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A dead time process controller

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A DEAD TIME PROCESS CONTROLLER

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ABSTRACT

The control of systems containing pure dead time elements has plagued the control engineer for many years. This thesis discusses a new controller developed by this writer which offers improved performance in first order processes dominated by a dead time element.

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I. INTRODUCTION

This writer contacted a number of control systems engineers at Taylor Instrument Company in Rochester, New York, in an attempt to find a suitable thesis topic which would satisfy the requirements of the Master of Science Degree in Electrical Engineering and also benefit Taylor Instrument Company of which this writer is an employee. In each discussion with various engineers, the difficulty in controlling processes with large dead time elements was mentioned and the desirability of a new method of control was expressed. As a result of these discussions, this writer spent a few months researching the problem of dead time process control. The result of this research is detailed in this thesis. It essentially consists of a new controller which itself contains a dead time element. The new controller provides improved response with respect to conventional controllers when dead time dominated processes are considered.

II. REVIEW OF LITERATURE

Available literature on dead time process control indicates a general awareness of the associated control difficulty. Detailed investigations into optimum controller settings when industry standard controllers are used have been undertaken. For instance, an article published in the July, 1965 issue of Control Engineering written by A. Haalman and titled "Adjusting Controllers For A Dead Time Process" suggests optimum standard controller types and associated settings for various plants which contain dead time elements. An article by G. H. Cohen and G. A. Coon titled "Theoretical Consideration of Retarded Control", which was published in the July, 1953 issue of ASME Transactions, suggests settings for standard controllers used in dead time systems. The famous Ziegler-Nichols settings which are documented in almost every process control text book also account for system dead time. However, for systems dominated by a dead time element, the Ziegler-Nichols settings are conservative, resulting in sluggish response.

Available literature indicates a general lack of nonstandard controller configurations which could provide better performance than standard controllers. An article by Masahiro Hori titled "Discrete Compensator Controls Dead Time Process" suggests a new sampled-data controller. However, performance with this controller is still poor. This thesis depicts a non-standard controller which attempts to bridge this gap.

Since the controller derived in this thesis contains a dead time element, various means of simulating a dead time element were investigated. The following articles were used as reference:

1) "Comparing Dead Time Approximations", F. G. Haag, Control Engineering, October, 1967.

2) "An Analysis of Transport Delay Simulation Methods", J. B. Knowles and D. W. Leggett, The Radio and Electronic Engineer, Vol. 42, No. 4, April, 1972.

3) "A Transport Delay Simulator Using Digital Techniques", A. B. Keats and D. W. Leggett, The Radio and Electronic Engineer, Vol. 42, No. 4, April, 1972.

4) "Transport Delay Simulation", K. Hogberg, Instruments and Control Systems, June, 1966.

III. GENERAL DISCUSSION OF DEAD TIME ELEMENTS AND SYSTEMS

Many process control systems contain dead time elements. The output of such an element is equivalent to the input delayed in time by a finite amount. Thus, if r is the input to a dead time element where r = 0 for $t \leq 0$, the output, c, can be expressed as

$$c(t) = r(t-T)\, u(t-T) \qquad (1)$$

where u(t-T) denotes a function which has a value of zero for $t < T$ and a value of unity for $t \geq T$.

The transfer function for a dead time element can be obtained by taking the Laplace transform of equation (1).

Thus $C(S) = \mathcal{L}\left[r(t-T)\, u(t-T)\right]$

or $C(S) = e^{-ST} R(S)$

and $$\frac{C(S)}{R(S)} = e^{-ST}. \qquad (2)$$

Therefore the transfer function for a dead time element is simply e^{-ST}, where T corresponds to the magnitude of the time delay. The magnitude and phase of this transfer function can be obtained by using Euler's equation as

$$\frac{C(jw)}{R(jw)} = X(jw) = e^{-jwT} = \cos wT - j\sin wT$$

from which $|X(jw)| = \sqrt{\cos^2 wT + \sin^2 wT} = 1$

and $\underline{/X(jw)} = \tan^{-1}\left[\frac{-\sin wT}{\cos wT}\right] = \tan^{-1}\left[\frac{\sin(-wT)}{\cos wT}\right]$

$$= \tan^{-1}\left[\frac{\sin(-wT)}{\cos(-wT)}\right] = \tan^{-1}\left[\tan(-wT)\right] = -wT.$$

Thus, the dead time element has unity gain throughout the frequency domain and a phase lag which is a linear function of frequency. The extreme difficulty in controlling dead time systems is a direct result of this phase lag which increases rapidly with increasing frequency.

An example of a system containing a dead time is shown in Figure 1.

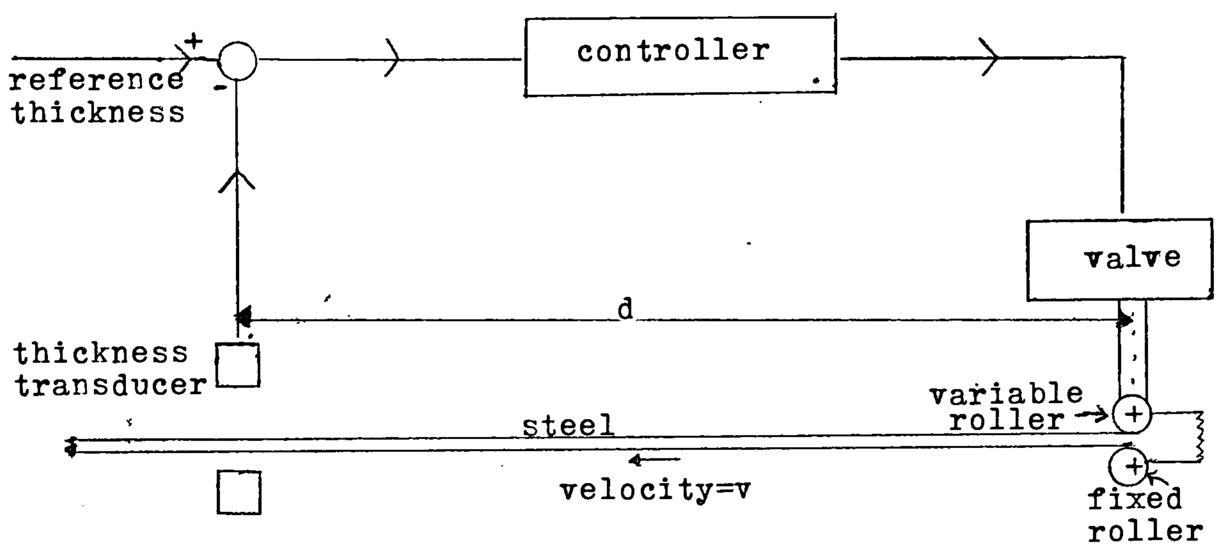

Figure 1: Steel Thickness Control System

The purpose of this control system is to keep the thickness of a strip of steel constant. The steel strip is fed between two rollers. The distance between the two rollers, and thus the thickness of the steel, can be adjusted by varying the air pressure to the movable roller. The feedback signal, which is obtained by a thickness transducer, is located a distance, d, from the rollers. Thus, the feedback signal has a pure delay associated with it due to the finite velocity of the steel and the distance, d, which must be traveled from the rollers to the sensing element. The magnitude of the time delay is simply the distance divided by the velocity ($T = \frac{d}{v}$). Similarly, many fluid flow control systems also contain a dead time due to the distance velocity lag along a pipe.

IV. DESCRIPTION OF SYSTEM

The type of dead time system which will be discussed in this paper is shown in Figure 2.

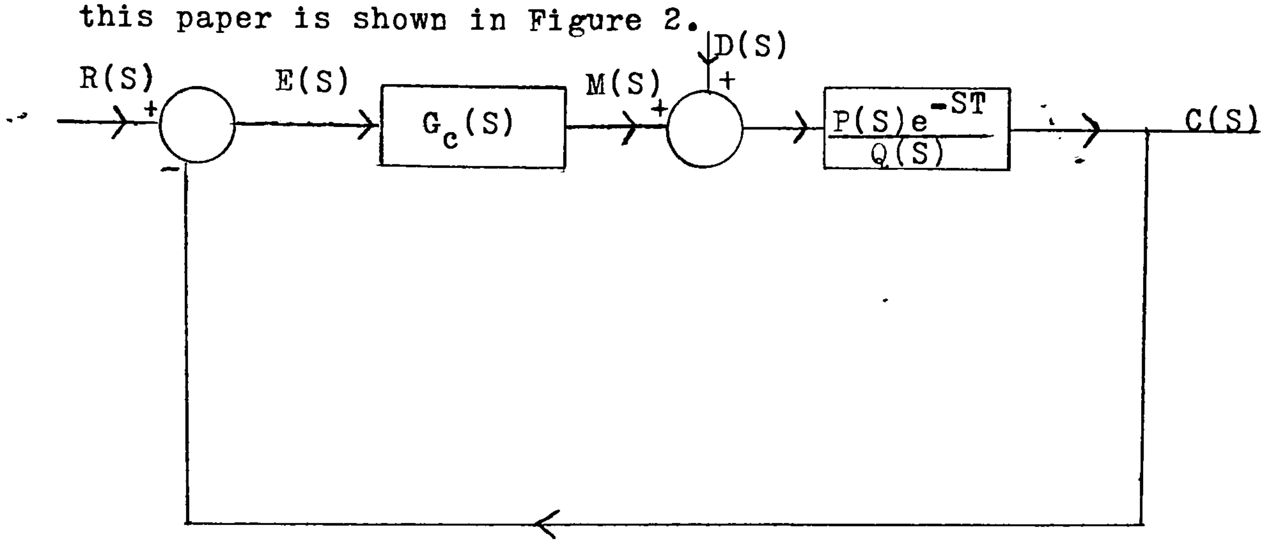

Figure 2: General system block diagram with dead time element in the forward path.

The following nomenclature is applicable:

R(S) = reference input,

E(S) = error signal,

M(S) = manipulated variable,

C(S) = controlled variable,

D(S) = disturbance input,

$G_c(S)$ = controller transfer function

and $\frac{P(S)e^{-ST}}{Q(S)}$ = plant transfer function, where P(S) and Q(S) are assumed to be polynomials in S.

It is shown below that the transient analysis of this system is analogous to that of a system having the dead time element in the feedback path, except that the time response of the system with the dead time element in the forward path will be delayed by one dead time, T. For the system shown in Figure 2 the transfer functions can be derived as

$$\frac{C(S)}{R(S)} = \frac{\dfrac{G_c(S)P(S)\ e^{-ST}}{Q(S)}}{1 + \dfrac{G_c(S)P(S)\ e^{-ST}}{Q(S)}} = \frac{G_c(S)P(S)e^{-ST}}{Q(S) + G_c(S)P(S)e^{-ST}}$$

and $$\frac{C(S)}{D(S)} = \frac{P(S)e^{-ST}}{Q(S) + G_c(S)P(S)e^{-ST}} .$$

For a system with the dead time in the feedback path as shown in Figure 3, the transfer functions can be derived as

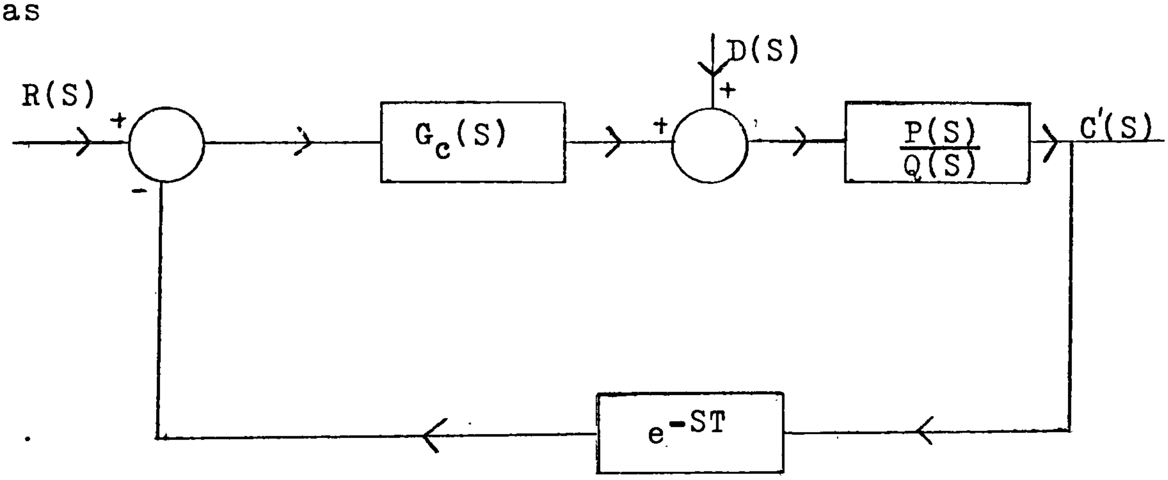

Figure 3: General system block diagram with dead time element in the feedback path.

$$\frac{C'(S)}{R(S)} = \frac{\dfrac{G_c(S)P(S)}{Q(S)}}{1 + \dfrac{G_c(S)P(S)e^{-ST}}{Q(S)}} = \frac{G_c(S)P(S)}{Q(S) + G_c(S)P(S)e^{-ST}}$$

and $$\frac{C'(S)}{D(S)} = \frac{P(S)}{Q(S) + G_c(S)P(S)e^{-ST}} .$$

Thus the two systems have response transforms which differ only in terms of an e^{-ST} term in the numerator. For identical setpoint or disturbance inputs, the two systems will have almost identical transient responses, differing only in that the response of the system having the dead time in the forward path will be delayed by time, T. This point is emphasized because many practical systems, such as the steel thickness control system previously discussed, have dead times in the feedback path. The responses which are presented later in this paper directly conform to a system with the dead time lag in the forward path. However, the response of a similar system with the dead time lag placed in the feedback loop can easily be obtained by shifting the time axis by T.

V. IDEAL CONTROLLER

Let us assume that the system depicted in Figure 2 is subjected to a unit step change in the setpoint, that is $R(S)=1/S$. Since the forward path has a dead time lag, the output of the system will not be able to change until at least one dead period interval, T, has elapsed. The best possible response for this system would be a unit step change in the output occurring T seconds after the input step was applied. Mathmatically, the best possible or ideal response to a unit step change in the setpoint would be expressed as

$$C_{Ideal} = C_I = \frac{e^{-ST}}{S} .$$

The form of the controller transfer function, $G_c(S)$, which will provide this ideal output is derived below.

$$\frac{C(S)}{R(S)} = \frac{\dfrac{G_c(S)P(S)e^{-ST}}{Q(S)}}{1 + \dfrac{G_c(S)P(S)e^{-ST}}{Q(S)}} = \frac{G_c(S)P(S)e^{-ST}}{Q(S) + G_c(S)P(S)e^{-ST}} \text{ and,}$$

since $R(S)=\frac{1}{S}$, the output transform can be written as

$$C(S) = \frac{G_c(S)P(S)e^{-ST}}{S\left[Q(S) + G_c(S)P(S)e^{-ST}\right]} . \tag{3}$$

Let $C(S) = C_I(S) = \frac{e^{-ST}}{S}$.

Then, $\frac{e^{-ST}}{S} = \frac{G_c(S)P(S)e^{-ST}}{S\left[Q(S) + G_c(S)P(S)e^{-ST}\right]}$ which, after cross multiplying and simplifying, yields

$$G_c(S)P(S) = Q(S) + G_c(S)P(S)e^{-ST}.$$

This can be rearranged as

$$G_c(S) = \frac{Q(S)}{P(S)\left[1-e^{-ST}\right]} = G_I(S), \qquad (4)$$

the ideal controller transfer function.

Let us assume that a good approximation to $\frac{Q(S)}{P(S)}$ can be obtained in the frequency range of interest, with the understanding that, in practice, it generally could not be exactly realized, since the order of the numerator would usually be higher than the order of the denominator. The system with the ideal controller is diagrammed in Figure 4. The variable T in equation 4 has been replaced by X in Figure 4 to account for differences between the dead time value in the controller and that of the plant.

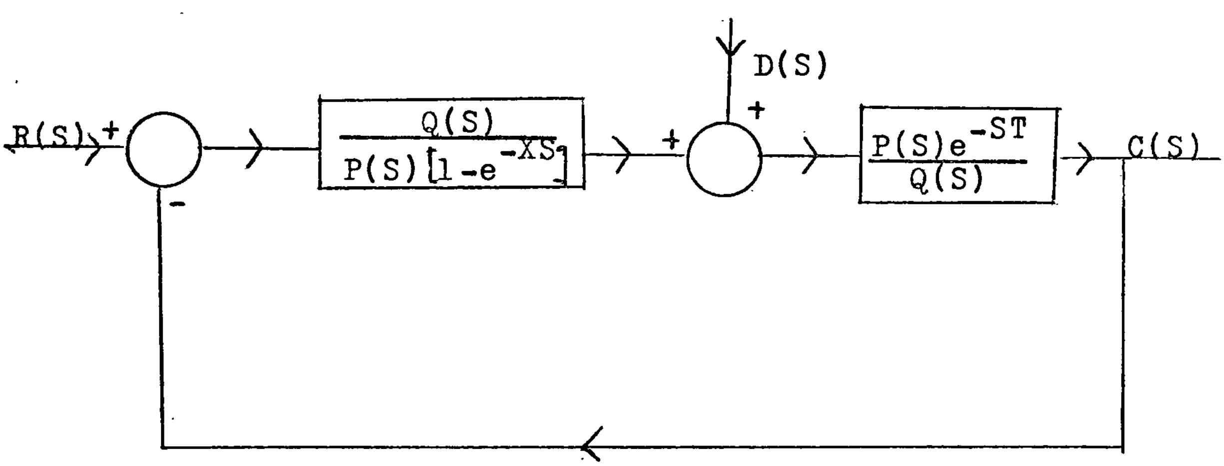

Figure 4: System block diagram with ideal controller.

The $\frac{1}{1 - e^{-XS}}$ portion of the ideal controller can be realized by an inner positive feedback loop as diagrammed in Figure 5. This can readily be seen since $H(S) = J(S) + e^{-XS}H(S)$ or $\frac{H(S)}{J(S)} = \frac{1}{1 - e^{-XS}}$.

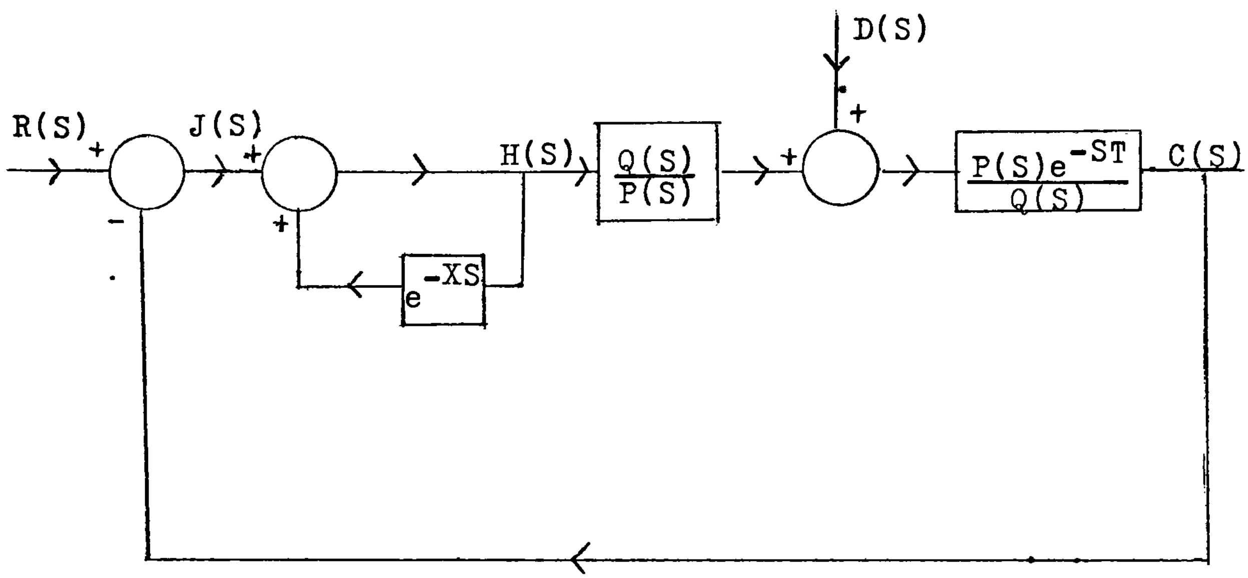

Figure 5: System block diagram with the ideal controller realized by an inner positive feedback loop.

Let us derive the output response of this system for a unit step change in the reference input, r.

$$\frac{C(S)}{R(S)} = \frac{\frac{e^{-ST}}{1 - e^{-XS}}}{1 + \frac{e^{-ST}}{1 - e^{-XS}}} = \frac{e^{-ST}}{1 - e^{-XS} + e^{-ST}}$$ and, since

$R(S) = \frac{1}{S}$, the output transform can be written as

$$C(S) = \frac{e^{-ST}}{S(1 + e^{-ST} - e^{-XS})} .$$

If the dead time in the controller, X, identically equals the process dead time, T, the output C(S) is $\frac{e^{-ST}}{S}$, the ideal response. However, let us assume a mismatch in T and X and calculate the output transient response. Then

$$C(S) = \frac{e^{-ST}}{S(1 + e^{-ST} - e^{-XS})} = \frac{e^{-ST}}{S} \cdot \frac{1}{1 + e^{-ST} - e^{-XS}}$$

and the $\frac{1}{1 + e^{-ST} - e^{-XS}}$ portion of the response can be expanded into a power series by application of the following formula

$$\frac{1}{1+Z} = \sum_{n=0}^{\infty}(-1)^n Z^n = 1 - Z + Z^2 - Z^3 + Z^4 \ldots .$$

Therefore,

$$C(S) = \frac{e^{-ST}}{S}\left[1 - (e^{-ST} - e^{-XS}) + (e^{-ST} - e^{-XS})^2 - (e^{-ST} - e^{-XS})^3 \ldots\right]$$

which can be further expanded to yield

$$C(S) = \frac{e^{-ST}}{S}\left[1 - e^{-ST} + e^{-XS} + e^{-2ST} - 2e^{-(X+T)S} + e^{-2XS} - e^{-3ST} + 3e^{-(X+2T)S} - 3e^{-(2X+T)S} + e^{-3XS} \ldots\right] .$$

Thus,

$$C(S) = \frac{e^{-ST}}{S} - \frac{e^{-2ST}}{S} + \frac{e^{-(X+T)S}}{S} + \frac{e^{-(3ST)}}{S} - \frac{2e^{-(X+2T)S}}{S} + \frac{e^{-(2X+T)S}}{S} - \frac{e^{-4ST}}{S} + \frac{3e^{-(X+3T)S}}{S} - \frac{3e^{-(2X+2T)S}}{S} + \frac{e^{-(3X+T)S}}{S} \ldots .$$

Letting X = 0.9 and T = 1.0, a 10% mismatch, the output equation can be written in the time domain as

$$c(t) = u(t-1.0) - u(t-2.0) + u(t-1.9) + u(t-3.0)$$
$$- 2u(t-2.9) + u(t-2.8) - u(t-4) + 3u(t-3.9) - 3u(t-3.8)$$
$$+ u(t-3.7) \ldots \quad .$$

A plot of c(t) versus time is diagrammed below.

