A Decomposition Methodology Applied to the Multi-Area Optimal Power Flow Problem

FRANCISCO J. NOGALES and FRANCISCO J. PRIETO* {fjnm,fjp}@est-econ.uc3m.es
Department of Statistics and Econometrics, University Carlos III de Madrid, Spain

ANTONIO J. CONEJO** Antonio.Conejo@uclm.es
Department of Electrical Engineering, University of Castilla-La Mancha, Campus Universitario, s/n, 13071 Ciudad Real, Spain

Abstract. This paper describes a decomposition methodology applied to the multi-area optimal power flow problem in the context of an electric energy system. The proposed procedure is simple and efficient, and presents some advantages with respect to other common decomposition techniques such as Lagrangian relaxation and augmented Lagrangian decomposition. The application to the multi-area optimal power flow problem allows the computation of an optimal coordinated but decentralized solution. The proposed method is appropriate for an Independent System Operator in charge of the electric energy system technical operation. Convergence properties of the proposed decomposition algorithm are described and related to the physical coupling between the areas. Theoretical and numerical results show that the proposed decentralized methodology has a lower computational cost than other decomposition techniques, and in large large-scale cases even lower than a centralized approach.

Keywords: electric energy systems, multi-area optimal power flow, nonlinear programming, decomposition methods, decentralized coordination

Introduction

In this paper, a decomposition procedure is described and applied to a multi-area Optimal Power Flow (OPF) problem in the context of an electric energy system that spans several interconnected areas. It is often desirable to preserve the autonomy of each area in these systems. A decentralized operation can be preserved while still attaining overall optimality by applying decomposition techniques to a centralized operation problem.

Decomposition techniques have often been used in the solution of many operations and planning problems with the above aim. Also, the application of these techniques may provide potential gains in computational efficiency and useful information as part of the decomposition process.

The multi-area OPF problem is an important problem for the secure and economic operation of an interconnected power system. The multi-area OPF determines, in a

* Research supported by Spanish grants PB98-0728 and BEC 2000-0167.

** Research partly supported by Ministerio de Ciencia y Tecnología of Spain, project CICYT DPI-2000-0654.

precise way, the active and reactive power that each generation unit in the system must generate. This is done to ensure that all demand and security constraints for the system are satisfied at a minimal cost for all interconnected areas. The resulting multi-area OPF problem is a large-scale nonconvex optimization problem [12,21].

The decomposition methodology proposed in this paper is simple and efficient. The procedure allows the company in each area to operate its system independently of the other areas, while obtaining an optimal coordinated but decentralized solution. A central agent in the model is necessary to collect and distribute information for the whole system. This agent ensures the coordination of the global system and therefore, the proposed methodology is appropriate for an Independent System Operator (ISO) in charge of the electric energy system technical operation.

The local convergence properties of the proposed algorithm depend on the spectral radius of a matrix related to the Newton matrix of the global system. If this spectral radius is smaller than one, local convergence is guaranteed. In many practical cases this condition is satisfied. However, if it is not, local convergence can be guaranteed using conjugate gradient techniques, see [10].

Several other decomposition techniques, such as Lagrangian relaxation [3,13,16], and relaxation techniques based on augmented Lagrangian functions [5,6,17], have been proposed for the solution of similar problems. Particularly remarkable is the theoretical decomposition framework based on the auxiliary problem principle analyzed in [6–8]. An application of Lagrangian relaxation to solve a multi-area OPF is described in [1,9], while in [2,14] an augmented Lagrangian relaxation procedure is used to solve a distributed OPF. In some cases, Lagrangian procedures may present drawbacks, such as difficulties to converge to an optimal solution for the global system (in the absence of convexity assumptions), and convergence rates that depend on the correct choice of the values for several parameters which may be difficult to update, and require the intervention of a central agent to update this information.

The proposed decomposition algorithm presents the following advantages:

1. A coordinated solution of the global problem is achieved in a decentralized manner. The coupling of the system is obtained through the Lagrange multipliers associated with certain constraints. However, the proposed technique is essentially different than Lagrangian decomposition techniques. In the proposed procedure, the Lagrange multipliers do not need to be estimated (as in most common Lagrangian relaxation approaches), because the proposed method provides efficient information to update these multipliers.

2. Computational efficiency is improved. In the proposed decomposition algorithm, an exact solution of each subproblem is not required in every iteration. The results from a single iteration of each subproblem are enough, resulting in considerable savings in computing time.

3. The implementation is simple and robust. The procedure generates subproblems that are slightly modified versions of the optimization problems for each area. Also, the

algorithm requires very few parameters, and their updating procedures are clearly specified.

4. The solution process is simple. The central agent does no need to update any information, only to distribute it. This information is updated by the different areas of the system.
5. It can be implemented in a distributed computation environment, given its reduced requirements for information exchanges between areas.

The advantages of the proposed decomposition technique with respect to Lagrangian relaxation and augmented Lagrangian decomposition are mainly two:

1. The proposed technique does not need to solve subproblems until optimality in very iteration and this results in relevant computational savings. On the contrary, most common Lagrangian procedures applied to practical problems do need to solve the subproblems until optimality in order to obtain multipliers updates.
2. In the proposed technique, the coordinator does not update information but collects and distributes it. On the contrary, the coordinator in most common Lagrangian procedures applied to practical problems, does need to update information.

The remaining of the paper is organized as follows. In section 1, a mathematical formulation of the multi-area OPF model is given. Section 2 describes the proposed decomposition methodology and shows its convergence properties. Section 3 presents computational results, and section 4 provides some conclusions. Finally, an appendix shows a simple example that illustrates the main ideas behind the proposed procedure.

