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A Cross-Province Comparison Of Okun's Coefficient For Canada

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Abstract

In this paper, we estimate Okun's coefficients for ten Canadian provinces using real GDP and unemployment rate data across the provinces. We obtain an average estimated Okun's coefficient of –1.58 under the Hodrick-Prescott detrending method and –1.32 under the quadratic detrending method. There is relative stability of the coefficients across the two detrending methods. Generally, the cost of unemployment in terms of the loss in real GDP is higher in the bigger and more industrialized provinces ranging from –2.14 for Ontario to less than -1 for the Maritime provinces.

Key Words: Okun's law, detrending, unit roots, provincial data.

JEL Classifications: C22, E32

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I. INTRODUCTION

The fundamental inverse relationship between the unemployment rate and the growth of real output has been known to economists for a very long time. Okun (1962) formalized this relationship into a statistical one, indicating the extent to which the unemployment rate is negatively related to real output growth. Using U.S. Gross National Product (GNP) data, Okun showed that for every percentage point that unemployment rate falls (increases) in excess of the natural unemployment rate, real output rises (falls) by approximately three per cent per year. Okun sought to use the relationship between the real GNP gap[1]and the unemployment rate gap to predict potential GNP given the past relationship between unemployment and GNP[2]. He was quick to point out that changes in unemployment rate *per se* cannot account for the reported magnitude of the change in real output realized when the unemployment rate changes. In other words, there are other intermediary factors linking the unemployment rate and real output in the above-specified relationship. For instance, a drop in the unemployment rate is expected to induce an increase in labor force participation, hours worked and productivity, thus resulting in an increase in real output.

Okun's law has been relatively consistent in explaining the relationship between the unemployment rate and real output and in forecasting full employment output for several decades now. Although the negative relationship between the unemployment rate gap and the growth of real output has remained quite stable, the absolute value of Okun's coefficient seems to be varying over time and from country to country. The stability of Okun's coefficient has been put to the test by several authors in recent times. Altig *et al.* (2002) have shown, using several sub-samples of U.S. GDP and unemployment rate, that Okun's coefficient is not constant over time. In fact, they found that GDP forecasts generated from the estimated model improve as more recent sub-samples are used in estimating the model. Also see Clark (1983) and Gordon and Coe (1989). Recent estimates of Okun's coefficient have ranged from as low as -0.6 by Prachowny (1993)[3] to around -2 (see Attfield and Silverstone, 1998). It is clear from an analysis of the current literature that estimates of Okun's coefficients tend to be sensitive to the model specification and the method of estimation. A review of the literature also shows that many of the Okun's coefficients estimated so far have been obtained from U.S. data (Weber, 1997; Prachowny, 1993; Freeman, 2000 and Altig *et al.* 2002).

A very simple regression specification of Okun's law can be represented as:

$$(Y\text{-}Y^*)_t = \alpha + \beta\ (U - U^*)_t + \varepsilon_{t,}$$

Where Y is the natural log of real output, U is the unemployment rate, and Y* and U* are the potential output and natural rate of unemployment respectively. Prachowny (1993) derived a log-linear relation from a Cobb-Douglas aggregate production function and showed that the true Okun's coefficient should be around -0.6 percent when other factors such as productivity, induced labor supply and weekly hours are included in the model. However, Prachowny's results have been widely criticized due to some lapses in the data modeling process. Attfield and Silverstone (1997), using the same data set as Prachowny, obtained an Okun's coefficient of –2.25 after performing co-integration tests and modeling the output-unemployment relation with a dynamic ordinary least squares (DOLS) model. The lower estimate by Prachowny can therefore be attributed to the fact that he estimated his model in first differences, accounting only for the fact that his variables are I (1), but ignoring the information that the data are cointegrated.

One other contentious issue associated with the estimation of Okun's coefficient has been the inclusion of additional regressors (as evidenced in Prachowny's work) to account for the fact that other intermediary factors are at play in explaining the relationship between output and the unemployment rate. However, some of these variables tend to correlate strongly with each other. Attfield and Silverstone (1997) noted that while there is evidence of a strong relationship between real output and the unemployment gap, the impact of the other variables included in the model by Prachowny was tenuous. The capacity utilization gap (CGAP) and the labour supply gap (LGAP) were found to be strongly collinear. Given the strong correlation that is likely to be observed among some of the variables that may be included in a model to estimate Okun's coefficient, it is obvious that an efficient way to proceed would be to estimate the model as a system using instrumental variables, or preferably full information maximum likelihood (FIML) which is efficient among all estimators since some of these additional variables are endogenous. However, it is also important to note that due to possible compounding of errors and measurement error associated with these additional variables, there is no guarantee that the estimated Okun's coefficients from the system approach would be any more accurate than estimates from the simple single equation approach.

Okun's law basically involves the deviation of real output and unemployment rates from their long run or full employment levels. A major step in the estimation of Okun's coefficient therefore is the determination of potential output and the natural rate of unemployment. Unfortunately however, these values are not observable and have to be estimated. Generally, there is no simple and straightforward way of doing this that guarantees the accuracy of the estimates. The accuracy of the method used to determine these two components of the law can, however, have an immense influence on the estimated coefficient. Several well known approaches exist for the derivation of these two variables. In isolating potential output, univariate techniques such as the removal of deterministic or quadratic trends, trend-cycle decomposition, as well as first differencing are all possible time series approaches that can be adopted[4]. Other relatively more sophisticated techniques include the Hodrick-Prescott (1997) approach and the Bandpass filter developed by Baxter and King (1995), which arguably is capable of removing both high (irregular) and low frequencies (stochastic or deterministic) from the series, leaving what is believed to approximate the output gap. The usual problem with some of the relatively simple methods is the fact that they fail to account adequately for the stochastic component of unemployment and real output in determining their potential components. As noted by Freeman (2000), the choice of the detrending methodology can account for the failure to reject non-stationarity in the variables being used in the regression, resulting in a misspecification of the regression model. Using the band-pass filter to determine the business cycle values, Freeman (2000) found that the estimate of Okun's coefficient has been quite stable around -1.2 to -2.0 for the U.S economy.