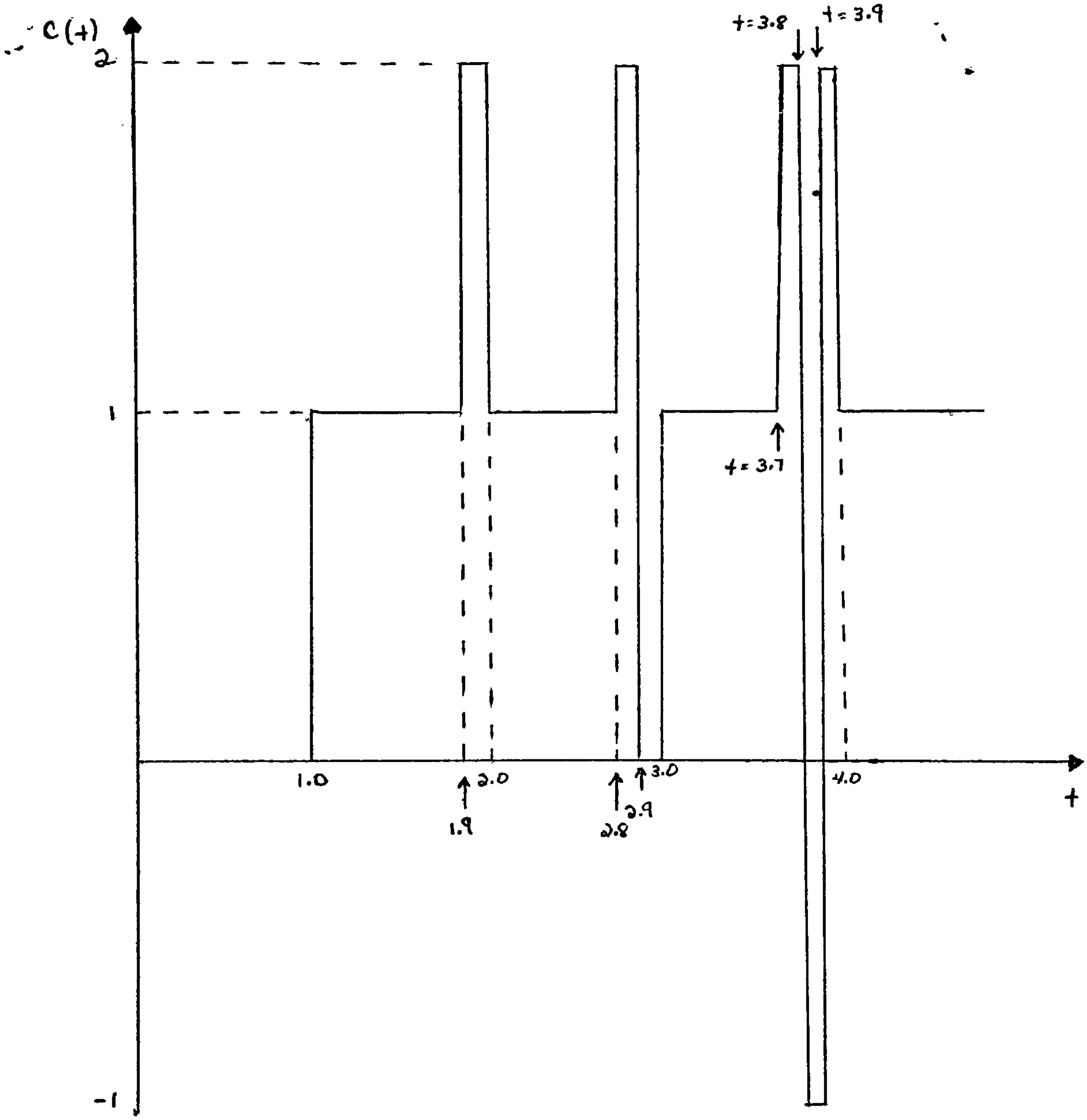

Figure 6: The output response to a unit step change in setpoint with the ideal controller having X=0.9 and T=1.0.

Thus, with the controller dead time equal to 90% of the process dead time, the system response is poor due to the large output peaks which deviate significantly from the desired output response. It is not difficult to show that the system response is unacceptable for any value of X unequal to T. Thus, in a practical situation, where it would be impossible to exactly match X and T, this controller form would give unacceptable response.

VI. NEW CONTROLLER

A new controller form can be obtained by relaxing the ideal response, $C(S) = \frac{e^{-ST}}{S}$ to $C(S) = \frac{ae^{-ST}}{S(S+a)}$. The addition of the pole at S = -a to the output response causes a rounding of the waveforms and eliminates the large peaks associated with the ideal controller. The value of a will be determined as a compromise between fast and smooth response for any given mismatch between X and T.

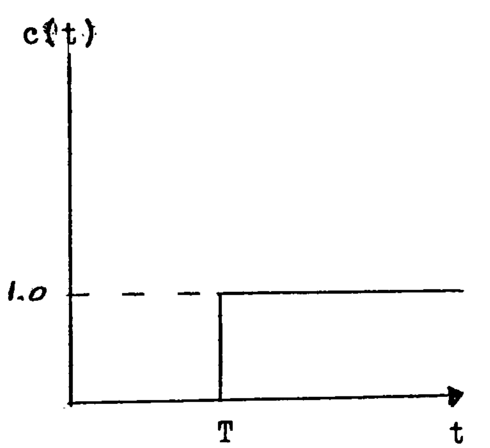

Figure 7a: Ideal unit step response

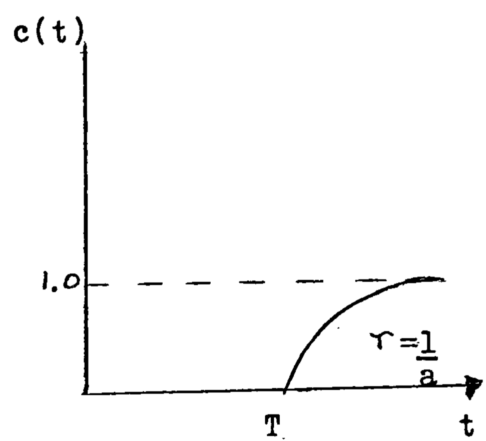

Figure 7b: Relaxed unit step response

The form of $G_c(S)$ resulting from the relaxed output response criteria can now be determined. Equation (3) is repeated below, as

$$C(S) = \frac{G_c(S)P(S)e^{-ST}}{S\left[Q(S) + G_c(S)P(S)e^{-ST}\right]}. \qquad (3)$$

Let $C(S) = \frac{ae^{-ST}}{S(S+a)}$, the relaxed output.

Then $\frac{ae^{-ST}}{S(S+a)} = \frac{G_c(S)P(S)e^{-ST}}{S\left[Q(S) + G_c(S)P(S)e^{-ST}\right]}$ which, after cross multiplying and simplifying can be written as

$$a\left[Q(S) + G_c(S)P(S)e^{-ST}\right] = G_c(S)P(S)(S+a).$$

Rearranging yields $Q(S)\ a = G_c(S)\left[P(S)(S+a) - a\ P(S)e^{-ST}\right]$

or $$G_c(S) = \frac{a\ Q(S)}{P(S)\left[S+a-ae^{-ST}\right]}. \qquad (5)$$

Equation (5) defines the new controller. The defining equation can be written as $G_c(S) = \frac{a\ Q(S)}{P(S)(S+a)\left[1-\frac{ae^{-ST}}{S+a}\right]}$

or $G_c(S) = \frac{\frac{a}{S+a}}{1 - \frac{ae^{-ST}}{S+a}} \frac{Q(S)}{P(S)}$ by factoring out the S+a term in the denominator.

A block diagram of the new controller is shown in Figure 8. The $\frac{a}{S+a-ae^{-ST}}$ portion of the new controller response is realized here by a positive feedback loop having $\frac{a}{(S+a)}$ in the forward path and a pure delay in the feedback path.

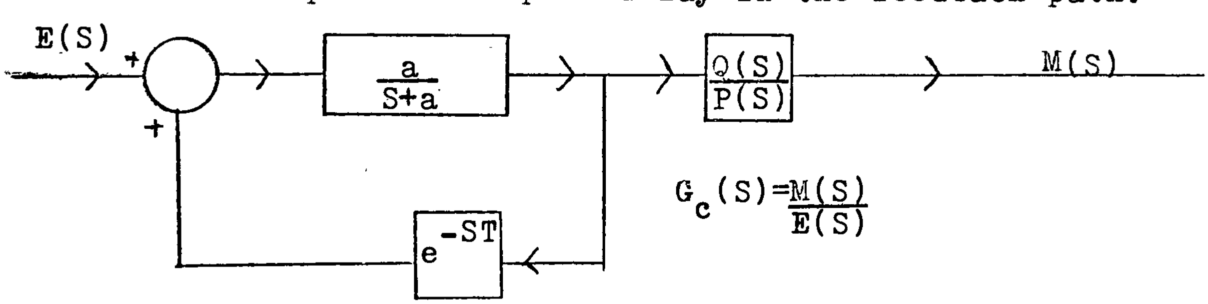

Figure 8: One possible realization of the new controller.

By using well-known block diagram reduction techniques the above diagram can be rearranged to the form shown in Figure 9.

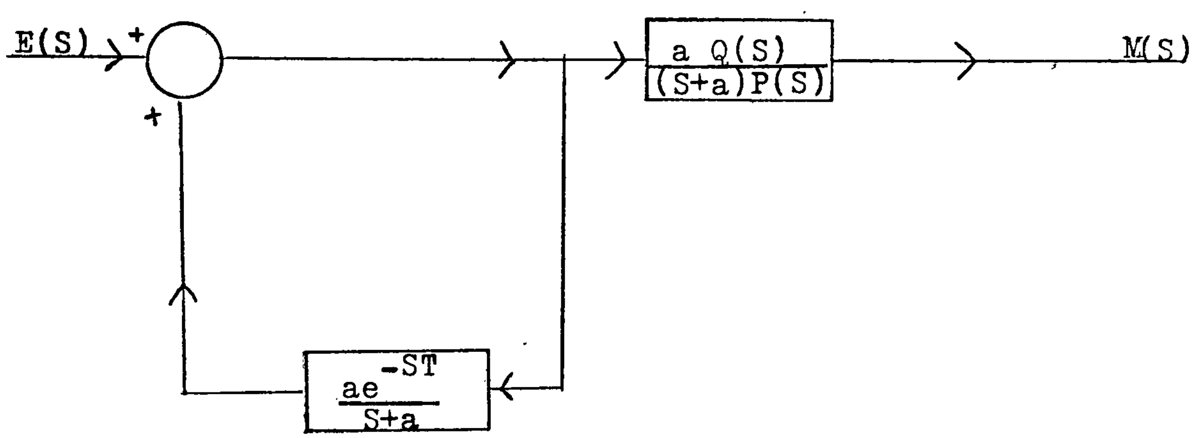

Figure 9: Another possible realization of the new controller.

The plant transfer function, $\frac{P(S)e^{-ST}}{Q(S)}$, dictates that the order of Q(S) must be equal to or greater than the order of P(S) for the plant to be physically realizable. Most physical systems do have the order of Q(S) greater than the order of P(S). The controller realization shown in Figure 8 requires the realization of $\frac{Q(S)}{P(S)}$ which cannot be achieved if the order of Q(S) is greater than the order of P(S). If an approximation to $\frac{Q(S)}{P(S)}$ over a limited frequency range proves unsatisfactory in terms of output response the form shown in Figure 9 could be used. The transfer function $\frac{a\,Q(S)}{(S+a)P(S)}$ can be realized even if the order of Q(S) does exceed the order of P(S) by one. The form shown in Figure 9 is not recommended, however, unless an approximation to $\frac{Q(S)}{P(S)}$ is not satisfactory because the term $\frac{a}{S+a}$ appears twice in this form. This means additional hardware and makes one more adjustment necessary.

VII. COMPARISON OF NEW CONTROLLER WITH A PROPORTIONAL-PLUS-INTEGRAL CONTROLLER

In the remainder of this paper the response of a plant having $\frac{P(S)}{Q(S)} = \frac{B}{S+B}$, that is a plant described by the transfer function $\frac{Be^{-ST}}{S+B}$, will be determined. This specific form of plant transfer function was chosen for two reasons. First, many practical systems are accurately represented by the combination of a dead time element and a simple first order lag such as $\frac{B}{S+B}$. This statement can be supported by the fact that the famous Ziegler-Nichols controller setting equations are based on the premise that most process control systems can be approximated by a transfer function of the form $\frac{Be^{-ST}}{S+B}$. Second, this paper deliberately considers systems which are relatively dominated by the dead time element and thus represent very difficult control problems. The addition of more poles or zeros to the plant transfer function would not appreciably change the results obtained because of the assumed dominance of the dead time element.

The standard proportional-plus-integral controller form is recommended in Haalman's paper for good response to a plant of the form $\frac{Be^{-ST}}{S+B}$ and, thus, will be used as a comparison to the new controller scheme.

The setpoint and disturbance response for a unit step change will now be determined for each controller configuration.

First, the setpoint response for the assumed plant using the new controller, as shown in Figure 10, will be calculated. It is assumed that $\frac{Q(S)}{P(S)}$ in the controller can be set exactly equal to $\frac{S+B}{B}$. Referring to Figure 9 it has been shown that this is possible, since $\frac{Q(S)}{P(S)}$ is cascaded with the $\frac{a}{S+a}$ lag.

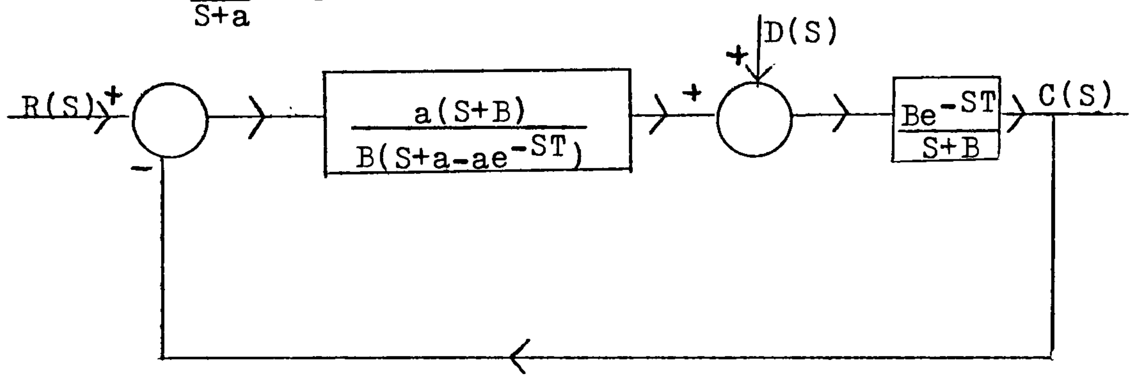

Figure 10: System diagram of an assumed $\frac{Be^{-ST}}{S+B}$ plant with new controller.

For this system $$\frac{C(S)}{R(S)} = \frac{\dfrac{a(S+B)\ Be^{-ST}}{B(S+a-ae^{-ST})(S+B)}}{1 + \dfrac{a(S+B)\ Be^{-ST}}{B(S+a-ae^{-ST})(S+B)}} = \frac{ae^{-ST}}{S+a}$$

and, when $R(S) = \frac{1}{S}$, the output transform becomes $C(S)=\frac{ae^{-ST}}{S(S+a)}$. Taking the inverse transform yields

$$c(t) = \left[1-e^{-a(t-T)}\right] u(t-T). \qquad (6)$$

It should be noted that the response is independent of B.

If the ideal response, $c_I(t)$, is considered to be a unit step delayed in time by an amount T, the integral absolute error can be written as

$$E_S = \int_0^\infty \left| u(t-T) - \left[1-e^{-a(t-T)}\right] u(t-T) \right| dt,$$ which can be simplified to $$E_S = \int_0^\infty \left| e^{-a(t-T)} u(t-T) \right| dt.$$

Since $u(t-T) = 0$ for $t < T$

$$E_S = \int_T^\infty e^{-a(t-T)} dt = -\frac{1}{a} e^{-a(t-T)} \Big|_T^\infty$$

$$= -\frac{1}{a}(0-1) = \frac{1}{a}. \qquad (7)$$

Thus, the integral absolute error (I.A.E.) for a step change in the setpoint is simply $\frac{1}{a}$. To minimize the I.A.E. it would be desirable to make the value of a as large as possible. However, for very large values of a the system would become unstable when small mismatches between the plant dead time and the controller dead time exist. Thus the value of a must be chosen to give small error with reasonable mismatches in controller settings. For example, later in this paper it will be shown that the value of 3.33 for a will result in good response when a 10% mismatch exists between the plant and controller dead time.

The response to a step disturbance will now be calculated. Referring to Figure 10 we find

$$\frac{C(S)}{D(S)} = \frac{\dfrac{Be^{-ST}}{S+B}}{1 + \dfrac{a(S+B)e^{-ST_B}}{B(S+a-ae^{-ST})(S+B)}}$$

or, after simplification,

$$\frac{C(S)}{D(S)} = \frac{Be^{-ST}}{S+B} \left[1 - \frac{ae^{-ST}}{S+a}\right] .$$

Assuming $D(S) = \frac{1}{S}$, the output transform can be written as

$$C(S) = \frac{Be^{-ST}}{S(S+B)} - \frac{aBe^{-2ST}}{S(S+a)(S+B)} .$$

Using partial fraction expansion techniques, the output can be rewritten as

$$C(S) = \left[\frac{Q1}{S} + \frac{Q2}{S+B}\right] e^{-ST} - \left[\frac{Q3}{S} + \frac{Q4}{S+a} + \frac{Q5}{S+B}\right] e^{-2ST} .$$

The coefficients can be found, using residues, to be

$$Q1 = \left.\frac{B}{S+B}\right|_{S=0} = 1,$$

$$Q2 = \left.\frac{B}{S}\right|_{S=-B} = -1,$$

$$Q3 = \left.\frac{aB}{(S+a)(S+B)}\right|_{S=0} = 1,$$

$$Q4 = \left.\frac{aB}{S(S+B)}\right|_{S=-a} = \frac{B}{(a-B)},$$

and $$Q5 = \left.\frac{aB}{S(S+a)}\right|_{S=-B} = \frac{a}{B-a} .$$

Thus,

$$C(S) = \left[\frac{1}{S} - \frac{1}{S+B}\right] e^{-ST} - \left[\frac{1}{S} + \frac{B}{(a-B)(S+a)} + \frac{a}{(B-a)(S+B)}\right] e^{-2ST} .$$

Taking the inverse transform yields

$$c(t) = \left[1 - e^{-B(t-T)}\right] u(t-T) - \left[1 + \frac{B}{a-B} e^{-a(t-2T)} + \frac{a}{B-a} e^{-B(t-2T)}\right] u(t-2T). \qquad (8)$$

Since this is the response to a disturbance, the ideal response, $c_I(t)$, is zero. The I.A.E. can be calculated as

$$E_d = \int_0^\infty \left| \left[1-e^{-B(t-T)}\right]u(t-T) - \left[1 + \frac{B}{a-B}e^{-a(t-2T)} + \frac{a}{B-a}e^{-B(t-2T)}\right]u(t-2T)\right| dt$$

$$\text{or } E_d = \int_T^{2T}\left[1-e^{-B(t-T)}\right]dt + \int_{2T}^\infty\left[-e^{-B(t-T)} - \frac{B}{a-B}e^{-a(t-2T)} - \frac{a}{B-a}e^{-B(t-2T)}\right]dt.$$

$$\text{Thus } E_d = \left[t + \frac{1}{B}e^{-B(t-T)}\right]_T^{2T} + \left[\frac{1}{B}e^{-B(t-T)} + \frac{B}{a(a-B)}e^{-a(t-2T)} + \frac{a}{B(B-a)}e^{-B(t-2T)}\right]_{2T}^\infty$$

$$\text{or } E_d = T + \frac{1}{B}e^{-BT} - \frac{1}{B} - \frac{1}{B}e^{-BT} - \frac{B}{a(a-B)} - \frac{a}{B(B-a)}.$$

This can be rewritten as

$$E_d = T - \frac{1}{B} + \frac{B^2 - a^2}{aB(B-a)}. \qquad (9)$$

The setpoint and disturbance responses with a proportional-plus-integral controller are difficult to obtain by hand calculation. An analog computer simulation could be used, however, considerable effort would be required to obtain a satisfactory simulation for the dead time element. A simulation using a second order Pade' approximation to the dead time element was tried but the recorded response did not accurately match the calculated response, indicating that a higher order Pade' approximation was necessary. Since the second order approximation used all the integrators conveniently available, the analog simulation approach was not pursued further. Instead, to eliminate errors due to dead time element approximations, a digital computer simulation was used. A stored analog computer simulation program called DYSIM***, which is available on a General Electric time shared computer, was employed. A detailed discussion of the DYSIM*** simulation of this system is given in Appendix A. This program can accurately simulate a dead time delay.

The settings for the proportional gain and reset rate of the proportional-plus-integral controller were calculated by formulas presented in an article by A. Haalman.[1] Mr. Haalman suggests that a proportional-plus-integral controller, having the form $G_c(S) = K(1+\frac{1}{S\tau_A})$, be used if the plant equation has the form $\frac{e^{-ST}}{1+S\tau_2}$. Mr. Haalman recommends that for this situation the equations $K = \frac{2\tau_2}{3T}$ and $\tau_A = \tau_2$ be used to calculate the controller settings.

By letting $\tau_2 = \frac{1}{B}$, the plant transfer function referred to by Mr. Haalman becomes the plant transfer function analyzed in this paper. The recommended settings thus become $K = \frac{2}{3TB}$ and $\tau_A = \frac{1}{B}$. The regular block diagram for the proportional-plus-integral control of a plant of the form $\frac{Be^{-ST}}{S+B}$ is shown in Figure 11.

1. Haalman, A., "Adjusting Controllers For A Dead Time Process", Control Engineering, July, 1965.

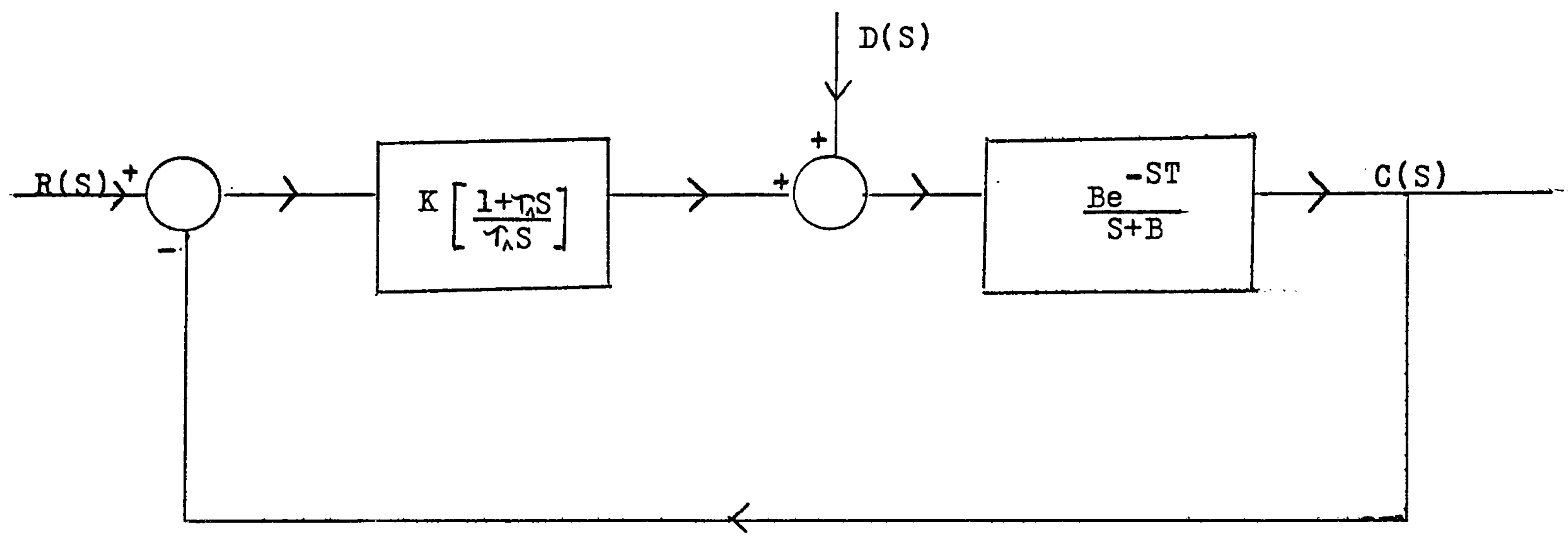

Figure 11: The block diagram of a $\frac{Be^{-ST}}{S+B}$ plant having proportional-plus-integral control.

The responses to a unit step change in the setpoint for both controller configurations are plotted in Figure 12. The value of T was arbitrarily set at 1. It should be noted that both setpoint response plots are independent of the value of B. This fact is obvious for the new controller by analysis of equation 6. It is shown below that the same is true for the proportional-plus-integral controller. Referring to Figure 11

$$\frac{C(S)}{R(S)} = \frac{K \frac{1+\tau_A S}{\tau_A S} \frac{Be^{-ST}}{S+B}}{1 + K \frac{1+\tau_A S}{\tau_A S} \frac{Be^{-ST}}{S+B}}.$$

Haalman's settings dictate that $K = \frac{2}{3TB}$ and $\tau_A = \frac{1}{B}$.