1. Problem formulation

The model for the multi-area OPF problem is described below. The notation used in the model is:

A	total number of areas,
B	total number of buses,
G	total number of generators,
L	total number of transmission lines,
Λ_j	set of indices of generators in bus j,
Ω_j	set of indices of buses connected to bus j,
Θ	set of indices of transmission lines,
Y_{jk}	admittance magnitude matrix,
δ_{jk}	admittance phase matrix,
p_{G_i}	active power produced by generator i,
q_{G_i}	reactive power produced by generator i,
v_j	voltage magnitude in bus j,
θ_j	voltage phase in bus j,
$P_{G_i}^{\max}$, $P_{G_i}^{\min}$	maximum and minimum active power production capacity of generator i,

$Q_{G_i}^{\max}$, $Q_{G_i}^{\min}$ maximum and minimum reactive power production capacity of generator i,

$V_j^{\max}$, $V_j^{\min}$ maximum and minimum voltage magnitude in bus j,

P_{D_j} active power demand in bus j,

Q_{D_j} reactive power demand in bus j,

$S_{jk}^{\max}$ maximum transmission capacity of line jk.

The multi-area OPF model can be formulated as

$$\begin{aligned}
&\text{minimize } f(p_G, q_G, v, \theta) && && (1)\\
&\text{subject to } \; a_j(p_G, v, \theta) = 0, && j = 1, \dots, B, && (2)\\
&\qquad r_j(q_G, v, \theta) = 0, && j = 1, \dots, B, && (3)\\
&\qquad t_j(v, \theta) \leqslant 0, && j = 1, \dots, L, && (4)\\
&\qquad P_{G_i}^{\min} \leqslant p_{G_i} \leqslant P_{G_i}^{\max}, && i = 1, \dots, G, && (5)\\
&\qquad Q_{G_i}^{\min} \leqslant q_{G_i} \leqslant Q_{G_i}^{\max}, && i = 1, \dots, G, && (6)\\
&\qquad V_j^{\min} \leqslant v_j \leqslant V_j^{\max}, && j = 1, \dots, B, && (7)\\
&\qquad -\pi \leqslant \theta_j \leqslant \pi, && j = 1, \dots, B. && (8)
\end{aligned}$$

Function (1) is the objective function. Different objective functions may be of interest for an ISO, such as: total system operation cost, total power transmission losses, total system emissions, congestion clearance, or the achievement of a feasible solution, for example.

The power flow equations are included in the model as constraints (2), (3); there are two equations for each bus of the global system, representing the active and reactive power balance in each node,

$$\sum_{i\in\Lambda_j^a} p_{G_i}^a - P_{D_j}^a = v_j^a \sum_{k\in\Omega_j^a} Y_{jk}^a v_k^a \cos\bigl(\theta_j^a - \theta_k^a - \delta_{jk}^a\bigr), \quad j = 1, \dots, N^a, \tag{9}$$

$$\sum_{i\in\Lambda_j^a} q_{G_i}^a - Q_{D_j}^a = v_j^a \sum_{k\in\Omega_j^a} Y_{jk}^a v_k^a \sin\bigl(\theta_j^a - \theta_k^a - \delta_{jk}^a\bigr), \quad j = 1, \dots, N^a, \tag{10}$$

where $a = 1, \dots, A$, and the superscripts ^{a} indicate the area for each constant and variable.

Constraints (4) are the transmission capacity limits for each line of the global system,

$$\begin{aligned}
&\bigl(v_j^a v_k^a Y_{jk}^a \cos\bigl(\theta_j^a - \theta_k^a - \delta_{jk}^a\bigr)\bigr)^2 + \bigl(v_j^a v_k^a Y_{jk}^a \sin\bigl(\theta_j^a - \theta_k^a - \delta_{jk}^a\bigr)\bigr)^2 \leqslant \bigl(S_{jk}^{\max}\bigr)^2,\\
&\quad (j,k) \in \Theta^a, \quad a = 1, \dots, A.
\end{aligned} \tag{11}$$

Constraints (5)–(8) represent technical limits over variables. Model (1)–(8) can be written in compact form as

$$\text{minimize } \sum_{a=1}^{A} f_a(x_a) \tag{12}$$

$$\begin{aligned} \text{subject to} \quad & h_a(x_1, \ldots, x_a) \leqslant 0, \quad && a = 1, \ldots, A, && (13) \\ & g_a(x_a) \leqslant 0, && a = 1, \ldots, A, && (14) \end{aligned}$$

where x_a are the state variables for each area a of the global system, that contain bus voltage magnitudes and bus phase angles. In addition, x_a also contains information on the control variables for area a, such as real and reactive power generation, phase-shifter angles, voltage control settings and transformer taps settings, for example. Constraints (13) represent the power flow equations and transmission capacity limits (9)–(11) for those lines and buses interconnecting different areas. Constraints (14) include the power flow equations and transmission capacity limits (9)–(11), only for those lines and buses lying within a given area, and limits over dependent and control variables (5)–(8). It should be noted that the sets of constrains (13) and (14) represent both equality and inequality constraints.

The multi-area OPF model (12)–(14) is a nonconvex large-scale optimization problem. Constrains (13) are commonly known as *complicating constraints*. These equations contain variables from different areas and prevent each system from operating independently from the others. If these equations are removed from problem (12)–(14), the resulting problem can be trivially decomposed into one subproblem for each area.

The complicating constraints (13) include the power balance equations at the interconnecting buses of area a (the buses from area a connected to buses from areas b different than area a). Also, the transmission capacity limits for the interconnecting lines of the global system are complicating constraints. It should be noted that, the only variables appearing in the complicating constraints are those corresponding to the interconnecting buses of the global system.

Constrains (14) contain only variables belonging to area a for $a = 1, \ldots, A$. These constraints represent balance equations, transmission limits, and technical constraints for area a.