Two of the very few studies undertaken with Canadian data are the cross-country studies of Harris and Silverstone (2001) and Moosa (1997). The former examined seven OECD countries to test for asymmetry in Okun's law. Using real GDP data spanning 1978:1 – 1999:4[5] Harris and Silverstone estimated Okun's coefficient for Canada to be –0.386. Moosa (1997) provides a similar cross-country study and obtained a coefficient value of -0.49 for Canada[6]. The objective of this paper is to estimate Okun's coefficients for various Canadian provinces and to examine the regional differences that exist in the response of the real output gap to changes in the unemployment rate gap.

In section II of this paper we describe the data being used and the estimation approach. In section III we present the results of our estimates of Okun's coefficients using appropriate estimation techniques to deal with the usual serial correlation problems associated with the estimation of Okun's law. The discussion of the results and conclusions are given in section IV. The Appendix provides information about the unit root and diagnostic tests that we have conducted.

II. DATA DESCRIPTION AND METHODOLOGY

This study uses data on the unemployment rate and real GDP for ten Canadian provinces to estimate provincial Okun's coefficients for Canada. The source of the data is Statistic Canada's "Cansim" data site. All series are annual and span twenty-one years (1981-2001). The data for the Maritimes provinces (New Brunswick, Prince Edward Island and Nova Scotia) have been combined (by simple averaging) to construct a Maritimes data.

Okun used GNP in his original work. However, many authors have since produced estimates of Okun's coefficient using real GDP (Harris and Silverstone, 2001; Moosa, 1997) and other measures of output, including nonfarm business sector output (Prachowny, 1993) and gross state product (Freeman, 2000). Walsh however found that the estimated Okun's coefficients tend to be sensitive to the choice of real output data as well.

This paper uses the both Hodrick-Prescott (HP) filter (with λ=100)[7] and the quadratic trend to decompose the two series into their trend and cyclical components. The purpose of using these two detrending methods is to enable us to observe the sensitivity of the estimated Okun's coefficients to the choice of detrending method. One advantage of using the HP filter in particular is that the resulting detrended series is stationary (see Cogley and Nason, 1991).

Figure 1 shows the cyclical unemployment (CUN) rates and cyclical log real GDP (CLG) for the various provinces.

[Figure 1 here]

The inverse relationship between the log real GDP gap and the unemployment gap is very obvious from these plots. Although there are provincial differences in the business cycle plots, the major recessions (1981-82 and 1991) are clear in all of them. Some lag relationships can also be inferred here, although the direction in which the relationship needs to be specified is not clear. Moosa (1997) included a one period lag of the unemployment rate in his long-run regression to introduce some dynamics into the model. However, the lag structure must be chosen appropriately to ensure that the optimal number of lags is included. The general-to-specific method (which we adopt here) is an effective way of determining this appropriate lag structure that is expected to reasonably deal with the persistent serial correlation problems associated with estimation of Okun's law. This is an approach to testing that has thrown light on the fact that some signals of serial correlation coming from a model can simply be attributed to dynamic misspecification in the model and need not be corrected using traditional autocorrelation correction measures. This can often be the case where lags are wrongly omitted from models without ensuring that common factor (COMFAC) restrictions are satisfied.

III. EMPIRICAL RESULTS

Unit root tests performed on the gaps of the unemployment rate and log real GDP series (see the Appendix) show that all of the series are I(0) except for the real GDP gap series for the Maritimes and Alberta. For each province, we specify our Okun's law regression as:

$$LGGAP_t = a + b\ UNGAP + \sum_{j=1}^{k} h_j LGGAP_{t-j} + e_t \quad ,$$

where LGGAP is the log of real GDP gap series, UNGAP is unemployment gap series and β is the Okun's coefficient to be estimated. As discussed in the Appendix, the 'gap' variables used in equation (1) are stationary, so no differencing of the data is needed

Starting with a maximum of five lags (k=5) of the log real GDP gap (reflecting sample size constraints), we adopt a sequential testing approach to determine the appropriate lag structure of the model. We perform this test with increasing restrictiveness, which simply means that we begin with the most general hypothesis, and successively test downwards with increasing order of restrictiveness until we reject the null at one stage in the test. The accepted hypothesis is then the one immediately above the one that produced a significant result. The ordered hypotheses to be tested are as follows:

H^1_0: $\eta_5=0$
H^2_0: $\eta_5=\eta_4=0$
H^3_0: $\eta_5=\eta_4=\eta_3=0$
H^4_0: $\eta_5=\eta_4=\eta_3=\eta_2=0$
H^5_0: $\eta_5=\eta_4=\eta_3=\eta_2=\eta_1=0$

In selecting the significance level for evaluating the tests, we follow the suggestions made by Mizon (1977). Mizon's point is that, if α_i is the chosen significance level of the ith test in the sequence of tests to be undertaken, then the overall significance level of the nth hypothesis against the maintained hypothesis is actually $1-\prod^n (1-\alpha_i)$. We use the Wald test statistic (asymptotically distributed as chi-square) to evaluate the restrictions and we peg the probability of making a type one error at 20% for the most restrictive test.

Table 1

ESTIMATED RESULTS

	Hodrick Prescott (HP)		Quadratic Trend (QT)	
Provinces	**b**	**Lags (k)**	**b**	**Lags (k)**
Alberta	- 2.00 (0.64)	3	-0.06 (0.35)	0
B. Columbia	-1.72 (0.26)	5	-1.41 (0.21)	5
Manitoba	-2.14 (0.55)	0	-2.13 (0.01)	1
Quebec	-1.89 (0.13)	0	-2.00 (0.12)	0
Saskatchewan	-1.44 (1.01)	2	-1.49 (0.81)	0
Ontario	-2.13 (0.20)	0	-2.37 (0.21)	0
Newfoundland	-0.93 (0.39)	1	-0.88 (0.42)	1
Maritimes	-0.39 (0.43)	1	-0.18 (0.46)	2

Note: Values in parentheses are standard errors.

