A Delay Metric for RC Circuits based on the Weibull Distribution

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Abstract

Physical design optimizations such as placement, interconnect synthesis, ¤oorplanning, and routing require fast and accurate analysis of RC networks. Because of its simple close form and fast evaluation, the Elmore delay metric has been widely adopted. The recently proposed delay metrics PRIMO and H-gamma match the £rst three circuit moments to the probability density function of a Gamma statistical distribution. Although these methods demonstrate impressive accuracy compared to other delay metrics, their implementations tend to be challenging. As an alternative to matching to the Gamma distribution, we propose to match the £rst two circuit moments to a Weibull distribution. The result is a new delay metric called Weibull based Delay (WED). The primary advantages of WED over PRIMO and H-gamma are its ef£ciency and ease of implementation. Experiments show that WED is robust and has satisfactory accuracies at both near- and far-end nodes.

1 Introduction

As CMOS technologies scale into nanometer regime, interconnect networks are becoming increasingly dominant in terms of total path delay or total path capacitance. The effect of interconnect networks on signal propagation delay and transition time degradation has to be considered in various physical design tools, e.g., placement, ¤oorplanning and routing. Model order reduction techniques [10] have been proven to be highly effective in analyzing interconnect delay. Essentially, these model order reduction techniques compute the £rst few dominant poles and the corresponding residues by matching the *moments* of the circuit impulse response (either explicitly or implicitly). The response of the interconnect network is then represented as the sum of exponential functions. In order to calculate the 50% signal delay, nonlinear solution methods such as Newton-Raphson have to be used to solve the transcendental equation. The overall computational cost of this approach is expensive and could have a negative impact on the overall speed of the physical optimization.

Given its explicit nature of a simple closed form and ease of calculation, Elmore delay[3] is widely used as the delay metric of choice within physical design optimization algorithms. Although it has been proven to be the upper bound of the propagation delay[4], Elmore delay is known to be extremely inaccurate in some cases because it ignores the effect of resistive shielding [9].

Various delay metrics have been proposed that achieve better accuracy than Elmore delay, such as those proposed in [1][5][6][7][8][14]. Among them, some metrics such as [6] and [14] try to construct a stable two-pole approximation and provide an explicit solution to the signal delay. Another metric in [5] simpli£es the two-pole approximation to a single pole by matching the transfer function. The resulting delay metric is simple but its accuracy is far from satisfactory. More recently, an empirical metric, D2M, was proposed[1]. Despite its simplicity, D2M has remarkably high accuracy at the far-end nodes.

The PRIMO[7] and H-gamma[8] metrics extend the observation of Elmore that the impulse response of an RC circuit can be treated as the *probability density function* (PDF) of a statistical distribution[3]. By using the Gamma distribution in particular as the underlying "template", the calculation of the signal delay is transformed to the calculation of the *median* of the Gamma distribution. Despite some problems at the near-end with PRIMO, these two methods demonstrate better overall accuracy than previous metrics, although their implementations require careful algorithmic tuning. A carefully constructed two-dimensional look-up table is required to calculate the incomplete gamma function, which is needed in order to obtain correct delay values.

In this paper, we propose a new delay metric for RC circuits called WED (**WE**ibull-based **D**elay). Like PRIMO and H-gamma, we interpret the circuit impulse as a PDF; however, we match the £rst two moments to a *Weibull* Distribution. Because of the characteristics of the Weibull distribution, WED does not require the evaluation of incomplete Gamma function. Although look-up tables are still needed to speed up the delay calculation, instead of carefully constructed multi-dimensional tables, only two small one-dimensional tables are necessary. Experiments on large industrial design demonstrate that WED achieves the same accuracy as PRIMO, but does not generate unrealistic results as PRIMO occasionally does at near-end nodes.