The proposed decomposition is as follows. Problem (12)–(14) is equivalent to the problem below

$$\begin{aligned} \text{minimize} \quad & \sum_{a=1}^{A} f_a(x_a) + \sum_{a=1}^{A} \lambda_a^{\mathrm{T}} h_a(x_1, \ldots, x_A) && && (15) \\ \text{subject to} \quad & h_a(x_1, \ldots, x_a) \leqslant 0, \quad && a = 1, \ldots, A, && (16) \\ & g_a(x_a) \leqslant 0, && a = 1, \ldots, A. && (17) \end{aligned}$$

Given trial values to all variables and multipliers (indicated by overlining) different than those in area a, problem (15)–(17) reduces to

$$\begin{aligned} \text{minimize} \; & k + \sum_{a=1}^{A} f_a(x_a) + \sum_{b=1, b \neq a}^{A} \bar{\lambda}_b^{\mathrm{T}} h_b(\bar{x}_1, \ldots, \bar{x}_{a-1}, x_a, \bar{x}_{a+1}, \ldots, \bar{x}_A) && (18) \\ \text{subject to} \quad & h_a(\bar{x}_1, \ldots, \bar{x}_{a-1}, x_a, \bar{x}_{a+1}, \ldots, \bar{x}_A) \leqslant 0, && (19) \\ & g_a(x_a) \leqslant 0, && (20) \end{aligned}$$

where $k = \sum_{b=1, b \neq a}^{A} f_b(\bar{x}_b)$ is a constant.

The dual variable vector corresponding to constraint (19) is denoted by λ_a.

The reduced problem (18)–(20) can be obtained for every area. The proposed decomposition technique is actually based on the solutions of these reduced area problems.

2. Proposed decomposition methodology

2.1. *Decomposition derivation*

The proposed method is based on the decomposition of the optimality conditions for the global problem (12)–(14), see [3]. Note that from standard optimization theory, the first-order optimality conditions for problem (12)–(14) are:

$$\nabla_{x_a} f_a(x_a^*) + \sum_{a=1}^{A} \nabla_{x_a}^{\mathrm{T}} h_a(x_1^*, \ldots, x_A^*)\lambda_a^* + \nabla_{x_a}^{\mathrm{T}} g_a(x_a^*)\sigma_a^* = 0, \quad a = 1, \ldots, A, \tag{21}$$

$$h_a(x_1^*, \ldots, x_A^*) \leqslant 0, \quad a = 1, \ldots, A, \tag{22}$$

$$h_a(x_a^*)^{\mathrm{T}}\lambda_a^* = 0, \quad a = 1, \ldots, A, \tag{23}$$

$$\lambda_a^* \geqslant 0, \quad a = 1, \ldots, A, \tag{24}$$

$$g_a(x_a^*) \leqslant 0, \quad a = 1, \ldots, A, \tag{25}$$

$$g_a(x_a^*)^{\mathrm{T}}\sigma_a^* = 0, \quad a = 1, \ldots, A, \tag{26}$$

$$\sigma_a^* \geqslant 0, \quad a = 1, \ldots, A. \tag{27}$$

These conditions have been constructed using the optimal values x_a^*, λ_a^* and σ_a^* that are assumed known. The values λ_a^* are the optimal Lagrange multipliers associated with constrains (13) and the values σ_a^* are the optimal Lagrange multipliers associated with constrains (14).

For convenience, the area reduced subproblem (18)–(20) is restated below for optimal values x_a^*, λ_a^* and σ_a^*:

$$\text{minimize } f_a(x_a) + \sum_{a=1, b\neq a}^{A} \lambda_b^{*\mathrm{T}} h_b(\boldsymbol{x}^a) \tag{28}$$

$$\text{subject to } \quad h_a(\boldsymbol{x}^a) \leqslant 0, \tag{29}$$

$$g_a(x_a) \leqslant 0, \tag{30}$$

where

$$\boldsymbol{x}^a = (x_1^*, \ldots, x_{a-1}^*, x_a, x_{a+1}^*, \ldots, x_A^*).$$

If the first-order optimality conditions of every area reduced subproblem (28)–(30) ($a = 1, \ldots, A$) are stucked together, it can be observed that they are identical to the first-order optimality conditions (21)–(27) of the global problem (12)–(14). It should be emphasized that this is a relevant result that is exploited in the algorithm below.

As previously stated, area subproblem (28)–(30) is obtained relaxing all the complicating constraints of others areas (i.e. adding them to the objective function of problem (12)–(14)) and maintaining its own complicating constraints. The reduction is possible once the optimization variables are given trial values. The main difference between the Lagrangian relaxation algorithm and the proposed decomposition one is that Lagrangian relaxation adds all the complicating constraints into the objective function. Therefore it needs auxiliary procedures to update the Lagrange multipliers. On the contrary, the proposed technique does not need any procedure to update the multipliers because this updating is automatic and results from keeping the complicating constraints (29) in every area subproblem.

The proposed approach has the advantage that convergence properties do not require an optimal solution of the subproblems at each iteration of the algorithm. It is enough to perform a single iteration for each subproblem, and then to update variable values. As a consequence, computation times can be significantly reduced with respect to other methods that require the computation of the optimum for the subproblems in order to attain convergence.

The coordination of the global system to ensure the satisfaction of the complicating constraints is achieved through the Lagrange multipliers associated with constrains (13).

2.2. Decomposition algorithm

An outline of the proposed algorithm is as follows:

Step 0.

Each area ($a = 1, \dots, A$) initializes its variables and parameters, $\bar{x}_a$, $\bar{\lambda}_a$.

Step 1.

Each area ($a = 1, \dots, A$) carries out one iteration for its corresponding subproblem

$$\text{minimize } f_a(x_a) + \sum_{b=1, b \neq a}^{A} \bar{\lambda}_b^{\mathrm{T}} h_b(\boldsymbol{x}^a) \tag{31}$$

$$\text{subject to } \quad h_a(\boldsymbol{x}^a) \leqslant 0, \tag{32}$$

$$g_a(x_a) \leqslant 0, \tag{33}$$

where $\boldsymbol{x}^a = (\bar{x}_1, \dots, \bar{x}_{a-1}, x_a, \bar{x}_{a+1}, \dots, \bar{x}_A)$, and obtains search directions Δx_a, $\Delta\lambda_a$.