Thus,

$$\frac{C(S)}{R(S)} = \frac{\frac{2}{3TB} \frac{1+\frac{1}{B}S}{\frac{1}{B}S} \frac{Be^{-ST}}{S+B}}{1 + \frac{2}{3TB} \frac{1+\frac{1}{B}S}{\frac{1}{B}S} \frac{Be^{-ST}}{S+B}}$$

$$= \frac{\frac{2}{3T} \frac{S+B}{S} \frac{e^{-ST}}{S+B}}{1 + \frac{2}{3T} \frac{S+B}{S} \frac{e^{-ST}}{S+B}} = \frac{2e^{-ST}}{3ST + 2e^{-ST}}.$$

Since $R(S) = \frac{1}{S}$, the output transform can be written as

$C(S) = \frac{1}{S} \frac{2e^{-ST}}{3ST + 2e^{-ST}}$, which is independent of B. Thus, if Haalman's settings are used, the setpoint response of the system with PI control will be independent of B.

Table 1 outlines the controller settings which were used for four different values of B. The ratio of B to T indicates the relative dominance of the dead time element. As the value of B increases the effect of the dead time element becomes more significant. It should be noted that the value of a has been set equal to 3.33. This value was found in a later part of this thesis to result in good output response when there is a 10% mismatch between plant and controller dead times. The choice of the value of a will be considered in detail later in this thesis.

B	P.I. Controller $G_c(S) = K \frac{1+\tau_i S}{\tau_i S}$		New Controller $G_c(S) = \frac{a(S+B)}{B(S+a-ae^{-S})}$	
	K	τ_i	a	B
4.0	0.1667	0.25	3.33	4.0
2.0	0.333	0.5	3.33	2.0
1.0	0.667	1.0	3.33	1.0
0.5	1.3334	2.0	3.33	0.5

Table 1. Controller settings for various values of B when dead time, T, is unity.

The setpoint response curves shown in Figure 12 indicate that the system performance is superior with the new controller. The I.A.E. has been improved by better than a factor of 4 and the settling time to within 2% of the final value is improved by a factor of 3.7. The I.A.E. for the response obtained when the new controller form is used was calculated using equation 7. The I.A.E. for the proportional-plus-integral controller response was calculated by DYSIM***. The output of block 32 on Figure A1 in Appendix A represents the I.A.E. calculated by DYSIM***.

The responses to a unit step disturbance are different for each controller form. The responses for various values of B are plotted in Figures 13, 14, 15, and 16 for T = 1.0. The output response and I.A.E. for the proportional-plus-integral controller were calculated by DYSIM*** and the output response and I.A.E. for the new controller were determined using equations 8 and 9. A digital computer program was used to evaluate equation 8. The same controller settings which were used for the setpoint response (listed in Table 1) were used to calculate the disturbance responses. Table 2 contains the I.A.E. and the 2% settling time, τ_S, for the various cases considered.

	Proportional-Plus-Integral Controller		New Controller		% improvement using new controller	
B	I.A.E.	τ_s (2%) seconds	I.A.E.	τ_s (2%) seconds	I.A.E.	τ_s (2%) seconds
4.0	2.26	9.0	1.24	3.6	43%	60%
2.0	2.15	9.0	1.31	4.5	39.1%	50%
1.0	1.85	9.0	1.32	5.6	28.6%	37.8%
0.5	1.51	9.0	1.325	9.0	11.9%	0%

Table 2. I.A.E. and τ_s comparison of the Proportional-Plus-Integral Controller and the new controller for various values of B with T = 1.0 for a unit step disturbance.

Table 2 shows that the new controller provides significant improvement in most disturbance responses. The degree of improvement increases as the value of B increases, indicating that the new controller would be most judiciously used on processes which are dominated by dead time elements.

The disturbance response for a pure dead time plant is plotted in Figure 17. Instead of a proportional-plus-integral controller, Mr. Haalman recommends only integral control with $\tau_2 = \frac{3T}{2}$. Thus, the disturbance response was calculated for a controller of the form $G_c(S) = \frac{1}{\tau_2 S}$, where $\tau_2 = \frac{3T}{2}$ and the new controller of the form $G_c(S) =$

$$\frac{3.33}{S+3.33-3.33e^{-S}} .$$

The new controller configuration resulted in a 41% decrease in the I.A.E. and a 67.7% reduction in the 2% settling time.

The integral controller response and associated I.A.E. were calculated by DYSIM***. A detailed discussion is given in Appendix B. The regular block diagram is shown in Figure 18.

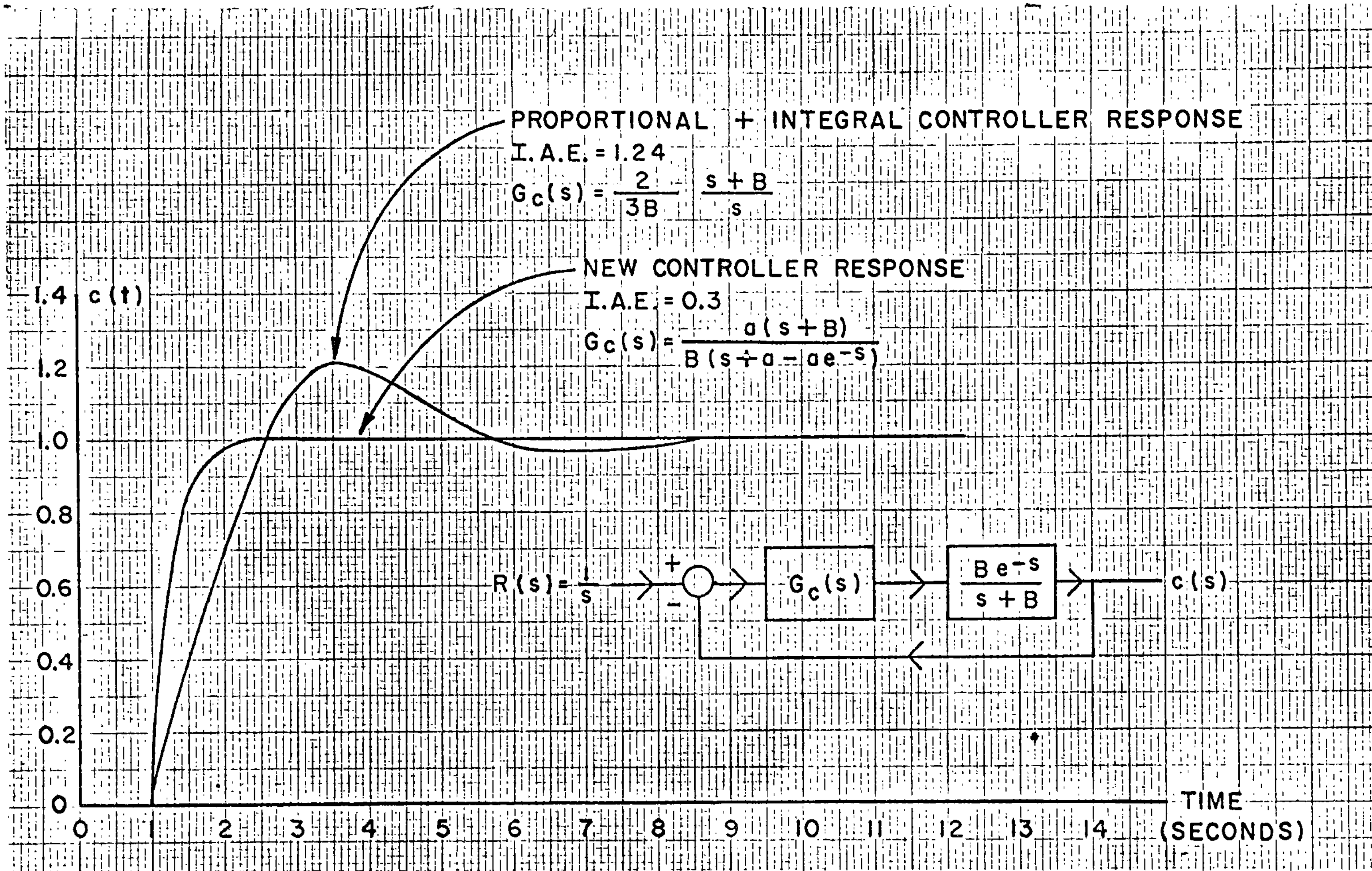

Figure 12: Setpoint Response with T=1.0.

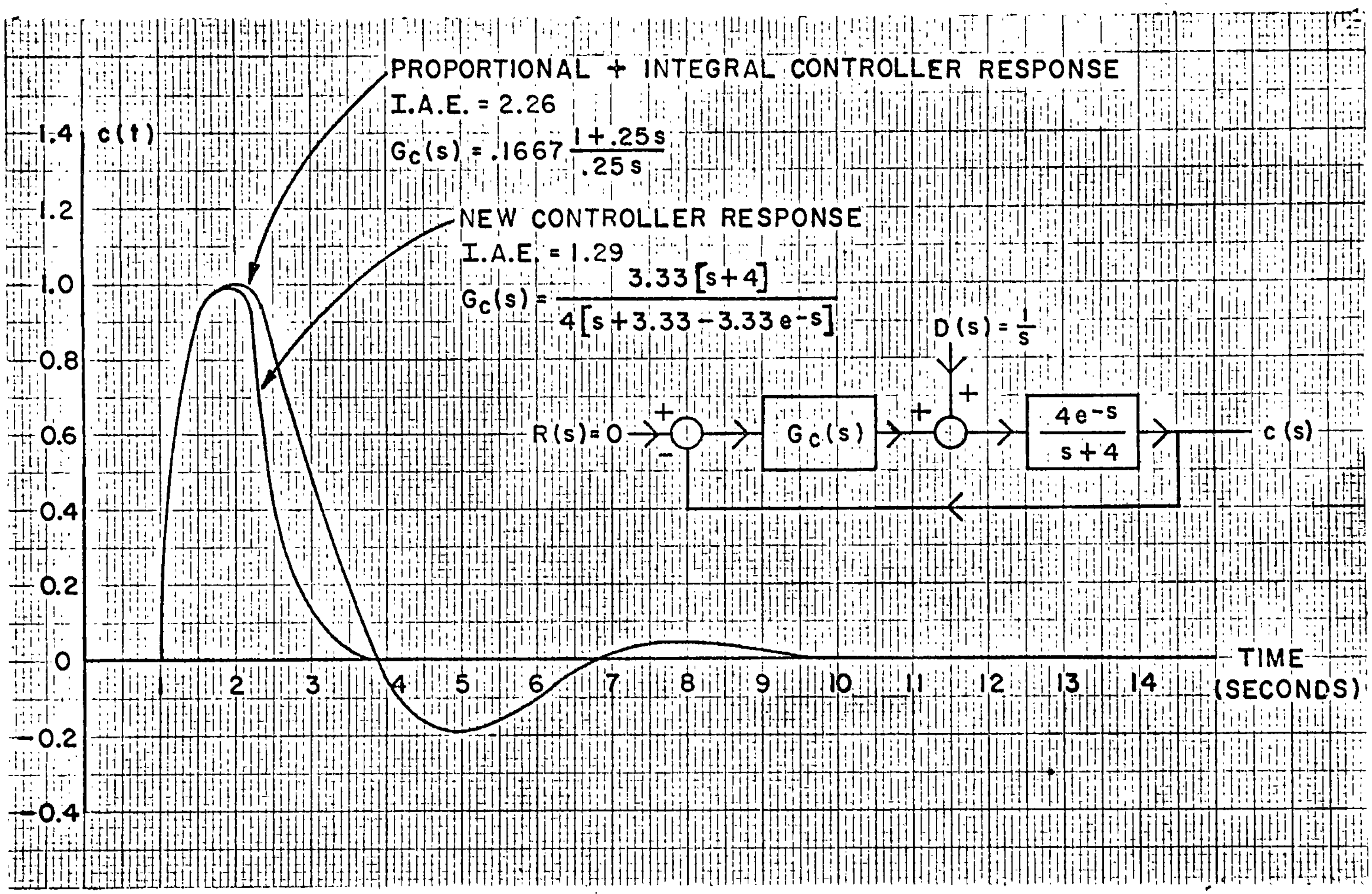

Figure 13: Disturbance Response with B=4 and T=1.

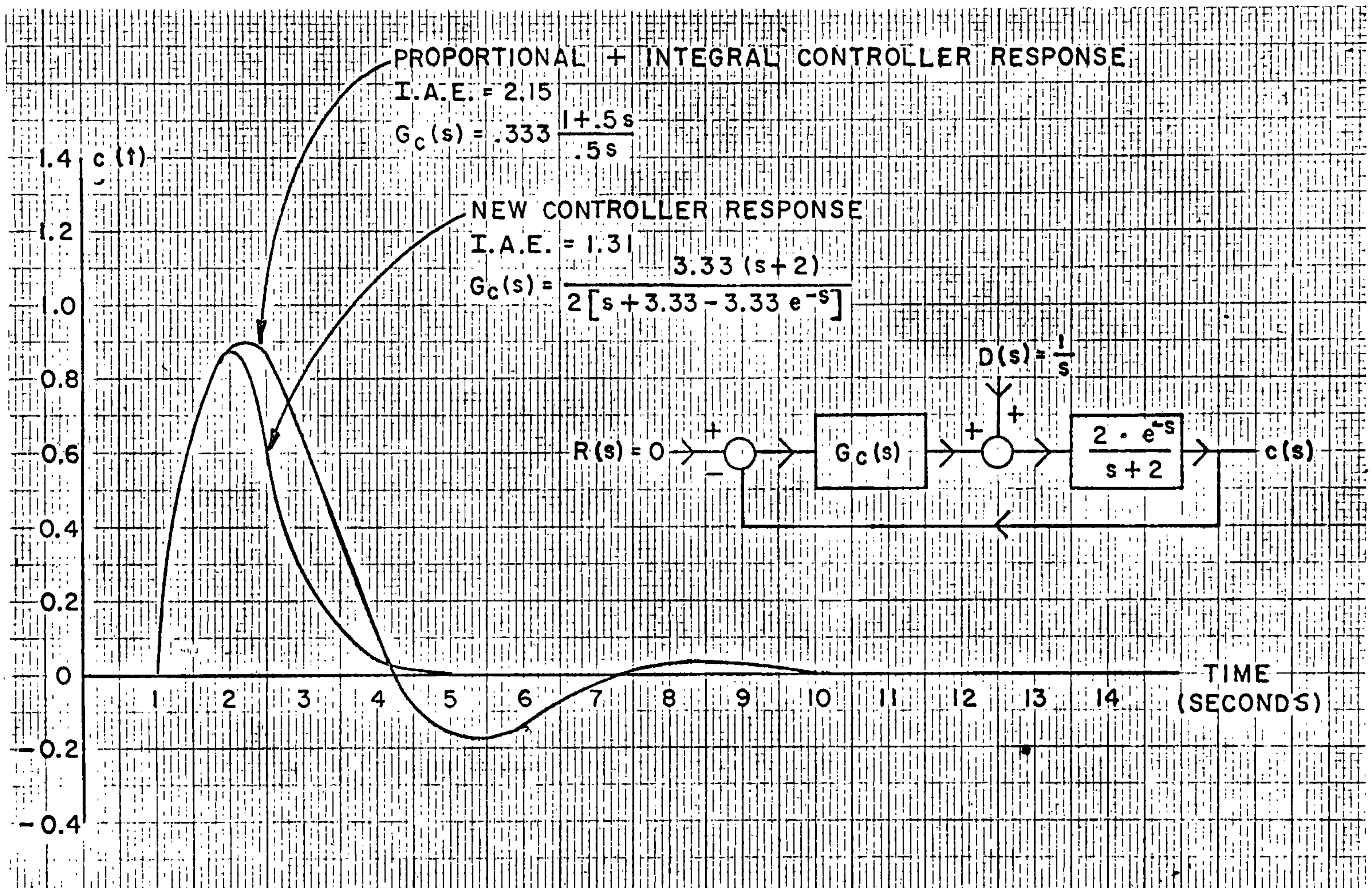

Figure 14: Disturbance Response with B=2 and T=1.

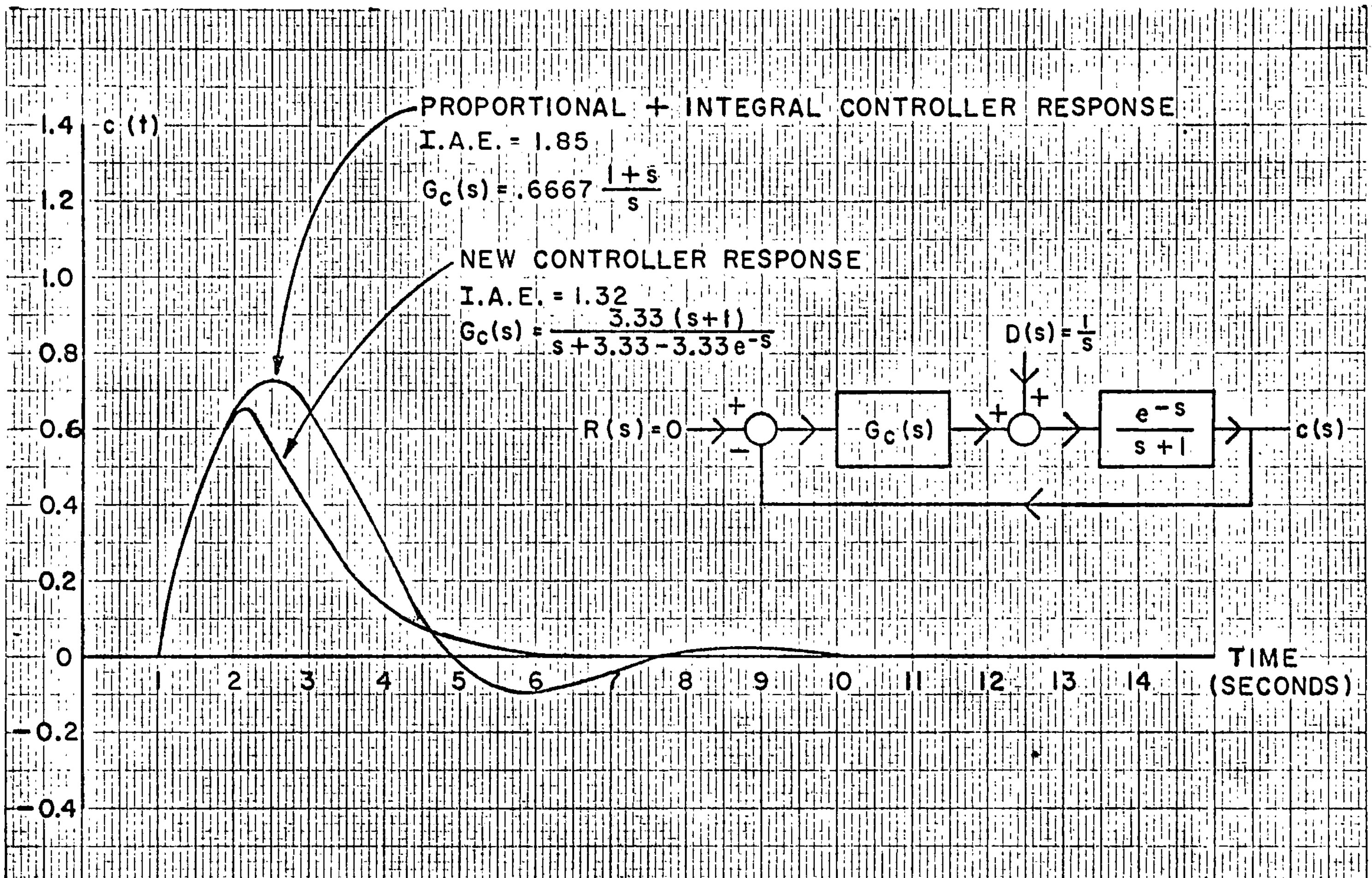

Figure 15: Disturbance Response with B=1 and T=1.

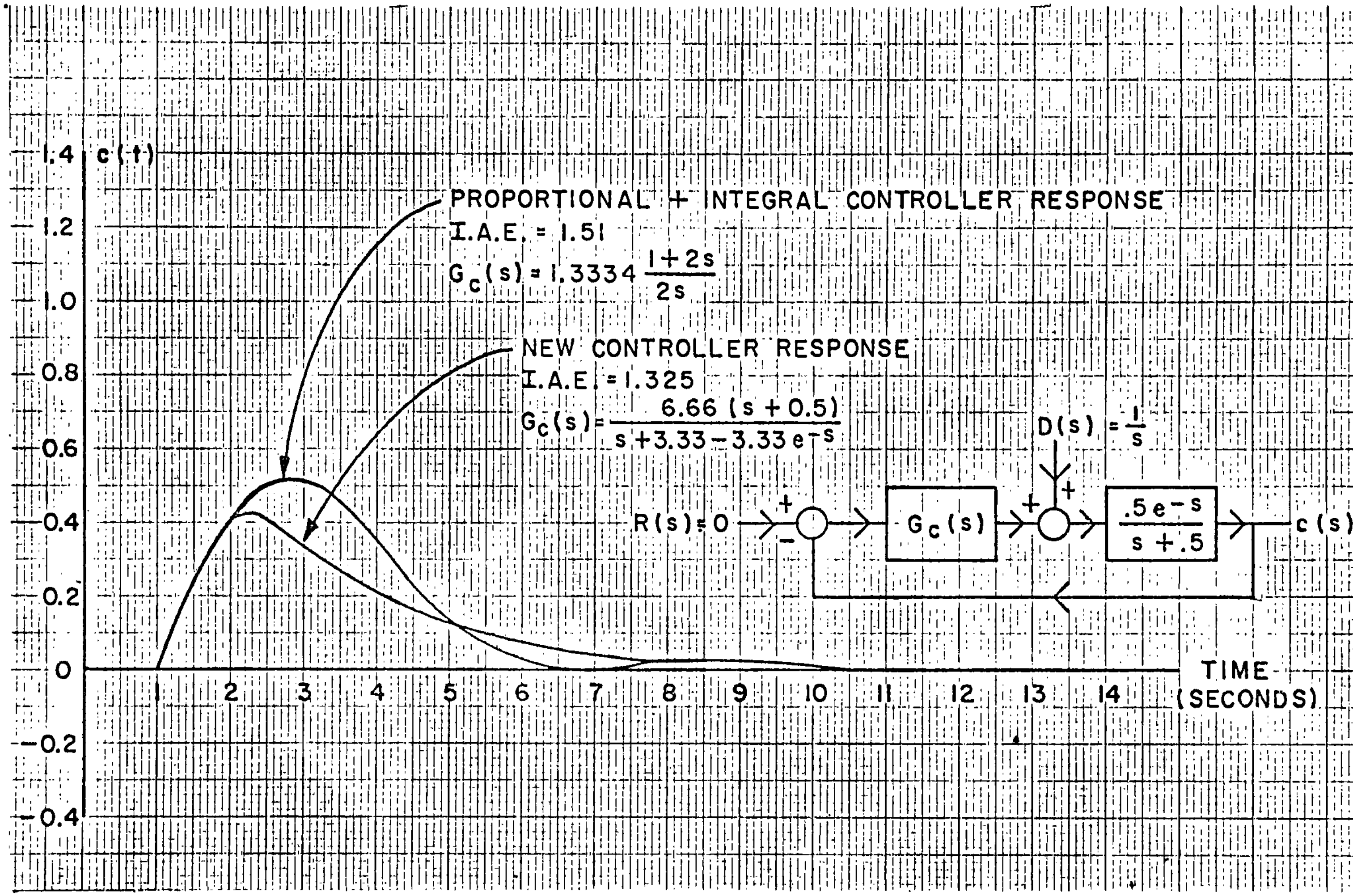

Figure 16: Disturbance Response with B=0.5 and T=1.

