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A DEA MODEL FOR TWO-STAGE PARALLEL-SERIES PRODUCTION PROCESSES

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Abstract. Data envelopment analysis (DEA) has been widely used to measure the performance of the operational units that convert multiple inputs into multiple outputs. In many real world scenarios, there are systems that have a two-stage network process with shared inputs used in both stages of productions. In this paper, the problem of evaluating the efficiency of a set of specialized and interdependent components that make up a large DMU is considered. In these processes the first stage consists of two parallel components which are connected serially with the process in the second stage. The paper develops a DEA approach for measuring efficiency of decision processes which can be divided into two stages. This application of parallel-series production process involves shared resources and the paper determines an optimal split of shared resources among two components.

Keywords. Data envelopment analysis, efficiency, production, two-stage.

Mathematics Subject Classification. 90B030.

1. Introduction

In the past few years, the data envelopment analysis (DEA) approach has become increasingly popular in the practice and research of efficiency analysis. Many DEA applications and research have led to new developments in concepts and

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methodologies related to the DEA-efficiency analysis. Traditional DEA models consider DMUs with multiple inputs and multiple outputs (see [3]). However, as discussed in many DEA studies, DMUs can have a two-stage structure where the first stage uses inputs to produce outputs that then become the inputs to the second stage. The second stage thus consumes these first stage outputs to produce its outputs.

The issue of network DEA has been extensively studied. Recently, important steps toward the development of two-stage DEA have been taken by Färe and Grosskopf [10], Seiford and Zhu [16], Lothgern and Tambour [15], Färe and Grosskopf [11], Cook *et al.* [8], Hoopes *et al.* [13], Zhu [18], Chen and Zhu [5], Kao [14], Chen *et al.* [4, 6], Tone and Tsutsi [17], Chen *et al.* [7], Cook *et al.* [9] and Amirteimoori [1]. Seiford and Zhu [16] introduced the two-stage processes and applied the standard DEA model to each stage. However, as noted in [5, 18], such an approach may conclude that two inefficient stages lead to an overall efficient DMU with the inputs of the first stage and outputs of second stage. Golany *et al.* [12] developed an efficiency measurement framework for systems composed of two subsystems arranged in series. Their approach expands the technology sets of each subsystem by allowing each to acquire resources from the other in exchange for delivery of the appropriate products, and to form composites from both subsystems.

Kao (2008) developed a parallel DEA model to measure the efficiency of the system which is composed of parallel production units. Chen *et al.* [4] proposed an additive efficiency decomposition approach wherein the overall efficiency is expressed as a weighted sum of the efficiencies of the individual stage. Chen *et al.* [6] examined relations and equivalent between the existing DEA approaches for measuring the performance of two-stage processes. Tone and Tsutsui (2009) proposed a network DEA model based on the weighted slack-based measure approach which accounts for the importance of each component. Chen *et al.* [7] developed a set of DEA models for measuring the performance of two-stage network processes with non splittable shared inputs. Additive efficiency decomposition for the two-stage network process was presented. Cook *et al.* [9] examined the more general problem of an open multistage process. In their paper, some outputs from a given stage may leave the system while others become inputs to the next stage. As well, new inputs can enter at any stage. They represented the overall efficiency of such a structure as an additive weighted average of the efficiencies of the individual components or stages that make up that structure. Chen *et al.* [7] developed a set of DEA models for measuring the performance of two-stage network processes with non splittable shared inputs.