Step 2.

Each area ($a = 1, \dots, A$) updates its variables and parameters

$$\bar{x}_a \leftarrow \bar{x}_a + \Delta x_a, \qquad \bar{\lambda}_a \leftarrow \bar{\lambda}_a + \Delta\lambda_a, \qquad \text{for } a = 1, \dots, A.$$

The central agent distributes updated information of border buses and lines.

Step 3.

The algorithm stops if variables do not change significantly in two consecutive iterations. Otherwise, it continues in step 1.

To speed convergence, the search directions obtained in step 1 can be refined using a Conjugate Gradient procedure as stated in [10].

Observe that problem (31)–(33) is a modified OPF for area a. The only departure from a standard OPF is the Lagrangian term in the objective function (31). This property allows a robust and simple implementation of the decomposition procedure, as specialized codes can be used for the subproblems.

The search directions, $(\Delta x_a, \Delta\lambda_a)$, for subproblems (31)–(33) can be computed independently of each other, allowing a parallel implementation in a distributed computation environment. In this paper, a modified Newton procedure is used, in conjunction with a nonlinear interior point treatment of the inequality constraints [22]. This treatment requires adding slack variables to the inequality constraints in (32), to convert them into equality constraints. These slack variables are then incorporated to the objective function through logarithmic barrier terms, to ensure their positivity.

Step 2 requires a central agent to coordinate the process; an ISO could perform this role. This agent receives certain information from the areas and returns it to the appropriate areas. This information consists of some of the values $\bar{x}_a$, $\bar{\lambda}_a$, for $a = 1, \ldots, A$. The values $\bar{x}_a$ that have to be distributed are the updated values of the variables associated with the interconnecting buses and lines, after one iteration of Newton's method. The values $\bar{\lambda}_a$ that need to be distributed are the updated multipliers corresponding to equation (32) of each area, again after one iteration of Newton's method.

It can be noted that the information exchanged between the areas and the central agent is minimal. Moreover, in the proposed decomposition algorithm the central agent only distributes information and checks the convergence condition. In other decomposition techniques (such as most common Lagrangian relaxation or augmented Lagrangian decomposition procedures) the central agent needs to update the exchanged information before distributing it to the different areas. In the proposed decomposition algorithm the central agent does not need to update any information, as this information is updated by the areas of the system, implying a simpler process.

2.3. Convergence properties

The convergence properties of the proposed decomposition algorithm are analyzed below. For the sake of simplicity in this discussion, and without loss of generality, separable constraints (14) are omitted. These constraints can be introduced into the objective function by means of an interior point procedure. Also, the systems will be represented using only two areas, a and b. It should be immediate to generalize the following results to more than two areas.

For the centralized approach, the search directions for areas a and b, $(\Delta_a^{\mathrm{N}}, \Delta_b^{\mathrm{N}})$ are computed by solving in each iteration a system of linear equations of the form

$$\mathrm{KKT} = \begin{bmatrix} \mathrm{KKT}_a & \mathrm{KKT}_{ba} \\ \mathrm{KKT}_{ab} & \mathrm{KKT}_b \end{bmatrix} \begin{bmatrix} \Delta_a^{\mathrm{N}} \\ \Delta_b^{\mathrm{N}} \end{bmatrix} = - \begin{bmatrix} \nabla_{x_a,\lambda_a} L \\ \nabla_{x_b,\lambda_b} L \end{bmatrix}, \tag{34}$$

where KKT_a, KKT_b, KKT_{ab} and KKT_{ba} are the Newton matrices [4] for areas a and b, defined as

$$\mathrm{KKT}_a = \begin{bmatrix} \nabla^2_{x_a,x_a} L & \nabla_{x_a} h_a^{\mathrm{T}} \\ \nabla_{x_a} h_a & 0 \end{bmatrix}, \qquad \mathrm{KKT}_b = \begin{bmatrix} \nabla^2_{x_b,x_b} L & \nabla_{x_b} h_b^{\mathrm{T}} \\ \nabla_{x_b} h_b & 0 \end{bmatrix},$$

$$\mathrm{KKT}_{ab} = \begin{bmatrix} \nabla^2_{x_a,x_b} L & \nabla_{x_b} h_a^{\mathrm{T}} \\ \nabla_{x_a} h_b & 0 \end{bmatrix}, \qquad \mathrm{KKT}_{ba} = \mathrm{KKT}_{ab}^{\mathrm{T}},$$

the superscript N indicates Newton directions, and L is the *Lagrangian function* for problem (12)–(14), defined as

$$L(x_1, \dots, x_A, \lambda_1, \dots, \lambda_A) = \sum_{a=1}^{A} f_a(x_a) + \sum_{a=1}^{A} \lambda_a^{\mathrm{T}} h_a(x_a, \dots, x_A). \tag{35}$$

Correspondingly, movement directions for areas a and b, (Δ_a, Δ_b), in step 1 of the decomposition algorithm can be obtained by solving the decomposable and approximate linear system of equations

$$\overline{\mathrm{KKT}} = \begin{bmatrix} \mathrm{KKT}_a & 0 \\ 0 & \mathrm{KKT}_b \end{bmatrix} \begin{bmatrix} \Delta_a \\ \Delta_b \end{bmatrix} = - \begin{bmatrix} \nabla_{x_a,\lambda_a} L \\ \nabla_{x_b,\lambda_b} L \end{bmatrix}. \tag{36}$$

From these definitions and from performing step 1 of the proposed algorithm in parallel, the sufficient condition for convergence of the proposed decomposition algorithm is given below. If at the optimal solution of problem (12)–(14) it holds that

$$\rho\left(I - \overline{\mathrm{KKT}}^{-1} \mathrm{KKT}\right) < 1, \tag{37}$$

then the proposed decomposition algorithm converges locally to the solution at a linear rate. Here $\rho(A)$ denotes the spectral radius of matrix A, matrix I is the identity matrix and it is assumed that functions in (12)–(14) are twice continuously differentiable. Condition (37) is related to the many results reported in the technical literature for the distributed solution of linear systems of equations, see, for example, [11,18]. Finally, note that by using Newton's method, the local rate of convergence for a centralized approach can be quadratic.