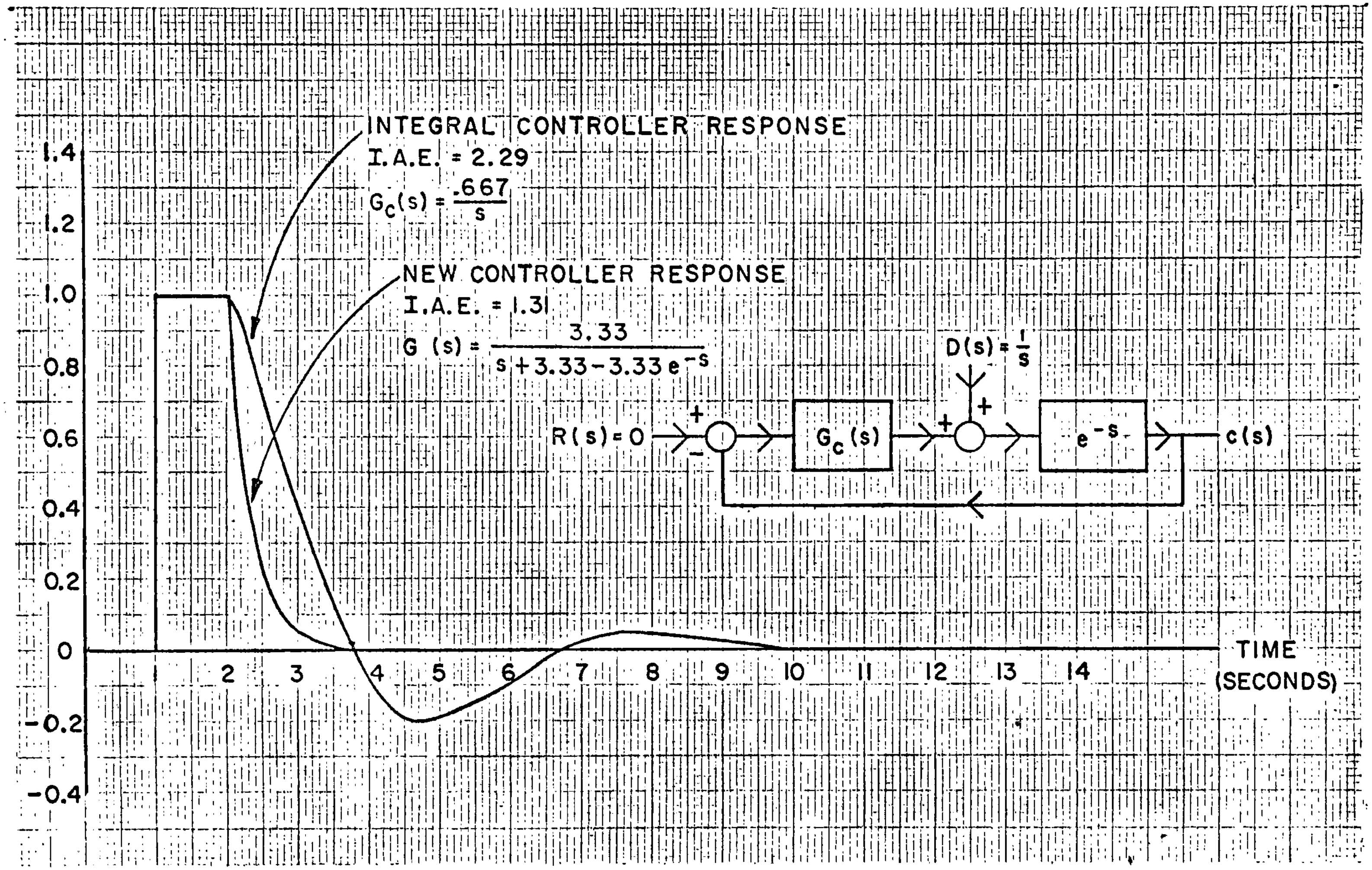

Figure 17: Disturbance Response with B=∞ and T=1.

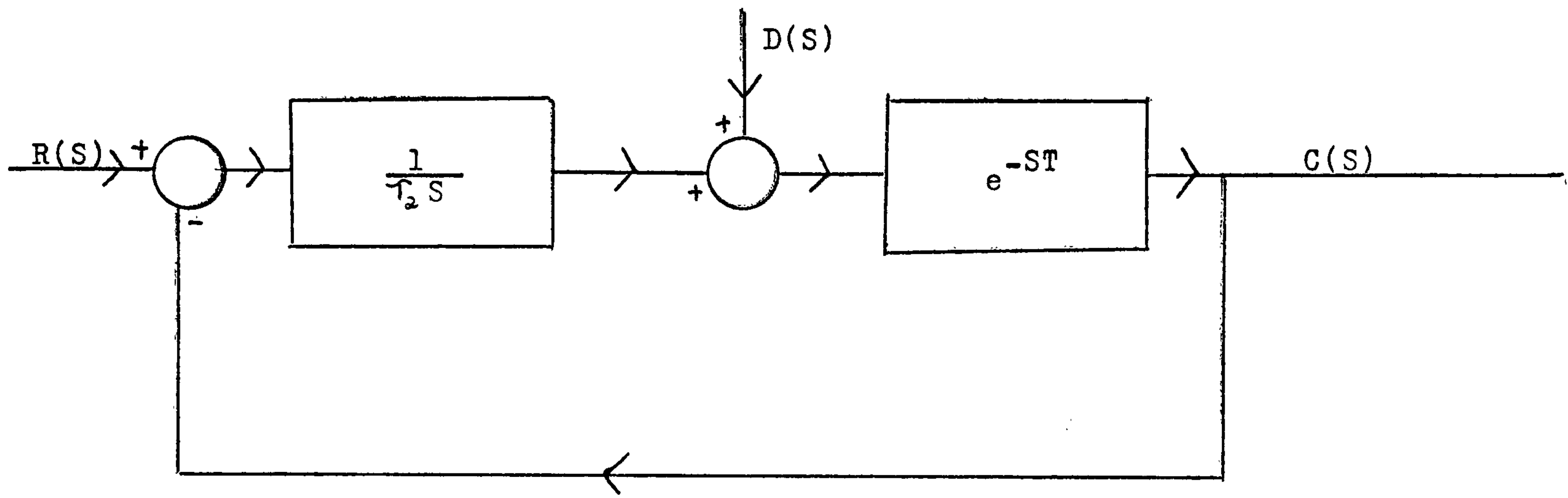

Figure 18: Integral control of pure dead time plant.

VIII. EFFECTS OF MISMATCH IN PROCESS AND CONTROLLER DEAD TIME VALUES

The effect on the setpoint response of a mismatch between the process dead time and the dead time value in the new controller can be determined by analysis of the system diagrammed below. The dead time value in the new controller is labeled X.

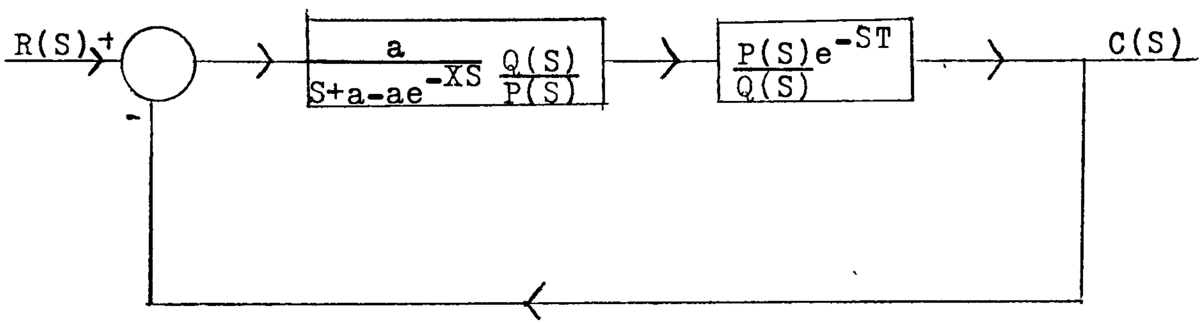

Figure 19: Block diagram of a plant having a dead time element and controlled by the new controller.

The transfer function between the setpoint and output can be written as

$$\frac{C(S)}{R(S)} = \frac{\dfrac{a}{S+a-ae^{-XS}}\,\dfrac{Q(S)}{P(S)}\,\dfrac{P(S)}{Q(S)}\,e^{-ST}}{1+\dfrac{a}{S+a-ae^{-XS}}\,\dfrac{Q(S)}{P(S)}\,\dfrac{P(S)}{Q(S)}\,e^{-ST}}$$

$$= \frac{ae^{-ST}}{S+a-ae^{-XS}+ae^{-ST}}$$

$$= \frac{ae^{-ST}}{(S+a)\left[1+\dfrac{a(e^{-ST}-e^{-XS})}{S+a}\right]}\,.$$

By application of the expansion formula

$$\frac{1}{1+Z} = \sum_{n=0}^{\infty} (-1)^n Z^n$$

the previous equation can be rewritten as

$$\frac{C(S)}{R(S)} = \frac{ae^{-ST}}{S+a}\left[1 - \frac{a(e^{-ST}-e^{-XS})}{S+a} + \frac{a^2(e^{-ST}-e^{-XS})^2}{(S+a)^2} - \frac{a^3(e^{-ST}-e^{-XS})^3}{(S+a)^3} + \frac{a^4(e^{-ST}-e^{-XS})^4}{(S+a)^4} \cdots\right].$$

Letting $R(S) = \frac{1}{S}$, the transform of the output, after expansion, becomes

$$C(S) = \frac{ae^{-ST}}{S(S+a)}\left[1 - \frac{a(e^{-ST}-e^{-XS})}{S+a} + \frac{a^2(e^{-2ST}-2e^{-(X+T)S}+e^{-2XS})}{(S+a)^2} - \frac{a^3(e^{-3ST}-3e^{-(X+2T)S}+3e^{-(2X+T)S}-e^{-3XS})}{(S+a)^3} + \cdots\right],$$

which can be rewritten as

$$C(S) = \frac{ae^{-ST}}{S(S+a)} - \frac{a^2(e^{-2ST}-e^{-(X+T)S})}{S(S+a)^2} + \frac{a^3(e^{-3ST}-2e^{-(X+2T)S}+e^{-(2X+T)S})}{S(S+a)^3} - \frac{a^4(e^{-4ST}-3e^{-(X+3T)S}+3e^{-(2X+2T)S}-e^{-(3X+T)S})}{S(S+a)^4} + \cdots .$$

Performing a partial fraction expansion yields

$$C(S) = \left[\frac{A}{S} + \frac{B}{S+a}\right]e^{-ST} + \left[\frac{C}{S} + \frac{D}{(S+a)^2} + \frac{E}{S+a}\right] \cdot$$

$$\left[e^{-2ST} - e^{-(X+T)S}\right] + \left[\frac{F}{S} + \frac{G}{(S+a)^3} + \frac{H}{(S+a)^2} + \frac{I}{S+a}\right] \cdot$$

$$\left[e^{-3ST} - 2e^{-(X+2T)S} + e^{-(2X+T)S}\right] +$$

$$\left[\frac{J}{S} + \frac{K}{(S+a)^4} + \frac{L}{(S+a)^3} + \frac{M}{(S+a)^2} + \frac{N}{S+a}\right] \cdot$$

$$\left[e^{-4ST} - 3e^{-(X+3T)S} + 3e^{-(2X+2T)S} - e^{-(3X+T)S}\right] \ldots .$$

The residues can be calculated as

$$A = \left.\frac{a}{S+a}\right|_{S=0} = 1,$$

$$B = \left.\frac{a}{S}\right|_{S=-a} = -1,$$

$$C = \left.\frac{-a^2}{(S+a)^2}\right|_{S=0} = -1,$$

$$D = \left.\frac{-a^2}{S}\right|_{S=-a} = a,$$

$$E = \left.\frac{a^2}{S^2}\right|_{S=-a} = 1,$$

$$F = \left.\frac{a^3}{(S+a)^3}\right|_{S=0} = 1,$$

$$G = \left.\frac{a^3}{S}\right|_{S=-a} = -a^2$$

$$H = \left.\frac{-a^3}{S^2}\right|_{S=-a} = -a,$$

$$I = \left.\frac{a^3}{S^3}\right|_{S=-a} = -1,$$

$$J = \left.\frac{-a^4}{(S+a)^4}\right|_{S=0} = -1,$$

$$K = \left.\frac{-a^4}{S}\right|_{S=-a} = a^3,$$

$$L = \left.\frac{a^4}{S^2}\right|_{S=-a} = a^2,$$

$$M = \left.\frac{-a^4}{S^3}\right|_{S=-a} = a,$$

and $N = \left.\frac{+a^4}{S^4}\right|_{S=-a} = 1.$

Thus, $c(t) = y_1(t-T)u(t-T) + y_2(t-2T)u(t-2T) -$

$y_2(t-X-T)u(t-X-T) + y_3(t-3T)u(t-3T) -$

$2y_3(t-X-2T)u(t-X-2T) + y_3(t-2X-T)u(t-2X-T) +$

$y_4(t-4T)u(t-4T) - 3y_4(t-X-3T)u(t-X-3T) +$

$3y_4(t-2X-2T)u(t-2X-2T) - y_4(t-3X-T)u(t-3X-T)$

$- - - -,$

where $y_1(t) = \mathcal{L}^{-1}\left[\frac{A}{S} + \frac{B}{S+a}\right] = 1-e^{-at}$,

$$y_2(t) = \mathcal{L}^{-1}\left[\frac{C}{S} + \frac{D}{(S+a)^2} + \frac{E}{S+a}\right] = -1+ate^{-at}+e^{-at},$$

$$y_3(t) = \mathcal{L}^{-1}\left[\frac{F}{S} + \frac{G}{(S+a)^3} + \frac{H}{(S+a)^2} + \frac{I}{S+a}\right] =$$

$$1 - \frac{1}{2}a^2t^2e^{-at} - ate^{-at} - e^{-at},$$

and $$y_4(t) = \mathcal{L}^{-1}\left[\frac{J}{S} + \frac{K}{(S+a)^4} + \frac{L}{(S+a)^3} + \frac{M}{(S+a)^2} + \frac{N}{S+a}\right] =$$

$$-1 + \frac{1}{6}a^3t^3e^{-at} + \frac{1}{2}a^2t^2e^{-at} + ate^{-at} + e^{-at}.$$

A digital computer was used to generate plots of c as a function of time for various values of a and X with T set equal to unity. The program was written in Basic language. A printout of this program is listed in Figures 20 and 21. Plots of c(t) for T=1 and a = 10, 5, 3.33, 2.5, and 2 are diagrammed in Figures 22, 23, 24, 25, and 26, respectively. In each plot the value of X was set equal to both .95 and 1.05, representing a plus and minus 5% mismatch between the process dead time and the controller dead time. Plots of c(t) for plus and minus 10% mismatches in the value of the dead time using the same values of a as above are shown in Figures 27, 28, 29, 30, and 31. For T=1, a=2.5, and X=1.2 and 0.8, the output response of the same system with a

20% mismatch is shown in Figure 32.

Analysis of these response plots indicates that the value of the parameter a considerably effects the shape of the response. In general, as a is increased in value the system exhibits larger peaks in the transient response, indicating that the system stability is reduced.

Using minimum I.A.E. to define an optimum criteria can be seen to be deficient by observing the curve for a=10 in Figure 22. This response as can readily be determined from observation of the plots has the smallest value of I.A.E., but the degree of smoothness is poor. An operator observing this response could think the process was in control after a time duration of about 1.5 seconds. However, an unexpected peak on the order of 20% at t=2.1 seconds and another peak on the order of 5% at t=3.0 seconds actually results in the output remaining outside a 2% error band for about 3.5 seconds. Consequently, the optimum value of a was defined as the value which minimizes the time for the output to come and stay within 2% of its final value.

In Figure 33 a plot of 2% settling time versus 1/a is shown for both 5% and 10% mismatches between plant and controller dead time. The plotted values of settling time are the worst case values when both plus and minus percentages of mismatch are considered. This graph indicates that a value of a=5 is optimum for a minimum 2% settling when a 5% mismatch is present while a value of a=3.33 is optimum when a 10% mismatch exists. The operator would be required to make an estimate of the probable mismatch variation for the particular plant under consideration and select a value of a based on this.

```
100 X=1.05
105 PRINT "X=",X
107 PRINT
110 T1=0
120 T1=T1+0.1
130 IF T1>=.75 THEN 750
140 PRINT
150 PRINT "T1=",T1
160 PRINT
170 A=1.0/T1
180 T=0.8
190 T=T+0.2
195 Q=0
200 Y1=0
210 Y2=0
220 Y3=0
230 Y4=0
240 Y5=0
250 Y6=0
260 Y7=0
270 Y8=0
280 Y9=0
290 IF T-1>=-.005 THEN 420
300 IF T-2>=-.005 THEN 440
310 IF T-1-X>=-.005 THEN 460
320 IF T-3>=-.005 THEN 480
330 IF T-X-2>=-.005 THEN 510
340 IF T-2*X-1>=-.005 THEN 540
350 IF T-4>=-.005 THEN 570
380 IF T-3-X>=-.005 THEN 600
390 IF T-2*X-2>=-.005 THEN 640
400 IF T-3*X-1>=-.005 THEN 690
410 GO TO 710
```

Figure 20: Computer program used to calculate c(t) when T and X are unequal. This listing is continued on the next page.

```
420 Y1=1-EXP(-A*(T-1))
430 GØ TØ 300
440 Y2=A*(T-2)*EXP(-A*(T-2))+EXP(-A*(T-2))-1
450 GØ TØ 310
460 Y3=-A*(T-1-X)*EXP(-A*(T-1-X))-EXP(-A*(T-1-X))+1
470 GØ TØ 320
480 X1=(-.5*A↑2*(T-3)↑2-A*(T-3)-1)*EXP(-A*(T-3))
490 Q=X1+1.0
500 GØ TØ 330
510 X2=(-.5*A↑2*(T-X-2)↑2-A*(T-X-2)-1)*EXP(-A*(T-X-2))
520 Y4=(X2+1.0)*(-2)
530 GØ TØ 340
540 X3=-.5*A↑2*(T-2*X-1)↑2-A*(T-2*X-1)-1
545 R5=X3*EXP(-A*(T-2*X-1))
550 Y5=R5+1.0
560 GØ TØ 350
570 X4=(A↑3/6)*(T-4)↑3+A↑2*0.5*(T-4)↑2+A*(T-4)+1
580 Y6=X4*EXP(-A*(T-4))-1
590 GØ TØ 380
600 X5=(A↑3/6)*(T-3-X)↑3+A↑2*0.5*(T-3-X)↑2+A*(T-3-X)+1
610 X6=X5*EXP(-A*(T-3-X))-1
620 Y7=-3*X6
630 GØ TØ 390
640 X7=(A↑3/6)*(T-2*X-2)↑3+A↑2*0.5*(T-2*X-2)↑2+A*(T-2*X-2)+1
650 X8=X7*EXP(-A*(T-2*X-2))-1
670 Y8=3*X8
680 GØ TØ 400
690 R1=(A↑3/6)*(T-3*X-1)↑3+A↑2*0.5*(T-3*X-1)↑2+A*(T-3*X-1)+1
700 Y9=-R1*EXP(-A*(T-3*X-1))+1
710 Y=Y1+Y2+Y3+Y4+Y5+Y6+Y7+Y8+Y9+Q
720 PRINT T,Y
730 IF T<=5.1 THEN 190
740 GØ TØ 120
750 END
```

Figure 21: Continued listing of Figure 20.

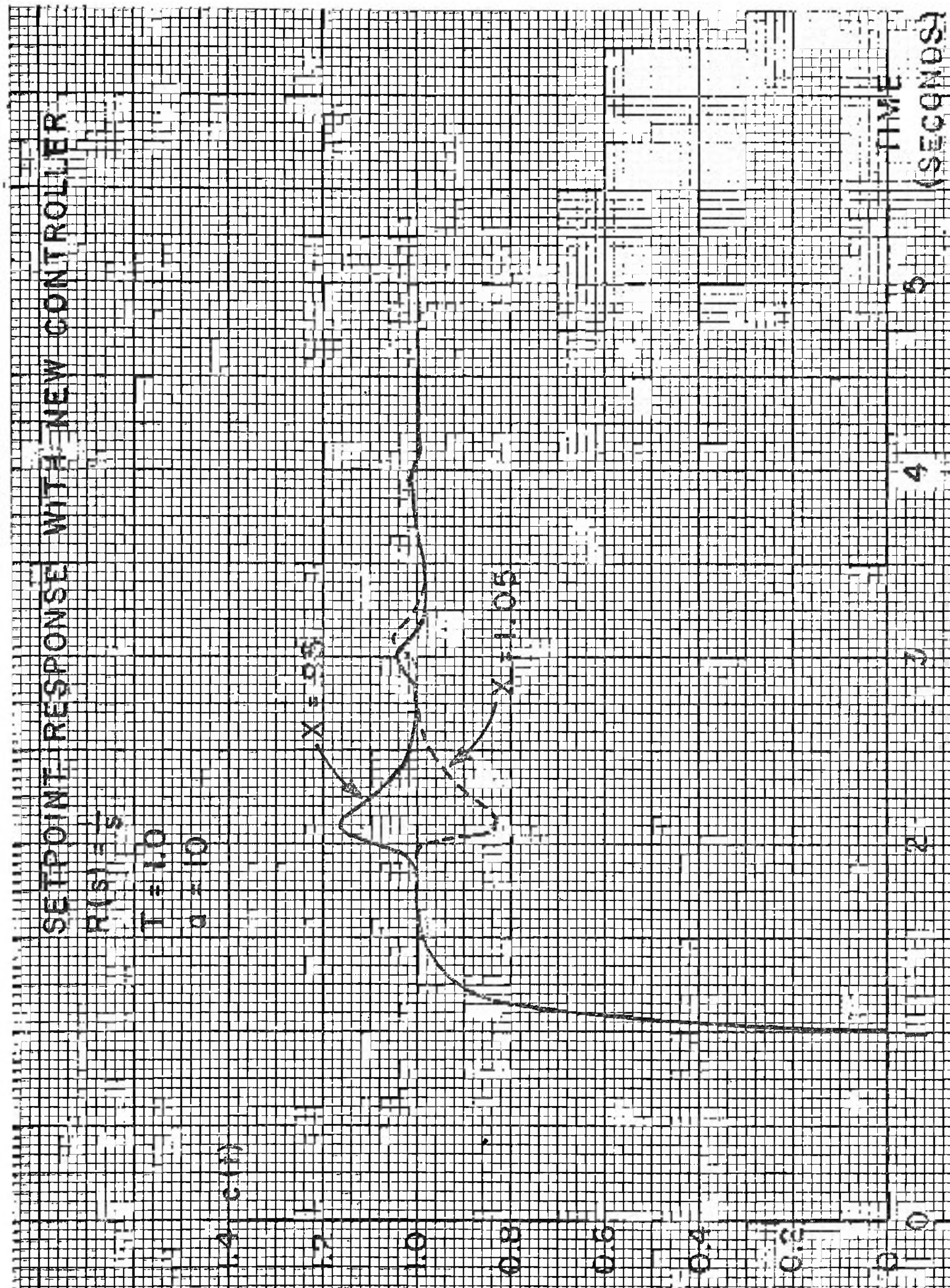
SETPOINT RESPONSE WITH NEW CONTROLLER
R(s) = 1/s
T = 1.0
a = 10
c(t)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
X = .95
X = 1.05
0
2
3
4
5
TIME
(SECONDS)