This paper develops a parallel-series DEA model for measuring efficiency of decision processes which can be divided into two stages. In these processes, the first stage consists of two parallel production lines connected serially with the process in an assembly line. The two production lines use their own inputs and a shared input to generate two types of outputs which become inputs to the assembly line. The assembly line is then fed by a mixture of these two outputs and its own inputs

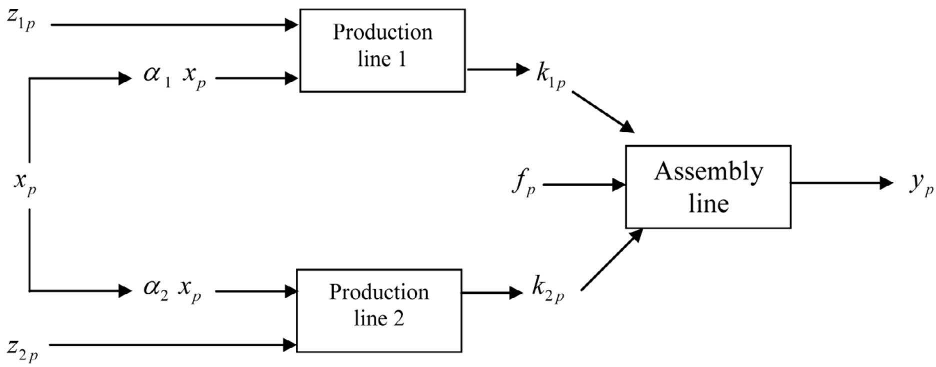

FIGURE 1. The production system.

to produce the final products. The structure in the current paper is different from those two-stage processes in the papers mentioned above, and the previous models are not suitable here. Additive efficiency decomposition for production lines and the assembly line is presented. The model proposed in this paper, determines a best resource split to optimize the additive efficiency of the whole system. We show herein that the presence of shared resources leads to a linear model rather than a nonlinear model.

The structure of the paper is organized as follows. The next section presents the proposed two-stage model. Section 3 applies the new approach to the 17 pre-fabricated cabin plants. Conclusions follow in Section 4.

2. A two stage model

Consider a two-stage production process shown in Figure 1. Suppose we have n DMUs and each $DMU_p : \ p = 1, \ldots, n$ consists of two parallel production lines and an assembly line. The first and second production lines use their own inputs $z_p^{(1)} = (z_{1p}^{(1)}, z_{2p}^{(1)}, \ldots, z_{Dp}^{(1)})^T$ and $z_p^{(2)} = (z_{1p}^{(2)}, \ldots, z_{2p}^{(2)}, \ldots, z_{Hp}^{(2)})^T$, respectively. We also assume that DMU_p has m inputs $x_p = (x_{1p}, x_{2p}, x_{mp})^T$ that should be shared among the two production lines. The observed shared input to production lines 1 and 2 are respectively, $x_p^{(1)} = (x_{1p}^{(1)}, x_{2p}^{(1)}, \ldots, x_{mp}^{(1)})^T$ and $x_p^{(2)} = (x_{1p}^{(2)}, x_{2p}^{(2)}, \ldots, x_{mp}^{(2)})^T$ and obviously we have $x_p = x_p^{(1)} + x_p^{(2)}$. In optimality, some portion $0 \leq \alpha_i^{(1)} < 1$ of the shared inputs x_{ip} is allocated to the first line, and the remainder $0 \leq \alpha_i^{(2)} < 1$ is allocated to the second line with $\alpha_i^{(1)} + \alpha_i^{(2)} = 1$. So, the first component uses inputs $\alpha_i^{(1)} x_{ip}$: $i = 1, 2, \ldots, m$ and $z_{dp}^{(1)}$: $d = 1, 2, \ldots, D$ to produce $k_{lp}^{(1)}$: $l = 1, 2, \ldots, L$ and the second line uses $\alpha_i^{(2)} x_{ip}$: $i = 1, 2, \ldots, m$ and $z_{hp}^{(2)}$: $h = 1, 2, \ldots, H$ to produce $k_{bp}^{(2)}$: $b = 1, 2, \ldots, B$. The assembly line is fed by a mixture of inputs $k_p^{(1)}$, $k_p^{(2)}$ and an external input $f_p = (f_{1p}, f_{2p}, \ldots, f_{Qp})^T$

and the final product is $y_p = (y_{1p}, y_{2p}, \ldots, y_{sp})^T$. Let $\alpha^{(d)} = (\alpha_1^{(d)}, \alpha_2^{(d)}, \ldots, \alpha_m^{(d)})^T$: $i = 1,\ 2$.