The estimated coefficients are fairly stable across the HP and QT approaches, with the exceptions of Alberta and the Maritimes, where the coefficient under the QT method is far smaller than that obtained under the HP approach. All of the estimated coefficients are significant at the10% level except for the Maritimes provinces, Alberta (under QT) and Saskatchewan (under HP). The average coefficient estimate under HP is around -1.58, and -1.32 under QT.

These estimated Okun's coefficients are simply a reflection of the variation in the cost of unemployment across the provinces. The results under the HP detrending method suggest that the cost of a unit increase in the unemployment rate in terms of a reduction in real GDP is highest in Manitoba, followed closely by Ontario and Alberta. On the other hand, the results under the QT detrending method show the cost of a unit increase in the unemployment rate to be highest in Ontario followed closely by Manitoba and Quebec. While the objective of this paper is not to explain why the cost of unemployment is higher in some provinces than others, it is obvious that the estimated Okun's coefficients are higher in the relatively more industrialized provinces that also have a fairly higher population and output. This could be pointing to an expected positive relationship between productivity and the cost of unemployment, where naturally the loss in real GDP expected when a trained person working in a high-knowledge based (manufacturing and business services) sector loses a job exceeds that of a person working in a less technologically developed sector. These results are close on average to the estimates of Harris and Silverstone (2001) and Moosa (1996) who obtained –0.386 as the Okun's coefficient for Canada.

IV. CONCLUSION

In this paper we have estimated Okun's coefficients for ten Canadian provinces using real GDP and unemployment rate data across the provinces. The purpose of this paper is to examine the regional variation of Okun's coefficient across Canada for the first time. Evaluating the results at the 10% level of significance, we obtain significant results for all of the provinces except for except for the Maritimes provinces, Alberta (under QT) and Saskatchewan (under HP). The average estimated Okun's coefficient is –1.58 under the HP detrending method and –1.32 under the QT method. There is however relative stability of the coefficients across the two detrending methods. Generally, the cost of unemployment in terms of the loss in real GDP is higher in the bigger and more industrialized provinces ranging from –2.14 for Ontario to less than -1.0 for the Maritimes provinces.

APPENDIX

We have used both the ADF test, in which the null hypothesis is non-stationarity, and the KPSS test of which the null hypothesis is trend stationarity. We specify our ADF regression for the real GDP gap as:

$$\Delta_1 LGGAP_t = a + r\ LGGAP_{t-1} + \sum_{j=1}^{8} \Delta_j LGGAP_{t-1} + e_t\ ,$$

where LGGAP = the log of real GDP gap series.

For the unemployment rate gap series, we specify the ADF regression as:

$$\Delta_1 UNGAP_t = a + r\ UNGAP_{t-1} + \sum_{j=1}^{8} \Delta_j UNGAP_{t-1} + e_t\ ,$$

where UNGAP = the unemployment gap series.

Table 2

UNIT ROOT TESTS ON UNEMPLOYMENT RATE GAP

PROVINCE	ADF	LAGS	KPSS	BANDWIDTH
Alberta	-3.158*	6	0.061	2
B.Columbia	-4.330*	0	0.069	1
Manitoba	-3.326*	5	0.071	1
Quebec	-3.463*	1	0.072	2
Saskatchewan	-4.016*	0	0.091	0
Ontario	-3.777*	1	0.071	2
Newfoundland	-4.964*	2	0.055	2
Maritimes	-3.220*	1	0.070	2

* Significant at the 10% level.

The results in Tables 2 and 3 are based on the HP detrended data. For the ADF test the null hypothesis is that the data are I (1) and the alternative hypothesis is that they are I(0). The converse situation applies to the KPSS test. So, in the case of the unemployment gap data, the results in Table 2 strongly imply that all of the series are stationary.

Table 3

UNIT ROOT TESTS ON REAL GDP GAP

PROVINCE	ADF	LAGS	KPSS	BANDWIDTH
Alberta	-4.667*	0	0.397*	16
B.Columbia	-3.033*	5	0.063	1
Manitoba	-4.265*	6	0.092	1
Quebec	-2.015*	8	0.079	2
Saskatchewan	-5.849*	1	0.500*	20
Ontario	-2.849*	5	0.088	2
Newfoundland	-2.372*	7	0.083	1
Maritimes	-2.831*	6	0.105	2

* Significant at the 10% level.

In the case of Alberta and Saskatchewan, both the ADF and KPSS tests on the real GDP gap reject their respective null hypotheses, making it difficult to make a conclusive decision on the unit root status of these series. However, the results are stronger for the ADF test where the null hypothesis for a unit root is rejected at the 1% level of significance for both Alberta and Saskatchewan while the KPSS cannot reject the null hypothesis of trend stationarity at the 5% level for Alberta and the 1% level for Saskatchewan. We thus model the data based on the ADF results. Unit root tests performed on the QT detrended data produced almost identical results to those shown above.

Table 4

BREUSCH–GODFREY SERIAL CORRELATION LM TEST AND THE R^2

PROVINCE	R-square	LM	R-square	LM
Alberta	0.59	0.12 (0.73)	0.01	0.25 (0.62)
B.Columbia	0.94	0.18 (0.67)	0.94	0.26 (0.61)
Manitoba	0.44	5.73 (0.02)	0.60	4.89 (0.03)
Quebec	0.92	0.77 (0.38)	0.93	1.12 (0.29)
Saskatchewan	0.47	0.04 (0.84)	0.15	0.20 (0.66)
Ontario	0.86	5.87 (0.02)	0.87	3.71 (0.05)
Newfoundland	0.51	4.68 (0.03)	0.74	3.26 (0.07)
Maritimes	0.55	1.24 (0.27)	0.72	0.01 (0.93)

Note: Values in parentheses are p-values.

We see that the adoption of Mizon's (1977) approach to determine the appropriate lag structure of the model has been relatively effective in dealing with the serial correlation problem. On the basis of the Breusch-Godfrey LM test, the null hypothesis of serial independence cannot be rejected (against the alternative of first-order autoregressive or moving average errors) at the 1% level for all provinces.