Condition (37) can be interpreted as a measurement of the coupling between the areas in the global system. This measure tends to be smaller for systems with a small number of interconnecting lines. It has been verified that condition (37) holds for all multi-area OPF cases that have been found available to test the procedure. In this regard, these convergence properties seem to be satisfied for most practical cases of interest.

If condition (37) does not hold, it is possible to modify the proposed decomposition algorithm to attain convergence [10]. For example, a preconditioned Conjugate Gradient method [11] can be applied. This approach would still preserve the property that the operation could be performed allowing each area to maintain its autonomy, i.e. in a decentralized manner.

3. Numerical results

This section presents numerical results obtained by applying the proposed decomposition algorithm to several test problems. Table 1 shows the most relevant characteristics for each problem. Case I correspond to the IEEE-9 system [15]; this system has been divided into two areas using two interconnecting lines. Case II is based on the IEEE-30 system [15]. Cases III and V are based on the IEEE RTS-24 [19]; this system has been duplicated for case III, and replicated three times in case V, following [23]. Case IV is based on the IEEE-57 system; this system has been divided into two areas, connected by eleven interconnecting lines; the division has been chosen on purpose to force a large value for the spectral radius. Finally, cases VI and VII are based on the IEEE-118 system [15]; this system has been replicated three times in case VI and six times in case VII using 10 and 22 interconnecting lines, respectively.

In all cases, the objective function (1) is taken to be the total operation cost for the system,

$$f(p_G) = \sum_{a=1}^{A} \sum_{i=1}^{G^a} c_i^a(p_{G_i}^a), \tag{38}$$

where functions $c_i^a(p_{G_i}^a)$ are quadratic and convex.

Table 1 shows the most relevant characteristics for each of the cases. The first column gives the case name. The second column shows the total number of buses for the global system. The third column provides the number of generation units for each case. The fourth column indicates the number of areas for the global system, and the fifth one the total number of lines. The sixth column shows the number of interconnecting lines between different areas, i.e. tie-lines. The seventh and eighth columns present the total number of variables and functional constraints, respectively. The ninth column shows the number of complicating constraints. Finally, the last column provides the spectral radius in condition (37), evaluated at the optimal solution of problem (12)–(14).

All cases have been solved by a centralized approach and a decentralized one. The decentralized approach uses the proposed decomposition algorithm presented in section 2.2, including and not including the search direction refinements based on the conjugate gradient procedure.

Table 1
Main characteristics of the case studies.

Case	Buses	Generators	Areas	Lines	Tie-lines	n	m	c	ρ
I	9	3	2	9	2	24	27	10	0.8
II	30	6	3	41	7	72	101	35	0.8
III	48	64	2	71	3	224	167	15	0.6
IV	57	7	2	77	10	128	191	50	207.3
V	72	96	3	107	5	336	251	25	0.6
VI	354	162	3	636	10	1032	1344	50	0.5
VII	708	324	6	1556	22	2064	2972	110	0.4

The solutions for both approaches have been computed using a nonlinear interior point procedure, based on a version of the algorithm reported in [22]. The algorithms have been implemented in MATLAB [20] and run on a PC Pentium II processor with 333 MHz and 128 MB of RAM. The two procedures have been run using the same starting point and stopping tolerance. Table 2 shows the numerical results for the case studies.

The second column of table 2 indicates the approach used to solved each case, a centralized procedure (centralized), the decomposed (decentralized) one and the decomposed one with the Conjugate Gradient search direction refinement (decentralized-CG). This subroutine has been used to improve convergence. The third column shows the total number of iterations required to reach the optimum for each problem. The fourth column provides the total number of inner iterations performed by the Conjugate Gradient subroutine in the decentralized-CG procedure. The fifth column shows CPU time in seconds needed to solve the linear systems. This CPU time has been chosen because it is in the solution of linear systems where the centralized and the decomposed procedures are different. Finally, the sixth column shows total CPU time.

It should be observed that most an important part the total CPU time for each case, as reported in the sixth column of table 2 (total CPU time), is spent in computing values

Table 2
Numerical results of the case studies.

Case	Algorithm	Iterations	Iterations CG	CPU time linear systems (sec.)	Total CPU time (sec.)
I	Centralized	13		0.002	0.004
	Decentralized	16		0.000	0.002
	Decentralized-CG	13	0	0.000	0.002
II	Centralized	22		0.153	0.463
	Decentralized	59		0.221	0.712
	Decentralized-CG	23	74	0.175	0.537
III	Centralized	21		0.472	0.897
	Decentralized	38		0.480	0.851
	Decentralized-CG	21	10	0.301	0.540
IV	Centralized	23		0.452	0.563
	Decentralized	–		–	–
	Decentralized-CG	42	391	2.178	2.560
V	Centralized	26		0.985	1.496
	Decentralized	58		1.438	2.148
	Decentralized-CG	26	57	0.954	1.420
VI	Centralized	36		7.405	17.571
	Decentralized	39		4.708	11.140
	Decentralized-CG	35	15	5.591	13.223
VII	Centralized	43		22.068	54.462
	Decentralized	52		8.536	22.194
	Decentralized-CG	39	9	7.707	20.006

associated to gradients and second derivative matrices. This computing time constitutes a significant part of total computing time in OPF problems and it is roughly equal for both procedures, centralized and decentralized. Therefore, for the sake of comparison of the centralized and decentralized procedures, CPU time to solve linear system, as reported in the fourth column (CPU time lin. systems), is a more appropriate measure than total CPU time, as reported in the sixth column.

