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A DEA- COMPROMISE PROGRAMMING MODEL FOR COMPREHENSIVE RANKING

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Abstract This paper addresses comprehensive ranking systems determining an ordering of entities by aggregating quantitative data for multiple attributes. We propose a DEA-CP (Data Envelopment Analysis - Compromise Programming) model for the comprehensive ranking, including preference voting (ranked voting) to rank candidates in terms of aggregate vote by rank for each candidate. Although the DEA-CP model once employs the flexible DEA weighting system that can vary by entity, it finally aims at regressing to the common weights across the entities. Therefore, the model can totally rank the entities by specifying nothing arbitrary, and can avoid to use the diverse DEA weights.

Keywords: DEA, ranking, compromise programming, common weighting

1. Introduction

This paper deals with *comprehensive ranking* systems, e.g., project ranking systems, in which we evaluate and rank n entities by aggregating quantitative data for t attributes. Although such a multi-attribute ranking is broadly used in various fields, we cannot but employ a weighted sum of the attribute values as an evaluation criterion. That is, the problem is to obtain a total score $Z_j = \sum_{r=1}^{t} u_r y_{rj}$ for entity j, $j = 1, ..., n$, where $y_{rj} (\geq 0)$ is the value to attribute r of entity j, and $u_r (\geq 0)$ is the weight given to attribute r. Consider a special case of the comprehensive ranking, and suppose that y_{rj} is the number of r-th place votes that candidate j receives. Then, this is a *preference voting* system, i.e., ranked voting system, in which each voter selects, ranks and votes the top t candidates among n, and we determine an ordering of all the n candidates by obtaining Z_j. Note that the comprehensive ranking includes the preference voting. Since it is not easy to determine a priori clear-cut weights in comprehensive ranking systems, we must say that any comprehensive ranking in terms of the weighted sum is somewhat *arbitrary* however the weights are specified.

In order to exclude the arbitrariness, we can consider to apply DEA (Data Envelopment Analysis) (e.g., Charnes et al. [3], Cooper et al. [5]) to the comprehensive ranking. Cook and Kress [4] first propose a DEA-based preference voting model, in which the candidates are regarded as DMUs (Decision Making Units) in DEA, and every DMU j is considered to have t DEA outputs, i.e., y_{rj}, $r = 1, ..., t$, and one DEA input, i.e., "unit input" whose amount is unity (Hashimoto [7]). This idea is applicable to the DEA-based comprehensive ranking. In the DEA comprehensive ranking, we evaluate entities using a total score Z_j, a weighted sum of multi-attribute values, but the weights u_r, $r = 1, ..., t$, can vary by entity. That is, each entity is rated in terms of the weights most favorable to itself.

Although the DEA comprehensive ranking can do without employing a priori weighting,

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there exist two kinds of criticism originated from the above-mentioned property peculiar to DEA: (1) Multiple *top-ties*. DEA usually judges multiple DMUs as DEA efficient. Therefore, DEA ranking usually has multiple entities tied for the first place; (2) Too *diverse* weights. Each entity evaluated in DEA ranking has the freedom to choose its own optimal weights. The ranking using such different weights by entity is not commonplace, so that it would rather be unacceptable than the traditional ranking using common weights across the entities.

To resolve the problem of multiple top-ties, several methods are addressed. For the preference voting, Cook and Kress [4] propose to discriminate the top-tied candidates, i.e., DEA efficient DMUs, by maximizing a discrimination intensity function, implying the minimum gap between successively ranked weights, subject to the condition that they remain DEA efficient. However, we must then specify the discrimination intensity function, which brings another arbitrariness than that in determining weights. Hashimoto [8] resolves this problem in also the preference voting by applying the DEA exclusion model (Andersen and Petersen [1]), that can discriminate DEA efficient DMUs, instead of the standard DEA model. But it should be noted that in the more general comprehensive ranking with no a priori information about weight distribution, as these authors also state, the DEA exclusion model ranks the DEA efficient DMUs *outlying* in the data space too high. Green et al. [6] develop another model that constructs the DEA/ cross-efficiency matrix (Sexton et al. [9]) and ranks the candidates by its eigenvector. This is applicable to the comprehensive ranking. But regrettably, we must say that this method also depends on the diverse DEA weights like [4] and [8].