Figure 22

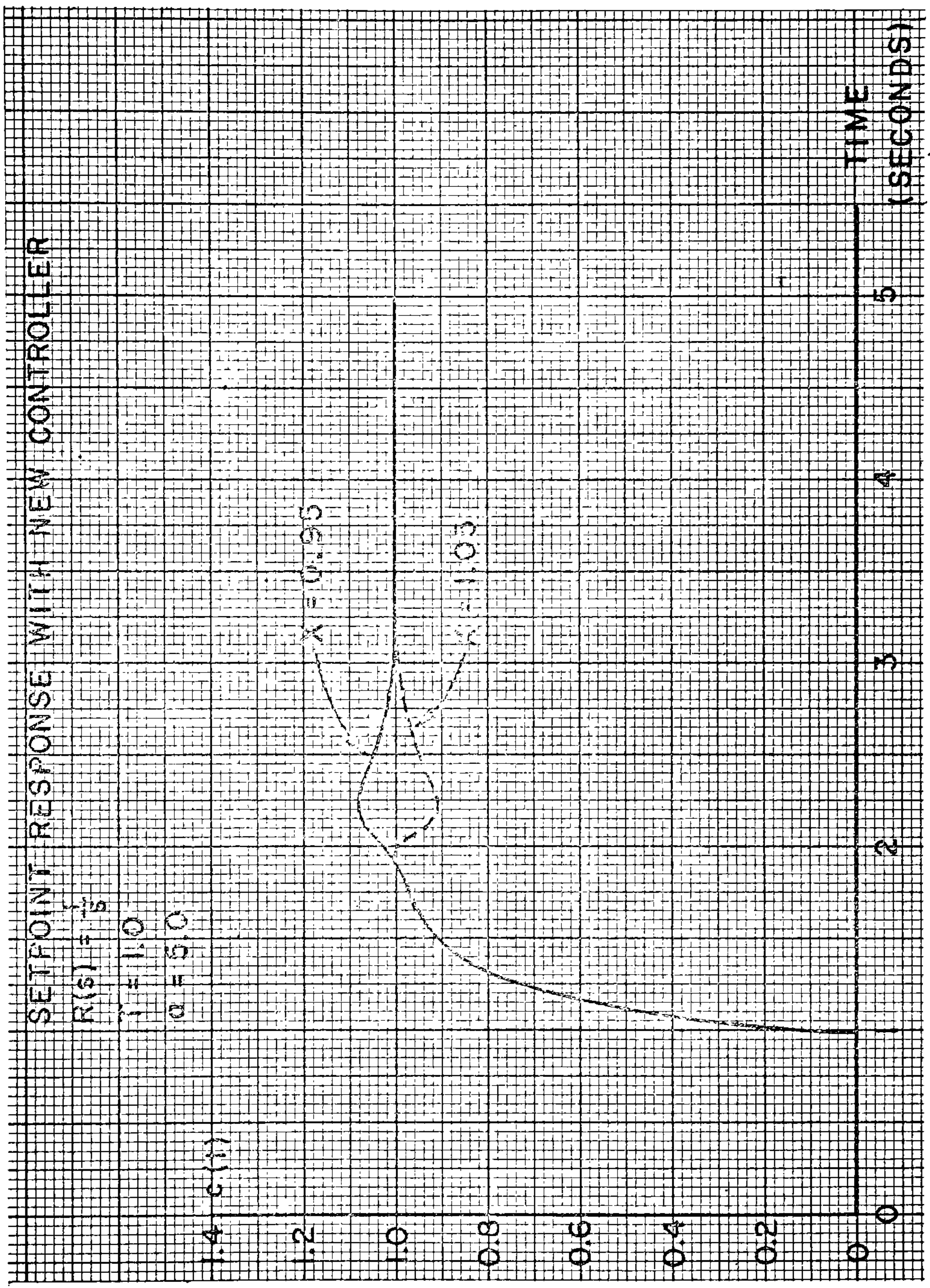
SETPOINT RESPONSE WITH NEW CONTROLLER
R(s) = 1/s
T = 1.0
a = 5.0
c(t)
K = 0.95
K = 1.05
TIME (SECONDS)
0
1
2
3
4
5
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Figure 23

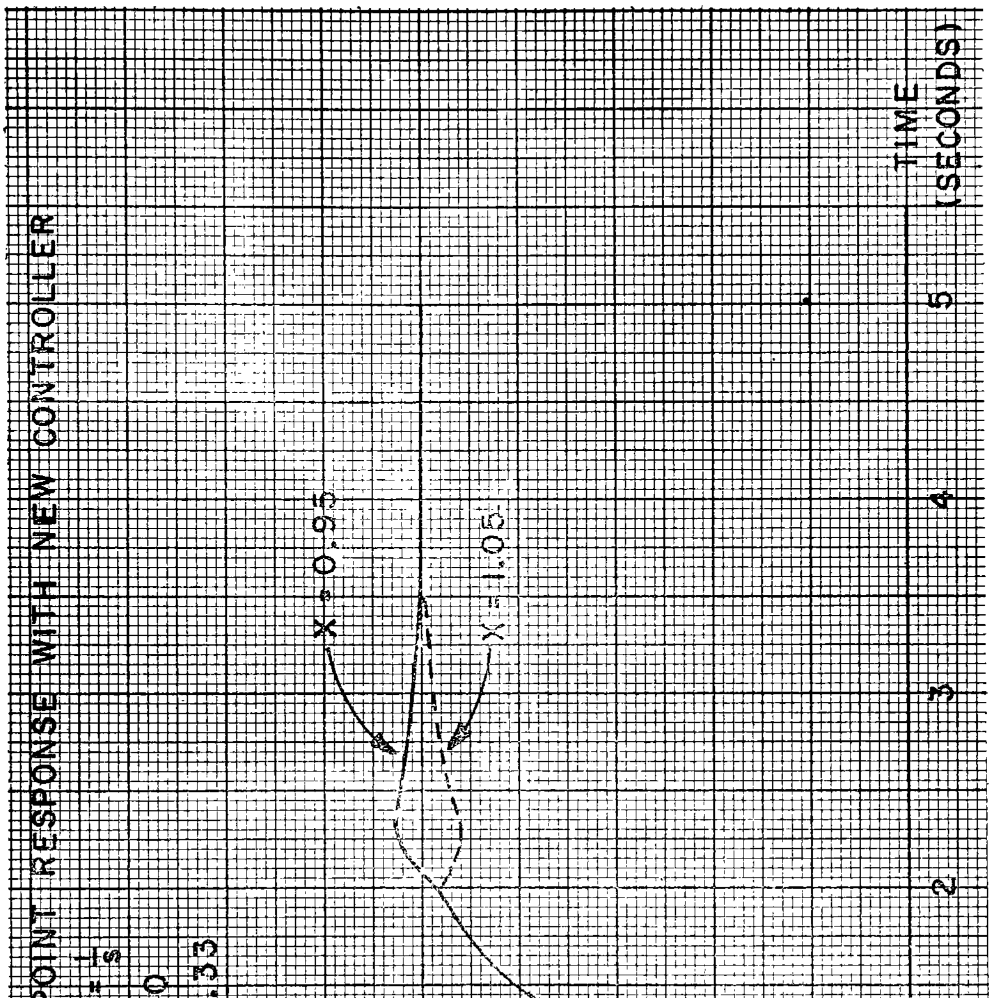
SETPOINT RESPONSE WITH NEW CONTROLLER
R(s) = 1/s
T = 1.0
a = 3.33
c(t)
X = 0.95
X = 1.05
TIME (SECONDS)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1
2
3
4
5

Figure 24

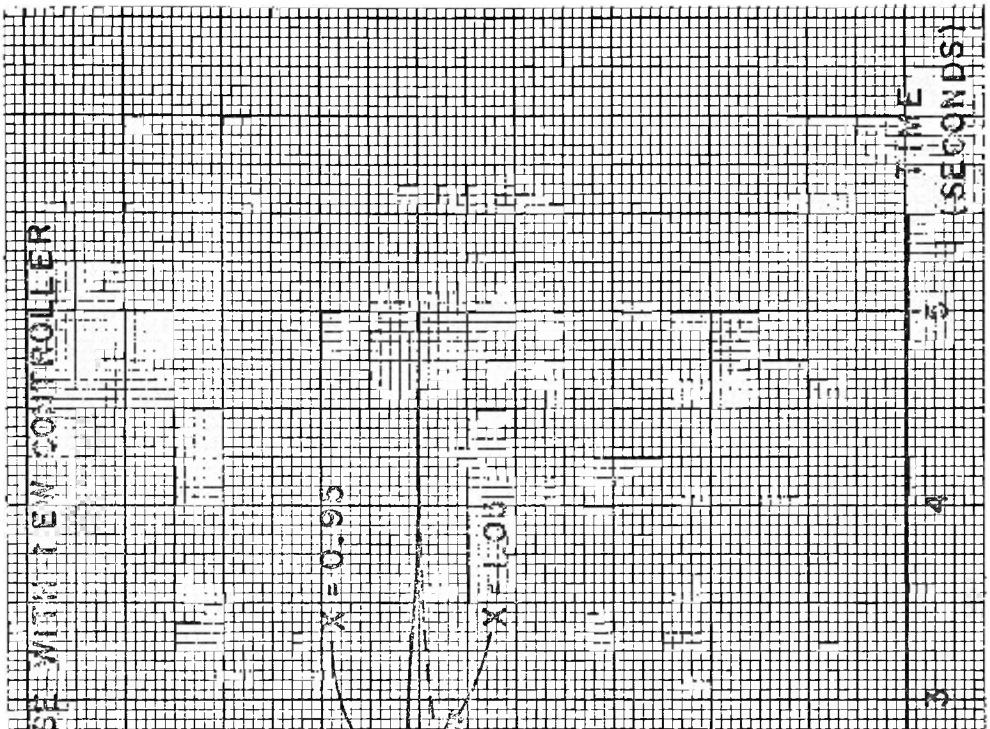
SETPOINT RESPONSE WITH NEW CONTROLLER
R(s) = 1/s
T = 1.0
a = 2.5
X = 0.95
c(t)
TIME (SECONDS)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
2
3
4
5

Figure 25

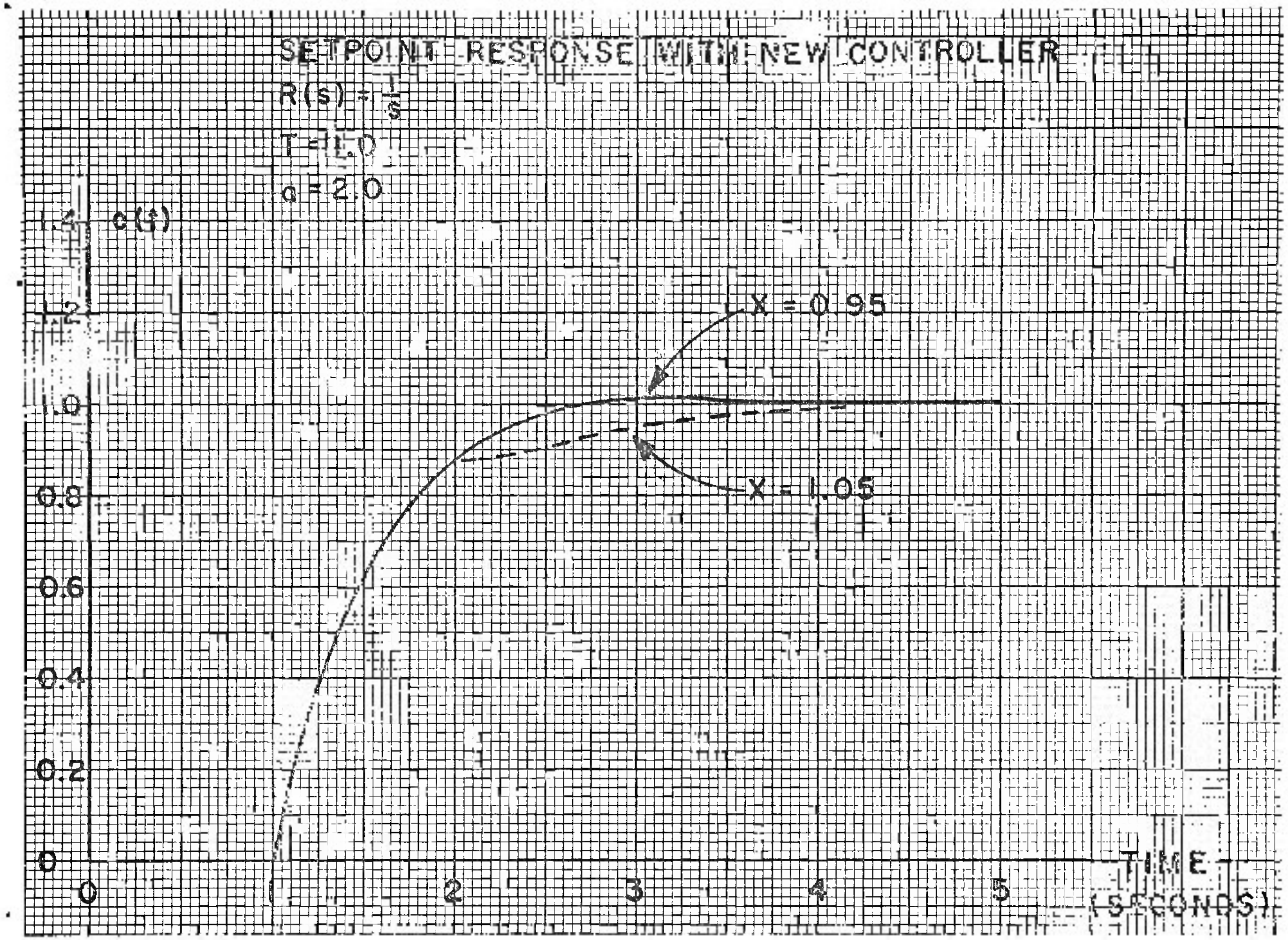
SETPOINT RESPONSE WITH NEW CONTROLLER
R(s) = 1/s
T = 1.0
a = 2.0
c(t)
X = 0.95
X = 1.05
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
2
3
4
5
TIME
(SECONDS)

Figure 26

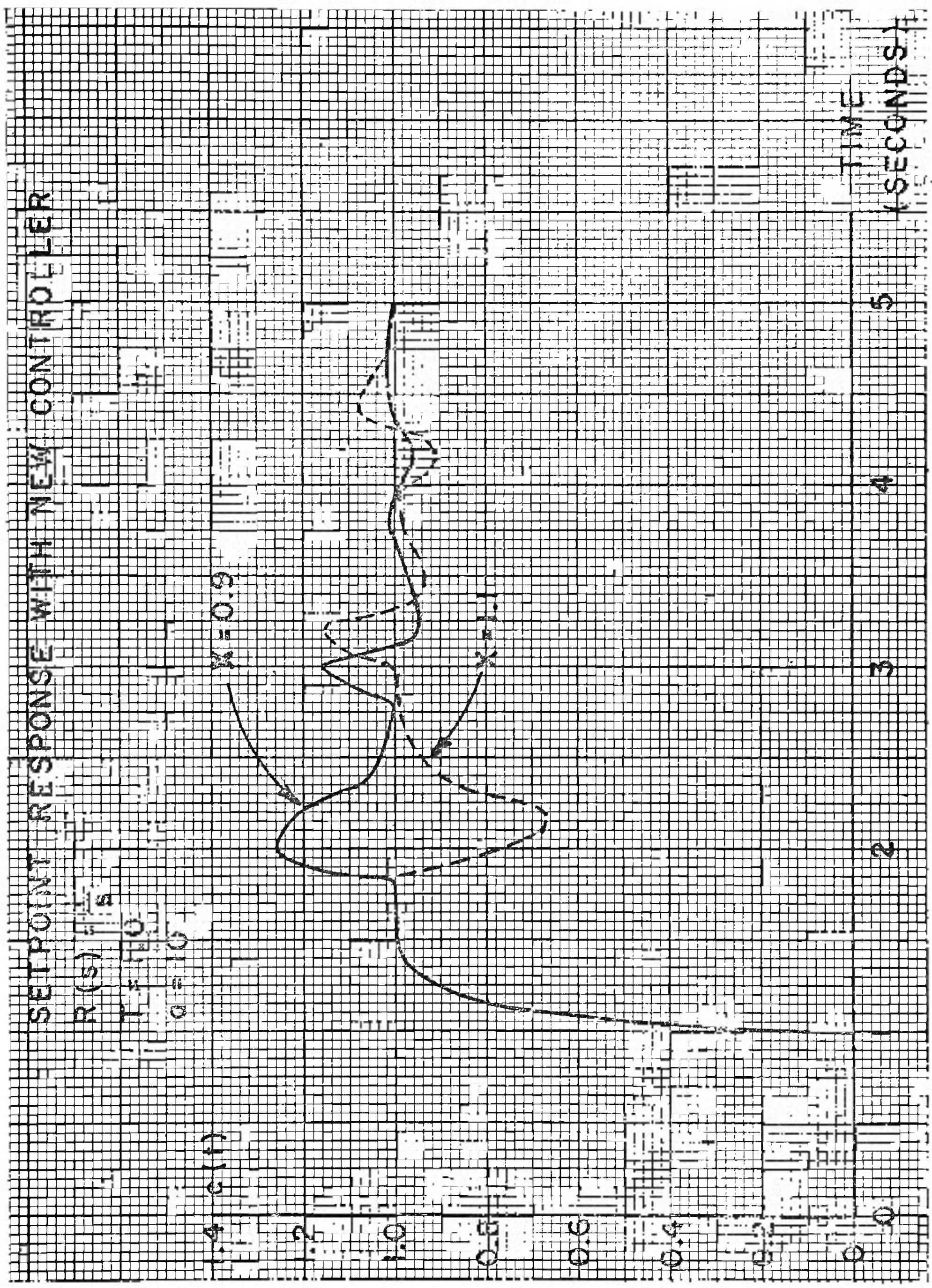
SETPOINT RESPONSE WITH NEW CONTROLLER
K=0.9
K=1.1
TIME (SECONDS)
c(t)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
2
3
4
5

Figure 27

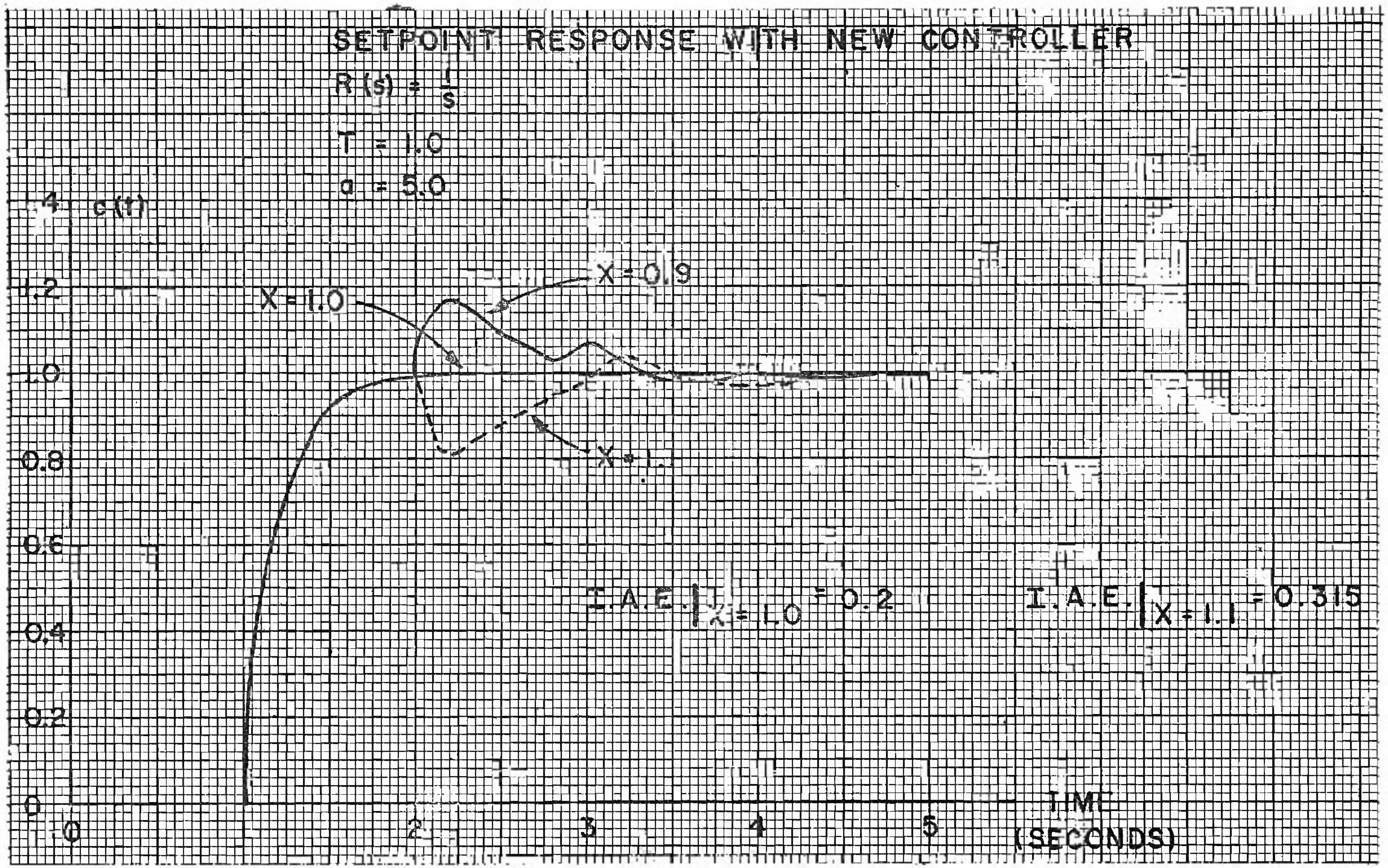
SETPOINT RESPONSE WITH NEW CONTROLLER
R(s) = 1/s
T = 1.0
a = 5.0
c(t)
X=1.0
X=0.9
X=1.1
I.A.E.|X=1.0 = 0.2
I.A.E.|X=1.1 = 0.315
TIME (SECONDS)
0
2
3
4
5
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Figure 28

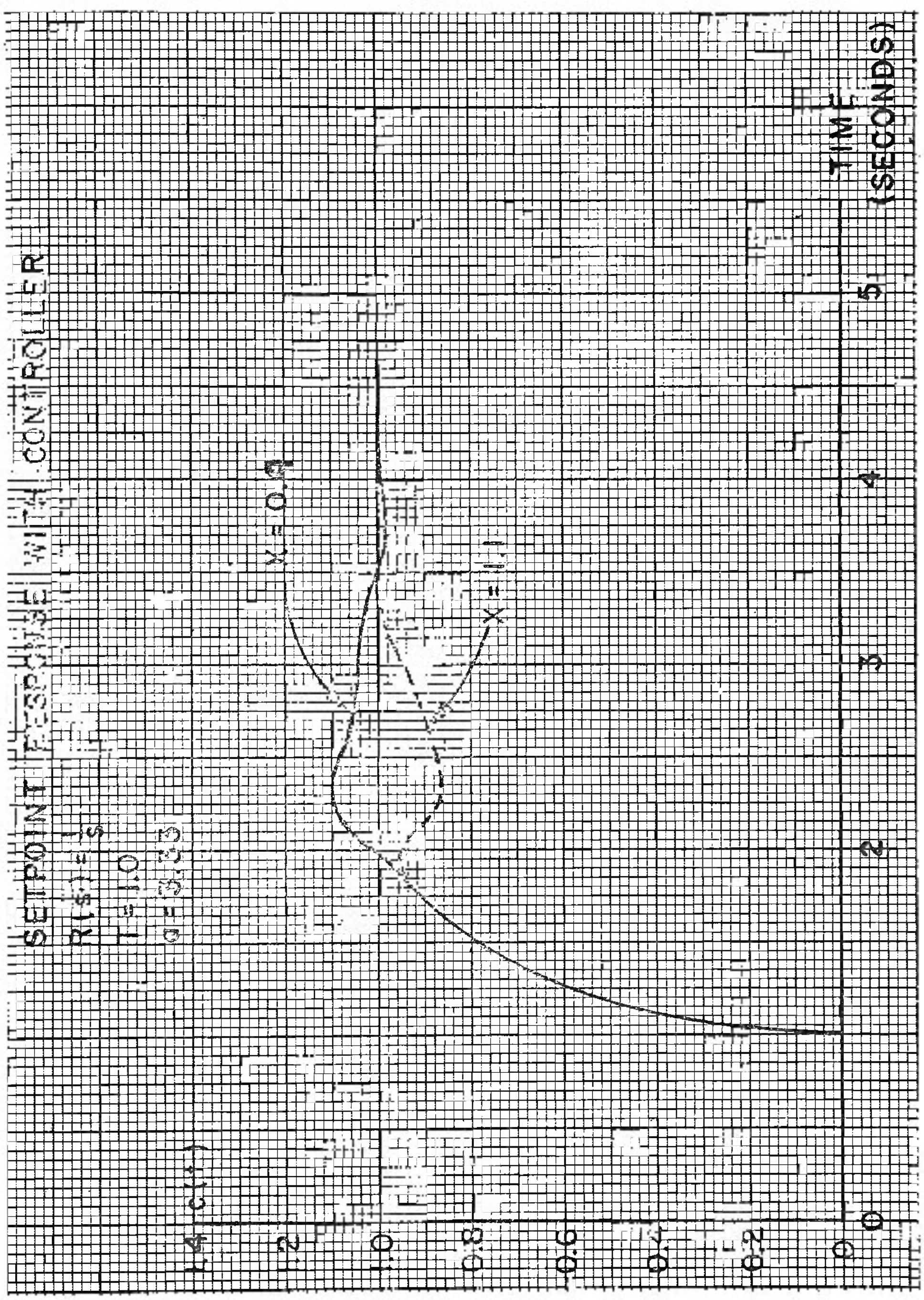
SETPOINT RESPONSE WITH CONTROLLER
R(s) = 1/s
T = 1.0
a = 3.33
C(t)
X = 0.9
X = 1.1
TIME (SECONDS)
0
1
2
3
4
5
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Figure 29

