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A decision support system for routing trains through railway stations

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Abstract

This paper deals with the problem of routing trains through railway stations. This problem is to be solved within the Decision Support System DONS: Designer of Network Schedules. This system is developed to support the strategic planning process related to the required future capacity of the Dutch railway infrastructure. The latter capacity will be assessed with the help of DONS by generating a number of plausible timetables, and by checking whether these timetables are feasible, given certain scenarios for the future railway infrastructure.

In this paper we give a description of the routing problem to be solved and of the relevant context. Then we formulate the problem as an integer linear programming model. The first objective is to maximize the number of trains that can be routed through a railway station, the second objective is to minimize the number of shunting movements and the third objective is to assign the trains to their most preferred platforms and routes.

We also describe a solution procedure based on preprocessing, the addition of valid inequalities, the application of heuristics, and a branch & cut approach. This solution procedure is used to solve real-life instances of the routing problem involving the Dutch railway stations Zwolle and Utrecht CS.

1 Introduction

This paper is motivated by the project DONS (see van den Berg & Odijk [1]) that is carried out by Railned in cooperation with Nederlandse Spoorwegen (Netherlands Railways). Railned is responsible for the planning of the required future layout of the Dutch railway infrastructure. Currently, this

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strategic capacity planning process is carried out by evaluating a number of scenarios for the improvement of the railway infrastructure with respect to their ability to handle a large variety of plausible future production plans (timetables) and the required investments for these improvements.

1.1 Scenario generation

In short, the process of generating these scenarios is as follows:

1. The expected demand for railway transportation in the future is determined. The Origin-Destination matrix gives estimates for the number of future travellers between each pair of railway stations.

2. A plausible design for the future railway infrastructure is determined. This design involves modifications and/or extensions of the current railway infrastructure.

3. The line system of the railway network is determined (see Claessens et al. [3]). Thus, for each line its origin and destination station, its frequency per hour, and its type (InterCity, Inter-Regional, Regional) is determined.

4. A compatible timetable is constructed. A timetable should be feasible and robust, and should allow for the most important transfer possibilities between pairs of trains at certain railway stations.

The last step of the scenario generating procedure turned out to be the bottle-neck, when carried out manually. Therefore, the DONS project was initiated recently. The objective of this project is to develop a Decision Support System (DSS) that will assist the planners in their capacity planning work.

1.2 DONS, CADANS and STATIONS

The DSS to be developed, also called DONS, will contain at least two complementary modules. The first module, called CADANS, will assist the planners in generating timetables. CADANS is being developed by Schrijver & Steenbeek [7]. The second module, called STATIONS, will assist the planners in checking whether a timetable generated by CADANS is feasible with respect to the routing of the trains through the railway stations, given the layout of the railway stations, and given the arrival and departure times of the trains generated by CADANS. The authors of the current paper are involved in the development of the system STATIONS.

The application of STATIONS is necessary, because CADANS considers the railway infrastructure within the railway stations only from a global point of view. Therefore it may happen that a timetable generated by CADANS is feasible with respect to the railway network *outside* the railway stations, but turns out to be infeasible if one also considers the detailed

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layout of the railway network *inside* the railway stations. If a routing for all trains through the railway stations does not exist, then STATIONS is expected to identify the blocking trains and to provide suggestions for the modification of the arrival and departure times of these blocking trains. A system with similar objectives was developed by Bourachot [2] for the Swiss railway system. All models within the latter system are solved heuristically, in contrast with the models within STATIONS. The developed solution procedures can also be used in the DSS RailEase (see Odijk [6]).

2 Problem description and notation

Within STATIONS always one railway station is considered at a time. The problem that is solved for this railway station can be stated as follows:

> Given the layout of the railway station, and given the planned arrival and departure times of a set of trains, STATIONS aims at routing as many trains from this set as possible through the railway station, taking into account restrictions set by the safety system, and taking into account (un-)coupling, connection and service requirements. The routing of the trains should minimize the number of shunting operations and the number of deviations from the planned arrival and departure times, and it should maximize the total preference for the platforms and routes.