Based on the considerations above, this paper proposes a new comprehensive ranking model named DEA-CP (DEA- Compromise Programming). This is also a DEA-based model, but aims at regressing to the common weights across the entities. For the ranking, we would prefer the common weights across the entities to the different weights by entity. We employ the DEA for resolving the difficulty to a priori determine clear-cut weights, not for obtaining the different weights. In order to regress from the diverse sets of weights gotten by the DEA to a set of the common weights, we employ the compromise programming (Yu [12], Zeleny [13]). We can find no other ranking models seeking the common weights across the entities by combining the DEA and the compromise programming. The DEA-CP model can rank the entities by specifying nothing *arbitrary*, and can avoid to use the *diverse* DEA weights. Further, it can resolve the problem of multiple *top-ties*, and produces no problem of *outliers* ranked too high.

2. DEA-CP Ranking

The DEA-CP ranking model proposed in this paper consists of two stages: (1) DEA comprehensive ranking and (2) compromise programming. We begin with describing the model for the preference voting. Therefore, the first stage of the model is here the DEA preference voting.

2.1. DEA preference voting model

We formulate the model of DEA preference voting, the same of Cook and Kress [4], as follows:

$$\text{Maximize} \qquad Z_{j_0} = \sum_{r=1}^{t} u_r y_{rj_0} \tag{2.1a}$$

$$\text{subject to} \qquad \sum_{r=1}^{t} u_r y_{rj} \le 1, \; j = 1, ..., n, \tag{2.1b}$$

$$u_r - u_{r+1} \geq \epsilon, \ r = 1, ..., t-1, \tag{2.1c}$$
$$u_t \geq \epsilon, \tag{2.1d}$$

where ϵ is a positive non-Archimedean infinitesimal.

This is a DEA/ multiplier form model with each candidate as a DMU. We solve this LP (Linear Programming) problem with decision variables u_r, $r = 1, ..., t$, for each candidate j_0, $j_0 = 1, ..., n$, and try to rank the candidates according to the maximum $Z^*_{j_0}$. Constraints (2.1c) form the DEA/ assurance region (Thompson et al. [11]) expressing a strict ordering of weights. Since each DMU can diversely select its own optimal weights, model (2.1) usually judges multiple DMUs as DEA efficient, i.e., $Z^*_{j_0} = 1$. In this way, although the DEA preference voting can certainly do without arbitrary specifying weights, it usually ranks multiple candidates as the top based on the diverse weights differently defined by candidate.

2.2. Compromise programming

Model (2.1) gives a DEA score vector $\boldsymbol{Z}^* = (Z^*_1, ..., Z^*_n)$ based on the optimal weights varying by candidate j, and there are usually multiple top-tied candidates with $Z^*_j = 1$. These diverse optimal weights imply different interpretations of the voters' preference by the candidates, which causes the problem of multiple top-ties. Moreover, although the idea of flexibly defined weights is peculiar to DEA vs other comprehensive evaluation methods, there are surely some people who think it unrealistic or unfair. To resolve this, at the second stage of the DEA-CP model, we consider regressing to the common weights across the candidates. However, we can neither employ any predetermined weights because of arbitrariness, nor employ optimal weights for any special candidate because of fairness. Therefore, we here apply the notion of compromise programming in the multiobjective decision-making, that minimizes the sum of each candidate's deviation from the ideal point.

Let $\boldsymbol{u} = (u_1, ..., u_t)$ be the vector of common weights across the candidates to be obtained, and let

$$\boldsymbol{U} = \left\{ (u_1, ..., u_t) \left| \sum_{r=1}^{t} u_r y_{rj} \leq 1, \ j = 1, ..., n, \ u_r - u_{r+1} \geq \epsilon, \ r = 1, ..., t-1, \ u_t \geq \epsilon \right. \right\} \tag{2.2}$$