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FOOTNOTES

1. The GNP gap is the difference between GNP and potential GNP while the unemployment rate gap is the difference between the unemployment rate and natural unemployment rate.
2. Okun's approach is a very novel way of getting round the problem of predicting potential GNP.
3. Prachowny used two main U.S. data sets that have been used by several other authors. These are the Gordon data set (1947:1 – 1986:2) and the Adams and Coe data set (1965:1 – 1988:4). The main difference between the two data sets is that Gordon's output gap refers to GNP, while the Adams and Coe measure is for the nonfarm business sector (*i.e.* about 80% of GNP).
4. Some are actually as simple as just drawing a line linking the peaks of the series.
5. Moosa used annual 1995 GDP (measured in 1985 prices) data covering the period 1960 – 1995 to estimate Okun's coefficient values for the G7 countries. No unit root test results were reported.
6. Both Moosa (1997) and Harris and Silverstone (2001) obtained the estimated Okun's coefficients using the reversed regression (regressing unemployment on real output) thus this estimated Okun's coefficient is usually inverted to obtain the true coefficient.
7. This is the smoothing parameter recommended for annual data series by Hodrick and Prescott.

Figure 1(a)

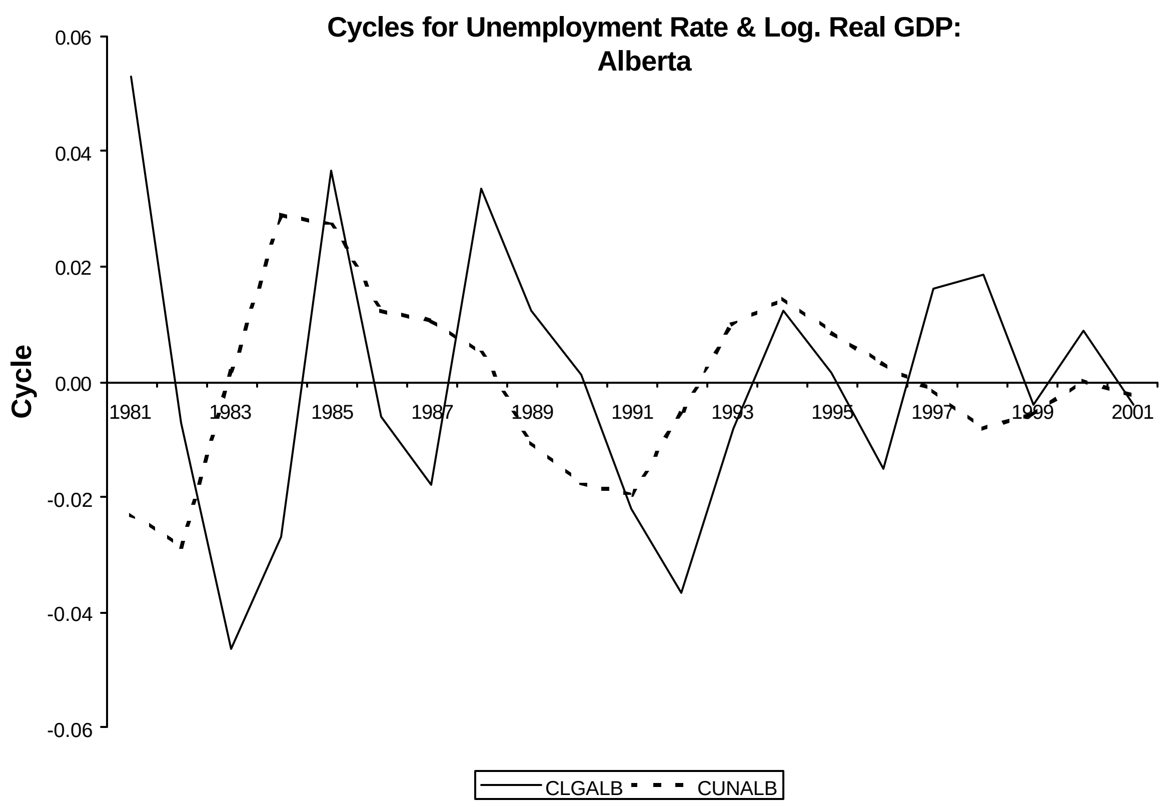
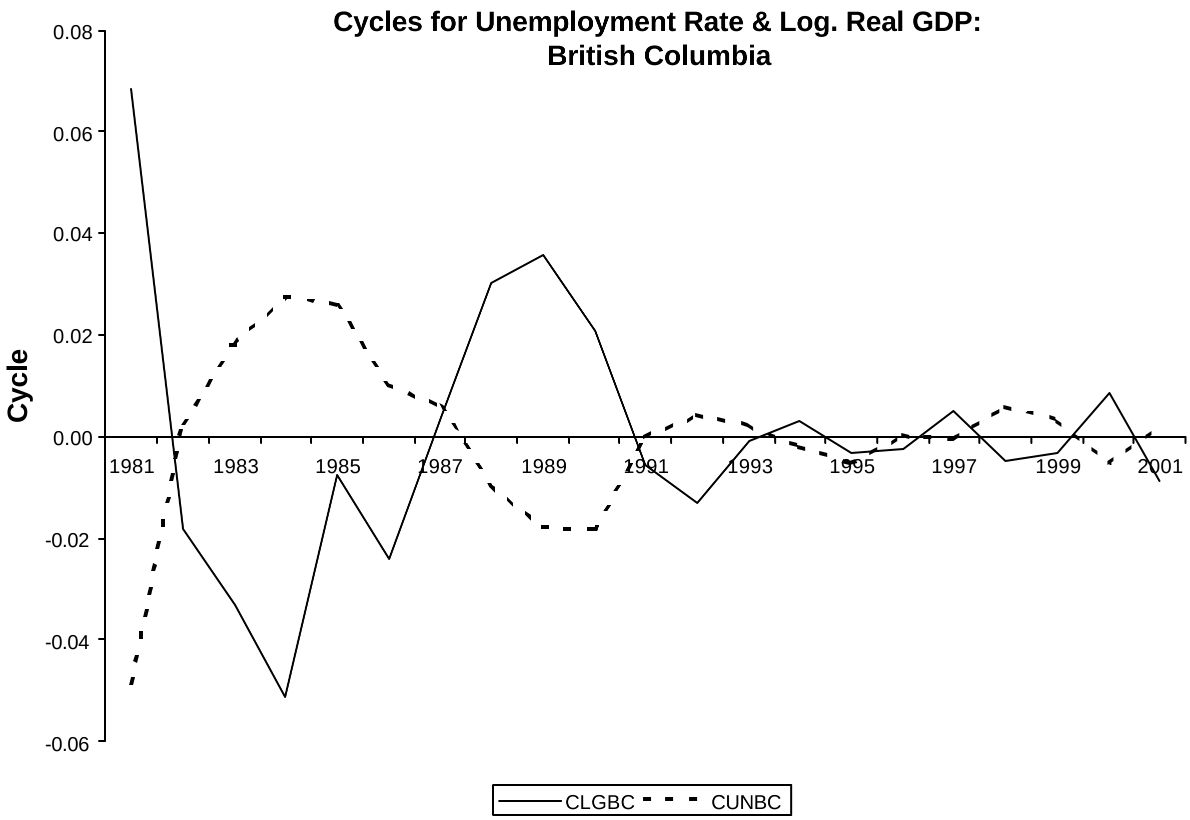