2 Background

2.1 Circuit Moments and Probability Moments

Assume that $h(t)$ is the impulse response of a node voltage in an RLC circuit and $H(s)$ is the corresponding Laplace transformation. The *(circuit) moments*, m_k, are de£ned as the coef£cient of the Taylor expansion of $H(s)$ at $s=0$:

$$H(s) = \int_0^\infty h(t) \cdot e^{-st} dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} s^k \int_0^\infty t^k h(t) dt \quad (1)$$

Therefore,

$$H(s) = m_0 + m_1 s + m_2 s^2 + m_3 s^3 + \cdots \quad (2)$$

with

$$m_k = \frac{(-1)^k}{k!} \int_0^\infty t^k h(t) dt \qquad \text{for } k = 0, 1, 2, \ldots \quad (3)$$

The circuit moments can be computed ef£ciently, e.g., by path tracing[12].

In [13], it was shown that for an RC circuit without resistive path to ground, the impulse response $h(t)$ satis£es the following conditions:

$$h(t) \geq 0 \quad \forall t \qquad and \quad \int_0^\infty h(t) dt = 1 \quad (4)$$

From probability theory[2], any continuous function which satis£es Equation (4) is a probability density function (PDF). Therefore, the impulse response of an RC circuit is a PDF; however, there is no known

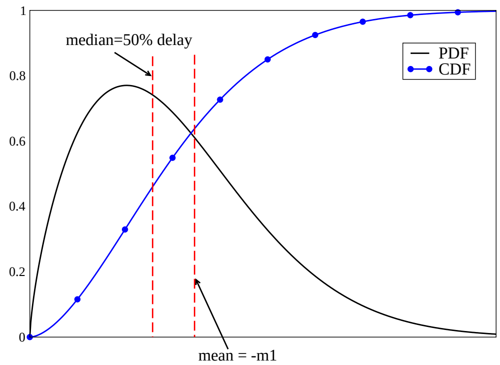

Figure 1: The CDF and PDF of a typical statistical distribution.

underlying statistical distribution for the impulse response PDF. Similarly, the step response of the same circuit is a cumulative density function (CDF). The de£nitions and properties of PDF and CDF can be found in any probability theory book such as [2]. Informally, the PDF describes how "dense" the statistical distribution is across the region, while the CDF is the area from $-\infty$ to a particular point underneath the PDF curve. The relationship between PDF ($P(t)$) and corresponding CDF ($C(t)$) is (see Fig. 1):

$$C(t) = \int_{-\infty}^{t} P(\tau)d\tau \tag{5}$$

Similar to a statistical distribution, the concept of the *mean* of an impulse response is de£ned as:

$$\mu = \int_{0}^{\infty} t \cdot h(t)dt \tag{6}$$

We can also calculate the *central moments* or *moments about the mean*, which is de£ned as:

$$\mu_k = \int_{0}^{\infty} (t-\mu)^k \cdot h(t)dt \tag{7}$$

The relationship between the central moments and the circuit moments listed in Equation (3) is [4]:

$$\begin{aligned} \mu_1 &= 0 \\ \mu_2 &= 2m_2 - m_1^2 \\ \mu_3 &= -6m_3 + 6m_1m_2 - 2m_1^3 \end{aligned} \tag{8}$$

According to probability theory, further characteristics of the distribution (or, the impulse response of an RC circuit) can be derived from the central moments. Besides the mean, two other important characteristics are *variance* (σ^2) and *skewness* (γ):

$$\begin{aligned} \mu &= -m_1 \\ \sigma^2 &= \mu_2 = 2m_2 - m_1^2 \\ \gamma &= \frac{\mu_3}{\sigma^3} = \frac{-6m_3+6m_1m_2-2m_1^3}{(\sqrt{2m_2-m_1^2})^3} \end{aligned} \tag{9}$$

For the circuit response, the mean (μ) describes the center of the gravity of a impulse response curve. The variance σ^2 (or its square root, which is referred as the *standard deviation*), describes the spread of the response curve. It also provides a measurement of the signal transition time. Finally the skewness describes the degree of asymmetry of the impulse response curve.