The layout of the railway station consists of a set S of track sections. The set of platforms is denoted by P. The set R of routes through the railway station can be determined from the set of sections. We distinguish the set $R^i \subset R$ of inbound routes leading towards a platform and the set $R^o \subset R$ of outbound routes departing from a platform.

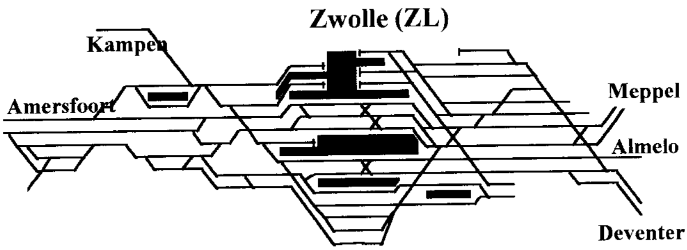

Figure 1: Infrastructure of the railway station Zwolle.

The set of trains to be routed through the railway station is denoted by T. Train t has arrival time A_t and departure time D_t, usually in minutes. In principle the involved timetables are assumed to be cyclic with a cycle time

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of one hour. That is, each hour the same pattern of arrivals and departures of trains is repeated. The cyclicity of the timetable is modelled by carrying out the calculations of the time instants *modulo 60 minutes*. Furthermore if a routing for all trains can not be obtained, given the planned arrival and departure times of the trains, then small deviations from these times may be allowed. In that case the second objective is to route as many trains as possible through the railway station with a minimum number of deviations. The allowed deviations for each train are determined by CADANS and correspond to allowed changes in the timetable without changing the order of the trains at the tracks between the railway stations. The set of allowed arrival and departure deviations for train t is denoted by Δ_t^a and Δ_t^d, respectively. Note that 0 is an element of all sets of deviations.

If the railway station is one of the final stations of train t, and the length of the train's standstill interval $D_t - A_t$ exceeds a certain lower bound, then the train may be shunted towards a certain parking area in order to release the arrival platform. Later on, the train will be shunted back towards its departure platform, which needs not be the same as its arrival platform. Thus, both the capacity and the flexibility within the railway station are increased by the shunting movements. On the other hand, it is desirable to minimize the number of shunting movements, since they also increase the complexity and the operating costs of the railway system within the station. The set of trains for which it has to be decided whether they are shunted or not is denoted by $T' \subset T$.

For each train $t \in T$ a set $P_t \subset P$ of allowed platforms is given, determined by the train's entering direction, its leaving direction, and a number of other relevant factors, like platform preferences, lengths of trains and platforms, etc. Based on this set of allowed platforms for train t, also a set $R_t^i \subset R$ of allowed inbound routes and a set $R_t^o \subset R$ of allowed outbound routes can be determined for train t. Based upon the platform and the route itself, the preference of a route can be determined. This preference can be based upon, for example, minimizing the passing of switches in non-preferred position, minimizing the travel time of the trains, or maximizing the speed of the trains. Finally, the set of all routes allowed for train t is denoted by $R_t \subset R$.

A combination of a route r and deviation δ is addressed in this paper as a *routing possibility* f. If the routing possibility represents an inbound (outbound) route, then the deviation represents an arrival (departure) deviation. The set of inbound and outbound routing possibilities for train t are denoted by F_t^i and F_t^o, respectively. A combination of a platform and an allowed combination of arrival and departure deviation is also referred to as a *routing possibility*. The set of platform routing possibilities is denoted by F_t^p. The set of all routing possibilities for train t is denoted by F_t.

Clearly, the routing of one train also depends on the routings of others. Most importantly, the Dutch safety rules dictate the following route reservation procedure. As soon as a train arrives in the neighbourhood of the railway station, it reserves a complete appropriate inbound route leading towards a platform. No section within the reserved inbound route may be reserved by another train until the section has been released again. The latter happens after a buffer time following the leaving by the train of the section. The buffer time is included for robustness reasons. Thus all sections within the inbound route are reserved at the same time, and they are released section-by-section. A similar procedure is followed for the reservation and release of an outbound route.