Cycles for Unemployment Rate & Log. Real GDP:
Alberta
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
Cycle
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
CLGALB
CUNALB

Figure 1(b)


Cycles for Unemployment Rate & Log. Real GDP:
British Columbia
Cycle
0.08
0.06
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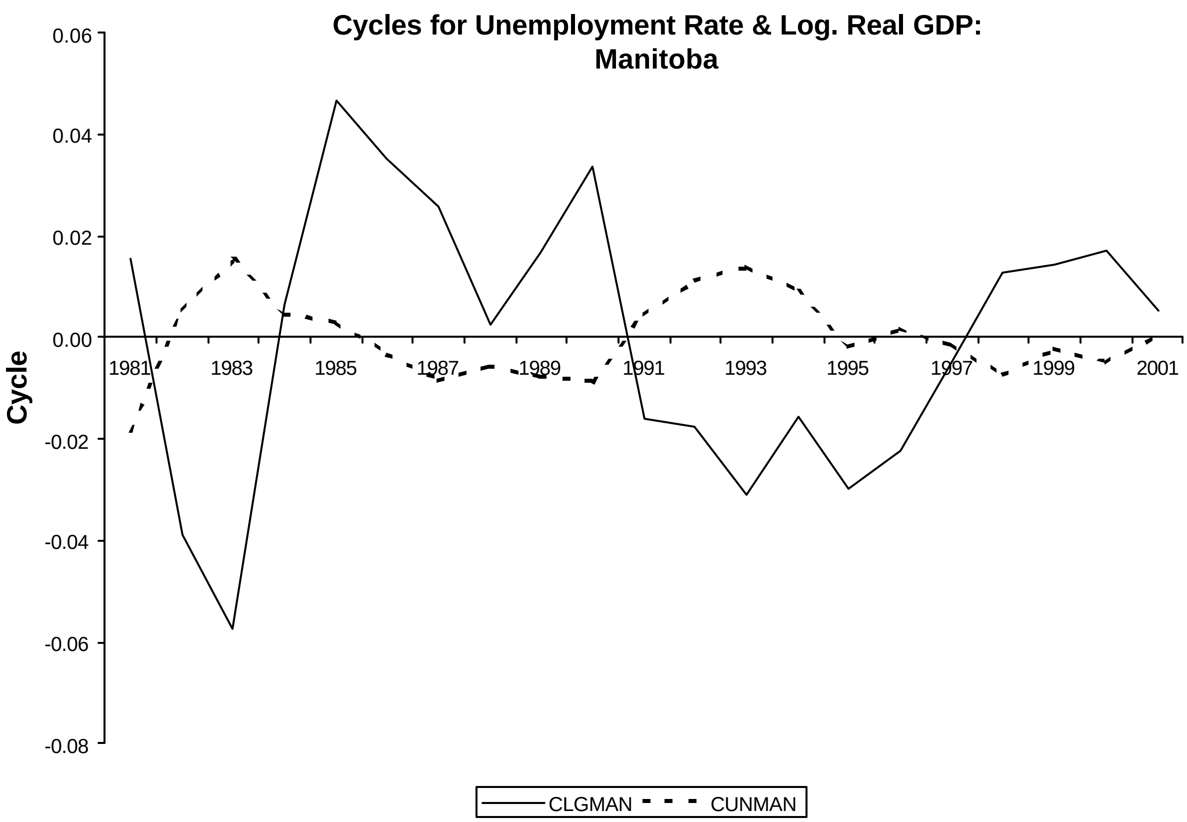
CUNBC

Figure 1(c)