Once the connection between the PDF function and impulse response curve is established, calculating signal delay at $100 \cdot \phi\%$ point is equivalent to £nding the ϕ percentile of the underlying statistical distribution. For example, £nding the 50% delay point is equivalent to £nding the *median* of the statistical distribution.

Elmore was the £rst to make the connection between the impulse response of an RC circuit and the statistical distribution. It turns out that the mean of an RC circuit can be calculated in a recursive way, and the resulting Elmore delay is a simple closed form. However, what we really want to £nd is not the mean but the *median* of the distribution since it corresponds to the 50% delay point (see Fig 1). Unfortunately, computing the median is quite elusive, hence Elmore approximates the median with the mean. Although Elmore delay is proven to be the upper bound of the propagation delay[4], in many cases, especially for the near-end nodes, there is signi£cant distance between the mean and the median of the impulse response waveform. For those nodes, using Elmore delay can generate tremendous amount of error. The overall accuracy of the Elmore delay can be far from satisfactory.

2.2 Probability Based Delay Metric: PRIMO/H-gamma

In PRIMO, the idea of statistical interpretation was extended by matching the impulse response to a shifted Gamma distribution. Once a particular statistical distribution is available, using the mean to approximate the median becomes unnecessary. Instead, the median can be calculated precisely from the parameters of the Gamma distribution. The PDF of a GAMMA distribution, denoted GAM(λ, n), is determined by two parameters λ and n:

$$P_{GAM}(t) = \frac{\lambda^n t^{n-1} e^{-\lambda t}}{\Gamma(n)} \quad \text{for } t > 0 \tag{10}$$

in which $\Gamma(n)$ is the (complete) Gamma function:

$$\Gamma(x) = \int_{0}^{\infty} \zeta^{x-1} e^{-\zeta} d\zeta \quad \text{for } x > 0 \tag{11}$$

which is the continuous extension to the discrete factorial function $(n-1)!$. For example $\Gamma(3) = 2! = 2$, $\Gamma(4) = 3! = 6$, while $\Gamma(3.5) = 3.323$. The Gamma function also has the recursive property which is quite useful:

$$\Gamma(1+x) = x \cdot \Gamma(x) \quad \text{for } x > 1 \tag{12}$$

In PRIMO, three circuit moments are used to calculate the three central moments as shown in Equation (8). Then the two parameters n and λ of the Gamma distribution are calculated by matching the second and third central moments. The Gamma parameters are given by:

$$\lambda = 2\mu_2/\mu_3 \quad n = 4\mu_2^3/\mu_3^2 \tag{13}$$

A shift is then added to the Gamma distribution so that its mean matches the mean of the circuit response. The amount of the shift is determined by:

$$\Delta = -m_1 - \frac{n}{\lambda} \tag{14}$$

In order to calculate the delay t_ϕ at $100 \cdot \phi\%$ point, the CDF of the Gamma distribution has to be calculated. However, the Gamma CDF is not explicit. As a result, the following nonlinear equation has to be solved numerically:

$$\int_{0}^{t_\phi} P_{GAM}(t)dt = \int_{0}^{t_\phi} \frac{\lambda^n t^{n-1} e^{-\lambda t}}{\Gamma(n)} dt = \phi \tag{15}$$

The resulting t_ϕ is then adjusted by Δ given in Equation (14) to compensate the shift.

Numerically solving Equation (15) is equivalent to evaluating the incomplete Gamma function, which can only be implemented by means of numerical iteration [11]. In order to alleviate the cost of the iterations involved, it was suggested that a pre-calculated look-up table be used[7]. Since the incomplete gamma function has two input variables, a two-dimensional look-up table has to be constructed in order to generate accurate delay values and careful algorithm tuning is required to make the method robust. Furthermore, due to the shift

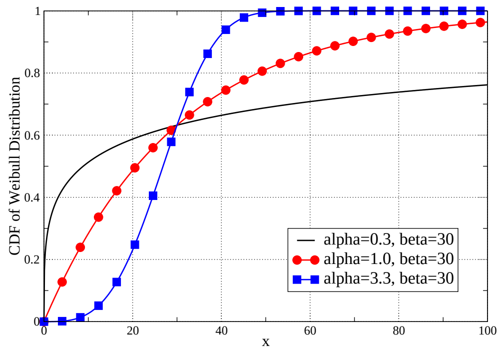

Figure 2: The CDF of Weibull distribution with same β but different shape parameter α.

described in Equation (14), at some near-end nodes, PRIMO returns negative delays, which are unrealistic.