These safety rules are represented by a set $F_{t,t'}$ for each pair of trains $t, t' \in T$. Such a set contains the allowed combinations of routing possibilities $\{(r,\delta),(r',\delta)\}$ for trains t and t'. Here $\{f, f'\} = \{(r,\delta),(p',\delta')\} \in F_{t,t'}$ implies that, if train t is assigned to route r with arrival or departure deviation δ and train t' is assigned to platform p' with the combination of arrival and departure deviation δ, then no section will be reserved simultaneously by both trains. The sets $F_{t,t'}$ are determined by calculating in detail all time instants corresponding to the reservation of a route and the release of a section, taking into account initial velocities, acceleration and deceleration characteristics, etc. This model can accommodate a large variety of safety rules (including the current Dutch ones).

Actually, many other constraints can be modelled in the same way. First of all, we have to guarantee that for each train t the selected inbound routes, outbound routes and platforms are *compatible*. These constraints are handled by sets $F_{t,t}$, which have the same interpretation as the sets $F_{t,t'}$. Indeed, only compatible pairs of routing possibilities are included in a set $F_{t,t}$. In a similar fashion, we can include (un-)coupling constraints, transfer possibilities for passengers and convenience considerations such as that all trains leaving in the same direction depart from the same platform.

3 Model formulation

The routing problem is formulated as a maximization problem, where the first objective is to maximize the number of trains that can be routed through the railway station, the second objective is to minimize the number of shunting movements and the third objective is to maximize the preferences for the platforms and routes.

In this paper the model of Zwaneveld et al. [8] is extended by also including the shunting decisions and the preferences for the platforms and routes. In our model we assume that a train $t \in T'$ that may be shunted towards a parking area is waiting at its arrival platform between the time instants A_t and $A_t + \tau$. Between $A_t + \tau$ and $D_t - \tau$ the train may be shunted towards a parking area, and between $D_t - \tau$ and D_t the train is waiting at its departure platform, which need not be the same as its arrival platform. Here the parameter τ denotes the time required by passengers to unboard

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and board a train.

In this approach the shunting movements themselves are not taken into account. However, this approach was adopted after ample discussions with the involved planners in practice. It is justified by the fact that the model is used in strategic capacity planning studies, where one is primarily interested in the capacity implications of the shunting movements and not in the detailed shunting movements themselves. Moreover, at this moment the detailed future capacities and locations of the parking areas are not yet known. Furthermore, if this would be desirable at some time in the future, then the model may be extended in such a way that also the shunting movements themselves are taken into account.

In order to model the described problem, we introduce a binary decision variable $X_{t,f}$ for each train $t \in T$ and each routing possibility $f \in F_t^i$ or F_t^o. The decision variable $X_{t,f}$ assumes the value 1 if train t is assigned to routing possibility $f = (r, \delta)$ and the value 0 otherwise. If $f \in F_t^i$, then this inbound routing possibility represents the inbound route r and the arrival at the platform at $A_t + \delta^a$. If $f \in F_t^o$, then this outbound routing possibility represents the outbound route r and the departure from the platform at $D_t + \delta^d$.

Furthermore, for each train $t \in T$ and each allowed platform routing possibility $f = (p, \delta) \in F_t^p$, we introduce the binary decision variable $X_{t,f}$. This decision variable $X_{t,f}$ assumes the value 1 if train t is waiting at platform p between $A_t + \delta^a + \tau$ and $D_t + \delta^d - \tau$, and it assumes the value 0 otherwise. The value 0 for all platform routing possibilities for a train $t \in T'$ corresponds to the shunting of the train towards a parking area.