Our object is to determine the relative efficiencies of the two production lines and assembly line along with an overall efficiency of the whole system. An Algebraic representation of the production possibility set of technology under consideration for the production line $d:\ d = 1,2$ in constant returns to scale environment is defined as:

$$T_d = \left\{ \left(\alpha^{(d)}\mathrm{diag}(x), z^{(d)}, k^{(d)}\right):\ \left(\alpha^{(d)}\mathrm{diag}(x)\right)^T \geq \sum_{j=1}^{n}\lambda_j x_j^{(d)},\ z^{(d)} \geq \sum_{j=1}^{n}\lambda_j z_j^{(d)}, \right.$$
$$\left. k^{(d)} \leq \sum_{j=1}^{n}\lambda_j k_j^{(d)},\ 0 \leq \alpha^{(d)} \leq e^T,\ \lambda_j \geq 0,\ j = 1,2,\ldots,n \right\}$$

in which $\mathrm{diag}(x)$ is a diagonal matrix with diagonal elements $x_1,\ x_2,\ \ldots, x_m$ and $e^T = (1,1,\ldots,1)$.

The symbol $\mathrm{diag}(x)$ is used to define the products $\alpha_i^{(d)}x_i$ in matrix form as follows:

$$\alpha^{(d)}\mathrm{diag}(x) = \left(\alpha_1^{(d)}, \alpha_2^{(d)}, \ldots, \alpha_m^{(d)}\right)\begin{bmatrix} x_1 & 0 & \ldots & 0 \\ 0 & x_2 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & x_m \end{bmatrix} = \left(\alpha_1^{(d)}x_1, \alpha_2^{(d)}x_2, \ldots, \alpha_m^{(d)}x_m\right).$$

Also, the superscript t in $(\alpha^{(d)}\mathrm{diag}(x))^t$ is used to transpose of the matrix $\alpha^{(d)}\mathrm{diag}(x)$. Now, let T_A be the production possibility set of technology under consideration for the assembly line. T_A is defined as follows:

$$T_A = \left\{ \left(k^{(1)}, k^{(2)}, f, y\right):\ k^{(1)} \geq \sum_{j=1}^{n}\lambda_j k_j^{(1)},\ k^{(2)} \geq \sum_{j=1}^{n}\lambda_j k_j^{(2)}, \right.$$
$$\left. f \geq \sum_{j=1}^{n}\lambda_j f_j,\ y \leq \sum_{j=1}^{n}\lambda_j y_j,\ \lambda_j \geq 0,\ j = 1,2,\ldots,n \right\}.$$

In applying the model described herein, attention is paid to additive model. In the assessment of production lines 1 and 2, the output measures $k^{(1)}$ and $k^{(2)}$ should be increased. On the other hand, these measures are considered as inputs to the assembly line and they should be decreased. If we treat the system's operation as a black-box, ignoring the intermediate measures may yield to an efficient DMU with inefficient production lines and/or assembly line. In model we proposed, the intermediate measures $k^{(1)}$ and $k^{(2)}$ are considered to be free variables, and they will be increased or decreased to make the whole system as efficient. To provide for a realistic picture of DMU's performance, some restrictions are imposed on the variables $\alpha^{(1)}$ and $\alpha^{(2)}$. Ratio constraints of the form $l_i^{(\alpha)} \leq \frac{\alpha_i^{(1)}}{\alpha_i^{(2)}} \leq u_i^{(\alpha)}$ on the portion variables $\alpha_i^{(1)}$ and $\alpha_i^{(2)}$ are imposed. These constraints reflect the relative importance of the shared resources that are split between two production lines.