It should be noted that the Conjugate Gradient subroutine is only necessary for case IV. In this case, the spectral radius in (37) is greater than 1 and the decomposition algorithm without Conjugate Gradient refinement does not converge.

The results show the good behavior of the proposed procedure: there is a reduction in running times for nearly all cases from a centralized solution to a decentralized one. However, this is not what happens in case IV because the spectral radius in this case is larger than 1, see table 1. Note that these results have been obtained in a sequential computational environment.

It should be noted that the total number of iterations for the proposed decomposition algorithm is higher than the corresponding total number of iterations for the centralized approach. This is due to the linear convergence presented by the decomposition approach versus the superlinear convergence presented by the centralized one. Moreover, the total number of iterations for the proposed decomposition algorithm with Conjugate Gradient refinement is similar to the total number of iterations for the centralized procedure. This results from the fact that the Conjugate Gradient refinement of the search directions in the decomposition algorithm approximates those directions to Newton directions. However, the total computing time for the proposed decomposition algorithm may be lower than the corresponding computing time for the centralized procedure. This is due to the size of the linear systems that need to be solved by each procedure. A centralized approach must solve a smaller number of systems that are larger by a factor equal to the number of areas than those solved by the decomposition algorithm.

4. Conclusions

In this paper, a decomposition methodology is presented and applied to the multi-area OPF problem that arises in the operation of electric energy systems. The proposed methodology preserves the autonomy of each area in the global system by means of a coordinated but decentralized procedure. This is a crucial fact in nowadays competitive electricity markets. Therefore, the decomposition method is appropriate for an Independent System Operator in charge of the electric energy system technical operation.

Unlike most common Lagrangian based algorithms, the proposed technique does not need to solve subproblems until optimality which results in computational savings. Furthermore, the central agent of the proposed procedure does not update information, it just distributes it. This is not the case of most common Lagrangian-based algorithms. As a result of these properties, the proposed methodology is very well-suited for its use in the solution of large-scale multi-area OPF problems by the Independent System

Operator. Numerical results suggest that the method has less computational cost than a centralized approach when applied to large-scale problems.

Appendix: example

In this appendix, a simple example that clarifies how the proposed decomposition algorithm works is presented. In order to enhance clarity, it is not based on any small dimension OPF.

The global problem to be solved is

$$\text{minimize} \quad x_1^2 + x_2^2 + y_1^2 + y_2^2 \tag{A.1}$$

$$\text{subject to} \quad 4x_1 + y_2 - 1 = 0, \tag{A.2}$$

$$x_1 + 4y_2 - 1 = 0. \tag{A.3}$$

This problem has the form of model (12)–(14). For the sake of simplicity, only two areas have been considered. Variables belonging to the first area ($a = 1$) are denoted by x and variables belonging to the second area ($a = 2$) are denoted by y. Equations (A.2) and (A.3) represent the complicating constraints (13) in the general model, for areas 1 and 2, respectively. Separable constraints of the form (14) have not been included in this simple example. There are only two variables implied in the complicating equations, x_1 and y_2.

The solution of this problem is

$$x^* = \begin{bmatrix} 0.2 \\ 0.0 \end{bmatrix}, \qquad y^* = \begin{bmatrix} 0.0 \\ 0.2 \end{bmatrix}, \qquad \lambda^* = \begin{bmatrix} -0.08 \\ -0.08 \end{bmatrix}.$$

The constraint vector (A.2)–(A.3) is denoted by h:

$$h(x, y) = \begin{bmatrix} 4x_1 + y_2 - 1 \\ x_1 + 4y_2 - 1 \end{bmatrix}.$$

Using the proposed methodology, the subproblems to be solved in step 1 of the decomposition algorithm are, respectively,

$$\text{minimize} \quad x_1^2 + x_2^2 + \bar{\lambda}_2 x_1 \tag{A.4}$$

$$\text{subject to} \quad 4x_1 + \bar{y}_2 - 1 = 0.$$

and

$$\text{minimize} \quad y_1^2 + y_2^2 + \bar{\lambda}_1 y_1 \tag{A.5}$$

$$\text{subject to} \quad \bar{x}_1 + 4y_2 - 1 = 0.$$

The algorithm is applied below.

Iteration $k = 0$:

Step 0.

Variables and multipliers are initialized, i.e.

$$\bar{x} = \begin{bmatrix} 0.4 \\ 0.4 \end{bmatrix}, \qquad \bar{y} = \begin{bmatrix} 0.4 \\ 0.4 \end{bmatrix}, \qquad \bar{\lambda} = \begin{bmatrix} -0.01 \\ -0.01 \end{bmatrix}.$$

Iteration $k = 1$:

Step 1.

System X computes a movement direction for problem (A.4), using Newton's method, for $x = \bar{x}$. The Lagrangian function for problem (A.4) is

$$L_x(x_1, x_2, \lambda_1) = x_1^2 + x_2^2 - 0.01x_1 + \lambda_1(4x_1 + 0.4 - 1),$$

then

$$\nabla_{x_1,x_2,\lambda_1} L_x(0.4, 0.4, -0.01) = \begin{bmatrix} 0.75 \\ 0.80 \\ 1.00 \end{bmatrix},$$

$$\nabla^2_{x_1,x_2,\lambda_1} L_x(0.4, 0.4, -0.01) = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 2 & 0 \\ 4 & 0 & 0 \end{bmatrix}.$$

If Newton's method is applied:

$$\begin{bmatrix} 2 & 0 & 4 \\ 0 & 2 & 0 \\ 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta \lambda_1 \end{bmatrix} = - \begin{bmatrix} 0.75 \\ 0.80 \\ 1.00 \end{bmatrix}, \quad \text{then} \quad \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta \lambda_1 \end{bmatrix} = \begin{bmatrix} -0.25 \\ -0.40 \\ -0.0625 \end{bmatrix},$$

and it is obtained

$$x = x + \Delta x = \begin{bmatrix} 0.4 \\ 0.4 \end{bmatrix} + \begin{bmatrix} -0.25 \\ -0.40 \end{bmatrix} = \begin{bmatrix} 0.15 \\ 0.00 \end{bmatrix}$$

and

$$\lambda_1 = \lambda_1 + \Delta\lambda_1 = -0.01 + (-0.0625) = -0.0725.$$

Step 2.

