3) space and then use the above relationship to convert these points into the (x_1, x_2, x_3) space so that the runs may be made. But, clearly, this procedure ultimately depends on the experience or intuition of the process engineer, which may be wrong.

3 Deficiency of the Foundational RSM Approach

Using the existing RSM procedure ultimately requires the conversion of points in the canonical variable space to points in the design variable space in other to explore the optimal region for near-optimal factors setting. How can one precisely select points in the canonical variable space if not by gambling or intuition based on the experience of the process engineers?

More so, the selected points in the canonical variable space are converted to points in the design variable space using equation (6) that has no unique solution. This procedure of the fundamental response methodology invariably implies that the process engineer can only but gamble with options of near-optimal factors settings should it become difficult to operate at the optimum operating condition owing to machine deterioration, drop in machine efficiency, voltage fluctuations, etc. This process of gambling is often time-consuming and does not guarantee precision in the selection of near-optimal factors settings. Need therefore arises for the advancement of the existing fundamental response methodology to curb this problem.

4 Conclusion

In conclusion, the exploration of optimal region for near-optimal settings via the foundational response surface methodology is achieved with canonical transformation procedure. This procedure is more or less an intuitive-dependent process; and, hence, gives little or no assurance of precision. This trend, therefore, prompts the need for alteration/advancement of the foundational response surface methodology in order to enable process engineer access near-optimal factors settings within the optimal region. We, therefore, recommend the procedure of [23] as one means to overcoming this pitfall.

Competing Interests

Apart from the author JEU, being the first and corresponding author, all other authors have no competing interests existing amongst them.

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