be the set of all feasible $\boldsymbol{u}$, the feasible weight set. Further, let $Z_j(\boldsymbol{u}) = \sum_{r=1}^{t} u_r y_{rj}$ be the total score function to candidate j, let $\boldsymbol{Z}(\boldsymbol{u}) = (Z_1(\boldsymbol{u}), ..., Z_n(\boldsymbol{u}))$ be the vector of $Z_j(\boldsymbol{u})$, and let $\boldsymbol{S} = \{\boldsymbol{Z}(\boldsymbol{u}) | \boldsymbol{u} \in \boldsymbol{U}\}$ be the total score set. For any candidate j, score Z^*_j model (2.1) gives is the maximum total score that he/she can obtain. Therefore, vector $\boldsymbol{Z}^*$ is the ideal point in the sense that every candidate is evaluated in terms of his/her own optimal weights. If $\boldsymbol{Z}^*$ is feasible, i.e., there exists $\boldsymbol{u}^0 \in \boldsymbol{U}$ such that $\boldsymbol{Z}(\boldsymbol{u}^0) = \boldsymbol{Z}^*$, the weight vector $\boldsymbol{u}^0$ would be acceptable. Since such a fortunate case is rare, we aim at reaching to the closest point $\boldsymbol{Z}(\boldsymbol{u})$ to the ideal point $\boldsymbol{Z}^*$. For this purpose, we need a distance function that measures the closeness between the points $\boldsymbol{Z}^*$ and $\boldsymbol{Z}(\boldsymbol{u})$. We introduce the well-known distance function $(\sum_{j=1}^{n} w_j |Z^*_j - Z_j(\boldsymbol{u})|^p)^{1/p}$, where w_j (≥ 0) is the weight given to candidate j, and p, $1 \leq p \leq \infty$, is a parameter (e.g., Chankong and Haimes [2]).

Since this distance function implies the total sum of each candidate's regret that he/she cannot achieve the ideal point $\boldsymbol{Z}^*$, we search the point $\boldsymbol{Z}(\boldsymbol{u})$ by minimizing the function. Noting that we should regard the weight to each candidate as equal because of fairness, and noting that $\boldsymbol{Z}^* \geq \boldsymbol{Z}(\boldsymbol{u})$, we can formulate the problem for seeking the closest point to $\boldsymbol{Z}^*$

as follows:

$$\text{Minimize} \quad D_p(\boldsymbol{Z}(\boldsymbol{u})) = \left[\sum_{j=1}^{n}\left(Z_j^* - \sum_{r=1}^{t} u_r y_{rj}\right)^p\right]^{1/p} \tag{2.3a}$$

$$\text{subject to} \quad \boldsymbol{u} \in \boldsymbol{U}. \tag{2.3b}$$

Obtaining the optimal solution $\boldsymbol{u}^*$ to model (2.3), we rank the candidates by $Z_j(\boldsymbol{u}^*)$.

In model (2.3), we can, in theory, specify the parameter p any value in the range $1 \leq p \leq \infty$. However, in computation, i.e., from a viewpoint of algorithms to obtain the optimal solution, and because of geometrical concepts of distance, we cannot but consider only the following three values: $p = 1$, the L_1 (absolute value) norm; $p = 2$, the L_2 (Euclidean) norm; and $p = \infty$, the L_∞ (Tchebycheff) norm.

(1) The L_1 norm. When $p = 1$, the model (2.3) is equivalent to the following LP problem:

$$\text{Maximize} \quad \sum_{j=1}^{n}\sum_{r=1}^{t} u_r y_{rj} \tag{2.4a}$$

$$\text{subject to} \quad \boldsymbol{u} \in \boldsymbol{U}. \tag{2.4b}$$

(2) The L_2 norm. When $p = 2$, let $\sum_{j=1}^{n}(Z_j^* - \sum_{r=1}^{t} u_r y_{rj})^2 = \widetilde{D}_2(\boldsymbol{Z}(\boldsymbol{u}))$. Then, $D_2(\boldsymbol{Z}(\boldsymbol{u})) = [\widetilde{D}_2(\boldsymbol{Z}(\boldsymbol{u}))]^{1/2}$ is a strictly increasing function of $\widetilde{D}_2(\boldsymbol{Z}(\boldsymbol{u}))$. Therefore, since minimizing $D_2(\boldsymbol{Z}(\boldsymbol{u}))$ is equivalent to minimizing $\widetilde{D}_2(\boldsymbol{Z}(\boldsymbol{u}))$, we can solve the following QP (Quadratic Programming) problem instead of the model (2.3) for $p = 2$:

$$\text{Minimize} \quad \widetilde{D}_2(\boldsymbol{Z}(\boldsymbol{u})) = \sum_{j=1}^{n}\left(Z_j^* - \sum_{r=1}^{t} u_r y_{rj}\right)^2 \tag{2.5a}$$

$$\text{subject to} \quad \boldsymbol{u} \in \boldsymbol{U}. \tag{2.5b}$$

(3) The L_∞ norm. As is well-known, we can transform the model (2.3) for $p = \infty$ to the following:

$$\text{Minimize} \quad \max_{j=1,\ldots,n}\left(Z_j^* - \sum_{r=1}^{t} u_r y_{rj}\right) \tag{2.6a}$$

$$\text{subject to} \quad \boldsymbol{u} \in \boldsymbol{U}. \tag{2.6b}$$

This can further be transformed as an LP formulation.

2.3. Selecting a norm

We must here choose one out of the L_1, L_2 and L_∞ norms. The effect of parameter p is to place more or less emphasis on the relative contribution of individual regret. The larger the p value is chosen, the more importance is given to the largest regret. Ultimately, the L_∞ treats only the maximal regret. On the contrary, the L_1 norm takes all regrets into account in direct proportion to their magnitudes. The L_2 norm measures the shortest geometric distance from the ideal point to the actual evaluation point. Notwithstanding, we have no rationale to choose the value of p.

We here select the L_2 norm from the reason of the unique optimal solution of total score vector $\boldsymbol{Z}(\boldsymbol{u})$. If model (2.3) has multiple optimal solutions of $\boldsymbol{Z}(\boldsymbol{u})$, the ranking in terms of $Z_j(\boldsymbol{u}^*)$ might be indefinite. Hence, we prove the following property.

Property. For $1 < p < \infty$, model (2.3) has a unique optimal solution of $\boldsymbol{Z}(\boldsymbol{u})$.
Proof. For $1 < p < \infty$, since $D_p(\boldsymbol{Z}(\boldsymbol{u}))$ is a strictly increasing function of

$$\widetilde{D}_p\left(\boldsymbol{Z}(\boldsymbol{u})\right) = \sum_{j=1}^{n}\left(Z_j^* - \sum_{r=1}^{t} u_r y_{rj}\right)^p,$$

minimizing $D_p(\boldsymbol{Z}(\boldsymbol{u}))$ is equivalent to minimizing $\widetilde{D}_p(\boldsymbol{Z}(\boldsymbol{u}))$. $\widetilde{D}_p(\boldsymbol{Z}(\boldsymbol{u}))$ is a strictly convex function of $\boldsymbol{Z}(\boldsymbol{u})$ because each $(Z_j^* - \sum_{r=1}^t u_r y_{rj})^p$, $j = 1, ..., n$, is strictly convex. Therefore, letting $\boldsymbol{Z}^1, \boldsymbol{Z}^2 \in \boldsymbol{S}$ and letting λ be a scalar $0 \le \lambda \le 1$,

$$\widetilde{D}_p\left((1-\lambda)\boldsymbol{Z}^1 + \lambda\boldsymbol{Z}^2\right) < (1-\lambda)\widetilde{D}_p(\boldsymbol{Z}^1) + \lambda\widetilde{D}_p(\boldsymbol{Z}^2). \tag{2.7}$$

Suppose that model (2.3) has two optimal solutions $\widehat{\boldsymbol{Z}}^1$ and $\widehat{\boldsymbol{Z}}^2$, then $\widetilde{D}_p(\widehat{\boldsymbol{Z}}^1) = \widetilde{D}_p(\widehat{\boldsymbol{Z}}^2) = \widetilde{D}_p^*$, where $\widetilde{D}_p^*$ is the minimum to $\widetilde{D}_p(\boldsymbol{Z}(\boldsymbol{u}))$, because minimizing $D_p(\boldsymbol{Z}(\boldsymbol{u}))$ is equivalent to minimizing $\widetilde{D}_p(\boldsymbol{Z}(\boldsymbol{u}))$. Since $\boldsymbol{S}$ is convex (see Appendix A), any convex combination of $\boldsymbol{Z}^1$ and $\boldsymbol{Z}^2$ also belongs to $\boldsymbol{S}$. But from (2.7),

$$\widetilde{D}_p\left((1-\lambda)\widehat{\boldsymbol{Z}}^1 + \lambda\widehat{\boldsymbol{Z}}^2\right) < (1-\lambda)\widetilde{D}_p(\widehat{\boldsymbol{Z}}^1) + \lambda\widetilde{D}_p(\widehat{\boldsymbol{Z}}^2) = \widetilde{D}_p^*.$$