System Y computes a movement direction for problem (A.5), using Newton's method, for $y = \bar{y}$. The Lagrangian function for problem (A.5) is

$$L_y(y_1, y_2, \lambda_2) = y_1^2 + y_2^2 - 0.01y_1 + \lambda_2(0.4 + 0.4y_2 - 1),$$

then

$$\nabla_{y_1,y_2,\lambda_2} L_y(0.4, 0.4, -0.01) = \begin{bmatrix} 0.80 \\ 0.75 \\ 1.00 \end{bmatrix},$$

$$\nabla^2_{y_1,y_2,\lambda_2} L_y(0.4, 0.4, -0.01) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 4 & 0 \end{bmatrix}.$$

If Newton's method is applied:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \\ \Delta\lambda_2 \end{bmatrix} = - \begin{bmatrix} 0.80 \\ 0.75 \\ 1.00 \end{bmatrix}, \quad \text{then} \quad \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \\ \Delta\lambda_2 \end{bmatrix} = \begin{bmatrix} -0.40 \\ -0.25 \\ -0.0625 \end{bmatrix},$$

and it is obtained

$$y = y + \Delta y = \begin{bmatrix} 0.4 \\ 0.4 \end{bmatrix} + \begin{bmatrix} -0.40 \\ -0.25 \end{bmatrix} = \begin{bmatrix} 0.00 \\ 0.15 \end{bmatrix}$$

and

$$\lambda_2 = \lambda_2 + \Delta\lambda_2 = -0.01 + (-0.0625) = -0.0725.$$

Step 3.

Convergence: the central agent checks if the selected convergence condition $\|h(x, y) < 10^{-4}\|$ is satisfied:

$$h = \begin{bmatrix} -0.25 \\ -0.25 \end{bmatrix}, \qquad \|h\| = 0.3536 > 10^{-4}.$$

As the convergence condition is not satisfied, variables

$$\bar{x} = x = \begin{bmatrix} 0.15 \\ 0.0 \end{bmatrix}, \qquad \bar{y} = y = \begin{bmatrix} 0.0 \\ 0.15 \end{bmatrix},$$

and multipliers

$$\bar{\lambda} = \lambda = \begin{bmatrix} -0.0725 \\ -0.0725 \end{bmatrix}$$

are fixed, the iteration counter is updated, $k = k + 1 = 2$, and steps 1–3 of the algorithm are repeated until convergence is achieved.

The algorithm stops for $k = 8$, with a tolerance $\|h\| = 8.6317 \times 10^{-5}$. The solution is

$$x = \begin{bmatrix} 0.2 \\ 0.0 \end{bmatrix}, \qquad y = \begin{bmatrix} 0.0 \\ 0.2 \end{bmatrix}, \qquad \lambda = \begin{bmatrix} -0.08 \\ -0.08 \end{bmatrix}.$$

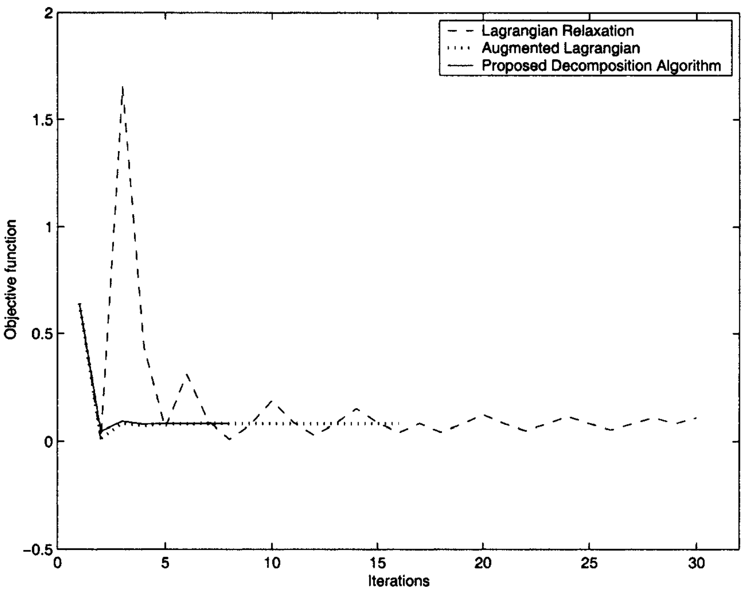

Figure 1. Example. Evolution of the objective function.

Problem (A.1)–(A.3) has also been solved using a Lagrangian relaxation procedure [16] and an augmented Lagrangian relaxation one [3,7]. The Lagrangian relaxation procedure uses a simple subgradient updating of multipliers, and the augmented Lagrangian decomposition procedure uses a progressively increasing penalty term and a simple gradient multiplier updating technique.

The Lagrangian relaxation procedure stopped after $k = 53$ iterations, with $\|h\| = 4.1772 \times 10^{-5}$. The augmented Lagrangian relaxation procedure stopped after $k = 16$ iterations, with $\|h\| = 9.9172 \times 10^{-5}$. Figure 1 shows the evolution of the objective function (A.1) as a function of the iteration number, for each of the three procedures. The dashed line represents the evolution of the objective function evaluated at the iterates for the Lagrangian relaxation procedure. The dotted line represents the evolution of the objective function evaluated at the iterates for the augmented Lagrangian decomposition procedure. Lastly, the solid line represents the evolution of the objective function evaluated at points computed by the proposed decomposition algorithm.