Consider the assessment of $DMU_p: \ p \in \{1, 2, \ldots, n\}$ in additive form. Taking in to consideration the forms of $T_1, \ T_2$ and T_A and considering $\alpha_i^{(1)} + \alpha_i^{(2)} = 1 : i = 1, 2, \ldots, m$, this is obtained as the optimal value of the following model:

$$
\begin{aligned}
&\text{Max } E_p = e^T s^{(x^{(1)})} + e^T s^{(x^{(2)})} + e^T s^{(k_1)} + e^T s^{(z_1)} + e^T s^{(z_2)} + e^T s^{(k_2)} \\
&\qquad\qquad + e^T s^{(f)} + e^T s^{(y)} \\
&s.t. \\
&\textit{Line } 1: && \sum_{j=1}^{n} \lambda_j x_j^{(1)} + s^{\left(x^{(1)}\right)} = \left(\alpha^{(1)} \text{diag}\left(x_p^{(1)} + x_p^{(2)}\right)\right)^t, \\
& && \sum_{j=1}^{n} \lambda_j z_j^{(1)} + s^{(z^{(1)})} = z_p^{(1)}, \\
& && \sum_{j=1}^{n} \lambda_j k_j^{(1)} + s^{(k^{(1)})} = k_p^{(1)}. \\
&\textit{Line } 2: && \sum_{j=1}^{n} \lambda_j x_j^{(2)} + s^{\left(x^{(2)}\right)} = \left(\alpha^{(2)} \text{diag}\left(x_p^{(1)} + x_p^{(2)}\right)\right)^t, \\
& && \sum_{j=1}^{n} \lambda_j z_j^{(2)} + s^{(z^{(2)})} = z_p^{(2)}, \\
& && \sum_{j=1}^{n} \lambda_j k_j^{(2)} + s^{(k^{(2)})} = k_p^{(2)}. \\
&\textit{Assembly Line}: && \\
& && \sum_{j=1}^{n} \lambda_j k_j^{(1)} + s^{(k^{(1)})} = k_p^{(1)}, \\
& && \sum_{j=1}^{n} \lambda_j k_j^{(2)} + s^{(k^{(2)})} = k_p^{(2)}, \\
& && \sum_{j=1}^{n} \lambda_j f_j + s^{(f)} = f_p, \\
& && \sum_{j=1}^{n} \lambda_j y_j - s^{(y)} = y_p. \\
&\textit{General constraint}: && \\
& && l_i^{(\alpha)} \leq \frac{\alpha_i^{(1)}}{\alpha_i^{(2)}} \leq u_i^{(\alpha)}, \\
& && \alpha^{(1)} + \alpha^{(2)} = e, \\
& && \alpha^{(1)}, \ \alpha^{(2)}, \ \lambda_j \geq 0, \quad \textit{for all } j, \\
& && s^{\left(x^{(1)}\right)}, \ s^{\left(z^{(1)}\right)}, \ s^{\left(x^{(2)}\right)}, \ s^{\left(z^{(2)}\right)}, \ s^{(f)}, \ s^{(y)} \geq 0.
\end{aligned}
\tag{1.1}
$$

Theorem 2.1. *The LP model (1) is feasible.*

Proof. Clearly,

$$\lambda_p = 1,\ \lambda_j = 0,\ j = 1, 2, \ldots, n,\ j \neq p,$$
$$s^{\left(x^{(1)}\right)} = s^{\left(z^{(1)}\right)} = s^{\left(k^{(1)}\right)} = s^{\left(x^{(2)}\right)} = s^{\left(z^{(2)}\right)} = s^{\left(k^{(2)}\right)} = s^{(f)} = s^{(y)} = 0,$$
$$\alpha_i^{(1)} = \frac{x_{ip}^{(1)}}{x_{ip}^{(1)} + x_{ip}^{(2)}}, \quad i = 1, 2, \ldots, m,$$
$$\alpha_i^{(2)} = \frac{x_{ip}^{(2)}}{x_{ip}^{(1)} + x_{ip}^{(2)}}, \quad i = 1, 2, \ldots, m,$$

is a feasible solution to this problem. □

Definition 2.2. DMU_p is said to be additive efficient if and only if $E_p = 0$.