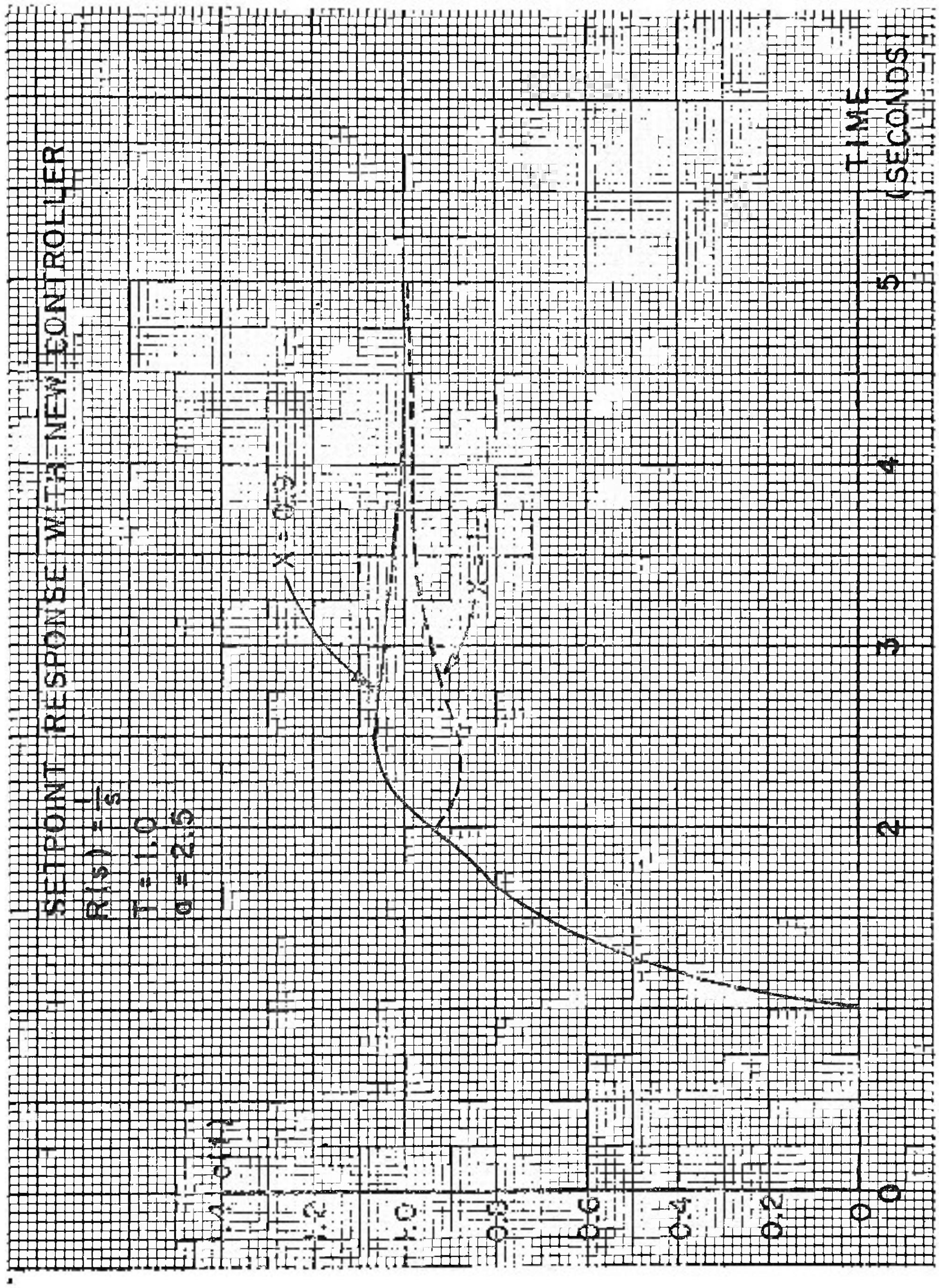
SETPOINT RESPONSE WITH A NEW CONTROLLER
T = 1.0
a = 2.5
X = 0.9
TIME (SECONDS)
c(t)
0
0.2
0.4
0.6
0.8
1.0
1.2
2
3
4
5

Figure 30

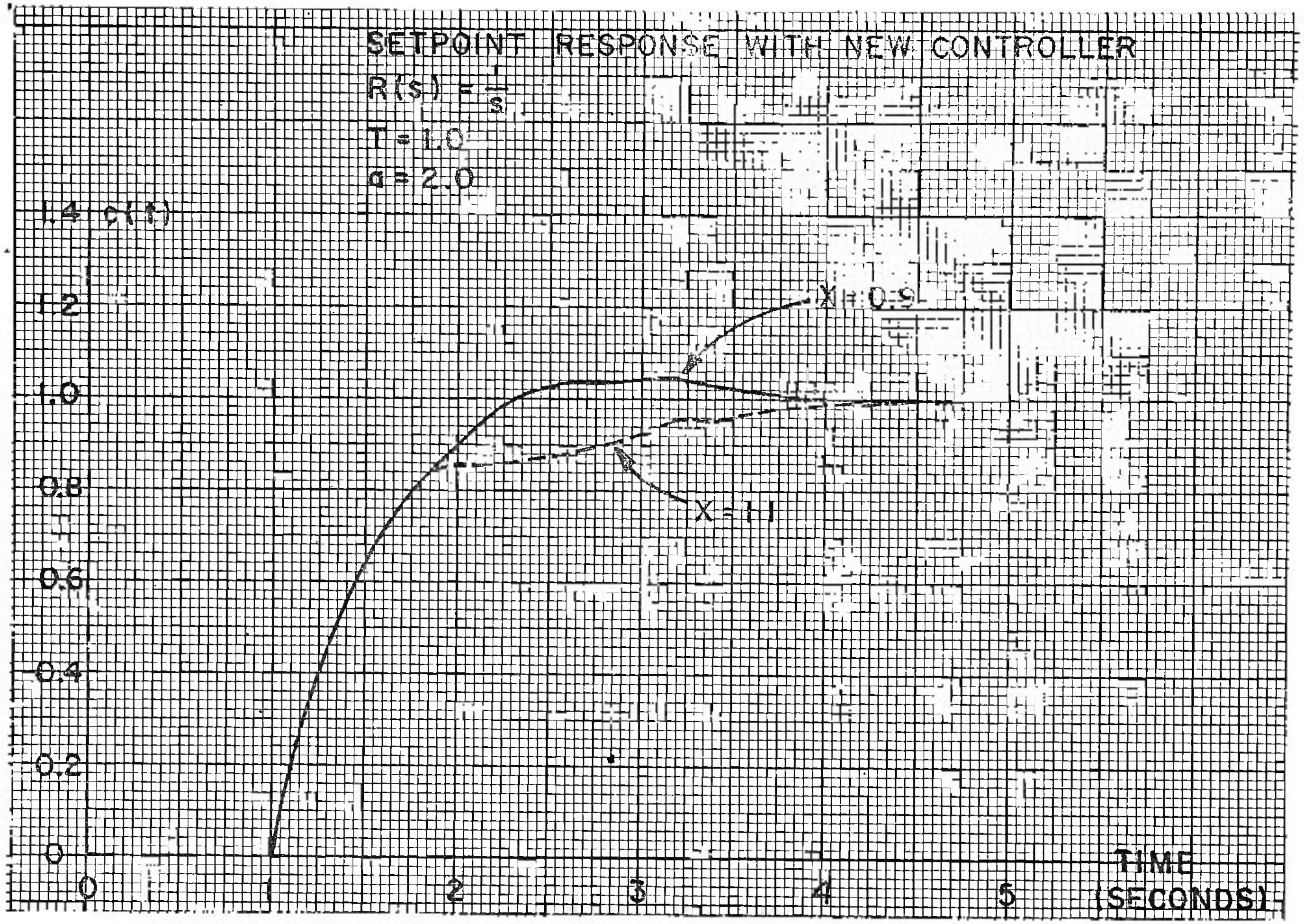
SETPOINT RESPONSE WITH NEW CONTROLLER
R(s) = 1/s
T = 1.0
a = 2.0
c(t)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
X = 0.9
X = 1.1
0
1
2
3
4
5
TIME
(SECONDS)

Figure 31

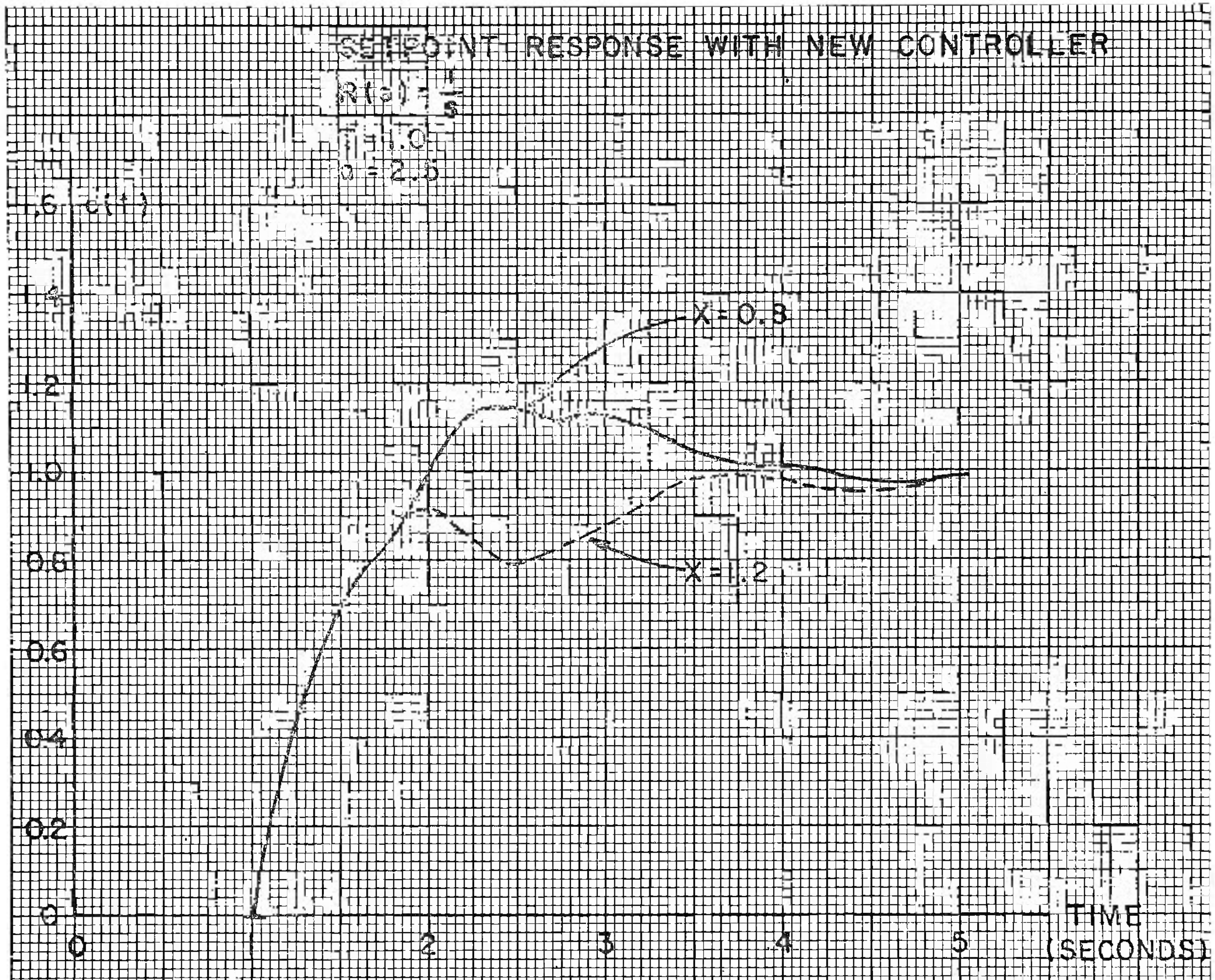
SETPOINT RESPONSE WITH NEW CONTROLLER
c(t)
X=0.8
X=1.2
0
0.2
0.4
0.6
0.8
1.0
1.2
0
2
3
4
5
TIME
(SECONDS)

Figure 32

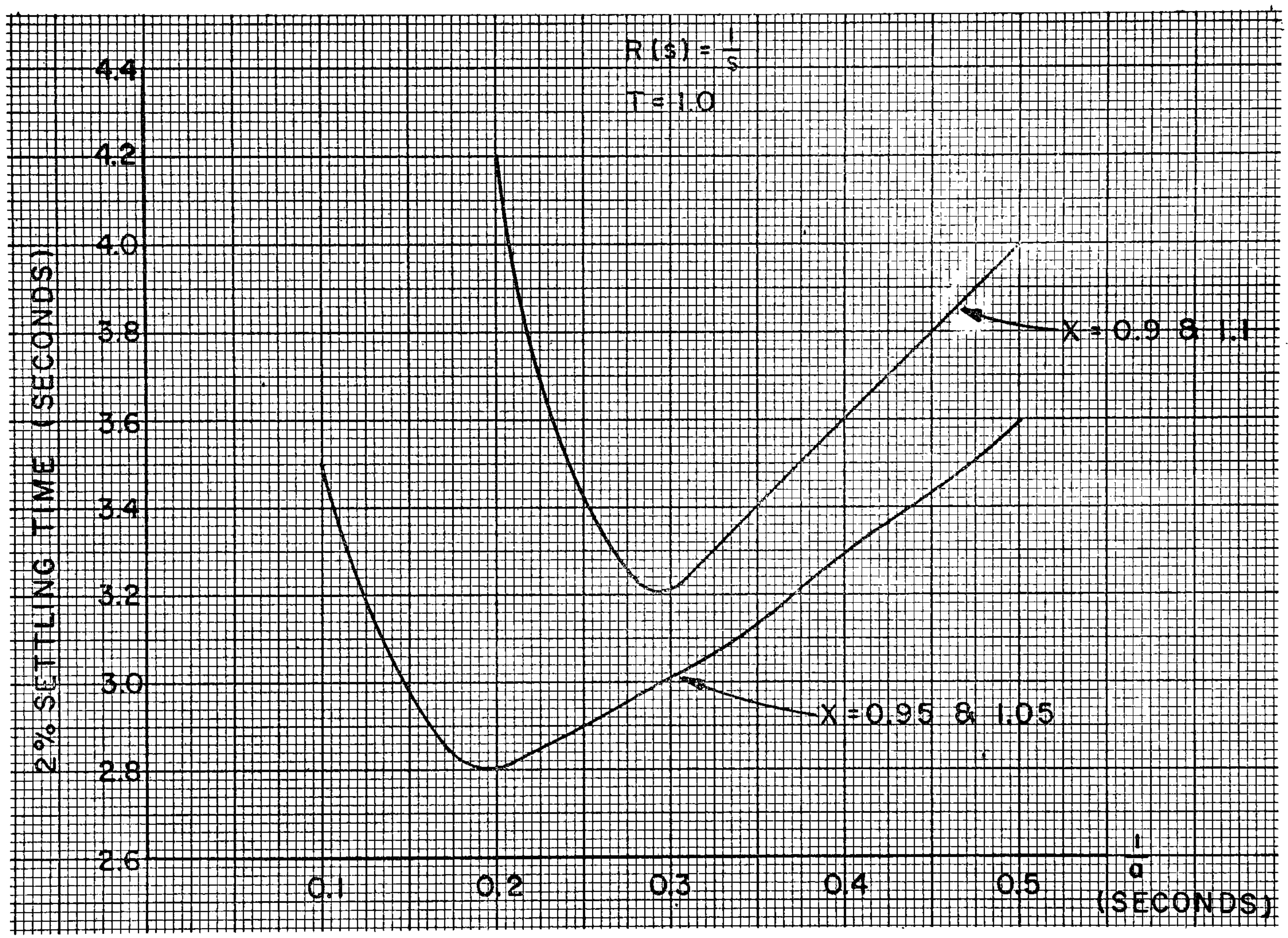

Figure 33: Two percent settling time versus 1/a for both 5% and 10% mismatches between T and X.

The previous analysis showed the effect of mismatches between process and controller dead time values on the setpoint step response. The following analysis shows the effect of this mismatch when a unit step disturbance occurs in the loop. For this analysis P(S) will again be set equal to B and Q(S) will again be set equal to S+B.

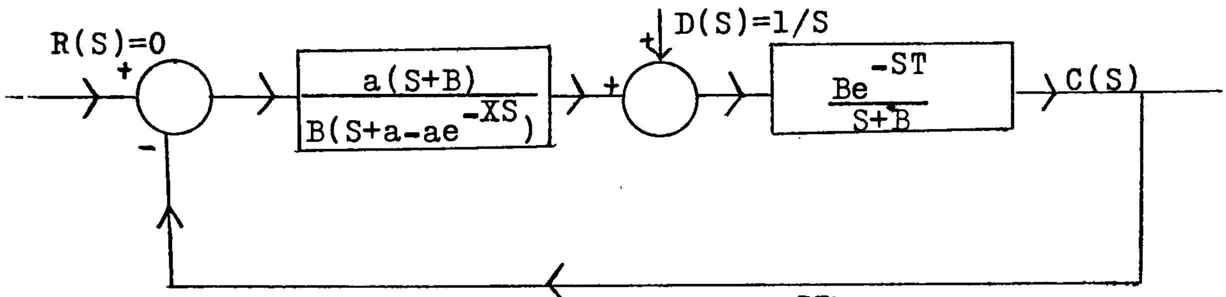

Figure 34: Block diagram of $\frac{Be^{-ST}}{S+B}$ plant controlled by new controller.

Referring to Figure 34, the transfer function from D(S) to C(S) can be written as

$$C(S) = \frac{\frac{Be^{-ST}}{S+B}}{1 + \frac{a(S+B)}{B(S+a-ae^{-XS})}\ \frac{Be^{-ST}}{S+B}}\ D(S)$$

$$= \frac{Be^{-ST}(S+a-ae^{-XS})}{(S+B)\left[S+a+a(e^{-ST}-e^{-XS})\right]}\ D(S) .$$

Factoring out an S+a term in both the numerator and denominator yields

$$C(S) = \frac{Be^{-ST}\left[1 - \frac{ae^{-XS}}{S+a}\right]}{(S+B)\left\{1 + \frac{a(e^{-ST}-e^{-XS})}{S+a}\right\}}\ D(S) .$$

Since $D(S) = \frac{1}{S}$, the output transform can be written as

$$C(S) = \frac{Be^{-ST}\left[1 - \frac{ae^{-XS}}{S+a}\right]}{S(S+B)\left[1 + \frac{a(e^{-ST}-e^{-XS})}{S+a}\right]} \,.$$

Applying the expansion formula,

$$\frac{1}{1+Z} = \sum_{n=0}^{\infty} (-1)^n Z^n \,,$$

the above expression can be rewritten as

$$C(S) = \frac{Be^{-ST}}{S(S+B)}\left[1 - \frac{ae^{-XS}}{S+a}\right]\left\{1 - \frac{a(e^{-ST}-e^{-XS})}{S+a} + \frac{a^2(e^{-ST}-e^{-XS})^2}{(S+a)^2} - \frac{a^3(e^{-ST}-e^{-XS})^3}{(S+a)^3} + \frac{a^4(e^{-ST}-e^{-XS})^4}{(S+a)^4} - \frac{a^5(e^{-ST}-e^{-XS})^5}{(S+a)^5} \cdots\right\}.$$

Expanding yields

$$C(S) = \frac{Be^{-ST}\left[1 - \frac{ae^{-XS}}{S+a}\right]}{S(S+B)}\left\{1 - \frac{ae^{-ST}}{S+a} + \frac{ae^{-XS}}{S+a} + \frac{a^2}{(S+a)^2}\left[e^{-2ST} - 2e^{-(X+T)S} + e^{-2XS}\right] - \frac{a^3}{(S+a)^3}\left[e^{-3ST} - 3e^{-(X+2T)S} + 3e^{-(2X+T)S} - e^{-3XS}\right] + \frac{a^4}{(S+a)^4}\left[e^{-4ST} - 4e^{-(X+3T)S} + 6e^{-(2X+2T)S} - 4e^{-(3X+T)S} + e^{-4XS}\right] - \right.$$

$$\frac{a^5}{(S+a)^5}\left[e^{-5ST}-5e^{-(X+4T)S}+10e^{-(2X+3T)S}-10e^{-(3X+2T)S}+ 5e^{-(4X+T)S}-e^{-5XS}\right]+\ldots\ldots\Big\}.$$

This can be rewritten as

$$C(S) = \frac{Be^{-ST}}{S(S+B)} - \frac{aBe^{-2ST}}{S(S+a)(S+B)} + \frac{aBe^{-(X+T)S}}{S(S+a)(S+B)} +$$

$$\frac{a^2B}{S(S+B)(S+a)^2}\left[e^{-3ST}-2e^{-(X+2T)S}+e^{-(2X+T)S}\right] -$$

$$\frac{a^3B}{S(S+B)(S+a)^3}\left[e^{-4ST}-3e^{-(X+3T)S}+3e^{-(2X+2T)S}- e^{-(3X+T)S}\right]+$$

$$\frac{a^4B}{S(S+B)(S+a)^4}\left[e^{-5ST}-4e^{-(X+4T)S}+6e^{-(2X+3T)S}- 4e^{-(3X+2T)S}+e^{-(4X+T)S}\right] + \ldots\ldots$$

$$- \frac{aBe^{-(X+T)S}}{S(S+B)(S+a)} + \frac{a^2Be^{-(2T+X)S}}{S(S+B)(S+a)^2} - \frac{a^2Be^{-(2X+T)S}}{S(S+B)(S+a)^2} -$$

$$\frac{a^3B}{S(S+B)(S+a)^3}\left[e^{-(X+3T)S}-2e^{-(2X+2T)S}+e^{-(3X+T)S}\right]+$$

$$\frac{a^4B}{S(S+B)(S+a)^4}\left[e^{-(4T+X)S}-3e^{-(2X+3T)S}+3e^{-(3X+2T)S}- e^{-(4X+T)S}\right]+\ldots\ldots$$

Combining terms results in

$$C(S) = \frac{Be^{-ST}}{S(S+B)} - \frac{aBe^{-2ST}}{S(S+a)(S+B)} + \frac{a^2Be^{-3ST}}{S(S+B)(S+a)^2} -$$

$$\frac{a^2Be^{-(X+2T)S}}{S(S+B)(S+a)^2} + \frac{a^3B}{S(S+B)(S+a)^3}\Big[-e^{-4ST} +$$

$$2e^{-(X+3T)S} - e^{-(2X+2T)S}\Big] +$$

$$\frac{a^4B}{S(S+B)(S+a)^4}\Big[e^{-5ST} - 3e^{-(X+4T)S} + 3e^{-(2X+3T)S} -$$

$$e^{-(3X+2T)S}\Big] + \ldots .$$

which, after a partial fraction expansion, can be rewritten as

$$C(S) = \left[\frac{J1}{S} + \frac{J2}{S+B}\right]e^{-ST} + \left[\frac{J3}{S} + \frac{J4}{S+B} + \frac{J5}{S+a}\right]e^{-2ST} +$$

$$\left[\frac{J6}{S} + \frac{J7}{S+B} + \frac{J8}{(S+a)^2} + \frac{J9}{(S+a)}\right]\left[e^{-3ST} - e^{-(X+2T)S}\right] +$$

$$\left[\frac{J10}{S} + \frac{J11}{S+B} + \frac{J12}{(S+a)^3} + \frac{J13}{(S+a)^2} + \frac{J14}{S+a}\right]$$

$$\left[-e^{-4ST} + 2e^{-(X+3T)S} - e^{-(2X+2T)S}\right] +$$

$$\left[\frac{J15}{S} + \frac{J16}{S+B} + \frac{J17}{(S+a)^4} + \frac{J18}{(S+a)^3} + \frac{J19}{(S+a)^2} + \frac{J20}{S+a}\right]$$

$$\left[e^{-5ST} - 3e^{-(X+4T)S} + 3e^{-(2X+3T)S} - e^{-(3X+2T)S}\right] + \ldots .$$