In H-gamma[8], an extension of PRIMO, the homogeneous component of the step response is matched to a Gamma distribution. Compared to PRIMO, H-gamma achieves better accuracy and overcomes PRIMO's problem at near-end nodes. However, its implementation is even more challenging.

3 WED: Weibull based Delay

Both PRIMO and H-gamma recognize the bene£t of matching the impulse response to the particular Gamma distribution. However, several other statistical distributions may be equally viable. In particular, as can be seen in Fig. 2, there is a large degree of similarity between the step response at different nodes in an RC circuit and the Weibull CDF. Hence, we sought to apply the same PDF matching principle to the Weibull distribution. The Weibull distribution has a distinct advantage over Gamma distribution in that the median (or any percentage delay point) can be expressed explicitly as a function of its parameters. This makes it much simpler for calculating delay or signal transition time (rise/fall time).

3.1 Weibull Distribution

A Weibull distribution, denoted as WEI(α,β), is determined by two parameters α and β. Its PDF is de£ned as[2]:

$$P_{WEI}(t) = \alpha\beta^{-\alpha}t^{\alpha-1}e^{-(t/\beta)^{\alpha}} \quad \text{for } t>0;\ \alpha,\ \beta>0 \tag{16}$$

while its CDF is de£ned as:

$$C_{WEI}(t) = 1-e^{-(t/\beta)^{\alpha}} \tag{17}$$

Compared to that of a Gamma distribution, the CDF of a Weibull distribution is much simpler. The parameter α is called a **shape parameter**. Depending on whether $\alpha<1$, $\alpha=1$, or $\alpha>1$, the Weibull CDF shows three different basic shapes (see Fig. 2). For convenience, we use parameter $\theta = 1/\alpha$ in the WED formulation.

The statistical characteristics of Weibull distribution involves Gamma function. The mean and the variance of Weibull distribution are:

$$\begin{aligned} \mu_{WEI} &= \beta\Gamma(1+\theta) \\ \sigma^2_{WEI} &= \beta^2(\Gamma(1+2\theta)-\Gamma^2(1+\theta)) \end{aligned} \tag{18}$$

Other characteristics such as skewness can also be written explicitly (see e.g. [2]).

3.2 WED Delay Metric

We seek to construct a Weibull distribution by matching the mean and variance of the RC step response to the mean and variance of a Weibull

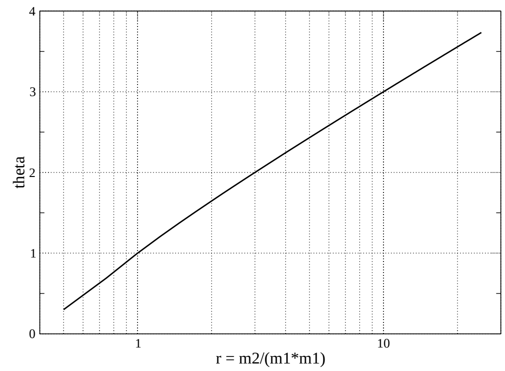

Figure 3: Values of θ with different values of r. Note that r is in logarithmic scale.

distribution. Dividing the second equation in Equation (18) by the square of the £rst equation, we have:

$$\frac{\Gamma(1+2\theta)}{\Gamma^2(1+\theta)} - 1 = \frac{\sigma^2_{WEI}}{\mu^2_{WEI}} \tag{19}$$