Definition 2.3. DMU_p is said to be additive efficient in stage 1 if and only if $E_p^{(1)} = e^T s^{(x^{(1)})} + e^T s^{(k_1)} + e^T s^{(z_1)} = 0$.

Definition 2.4. DMU_p is said to be additive efficient in stage 2 if and only if $E_p^{(2)} = e^T s^{(x^{(2)})} + e^T s^{(z_2)} + e^T s^{(k_2)} = 0$.

Definition 2.5. DMU_p is said to be additive efficient in assembly line if and only if $E_p^{(A)} = e^T s^{(k_1)} + e^T s^{(k_2)} + e^T s^{(f)} + e^T s^{(y)} = 0$.

For an inefficient production line $d: \ d = 1, 2$, we have $E_p^{(d)} > 0$. In this case, we must have

$$x_p^{(d)} = \sum_{j=1}^{n} \lambda_j x_j^{(d)} + s^{\left(x^{(d)}\right)} + \left[x_p^{(d)} - \alpha^{(d)} \text{diag}\left(x_p^{(1)} + x_p^{(2)}\right)\right],$$
$$z_p^{(1)} = \sum_{j=1}^{n} \lambda_j z_j^{(1)} + s^{\left(z^{(1)}\right)},$$
$$k_p^{(1)} = \sum_{j=1}^{n} \lambda_j k_j^{(1)} + s^{\left(k^{(1)}\right)},$$

When $E_p^{(A)} > 0$, the pth assembly line is inefficient and we have

$$k_p^{(1)} = \sum_{j=1}^{n} \lambda_j k_j^{(1)} + s^{\left(k^{(1)}\right)},$$

$$k_p^{(2)} = \sum_{j=1}^{n} \lambda_j k_j^{(2)} + s^{\left(k^{(2)}\right)},$$

$$f_p = \sum_{j=1}^{n} \lambda_j f_j + s^{(f)},$$

$$y_p = \sum_{j=1}^{n} \lambda_j y_j - s^{(y)}.$$

The production and assembly lines can be improved and become efficient by deleting the input excess and augmenting the output shortfalls.

It is to be noted that the intermediate measures $k^{(1)}$ and $k^{(2)}$ may be increased or decreased to make the whole system as efficient. These operations are called CRS projection (constant returns to scale) and make the inefficient process as efficient.

Generalization of the model.

In model 1, we assumed there are only two production stations in the first stage. Consider now that there are G parallel production stations and the gth station uses inputs $\alpha_i^{(g)} x_{ip} \; : \; i = 1, 2, \ldots, m$ and $z_{(hp)}^{(g)} \; : \; h = 1, 2, \ldots, H_g$ to generate $k_{(lp)}^{(g)} \; : \; l = 1, 2, \ldots, L_g$. The production possibility set of technology under consideration for production line $g : \; g = 1, 2, \ldots, G$ is as follows:

$$T_g = \left\{ \left(\alpha^{(g)} \mathrm{diag}(x), z^{(g)}, k^{(g)}\right) : \; \left(\alpha^{(g)} \mathrm{diag}(x)\right)^T \geq \sum_{j=1}^{n} \lambda_j x_j^{(g)}, \; z^{(g)} \geq \sum_{j=1}^{n} \lambda_j z_j^{(g)}, \right.$$
$$\left. k^{(g)} \leq \sum_{j=1}^{n} \lambda_j k_j^{(g)}, \; 0 \leq \alpha^{(g)} \leq e^T, \; \lambda_j \geq 0, \; j = 1, 2, \ldots, n \right\}.$$