The residues can be calculated as

$$J1 = \left.\frac{B}{S+B}\right|_{S=0} = 1,$$

$$J2 = \left.\frac{B}{S}\right|_{S=-B} = -1,$$

$$J3 = \left.\frac{-aB}{(S+a)(S+B)}\right|_{S=0} = -1,$$

$$J4 = \left.\frac{-aB}{S(S+a)}\right|_{S=-B} = \frac{a}{a-B},$$

$$J5 = \left.\frac{-aB}{S(S+B)}\right|_{S=-a} = \frac{-B}{a-B},$$

$$J6 = 1,$$

$$J7 = \frac{-a^2}{(a-B)^2},$$

$$J8 = \frac{aB}{a-B},$$

$$J9 = -\frac{(B-2a)B}{(a-B)^2},$$

$$J10 = 1,$$

$$J11 = -\frac{a^3}{(a-b)^3},$$

$$J12 = -\frac{a^2B}{B-a},$$

$$J13 = -\frac{aB(B-2a)}{(a-B)^2},$$

$$J14 = \frac{B(3a^2-3aB+B^2)}{(a-B)^3},$$

$$J15 = 1,$$

$$J16 = -\frac{a^4}{(a-B)^4},$$

$$J17 = \frac{a^3B}{a-B},$$

$$J18 = -\frac{a^2B(B-2a)}{(a-B)^2},$$

$$J19 = \frac{B(3a^3-3a^2B+aB^2)}{(a-B)^3},$$

and $$J20 = \frac{4a^3B+4aB^3-B^4-6a^2B^2}{(a-B)^4}.$$

Then, by taking the inverse transform,

$$c(t) = y_1(t-T)u(t-T) + y_2(t-2T)u(t-2T) +$$
$$y_3(t-3T)u(t-3T) - y_3(t-X-2T)u(t-X-2T) -$$
$$y_4(t-4T)u(t-4T) + 2y_4(t-X-3T)u(t-X-3T) -$$
$$y_4(t-2X-2T)u(t-2X-2T) +$$
$$y_5(t-5T)u(t-5T) - 3y_5(t-X-4T)u(t-X-4T) +$$
$$3y_5(t-2X-3T)u(t-2X-3T) - y_5(t-3X-2T)u(t-3X-2T)$$

where $y_1(t) = 1-e^{-Bt}$,

$$y_2(t) = -1 + \frac{a}{a-B}e^{-Bt} - \frac{B}{a-B}e^{-at},$$

$$y_3(t) = 1 - \frac{a^2}{(a-B)^2}e^{-Bt} + \frac{aB}{a-B}te^{-at} - \frac{(B-2a)Be^{-at}}{(a-B)^2},$$

$$y_4(t) = 1 - \frac{a^3}{(a-B)^3}e^{-Bt} - \frac{a^2B}{2(B-a)}t^2e^{-at} - \frac{aB(B-2a)}{(a-B)^2}te^{-at} +$$
$$\frac{B(3a^2-3aB+B^2)e^{-at}}{(a-B)^3}, \text{ and}$$

$$y_5(t) = 1 - \frac{a^4}{(a-B)^4}e^{-Bt} + \frac{a^3B}{6(a-B)}t^3e^{-at} - \frac{a^2B(B-2a)t^2e^{-at}}{2(a-B)^2}$$

$$+ \frac{B(3a^3-3a^2B+aB^2)te^{-at}}{(a-B)^3} + \frac{(4a^3B+4aB^3-B^4-6a^2B^2)e^{-at}}{(a-B)^4} .$$

The output disturbance response corresponding to the above equation was calculated by use of a digital computer. The response for X=1.0 and X=1.1 when a=5, T=1.0, and B=2 is plotted in Figure 35. The 10% mismatch in controller and plant dead times causes a 6% increase in I.A.E. with respect to the same system with no mismatch.

The setpoint response of the same system is shown in Figure 28 for X=1.1. The 10% mismatch causes 57.5% increase in I.A.E. (from .2 to .315). Thus, from the stand point of error due to plant and controller dead time mismatches, the response to a step change in setpoint is more sensitive and thus more critical than a step disturbance change. For this reason, the optimum value of a was determined by consideration of the setpoint response curves. The disturbance and setpoint responses, even with mismatches between the controller and plant dead times on the order of 10%, are generally superior to responses of the same plant with a proportional-plus-integral controller. For the disturbance response the degree of improvement becomes more substantial as the dominance of the dead time element increases.

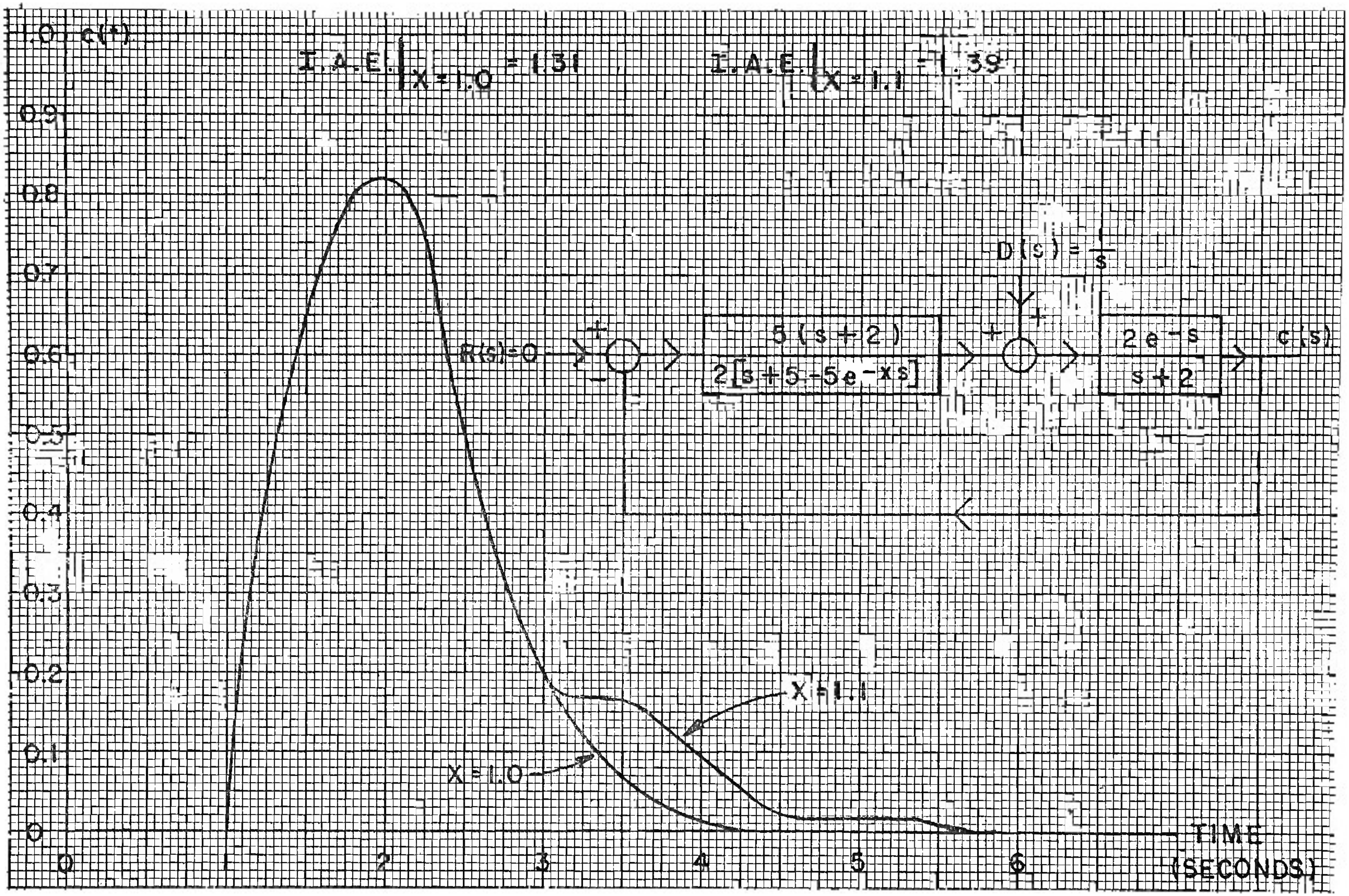

Figure 35: Disturbance Response with X=T& X=1.1T, B=2, a=5, and T=1.0.

IX. DEAD TIME LAG SIMULATION

To obtain the performance which the new controller is theoretically capable of, it is necessary to accurately simulate a dead time lag. In recent months the prices of analog to digital converters, digital to analog converters, and long shift registers have dropped significantly, indicating that a digital simulation may be economically achieved. One possible approach would be to sample and digitize the analog signal at a periodic rate. Each digital word could then be entered into a shift register and shifted at the same periodic rate. The digital information could be picked off at the end of the shift register and converted back to an analog signal by a digital to analog converter. The time delay would depend on the number of bits available in the shift register and the sampling frequency. This implementation was not undertaken as part of this thesis but is mentioned to show that the preceding results are of more than theoretical interest.

X. LOOP TUNING PROCEDURE

The loop tuning procedure for the new controller is extremely simple. One possible approach is to initially obtain the process reaction curve. This is done by breaking the loop, applying a unit step change to the plant, and recording the output response. This same technique is used to obtain the famous Ziegler-Nichols settings. From the process reaction curve, a plant transfer function approximation, $\frac{Ke^{-ST}}{S+B}$, can be obtained, as is shown in Figure 36.

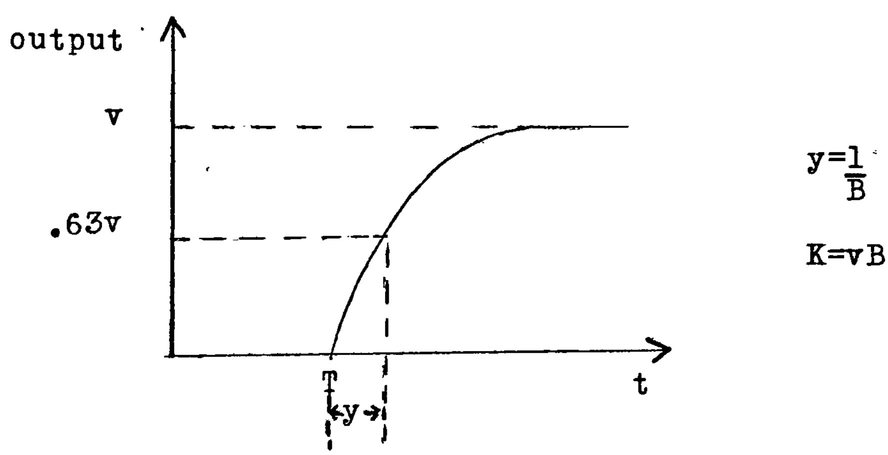

Figure 36: Typical Process Reaction Curve

This immediately gives the operator the value of $\frac{Q(S)}{P(S)}$ in the new controller. The value of a must be determined by consideration of the expected variation in the dead time T of the plant. If the controller is properly designed, the value of X should be capable of being set extremely close to the value of T and remain stable for various enviromental conditions. However, the value of T in the plant will typically not be constant but may have variations occurring from time to time. If expected variations combined with the uncertainty of the original measurement of T are on the order of 10%, a value of 1/a equal to 30% of T would give satisfactory performance. If variations plus measurement uncertainty are about 5%, then a value of 1/a equal to 20% of T would be satisfactory. These two results are based on calculations detailed in this thesis. For other possible variations other values of a should be determined.

It should be noted that the process reaction curve shown in Figure 35 can approximate most process control systems dominated by dead time element. Exceptions to this rule are plants which contain an integrating element which are termed non-self regulating.

XI. CONCLUSION

The new controller allows improved response over that of a proportional-plus-integral controller. The degree of improvement is significantly better for setpoint step changes and varies for disturbance step changes as a function of the ratio of plant dead time to lag, that is $T/\frac{1}{B}$.

As this ratio increases, indicating the plant is dominated by the dead time, the degree of improvement also increases so this controller would primarily be useful with plants having large dead time to lag ratios, that is a plant dominated by a pure dead time. From a practical standpoint the response to a disturbance change is usually of prime importance.

XII. APPENDIX A

DYSIM*** SIMULATION OF THE PROPORTIONAL-PLUS-INTEGRAL CONTROL OF $\frac{Be^{-ST}}{S+B}$ PLANT.

Figure A1 is a computer block diagram of the DYSIM*** simulation of the system shown in Figure 11 . The numbers above each element are the block numbers. The various symbols within each block indicate the block type. The following list summarizes the symbols used:

+	adder
K	constant
G	gain multiplier
I	integrator
-	inverter
u	dead time
m	absolute value block
P1	initial condition on an integrator
P2	gain term on an integrator
P3	gain term on an integrator.

The dead time block, which is symbolized by u, has a dead time equal to one-half the digital computer integration interval. To obtain accurate output data with T=1, the integration interval was specified as 0.1 and a fourth order Runge-Kutta integration was used. This integration interval thus requires 20 dead time blocks (block numbers 10 thru 29) to realize a unit delay. The printout shown in Figure A2 is a computer listing of the block diagram shown in Figure A1. The initial conditions and parameters at the bottom of the page determine the proportional gain, reset rate, plant time constant, and type of response (setpoint or disturbance) desired. For instance, in the printout shown, parameter one on block one is set at unity, indicating a unit step change in the setpoint. Parameter one on block number three sets the proportional gain at 0.1667, parameter two on block four sets the reciprocal of the reset time at four and parameters two and three on block eight set the value of B at 4. Figure A3 shows the computer program data file, which is used to operate DYSIM***.

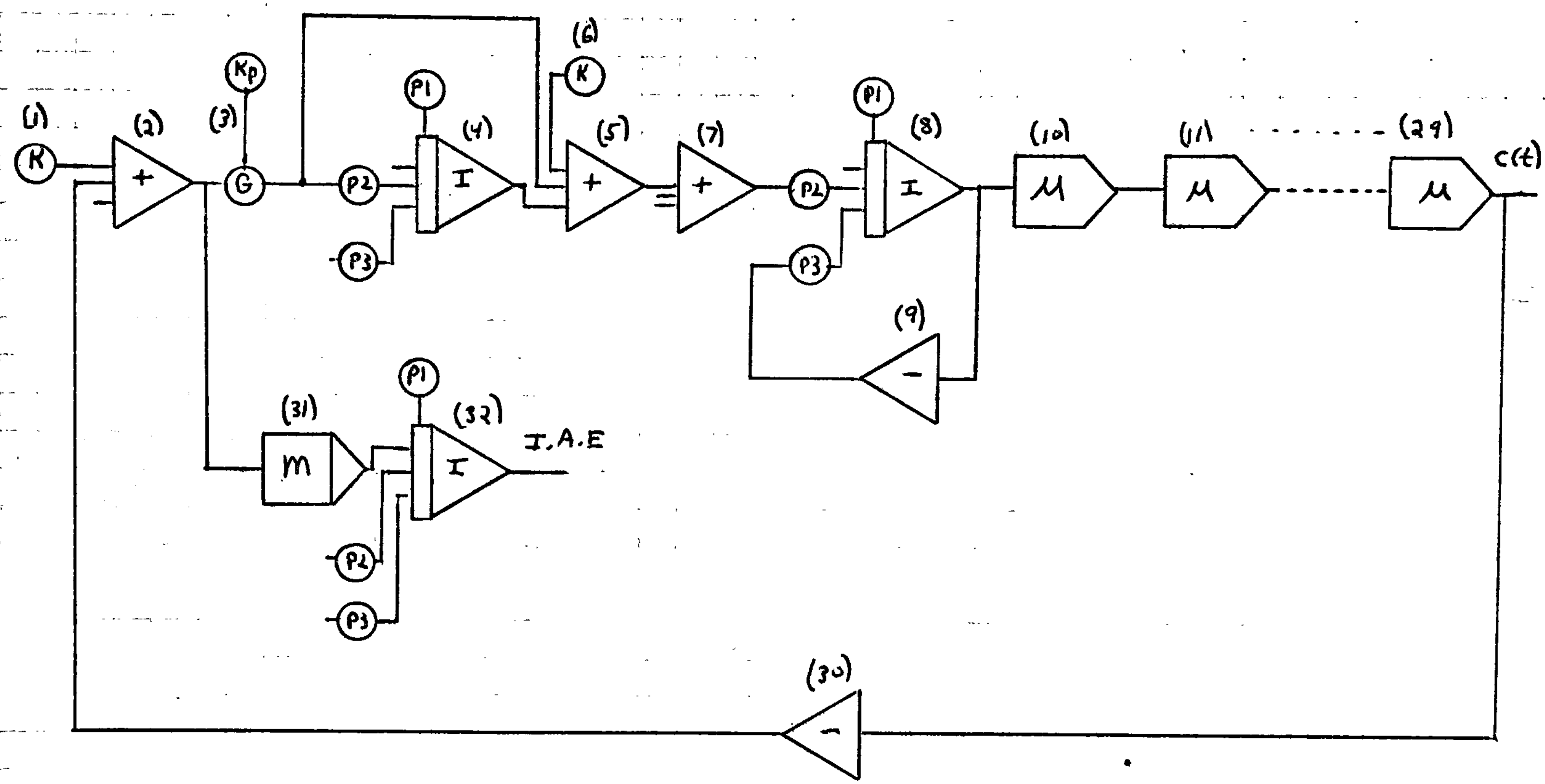

Figure A1: The DYSIM*** computer diagram for a $\frac{Be^{-ST}}{S+B}$ plant having proportional-plus-integral control.

BLØCK	TYPE	INPUT1	INPUT2	INPUT3
1	K	0	0	0
2	+	1	30	0
3	G	2	0	0
4	I	0	3	0
5	+	6	3	4
6	K	0	0	0
7	+	5	0	0
8	I	0	7	9
9	-	8	0	0
10	U	8	0	0
11	U	10	0	0
12	U	11	0	0
13	U	12	0	0
14	U	13	0	0
15	U	14	0	0
16	U	15	0	0
17	U	16	0	0
18	U	17	0	0
19	U	18	0	0
20	U	19	0	0
21	U	20	0	0
22	U	21	0	0
23	U	22	0	0
24	U	23	0	0
25	U	24	0	0
26	U	25	0	0
27	U	26	0	0
28	U	27	0	0
29	U	28	0	0
30	-	29	0	0
31	M	2	0	0
32	I	31	0	0

INITIAL CØNDITIØNS AND PARAMETERS

BLØCK	IC/PAR1	PAR2	PAR3
1	1.000000E+00	0.	0.
3	1.667000E-01	0.	0.
4	0.	4.000000E+00	0.
8	0.	4.000000E+00	4.000000E+00

Figure A2: DYSIM*** computer printout of block arrangement shown in Figure A1.

```
100 1,K,0,0,0
110 2,+,1,30,0
120 3,G,2,0,0
130 4,I,0,3,0
140 5,+,6,3,4
150 6,K,0,0,0
160 7,+,5,0,0
170 8,I,0,7,9
180 9,-,8,0,0
190 10,U,8,0,0
200 11,U,10,0,0
210 12,U,11,0,0
220 13,U,12,0,0
230 14,U,13,0,0
240 15,U,14,0,0
250 16,U,15,0,0
260 17,U,16,0,0
270 18,U,17,0,0
280 19,U,18,0,0
290 20,U,19,0,0
300 21,U,20,0,0
310 22,U,21,0,0
320 23,U,22,0,0
330 24,U,23,0,0
340 25,U,24,0,0
350 26,U,25,0,0
360 27,U,26,0,0
370 28,U,27,0,0
380 29,U,28,0,0
390 30,-,29,0,0
400 31,M,2,0,0
410 32,I,31,0,0
420 0,0,0,0,0
430 1,1,0,0
440 3,.1667,0,0
450 4,0,4,0
460 6,0,0,0
470 8,0,4,4
```

Figure A3: DYSIM*** computer program required to implement simulation shown in Figure A1.

APPENDIX B

DYSIM*** SIMULATION OF THE INTEGRAL CONTROL OF AN e^{-ST} PLANT.

The computer block diagram, block configuration listing, and DYSIM*** program are shown on Figures B1, B2, and B3, respectively.

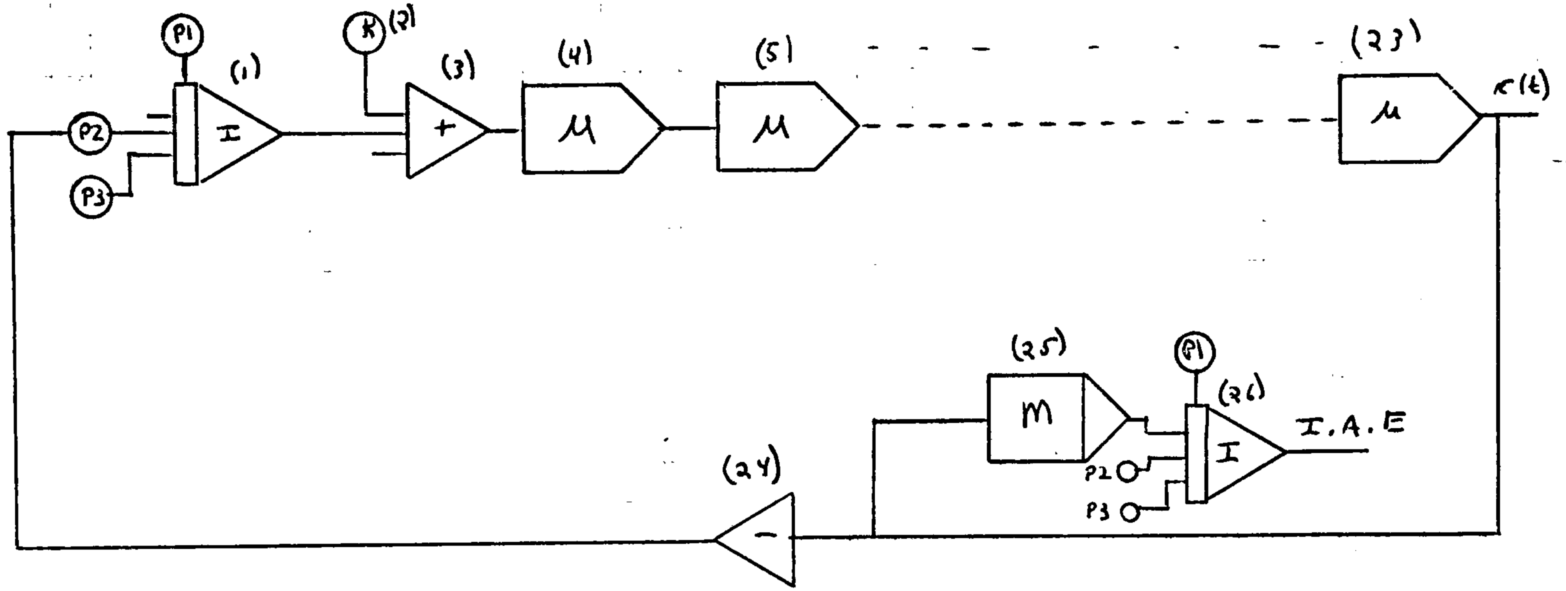

Figure B1:. DYSIM*** computer diagram for integral control of pure dead time process.