This is in contradiction to that $\widetilde{D}_p^*$ is the minimum to $\widetilde{D}_p(\boldsymbol{Z}(\boldsymbol{u}))$. Therefore, model (2.3) has a unique optimal solution $\widehat{\boldsymbol{Z}}$. □

This property implies that if an optimal solution $\boldsymbol{u}^*$ to the model (2.3) for $1 < p < \infty$ is obtained, then the unique optimal vector $\widehat{\boldsymbol{Z}}(\boldsymbol{u}^*)$ is also obtained, so that we can rank the candidates in terms of the total scores $\widehat{Z}_j(\boldsymbol{u}^*)$. This is guaranteed by only the L_2 norm among the three norms.

For the more general comprehensive ranking, not the preference voting, removing the constraints $u_r - u_{r+1} \ge \epsilon, r = 1, ..., t-1$, from the feasible weight set $\boldsymbol{U}$, we can follow the same discussion. Therefore, we employ the L_2 norm, i.e., employ solving the QP problem (2.5) as the second stage of the DEA-CP ranking model.

3. Ranking Cases

To demonstrate the performance of the proposed DEA-CP ranking model, we show two ranking cases corresponding to the preference voting and the comprehensive ranking.

3.1. Preference voting case

Table 1 shows a preference voting case where each voter selects, ranks and votes the top five candidates and we try to determine an ordering of all the fourteen candidates ($t = 5$, $n = 14$). We get the Borda order based on the weights a priori determined as $u_r = t-r+1 = 6-r, r = 1, ..., 5$ (Stein et al. [10]). The DEA/AR (DEA/ Assurance Region) score is that model (2.1), having constraint (2.1c) as the assurance region, produces. Here, candidates A and B are tied for the top. The DEA/AR exclusion model is by Hashimoto [8], and is different from the DEA/AR model by that candidate j_0 being evaluated is excluded from constraint (2.1b) as $\sum_{r=1}^5 u_r y_{rj} \le 1, j = 1, ..., 14, j \ne j_0$.

The DEA-CP obtains the results resolving the multiple top-ties like the DEA/AR exclusion. But unlike the diverse weights of the DEA/AR exclusion, the DEA-CP has the common weights across the candidates. That is, the optimal solution to model (2.5) is

Table 1: A preference voting case [a]

Candidate	Rank 1	2	3	4	5	DEA-CP Score	Order [c]	DEA/AR exclusion [b] Score	Order [c]	DEA/AR [b] Score	Borda Order
A	27	38	15	7	4	1.000	1	1.226	2	1.000	1
B	38	15	16	14	7	0.991	2	1.407	1	1.000	2
C	21	25	30	8	5	0.977	3	0.978	3	0.978	3
D	2	8	10	23	19	0.679	4	0.681	4	0.681	4
E	1	1	11	22	24	0.646	5	0.648	5	0.648	5
F	2	4	4	7	13	0.329	6	0.330	6	0.330	6
G	1	0	4	7	13	0.274	7	0.275	7	0.275	7
H	0	1	1	0	0	0.022	11	0.025	11	0.025	8
I	0	0	1	1	1	0.033	8	0.033	8	0.033	9
J	0	0	0	1	2	0.033	9	0.033	9	0.033	10
K	0	0	0	1	2	0.033	9	0.033	9	0.033	10
L	0	0	0	1	0	0.011	12	0.011	12	0.011	12
M	0	0	0	0	1	0.011	13	0.011	13	0.011	13
N	0	0	0	0	1	0.011	13	0.011	13	0.011	13

[a] Data of the aggregate votes $y_{rj}, r = 1, ..., 5, j = 1, ..., 14$, are quoted from Stein et al. [10], and are also used in Hashimoto [8].
[b] See [8]. But the AR (Assurance Region) here is different from that of [8].
[c] Different orders with the same score are according to the infinitesimal term. See Appendix B as to the computation of the DEA-CP model.