Moreover, the production possibility set of technology under consideration for the assembly line is defined as follows:

$$T_A = \left\{ \left(k^{(1)}, k^{(2)}, \ldots, k^{(G)}, f, y\right) : \; k^{(g)} \geq \sum_{j=1}^{n} \lambda_j k_j^{(g)}, \; g = 1, 2, \ldots, G, \right.$$
$$\left. f \geq \sum_{j=1}^{n} \lambda_j f_j, \; y \leq \sum_{j=1}^{n} \lambda_j y_j, \; \lambda_j \geq 0, \; j = 1, 2, \ldots, n \right\}.$$

Taking in to account the forms of $T_g : \ g = 1, 2, \ldots, G$ and T_A and considering $\sum_{g=1}^{G} \alpha_i^{(g)} = 1, \ i = 1, 2, \ldots, m$, this is obtained as the optimal value of the following model:

$$\text{Max} \ \ E_p = \sum_{g=1}^{G} e^T s^{(x^{(g)})} + \sum_{g=1}^{G} e^T s^{(k_g)} + \sum_{g=1}^{G} e^T s^{(z_g)} + e^T s^{(f)} + e^T s^{(y)}$$

$s.t.$

Production line $g : g = 1, 2, \ldots, G :$

$$\sum_{j=1}^{n} \lambda_j x_j^{(g)} + s^{\left(x^{(g)}\right)} = \left(\alpha^{(g)} \text{diag} \left(\sum_{g=1}^{G} x_p^{(g)}\right)\right)^t,$$

$$\sum_{j=1}^{n} \lambda_j z_j^{(g)} + s^{\left(z^{(g)}\right)} = z_p^{(g)},$$

$$\sum_{j=1}^{n} \lambda_j k_j^{(g)} + s^{\left(k^{(g)}\right)} = k_p^{(g)}.$$

Assembly line:

$$\sum_{j=1}^{n} \lambda_j k_j^{(g)} + s^{\left(k^{(g)}\right)} = k_p^{(g)}, \ g = 1, 2, \ldots, G,$$

$$\sum_{j=1}^{n} \lambda_j f_j + s^{(f)} = f_p,$$

$$\sum_{j=1}^{n} \lambda_j y_j - s^{(y)} = y_p.$$

General constraint:

$$l^{(\alpha)} \le \frac{\alpha^{(i)}}{\alpha^{(j)}} \le u^{(\alpha)}, \ for\ all\ i\ and\ j,$$

$$\sum_{g=1}^{G} \alpha^{(g)} = 1,$$

$$\alpha^{(g)}, \ \lambda_j \ge 0, \ for\ all\ g\ and\ j,$$

$$s^{\left(x^{(g)}\right)}, \ s^{\left(z^{(g)}\right)}, \ s^{(f)}, \ s^{(y)} \ge 0.$$

TABLE 1. The input-output measures used in this application.

Production line:	Inputs	Outputs
Structure production line	Steel (meter), wood (m^2)	Structure
D & W production line	Aluminum (meter), Glass (m^2), wood (m^2)	Door, window
Assembly line	Asbestos cement (m^2), Concrete (Tone)	Cabin

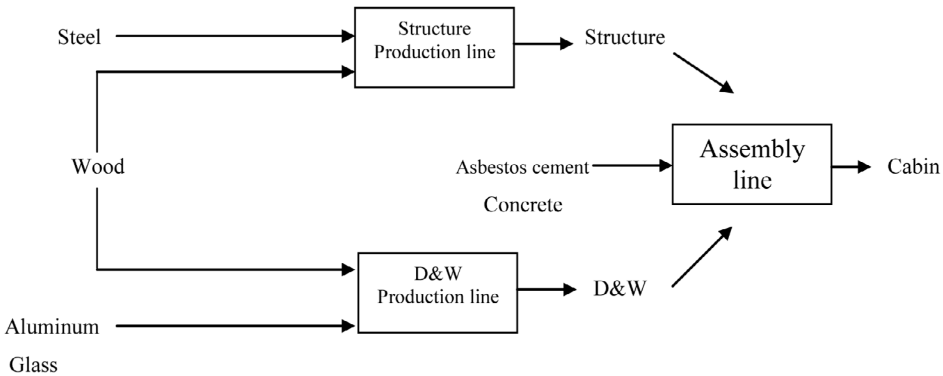

FIGURE 2. The production process of prefabricated cabin.