Cycles for Unemployment Rate & Log. Real GDP:
Manitoba
Cycle
0.06
0.04
0.02
0.00
-0.02
-0.04
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-0.08
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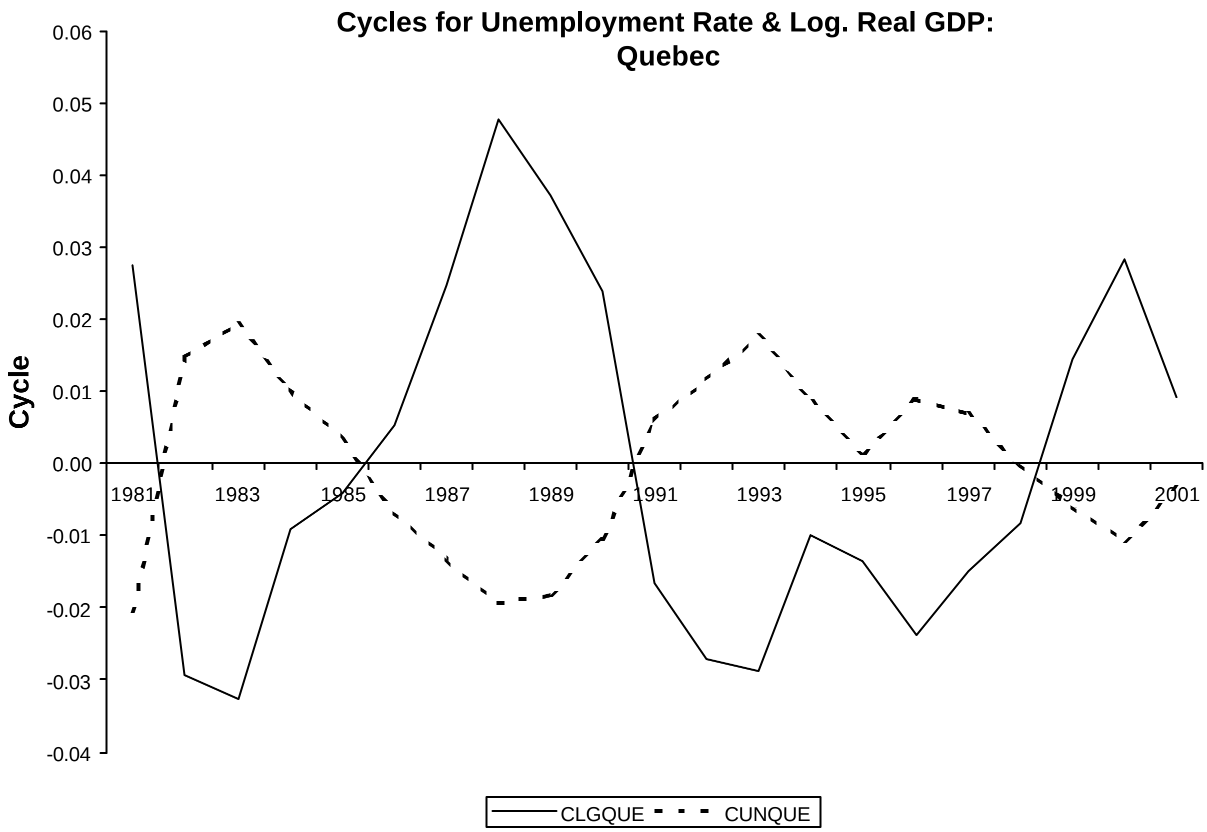
Figure 1(d)



Cycles for Unemployment Rate & Log. Real GDP:
Quebec
Cycle
0.06
0.05
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0.02
0.01
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-0.02
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2001
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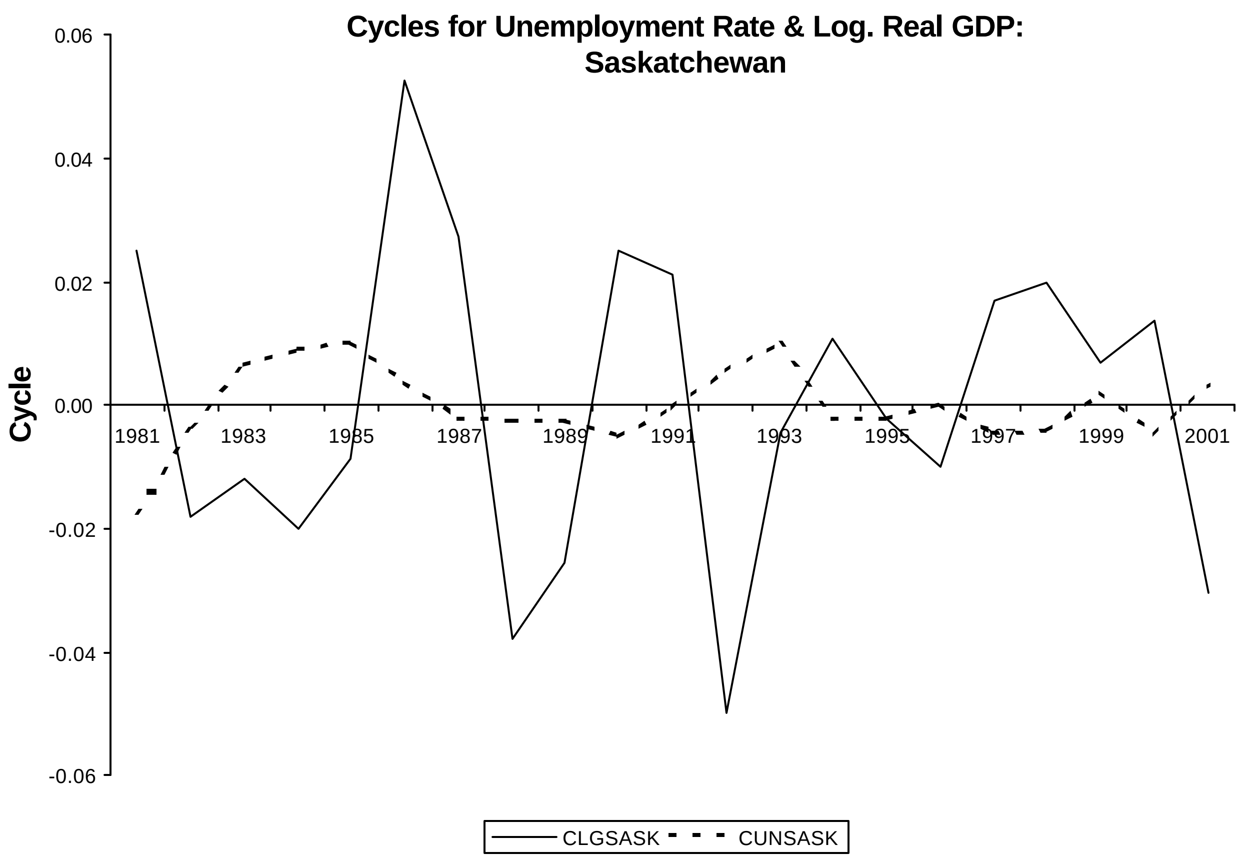
Figure 1(e)