The preference of a routing possibility f (and thus the platform involved) in combination with train t is denoted by $\gamma_{t,f}$. If f represents a routing possibility with a zero deviation from the original arrival or departure time, then $\gamma_{t,f}$ is sufficiently greater than the objective coefficients of routing possibilities representing non-zero deviations. Now the routing problem of trains through railway stations reads as follows.

$$\max \sum_{t \in T} \sum_{f \in F_t^i} (1 + \gamma_{t,f}) X_{t,f} + \sum_{t \in T} \sum_{f \in F_t^o} (1 + \gamma_{t,f}) X_{t,f} +$$

$$\sum_{t \in T \setminus T'} \sum_{f \in F_t^p} (1 + \gamma_{t,f}) X_{t,f} + \frac{1 + \gamma_{t,f}}{|T'| + 1} \sum_{t \in T'} \sum_{f \in F_t^p} X_{t,f} \tag{1}$$

subject to

$$\sum_{f \in F_t^i} X_{t,f} \leq 1 \qquad \text{for all } t \in T, \tag{2}$$

$$\sum_{f \in F_t^o} X_{t,f} \leq 1 \qquad \text{for all } t \in T, \tag{3}$$

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$$\sum_{f \in F_t^p} X_{t,f} \leq 1 \qquad \text{for all } t \in T, \tag{4}$$

$$X_{t,f} + X_{t',f'} \leq 1 \quad \text{for all } t, t' \in T;\ f \in F_t;\ f' \in F_{t'};\ (f, f') \notin F_{t,t'}, \tag{5}$$

$$X_{t,f} \in \{0, 1\} \qquad \text{for all } t \in T;\ f \in F_t. \tag{6}$$

The objective function (1) represents the fact that the first aim is to maximize the number of trains that can be routed through the railway station, that the second aim is to minimize the number of shunting movements and that the third objective is to maximize the total preferences for platforms and routes. Note that the penalty of a shunting movement is only $\frac{1}{|T'|+1}$ and that the values $\gamma_{t,f}$ have to be sufficiently small. Thus, in an optimal solution to the model, the number of trains that can be routed through the railway station is maximal.

Constraints (2), (3) and (4) ensure that each train t is assigned to at most one inbound, outbound, and platform routing possibility. Constraints (5) guarantee that only combinations of routing possibilities are selected that are allowed with respect to the safety rules, compatibility of inbound and outbound routes and platforms, transfer possibilities, etc. Finally, constraints (6) declare the decision variables $X_{t,f}$ as binary.

4 Solution procedure, results and DSS

Obviously,the integer linear program (1) to (6) allows a representation $\max \{ c^t X \mid MX \leq \mathbf{1}, X \text{ binary} \}$. Here X is a binary vector of decision variables, M is a zero/one matrix, and $\mathbf{1}$ is a vector of 1's. The latter is the general representation of a *Weighted Node Packing Problem* (WNPP) on the incidence graph of the zero/one matrix M. The interpretation of the routing problem as a WNPP allows one to deduce a number of valid inequalities that tighten the integer programming formulation, and thus make the LP-relaxation more accurate. For example, inequalities that are valid for the WNPP are *clique* inequalities, and *(lifted) odd hole* inequalities (cf. Hoffman & Padberg [4].) In our application we only consider clique inequalities.

In order to test the model and to experiment with it, we developed a solution procedure consisting of a number of consecutive steps. Below, these steps are described briefly.

Step 1: Preprocessing. First, we determine the sets $F_{t,t'}$ for all pairs of trains, based on the layout of the railway station, on the arrival and departure times of the trains, and on the previously mentioned criteria. Next, we try to reduce the problem instance a priori. This can be accomplished by removing superfluous routing possibilities. We have developed many preprocessing techniques to identify superfluous routing possibilities. Most of these preprocessing techniques are based only on the formulation of the problem as a WNPP problem. A preprocessing technique based on the infrastructure of the railway station follows from lemma 1 (cf. Kroon et al. [5]).

Lemma 1 *Given the present safety system of the Netherlands. Only the sections (i) containing a switch, (ii) corresponding to the entering and leaving points of a railway station, (iii) corresponding to the platforms, or (iv) corresponding to a crossing of routes have to be considered for determining feasibility from a safety point of view.*

Sections satisfying (i) – (iv) are called *important*. The set of important sections is denoted by S^*. The sections of route r are denoted by S_r. A route r is a *detour* route, if

$$\exists \tilde{r} \;:\; S_{\tilde{r}} \cap S^* \subset S_r \cap S^*.$$

An example of a *detour* route and corresponding *straight* route is given in figure 2. In most cases, variables involving a detour route are superfluous