Note the slow and oscillating behaviour of the Lagrangian relaxation procedure. The quadratic penalty term in the augmented Lagrangian procedure corrects this anomaly, although the convergence is still slower than that of the proposed decomposition algorithm.

Figure 2 shows the value of multiplier λ_1 at each iteration, for each of the three procedures. The value of multiplier λ_2, is the same for all procedures. As in figure 1, the dashed line represents the values of the multiplier computed by the Lagrangian re-

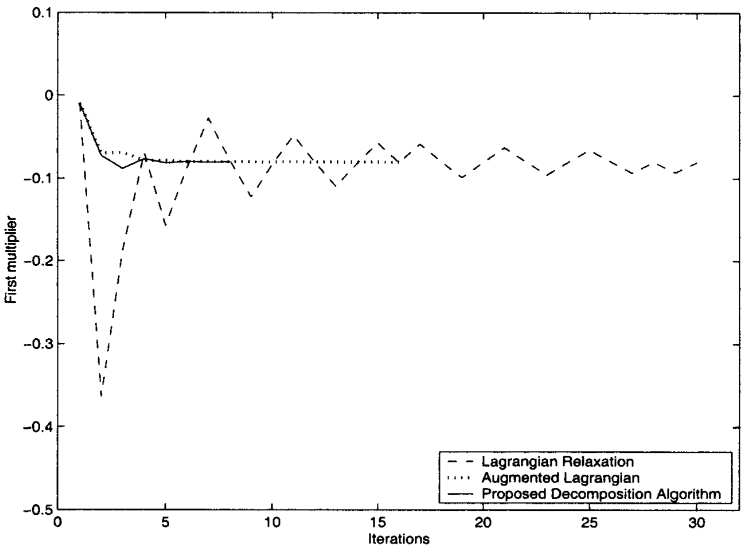

Figure 2. Example. Evolution of the first multiplier.

laxation procedure; the dotted line represents the evolution of the multiplier from the augmented Lagrangian procedure; lastly, the solid line represents the evolution of the multiplier as obtained by the proposed decomposition algorithm.

References

[1] J.A. Aguado, V.H. Quintana and A.J. Conejo, Optimal power flows of interconnected power systems, in: *Proc. of the IEEE Power Engineering Society Summer Meeting*, Vol. 2 (1999) pp. 814–819.

[2] R. Baldick, B.H. Kim, C. Chase and Y. Luo, A fast distributed implementation of optimal power flow, IEEE Trans. Power Systems 14(3) (1999) 858–864.

[3] M.S. Bazaraa, H.O. Sherali and C.M. Shetty, *Nonlinear Programming: Theory and Algorithms*, 2nd ed. (Wiley, New York, 1993).

[4] D.P. Bertsekas, *Constrained Optimization and Lagrange Multiplier Methods* (Academic Press, New York, 1982).

[5] P. Carpentier, G. Cohen, J.C. Culioli and A. Renaud, Stochastic optimization of unit commitment: A new decomposition framework, IEEE Trans. Power Systems 11(2) (1996) 1067–1073.

[6] G. Cohen, Optimization by decomposition and coordination: A unified approach, IEEE Trans. Automat. Control 23(2) (1978) 222–232.

[7] G. Cohen, Auxiliary problem principle and decomposition of optimisation problems, J. Optim. Theory Appl 32(3) (1980) 277–305.

[8] G. Cohen and B. Miara, Optimization with an auxiliary constraint and decomposition, SIAM J. Control Optim. 28(1) (1990) 137–157.

[9] A.J. Conejo and J.A. Aguado, Multi-area coordinated decentralized DC optimal power flow, IEEE Trans. Power Systems 13(4) (1998) 1272–1278.

[10] A.J. Conejo, F.J. Nogales and F.J. Prieto, An approximate procedure based on approximate Newton directions, Mathematical Programming, Series A 93 (2002) 495–515.

[11] G.H. Golub and C.F. Van Loan, *Matrix Computations*, 3rd ed. (Johns Hopkins Univ. Press, London, 1996).

[12] M. Huneault and F.D. Galiana, A survey of the optimal power flow literature, IEEE Trans. Power Systems 6(2) (1991) 762–770.

[13] J.E. Kelley, The cutting-plane method for solving convex programs, J. Soc. Industr. Appl. Math. 8 (1960) 703–712.

[14] B.H. Kim and R. Baldick, Coarse-grained distributed optimal power flow, IEEE Trans. Power Systems 12(2) (1997) 932–939.

[15] MATLAB power system simulation package, Matpower, `http://www.pserc.cornell.edu/matpower/` (2002).

[16] B.T. Polyak, *Introduction to Optimization* (Optimization Software, New York, 1987).

[17] A. Renaud, Daily generation management at Electricité de France: From planning towards real time, IEEE Trans. Automat. Control 38(7) (1993) 1080–1093.

[18] Y. Saad, *Iterative Methods for Sparse Linear Systems* (PWS Publishing, New York, 1996).

[19] The IEEE reliability test system 1996, Reliability Test System Task Force, IEEE Trans. Power Systems 14(3) (1999) 1010–1020.

[20] *Using Matlab. Version 5* (MathWorks, 1998).

[21] A.J. Wood and B.F. Wollenberg, *Power Generation, Operation and Control*, 2nd ed. (Wiley, New York, 1996).

[22] Y. Wu, A.S. Debs and R.E. Marsten, A direct nonlinear predictor-corrector primal–dual interior point algorithm for optimal power flow, IEEE Trans. Power Systems 9(2) (1994) 876–883.

[23] F. Xia and A.P. Sakis Meliopoulos, A methodology for probabilistic simultaneous transfer capability analysis, IEEE Trans. Power Systems 11(3) (1996) 1269–1276.