Cycles for Unemployment Rate & Log. Real GDP:
Saskatchewan
Cycle
0.06
0.04
0.02
0.00
-0.02
-0.04
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CLGSASK
CUNSASK

Figure 1(f)

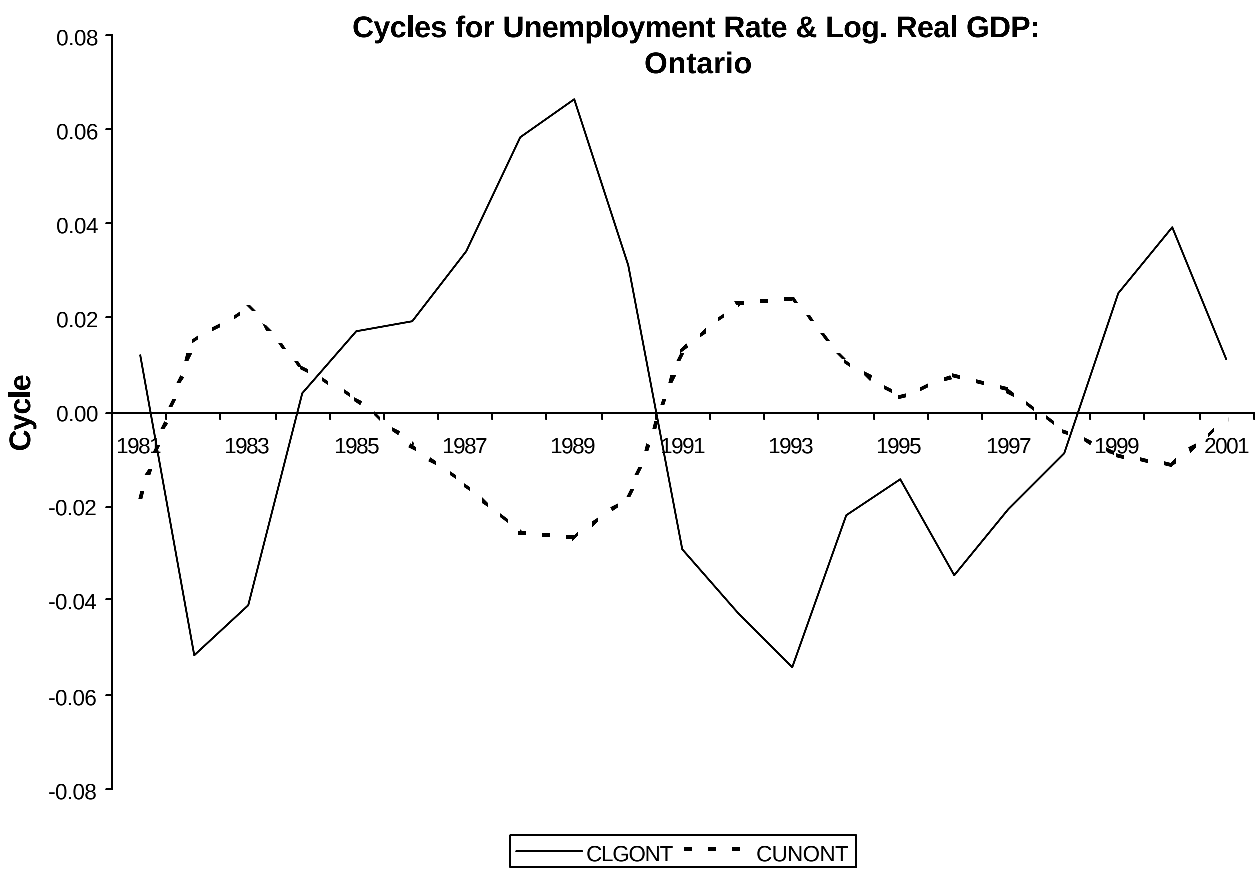
Cycles for Unemployment Rate & Log. Real GDP:
Ontario
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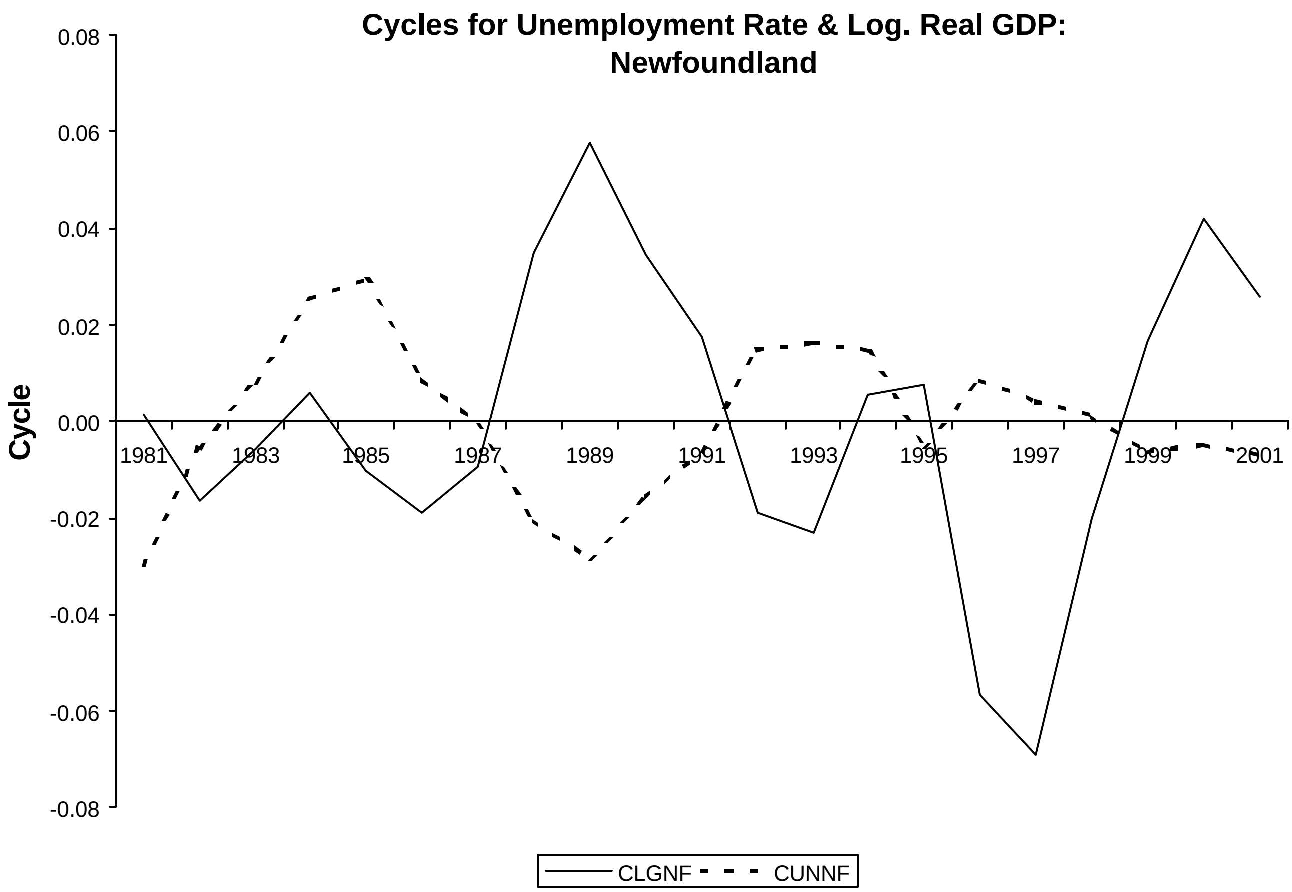
Figure 1(g)