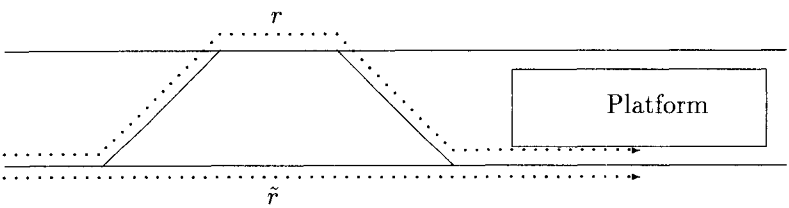

Figure 2: Example of a *detour* route r and a corresponding *straight* route $\tilde{r}$.

and can be removed from the problem instance.

Another preprocessing technique is the following: An inbound (outbound) routing possibility f for train t is dominated if there exists another inbound (outbound) routing possibility f' for train t that has less conflicts with all routing possibilities for all other trains.

Step 2: Generation. We generate the overall model based on the formulation of the problem as a WNPP. We generate a subset of the clique inequalities. For example, all inbound possibilities of train t' that are incompatible with routing possibility f for train t can be used to obtain the valid inequality

$$X_{t,f} + \sum_{f' \in F_{t'}^{i};(f,f') \notin F_{t,t'}} X_{t',f'} \leq 1. \qquad (7)$$

The constraint (7) replaces many of the equations (5) and tighten the formulation of the LP-relaxation. In fact, a similar summation over the routing possibilities of the same type as f for train t can be performed. The resulting problem formulation turns out to be rather compact and tight.

Step 3: Optimization. We use a heuristic to obtain an initial feasible solution, and we solve the LP-relaxation of the overall model. Thereby we obtain a lower bound and an upper bound to the optimal solution. Finally, we apply a specially designed branch & cut procedure to obtain an optimal solution to the overall model. For a comprehensive description of the branch & cut concept we refer to Hoffmann & Padberg [4].

4.1 Results

The above described procedure was implemented on a SUN workstation using CPLEX as the LP-solver. We first experimented with the railway station Zwolle, a medium-sized railway station in The Netherlands (cf. Figure 1). Since the results for this railway station were satisfying, the next challenge was the railway station Utrecht CS, which is the largest railway station in The Netherlands. The number of trains that have to be routed each hour through this railway station equals about 50. About 15 of these trains may be shunted towards a parking area. We experimented with a number of scenarios with respect to the arrival and departure times of the trains.

Initially, the model contains about 15.000 binary decision variables and about 20×10^6 constraints. The majority of the latter are constraints (5). After the application of the preprocessing techniques and the addition of the clique inequalities, a model is obtained with about 1.400 binary decision variables and about 3.000 constraints. On average, the application of the preprocessing techniques requires 2 minutes and the generation of the clique inequalities requires 1 minute. Finally, the optimization step requires about 3 minutes. In many cases the optimal solution to the LP-relaxation of the model is all-integer already. Although the average CPU-time is acceptable, more tests need to be carried out to give more sound conclusions. Especially the number of allowed deviations from the original arrival and departure times determine the initial problem size and thereby the overall computing time.

4.2 Implementation in DSS

Currently, a prototype of STATIONS is being implemented. The main functionalities of the system are : (1) a database for the infrastructure of the railway stations, (2) a link with the network planner CADANS, (3) the automatic routing of all trains, (4) interactive solution possibilities, and (5) a detailed report of the obtained solution. This report contains, among others, the detailed movements of all trains, platform track occupation charts (similar to Gantt charts), and information about the most intensively used track sections. If the required capacity is not yet sufficient for routing all trains through the railway station, the blocking trains are identified and reported.

5 Final remarks

It should be noted that we focused mainly on maximizing the number of trains that can be routed through the railway station, on minimizing the number of shunting movements and on maximizing the preferences of the platforms and routes. However, also more sophisticated performance indicators, such as the robustness of a solution with respect to disturbances in the arrival and departure times of the trains, may be used to judge the quality of a solution. This topic is a subject for further research.

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