$\boldsymbol{u}^* = (u_1^*, ..., u_5^*) = (0.0111, 0.0109, 0.0109, 0.0109, 0.0109)$. (Strictly, from constraints (2.2), the optimal value u_r^* is greater than u_{r+1}^*, $r = 2, ..., 4$, by an infinitesimal ϵ, respectively.) We should note that a total ordering different from the Borda one is obtained through the common weights across the candidates by specifying nothing arbitrary.

3.2. Comprehensive ranking example

Consider a general comprehensive ranking where we rank five entities by aggregating quantitative data for two attributes. The attribute data $y_{rj}, r = 1, 2, j = 1, ..., 5$, are given in Table 2, and Figure 1 displays the five entities on the attribute plane. Note that the comprehensive ranking model with no a priori information about weight distribution does not have constraint (2.1c) as the AR, i.e., it forms the DEA, not the DEA/AR, model.

Table 2 shows that the DEA exclusion can discriminate four DMUs (entities A-D) on the frontier in Figure 1. Here, entity D obtains the greatest DEA exclusion score 2.0 (= OD/OD′ in Figure 1) because the DEA exclusion computes the score in terms of its reference point

Table 2: A comprehensive ranking example

Entity	Attribute 1	Attribute 2	DEA-CP Score	DEA-CP Order	DEA exclusion Score	DEA exclusion Order	DEA Score
A	1	7	0.827	4	1.000	4	1.000
B	2	7	0.963	2	1.067	2	1.000
C	3	6	1.000	1	1.050	3	1.000
D	6	1	0.914	3	2.000	1	1.000
E	2	2	0.469	5	0.485	5	0.485

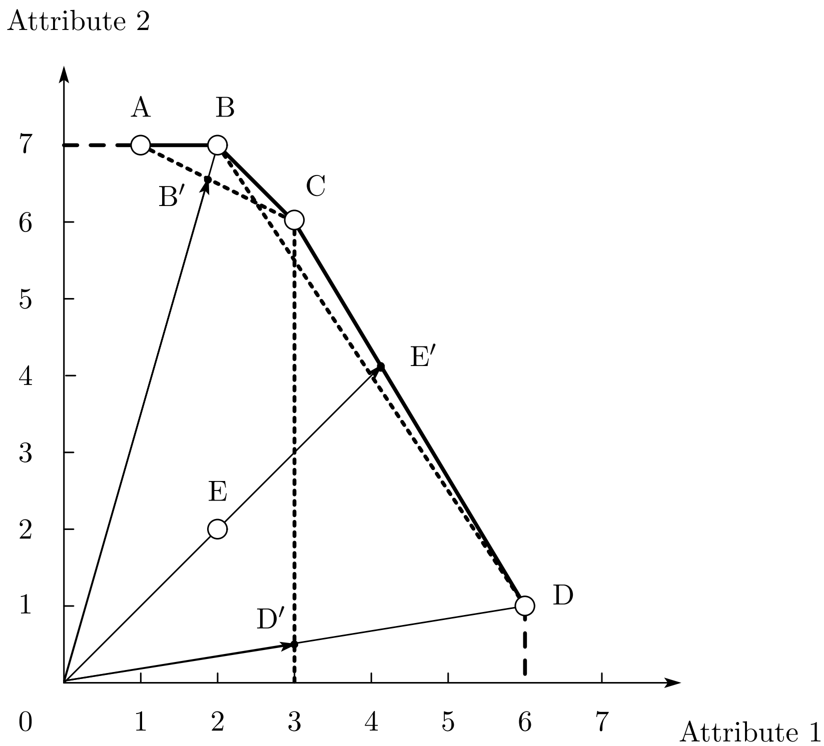

Figure 1: The five entities plotted on the attribute plane.

onto the shifted DEA exclusion frontier. This is the problem that the DEA exclusion ranks the DEA efficient DMUs outlying (entity D) too high.

On the other hand, the DEA-CP can provide more reasonable results with the optimal weights $\boldsymbol{u}^* = (0.136, 0.099)$ common across the entities. That is, the DEA-CP resolves the problem of outliers ranked too high as well.