Cycles for Unemployment Rate & Log. Real GDP:
Newfoundland
Cycle
0.08
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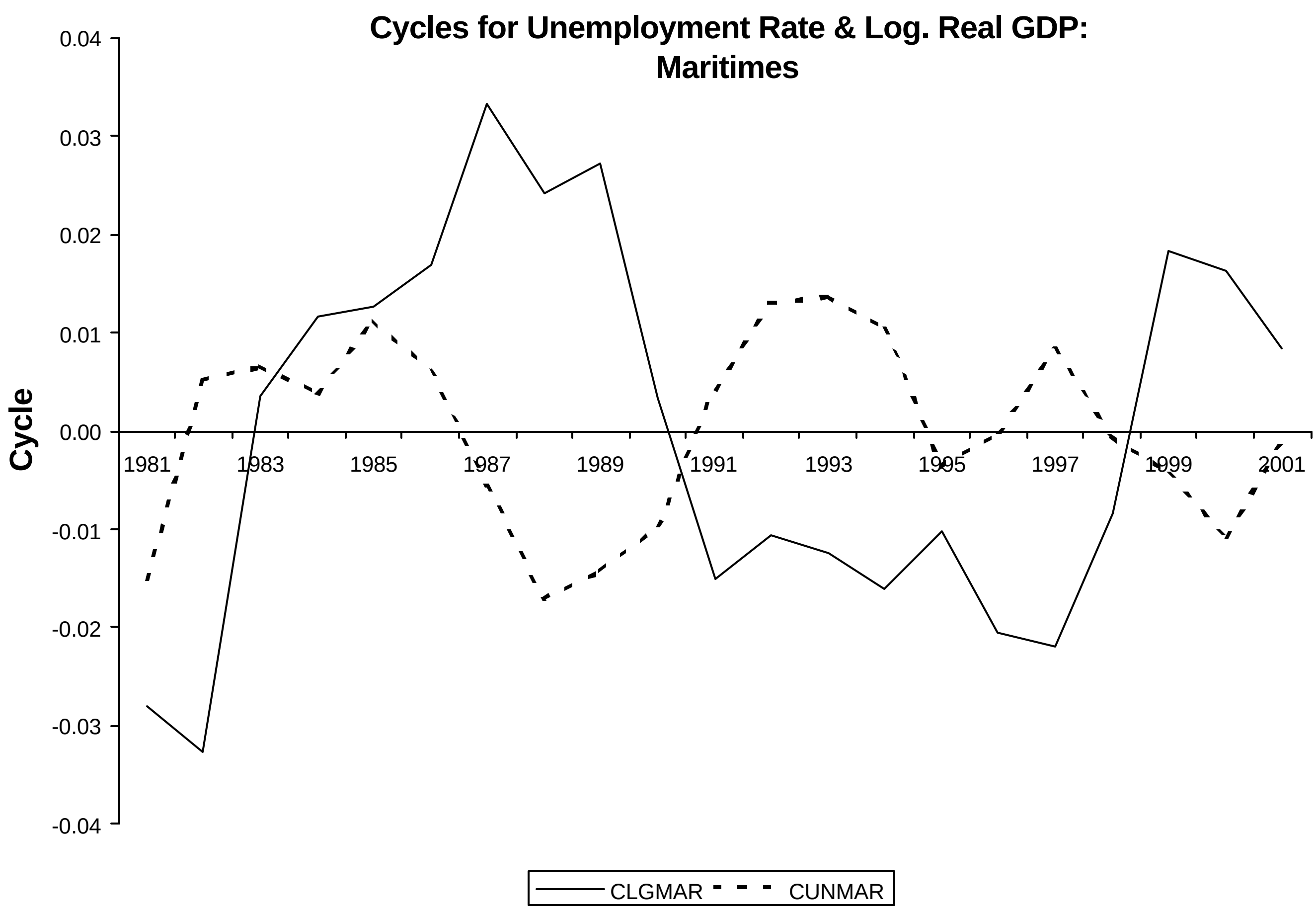
CUNNF

Figure 1(h)


Cycles for Unemployment Rate & Log. Real GDP: Maritimes
Cycle
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-0.04
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1997
1999
2001
CLGMAR
CUNMAR