BLØCK	TYPE	INPUT1	INPUT2	INPUT3
1	I	0	24	0
2	K	0	0	0
3	+	2	1	0
4	U	3	0	0
5	U	4	0	0
6	U	5	0	0
7	U	6	0	0
8	U	7	0	0
9	U	8	0	0
10	U	9	0	0
11	U	10	0	0
12	U	11	0	0
13	U	12	0	0
14	U	13	0	0
15	U	14	0	0
16	U	15	0	0
17	U	16	0	0
18	U	17	0	0
19	U	18	0	0
20	U	19	0	0
21	U	20	0	0
22	U	21	0	0
23	U	22	0	0
24	-	23	0	0
25	M	23	0	0
26	I	0	25	0

INITIAL CØNDITIØNS AND PARAMETERS

BLØCK	IC/PAR1	PAR2	PAR3
1	0.	6.670000E-01	0.
2	1.000000E+00	0.	0.
26	0.	1.000000E+00	0.

Figure B2: DYSIM*** computer printout of block arrangement shown in Figure B1.

```
LLL              14:43EST      11/20/72

100 1,I,0,24,0
110 2,K,0,0,0
120 3,+,2,1,0
130 4,U,3,0,0
140 5,U,4,0,0
150 6,U,5,0,0
160 7,U,6,0,0
170 8,U,7,0,0
180 9,U,8,0,0
190 10,U,9,0,0
200 11,U,10,0,0
210 12,U,11,0,0
220 13,U,12,0,0
230 14,U,13,0,0
240 15,U,14,0,0
250 16,U,15,0,0
260 17,U,16,0,0
270 18,U,17,0,0
280 19,U,18,0,0
290 20,U,19,0,0
300 21,U,20,0,0
310 22,U,21,0,0
320 23,U,22,0,0
330 24,-,23,0,0
340 25,M,23,0,0
350 26,I,0,25,0
360 0,0,0,0,0
370 1,0,.667,0
380 2,1,0,0
390 26,0,1,0
```

Figure B3: DYSIM*** computer program required to implement simulation shown in Figure B1.

APPENDIX C

DYSIM*** VERIFICATION OF NEW CONTROLLER RESPONSE.

The output responses which were plotted throughout this thesis using the new controller were obtained by hand calculation using the Laplace transform. Occasionally, a digital computer was used to evaluate the resultant time domain equations. This appendix outlines the simulation of the new controller in a system with a $\frac{Be^{-ST}}{S+B}$ plant using DYSIM***. Specifically, the response to a step disturbance when B=2, a=5, T=1, and X=1 was calculated by DYSIM***. The response of the same system was calculated by hand and the resultant output response was plotted in Figure 35. block diagram of the system which encorporates the new controller realization shown in Figure 9 is drawn in Figure C1. Figure C2 is a computer block diagram of the system. Figures C3 and C4 outline the DYSIM*** program, and Figure C5 is a table of the output values versus time. Comparison of this table with the output response plotted in Figure 35 indicate a close agreement, thus verifying the hand computation.

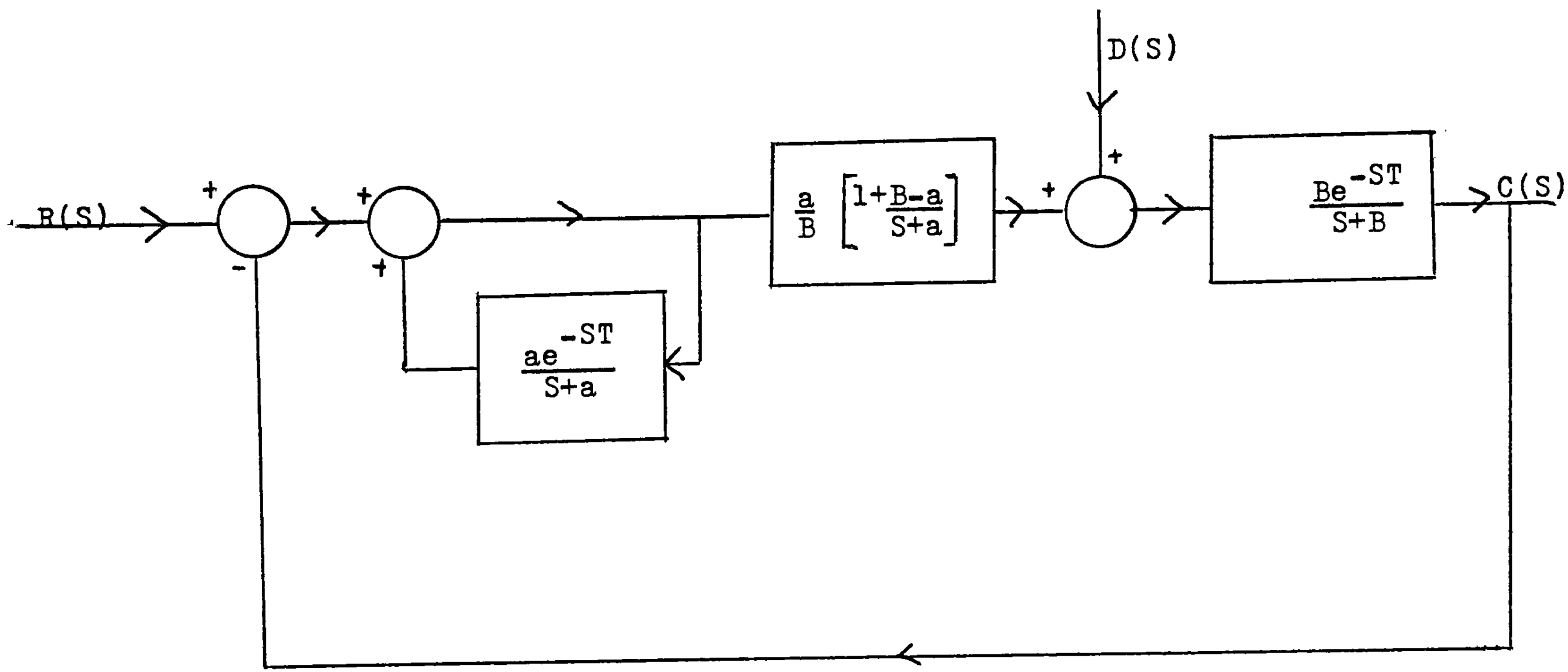

Figure C1: Block diagram of $\frac{Be^{-ST}}{S+B}$ plant with new controller.

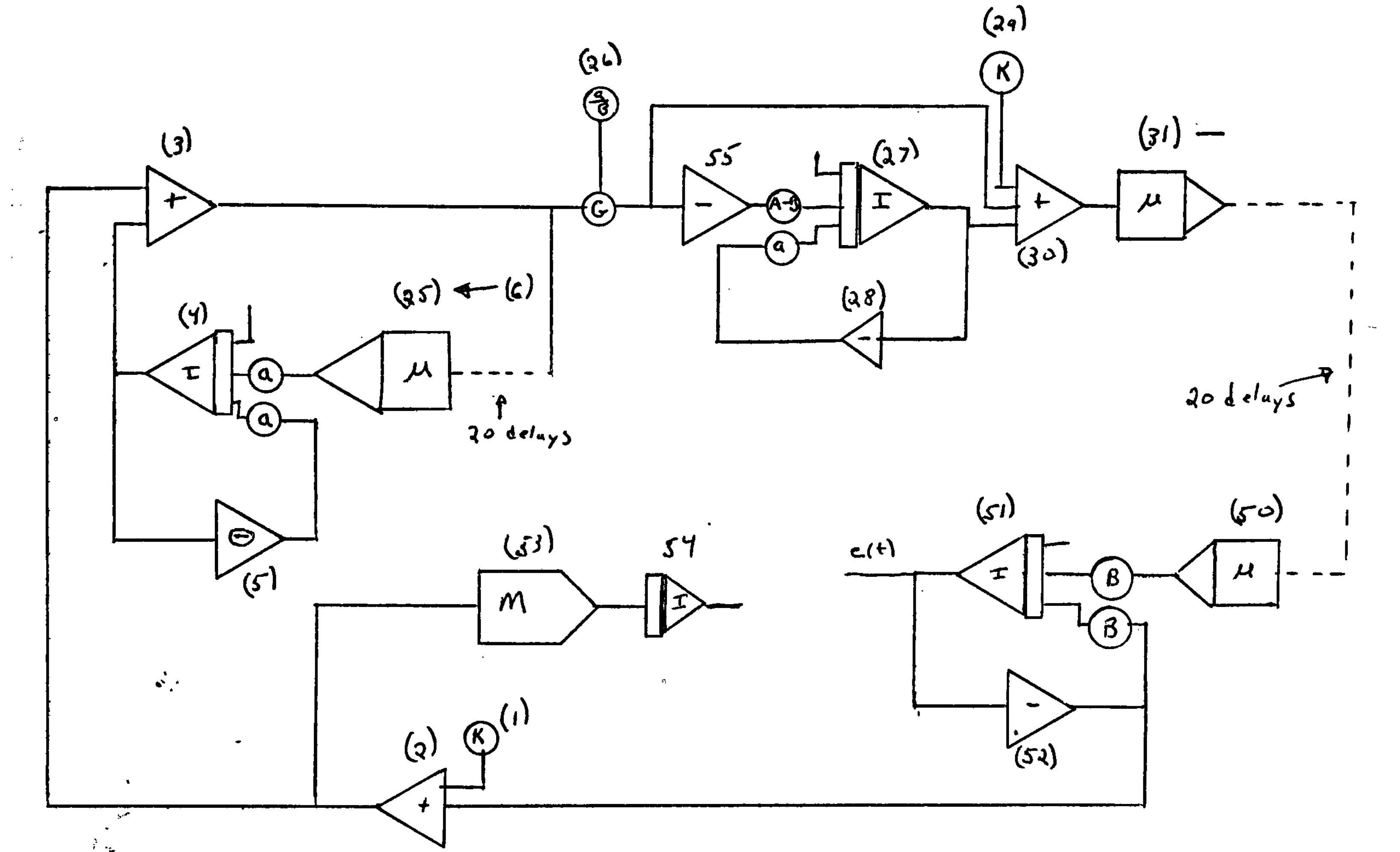

Figure C2: Computer diagram of system depicted in Figure C1.

CØMMAND1?CØNF

BLØCK	TYPE	INPUT1	INPUT2	INPUT3
1	K	0	0	0
2	+	52	1	0
3	+	2	4	0
4	I	0	25	5
5	-	4	0	0
6	U	3	0	0
7	U	6	0	0
8	U	7	0	0
9	U	8	0	0
10	U	9	0	0
11	U	10	0	0
12	U	11	0	0
13	U	12	0	0
14	U	13	0	0
15	U	14	0	0
16	U	15	0	0
17	U	16	0	0
18	U	17	0	0
19	U	18	0	0
20	U	19	0	0
21	U	20	0	0
22	U	21	0	0
23	U	22	0	0
24	U	23	0	0
25	U	24	0	0
26	G	3	0	0
27	I	0	55	28
28	-	27	0	0
29	K	0	0	0
30	+	29	26	27

Figure C3: DYSIM*** computer printout of block arrangement shown in Figure C2.

31	U	30	0	0
32	U	31	0	0
33	U	32	0	0
34	U	33	0	0
35	U	34	0	0
36	U	35	0	0
37	U	36	0	0
38	U	37	0	0
39	U	38	0	0
40	U	39	0	0
41	U	40	0	0
42	U	41	0	0
43	U	42	0	0
44	U	43	0	0
45	U	44	0	0
46	U	45	0	0
47	U	46	0	0
48	U	47	0	0
49	U	48	0	0
50	U	49	0	0
51	I	0	50	52
52	-	51	0	0
53	M	2	0	0
54	I	53	0	0
55	-	26	0	0

INITIAL CØNDITIØNS AND PARAMETERS

BLØCK	IC/PAR1	PAR2	PAR3
4	0.	5.000000E+00	5.000000E+00
26	2.500000E+00	0.	0.
27	0.	3.000000E+00	5.000000E+00
29	1.000000E+00	0.	0.
51	0.	2.000000E+00	2.000000E+00

Figure C4: Continued from Figure C3.

TIME	ØUTPUT 51	ØUTPUT 54
0.	0.	0.
2.0000E-01	0.	0.
4.0000E-01	0.	0.
6.0000E-01	0.	0.
8.0000E-01	0.	0.
1.0000E+00	0.	0.
1.2000E+00	3.0116662E-01	2.7910639E-02
1.4000E+00	5.3652860E-01	1.1311846E-01
1.6000E+00	6.9276334E-01	2.3702511E-01
1.8000E+00	7.9633228E-01	3.8658271E-01
2.0000E+00	8.6498830E-01	5.5314434E-01
2.2000E+00	8.1842845E-01	7.2548364E-01
2.4000E+00	6.2003665E-01	8.6939032E-01
2.6000E+00	4.3944063E-01	9.7495896E-01
2.8000E+00	3.0130450E-01	1.0484477E+00
3.0000E+00	2.0322908E-01	1.0983854E+00
3.2000E+00	1.3594058E-01	1.1319156E+00
3.4000E+00	8.8923280E-02	1.1539438E+00
3.6000E+00	6.2121045E-02	1.1689762E+00
3.8000E+00	4.2368869E-02	1.1793083E+00
4.0000E+00	2.8377164E-02	1.1862692E+00
4.2000E+00	1.8811392E-02	1.1908994E+00
4.4000E+00	1.2398392E-02	1.1939569E+00
4.6000E+00	8.3721547E-03	1.1960060E+00
4.8000E+00	5.4505781E-03	1.1973636E+00
5.0000E+00	3.5014067E-03	1.1982383E+00
5.2000E+00	2.2993480E-03	1.1988054E+00
5.4000E+00	1.5632999E-03	1.1991840E+00
5.6000E+00	1.1192468E-03	1.1994463E+00
5.8000E+00	7.7230113E-04	1.1996323E+00
6.0000E+00	5.1474880E-04	1.1997581E+00

CØMMAND2?CØMMAND1

Figure C5: Output response of system in Figure C1 with B=2, a=5, and T=1.0.

APPENDIX D

FREQUENCY RESPONSE OF THE NEW CONTROLLER

The controller derived in this thesis has a transfer function

$$G_c(S) = \frac{a}{S+a-ae^{-ST}} \frac{Q(S)}{P(S)}$$

which can be rewritten as

$$G_c(S) = X(S) \frac{Q(S)}{P(S)} .$$

The frequency response of $G_c(S)$ is thus determined by two factors, namely, $X(S)$ and $\frac{Q(S)}{P(S)}$. $\frac{Q(S)}{P(S)}$ is determined by the plant since the plant transfer is defined as $\frac{P(S)e^{-ST}}{Q(S)}$.

Since $\frac{Q(S)}{P(S)}$ was assumed to be a ratio of polynomials in S, the numerator and denominator could be factored and standard straight line approximations could easily be implemented on a Bode plot to obtain the magnitude and phase plots versus frequency. This same simple procedure cannot easily be applied to $X(S)$ since it is not a ratio of polynomials. If the magnitude and phase plots of $X(S)$ were known, it would require only a simple graphical procedure to add the effects of $\frac{Q(S)}{P(S)}$ and, thus, obtain the complete response plots of the controller. Thus, as a design guide, let us obtain the frequency plots of $X(S)$.

Letting $S=jw$ yields

$$X(jw) = \frac{a}{jw+a-ae^{-jwT}} .$$

Applying Euler's identity yields

$$X(jw)=\frac{a}{jw+a-a(coswT-jsinwT)}$$

which can be rewritten as

$$X(jw) = \frac{a}{a(1-coswT)+j(w+asinwT)} .$$

Thus $$|X(jw)| = \frac{a}{[a^2(1-coswT)^2+(w+asinwT)^2]^{\frac{1}{2}}}$$

and $$\underline{/X(jw)} = -tan^{-1} \frac{w+asinwT}{a(1-coswt)} .$$

Simplifying yields

$$|X(jw)| = \frac{a}{(2a^2-2a^2coswT+w^2+2awsinwT)^{\frac{1}{2}}}$$

and $$\underline{/X(jw)} = -tan^{-1} \frac{w+asinwT}{a(1-coswT)} .$$

Throughout this thesis, the value of T was typically set equal to 1 and the value of a was typically set equal to 3.34 to obtain transient output plots. With these same values of a and T, the $|X(jw)|$ and $\underline{/X(jw)}$ were obtained as a function of frequency on a digital computer. The response plots are shown in Figures D1 and D2. Both the magnitude and phase plots exhibit peaks at approximately multiples of w=5 radians per second due to the transcendental characteristics of the denominator of X(jw).

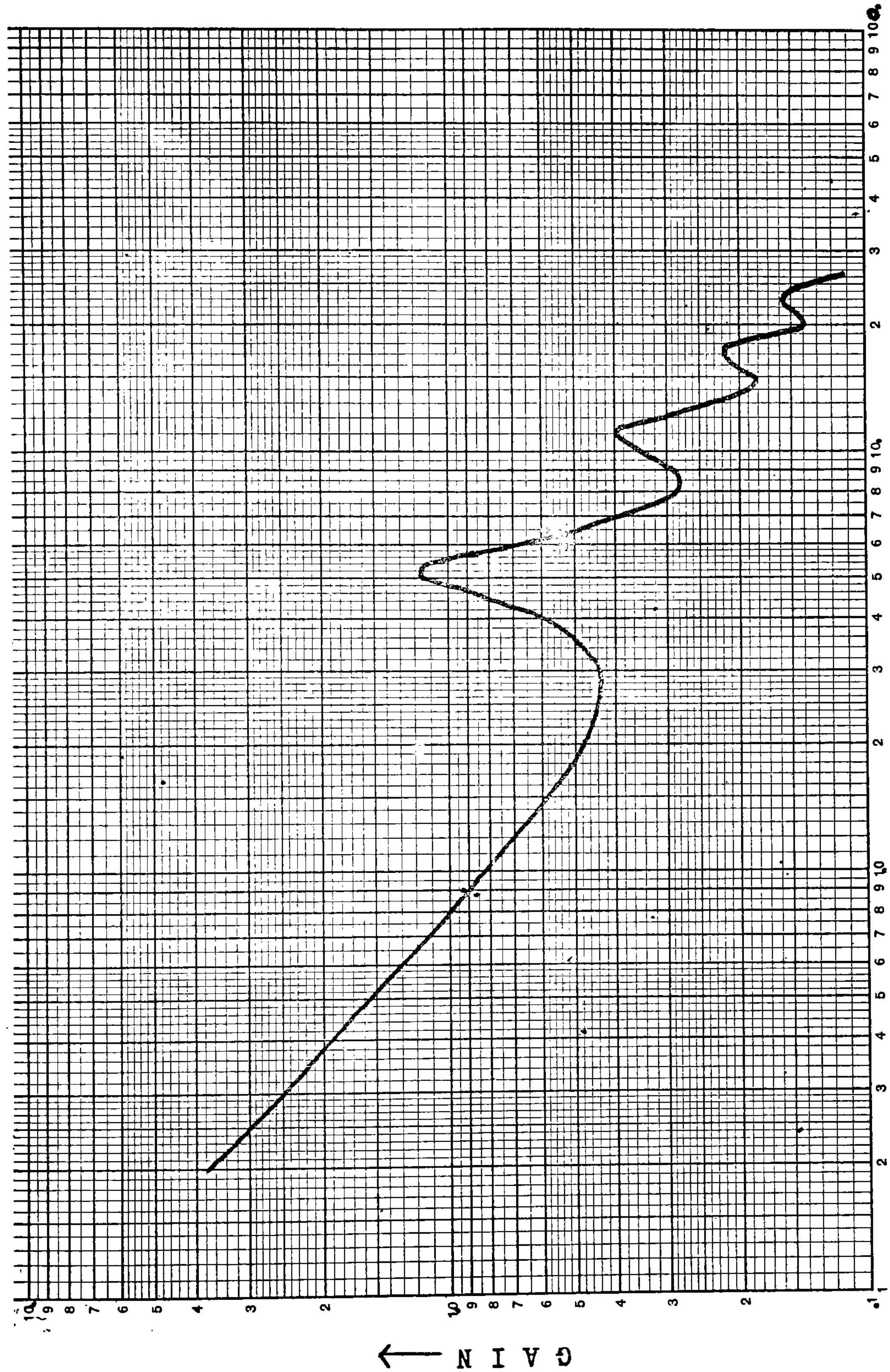

Figure D1: Magnitude vs. frequency plot of $X(jw) = \frac{a}{a(1-\cos wT)+j(w+a\sin wT)}$.

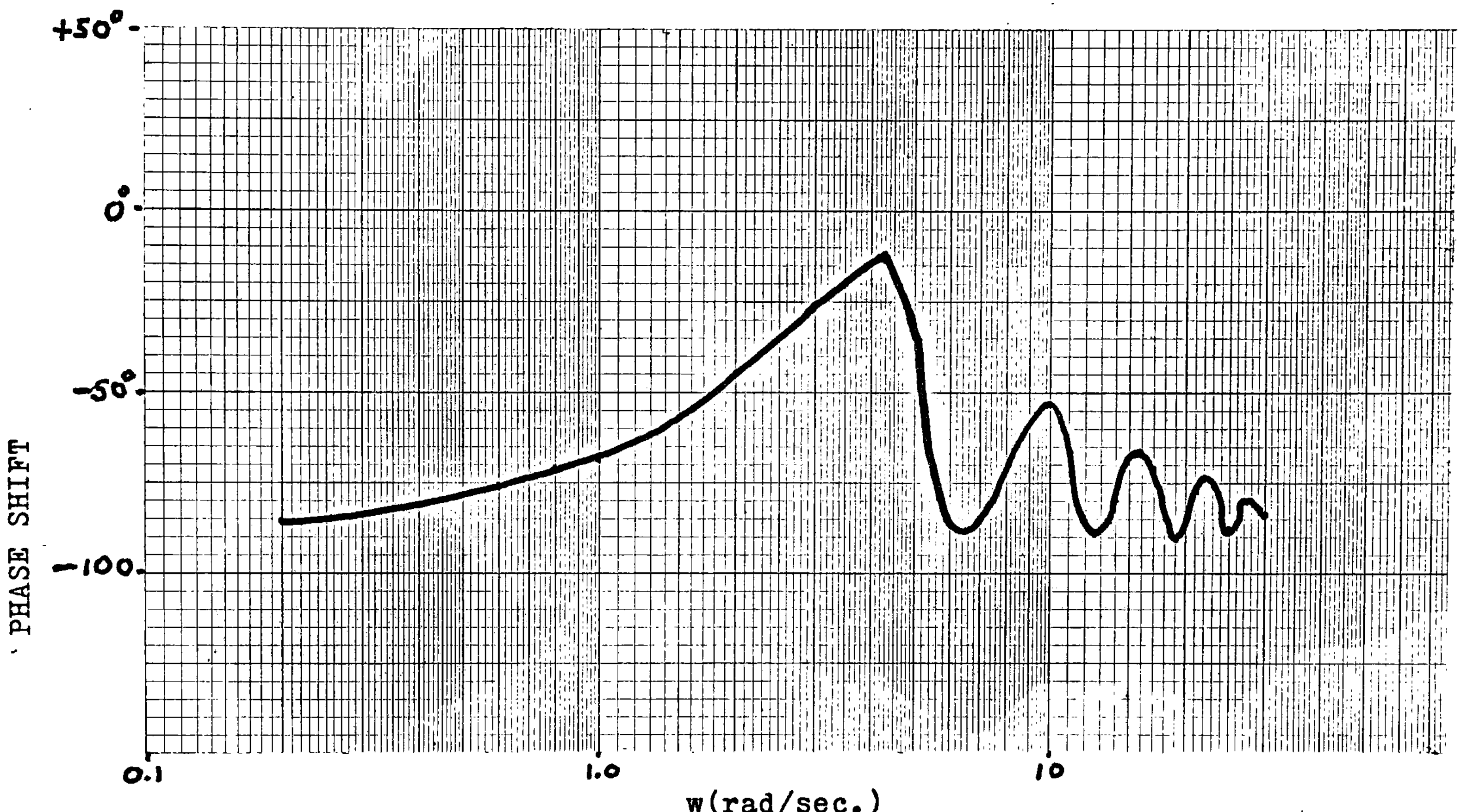

Figure D2: Phase shift vs. frequency plot of $X(jw) = \dfrac{a}{a(1-\cos wT)+j(w+a\sin wT)}$.

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