3. APPLICATION

In this section, we apply the two-stage production process discussed in this paper with the analysis of manufacturing company's activities. A limited company in Golestan, Iran, has 17 plants that produce prefabricated cabins. Each manufactory consists of two production lines arranged in series: structure production line and doors and windows ($D\&W$) production line. The structure production line uses steels ($z_1^{(1)}$) and some portion of woods (x_1) to produce structures ($k^{(1)}$). Parallel to this line, the $D\&W$ production line uses glasses ($z_2^{(2)}$), some portion of woods (x_2) and aluminums ($z_1^{(2)}$) to produce doors and windows ($k^{(2)}$). The produced structures, doors and windows will be assembled in the assembly line to produce the final products that are prefabricated cabins (y). The assembly line uses two external inputs: corrugated plate (Asbestos cement)(f_1) and concrete(f_2).

The input-output measures that are used in this application are summarized in Table 1.

The production process is depicted in Figure 2. The data for a six-month period is displayed in Table 2. The results from model (1) are reported in Table 3 where the columns are the inefficiency slacks obtained from model 1. As the table indicates, eight plants are efficient in overall sense. The projection points are listed in Table 4 (note that incomplete projects are acceptable by the board of management and hence, the input/output measures are not restricted to take integer values).

Table 2. Plants data.

DMU_j	$x^{(1)}$	$x^{(2)}$	$z_1^{(1)}$	$z_1^{(2)}$	$z_2^{(2)}$	$k^{(1)}$	$k^{(2)}$	y_1	f_1	f_2
1	5	5.5	12 700	9	8	168	328	160	310	1600
2	4.6	4.4	11 500	9	4	120	226	108	290	1100
3	3.5	5	11 300	11	7	84	220	84	160	800
4	2	3.2	8000	12	6	65	156	69	165	750
5	6.3	3.7	13 000	12	11	144	268	132	260	1400
6	4.7	5.8	13 500	22	23	158	280	149	290	1500
7	4.3	5.2	12 000	29	31	144	268	130	230	1350
8	6.8	4.2	13 450	13	9	168	326	158	300	1600
9	5	3.5	11 010	28	11	120	240	112	260	1100
10	4.1	3.4	10 500	19	12	89	178	84	160	900
11	4.8	5.2	12 350	10	9	144	284	132	235	1300
12	4.4	5.6	13 000	29	17	144	262	129	225	1350
13	3.8	4.2	11 505	9	11	108	200	99	215	1000
14	5	3.5	9550	22	21	96	178	82	165	850
15	5.2	6.3	13 800	24	11	168	330	157	315	1600
16	5.4	5.1	13 500	22	21	141	312	144	300	1500
17	6.8	5.7	13 505	24	11	153	318	150	295	1550

Table 3. Results from model 1.

DMU_j	E_p	$E_p^{(1)}$	$E_p^{(2)}$	$E_p^{(A)}$	α_1	α_2
DMU						
1	0	0	0	0	0.4762	0.5238
2	0	0	0	0	0.5111	0.4889
3	0	0	0	0	0.4118	0.5882
4	2632.8781	2516.3469	25.2187	98.4125	0.5439	0.4561
5	2619.8625	2529.2375	6.375	87.05	0.5462	0.4538
6	0	0	0	0	0.4476	0.5524
7	0	0	0	0	0.4526	0.5474
8	387.9808	361.9602	1.3839	23.4801	0.4884	0.5116
9	0	0	0	0	0.5882	0.4118
10	3698.46	3622.8657	21.88	59.1829	0.6027	0.3973
11	0	0	0	0	0.48	0.52
12	0	0	0	0	0.44	0.56
13	3692.1469	3652.4281	6.5312	34.2875	0.5746	0.4254
14	3134.5812	3054.2687	44.1875	55.925	0.6684	0.3316
15	1409.7531	1342.4719	26.4687	52.1125	0.5307	0.4693
16	2186.35	2060.85	44.5	87.6	0.5286	0.4714
17	1680.8438	1596.9063	29.5625	60.375	0.5875	0.4125

TABLE 4. Projection points.