4. Summary and Conclusions

This paper has proposed the DEA-CP model for comprehensive ranking, including preference voting, of entities by aggregating quantitative data for multiple attributes. This model avoids a priori fixed weights to the attributes in terms of the DEA, and aims at regressing to the common weights across the entities in terms of the CP. That is, the DEA-CP model can get a total ordering of the entities by specifying nothing arbitrary. Further, it can avoid to use the diverse DEA weights and produces no problems of outliers ranked too high. It is considered that the DEA-CP model can be a powerful method to comprehensively rank the entities with multiple attributes.

Appendix A

Proof of that the total score set $\boldsymbol{S}$ *is convex.* Let $\boldsymbol{Z}^1(\boldsymbol{u}^1), \boldsymbol{Z}^2(\boldsymbol{u}^2) \in \boldsymbol{S}$. From model (2.1), $\boldsymbol{Z}^1(\boldsymbol{u}^1) \leq (1, ..., 1)$ and $\boldsymbol{Z}^2(\boldsymbol{u}^2) \leq (1, ..., 1)$. Letting λ be a scalar $0 \leq \lambda \leq 1$, $(1-\lambda)\boldsymbol{u}^1 + \lambda \boldsymbol{u}^2 = \overline{\boldsymbol{u}} \in \boldsymbol{U}$ because the feasible region $\boldsymbol{U}$ of LP constraints is a convex set. Then,

$$\begin{aligned}
(1-\lambda)\boldsymbol{Z}^1(\boldsymbol{u}^1) + \lambda \boldsymbol{Z}^2(\boldsymbol{u}^2) &= (1-\lambda)\left(\sum_{r=1}^{t} u_r^1 y_{r1}, ..., \sum_{r=1}^{t} u_r^1 y_{rn}\right) + \lambda \left(\sum_{r=1}^{t} u_r^2 y_{r1}, ..., \sum_{r=1}^{t} u_r^2 y_{rn}\right) \\
&= (Z_1(\overline{\boldsymbol{u}}), ..., Z_n(\overline{\boldsymbol{u}})) \\
&\leq (1, ..., 1). \quad \square
\end{aligned}$$

Appendix B

We treat the positive non-Archimedean infinitesimal in solving model (2.5) as follows:

Setting the infinitesimal ($\epsilon > 0$) as $\epsilon = 0$ and computing model (2.5), we obtain the optimal solution $\boldsymbol{u}^*$ and the DEA-CP scores $\widehat{Z}_j(\boldsymbol{u}^*)$, $j = 1, ..., n$. This implies that we suppose boundaries of the constraints related to ϵ (we here call *ϵ-boundaries*) to be included in the feasible region to model (2.5).

(1) If $\boldsymbol{u}^*$ is not on any ϵ-boundary, $\boldsymbol{u}^*$ is also the optimal solution to model (2.5) in the case of $\epsilon > 0$. Therefore, we can rank the candidates by $\widehat{Z}_j(\boldsymbol{u}^*)$.

(2) If $\boldsymbol{u}^*$ is on the ϵ-boundary, $\boldsymbol{u}^*$ is not a feasible solution to (2.5). However, within the range of Archimedean numbers, $\boldsymbol{u}^*$ is the optimal solution to (2.5) and the ordering in terms of $\widehat{Z}_j(\boldsymbol{u}^*)$ is valid.

(3) In (2), and in the case of some ties in terms of the DEA-CP scores $\widehat{Z}_j(\boldsymbol{u}^*)$, we examine whether to discriminate the $\widehat{Z}_j(\boldsymbol{u}^*)$ scores using the non-Archimedean infinitesimal term with $\epsilon > 0$.

In the case of Table 1, letting $\epsilon = 0$, we get $u_r^* = 0.0109$, $r = 2, ..., 5$. This corresponds to (2) or (3) above. In terms of (2), we can rank ten candidate groups, i.e., candidates A-H, (I, J, K) and (L, M, N) using the DEA-CP scores $\widehat{Z}_j(\boldsymbol{u}^*)$. To discriminate candidates I, J and K, for example, we let $u_4^* = u_5^* + \epsilon$, $u_3^* = u_5^* + 2\epsilon$ and $u_2^* = u_5^* + 3\epsilon$ ($\epsilon > 0$), and can find $\widehat{Z}_{\mathrm{I}}(\boldsymbol{u}^*) > \widehat{Z}_{\mathrm{J}}(\boldsymbol{u}^*) = \widehat{Z}_{\mathrm{K}}(\boldsymbol{u}^*)$.

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