DMU_j	$x^{(1)}$	$x^{(2)}$	$z_1^{(1)}$	$z_1^{(2)}$	$z_2^{(2)}$	$k^{(1)}$	$k^{(2)}$	y_1	f_1	f_2
1	5	5.5	12 700	9	8	168	328	160	310	1600
2	4.6	4.4	11 500	9	4	120	226	108	290	1100
3	3.5	5	11 300	11	7	84	220	84	160	800
4	2.16	2.37	5476.88	3.88	3.45	72.45	141.45	69	133.69	690
5	4.13	4.54	10 477.5	7.43	6.6	138.6	270.6	132	255.75	1320
6	4.7	5.8	13500	22	23	158	280	149	290	1500
7	4.3	5.2	12 000	29	31	144	268	130	230	1350
8	5.13	5.63	13 088.76	9.99	9	167.53	327.63	158	300	1575.36
9	5	3.5	11 010	28	11	120	240	112	260	1100
10	2.65	2.98	6878.86	7.45	7.28	89.14	172.39	84	160	846.29
11	4.8	5.2	12350	10	9	144	284	132	235	1300
12	4.4	5.6	13 000	29	17	144	262	129	225	1350
13	3.09	3.4	7858.13	5.57	4.95	103.95	202.95	99	191.81	990
14	2.56	2.82	6508.75	4.61	4.1	86.1	168.1	82	158.88	820
15	4.91	5.4	12 461.88	8.83	7.85	164.85	321.85	157	304.19	1570
16	4.5	4.95	11 430	8.1	7.2	151.2	295.2	144	279	1440
17	4.69	5.16	11 906.25	8.44	7.5	157.5	307.5	150	290.63	1500

The interpretation of our model can be illustrated by considering a specific plant, say plant 10. The production lines 1 and 2 and the assembly line are inefficient in this plant. The shared resource to this plant should be reduced from its current level 7.5 (4.1 for production line 1 and 3.4 for the second line) to 5.63 (2.65 for production line 1 and 2.98 for the second line). The projection point to this plant is

$$\left(x^{(1)},\ x^{(2)},\ z_1^{(1)},\ z_1^{(2)},\ z_2^{(2)},\ k^{(1)},\ k^{(2)},\ y_1,\ f_1,\ f_2\right) =$$
$$(2.65,\ 2.98,\ 6878.86,\ 7.45,\ 7.28,\ 89.14,\ 172.39,\ 84,\ 160,\ 846.29).$$

Considering the optimal values to α_1 and α_2, we conclude that the first intermediate measure $k^{(1)}$ should be increased from 89 to 89.14, whereas, the second one, $k^{(2)}$, should be decreased from 178 to 172.39. These reductions make the whole chain as efficient.

4. Conclusion

This paper discusses the efficiency measurement of two-stage production processes with three processes where two parallel processes in the first stage are connected serially with the process in the second stage. For this type of production system, an additive efficiency measure has been defined. A method for determining the DEA frontier points for inefficient components in these parallel-series production systems has been faced. This application of parallel-series production process involves shared resources and the model proposed in this paper, determines an optimal split of shared resources. The case of prefabricated cabin plants is given using this newly developed approach.

The DEA model discussed in this paper is under CRS, in other words, an assumption of constant returns to scale is considered. The approach is also applicable to variable returns to scale under the BCC model of Banker *at al.* [2] by including the constraint $\sum_{j=1}^{n} \lambda_j = 1$ in the LP model 1.

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