Matching μ_{WEI} and σ^2_{WEI} to the circuit mean m_1 and variance $2m_2 - m_1^2$ de£ned in Equation (9), we have:

$$\frac{\Gamma(1+2\theta)}{\Gamma^2(1+\theta)} = \frac{2m_2}{m_1^2} \tag{20}$$

Let $r = m_2/m_1^2$. The parameter θ is determined by solving the following equation:

$$r = \frac{1}{2}\cdot\frac{\Gamma(1+2\theta)}{\Gamma^2(1+\theta)} \tag{21}$$

Although this equation is nonlinear and appears complicated, it actually can be solved with a simple table look-up operation (see Section 3.3.)

Once θ is known, parameter β is determined by Equation 18:

$$\beta = \frac{-m_1}{\Gamma(1+\theta)} \tag{22}$$

To calculate the signal delay at $100\cdot\phi$% point, we directly solve the CDF of Weibull distribution shown in Equation (17):

$$1-e^{-(t_\phi/\beta)^{1/\theta}} = \phi \tag{23}$$

which yields the solution:

$$t_\phi = \beta\cdot[ln(\frac{1}{1-\phi})]^{\theta} \tag{24}$$

In particular, the 50% delay point can be calculated as:

$$t_{0.5} = \beta\cdot(ln(2))^{\theta} \approx \beta\cdot(0.693)^{\theta} \tag{25}$$

3.3 WED Implementation

The seemingly complicated algorithm described above can be implemented quite easily. First, the nonlinear equation in Equation (21) can be easily solved by bi-section. We observe that the range of $r = m_2/m_1^2$ for an RC circuit generally lies between 0.8 and 5, with some extreme values reaching 10. If we plot the solution of Equation (21) against the logarithm of r, we observe an almost linear relationship (see Fig. 3). For such an "almost-linear" relationship, a look-up table is ideal and even a coarse one returns quite accurate results. From our experience, we found the table listed in Table 1 works very well in practice. Note

r	$log_{10}(r)$	θ
0.63096	-0.2	0.48837
0.79433	-0.1	0.76029
1.00000	0.0	1.00000
1.25892	0.1	1.22371
1.58489	0.2	1.43757
1.99526	0.3	1.64467
2.51189	0.4	1.84678
3.16228	0.5	2.04507
3.98107	0.6	2.24031
5.01187	0.7	2.43305
6.30957	0.8	2.62371
7.94328	0.9	2.81262
10.00000	1.0	3.00000
12.58925	1.1	3.18607
15.84893	1.2	3.37098

Table 1: The look-up table for the calculation of θ.

x	GAMMA(x)
1.0	1.00000
1.1	0.95135
1.2	0.91817
1.3	0.89747
1.4	0.88726
1.5	0.88623
1.6	0.89352
1.7	0.90864
1.8	0.93138
1.9	0.96176
2.0	1.00000

Table 2: The look-up table to evaluate Gamma function.

that because the relationship between θ and $log_{10}(r)$ is close to linear while the relationship between θ and r is exponential, using entries of $log_{10}(r)$ is more accurate.

Next, the evaluation of the Gamma function in Equation (22) is actually quite trivial. Gamma function is commonly available in many numerical packages and math libraries. If it becomes necessary to implement it, a detailed description of the procedure and a 10-line implementation in C can be found in the popular reference [11]. Further, the Gamma function may also be implemented as a look-up table. Notice that since the solution θ of Equation (21) is always positive, we only have to evaluate Gamma function $\Gamma(x)$ for $x > 1$. Due to the recursive property of the Gamma function (Equation (12)), we only need to store a look-up table for values between 1 and 2, which is shown in Table 2.

The ¤ow of carrying out WED calculation can be summarized as the following:

Algorithm for the WED Metric
1. Calculate the two circuit moments m_1, m_2.
2. Calculate $r = m_2/m_1^2$.
3. Use Table 1 to £nd θ from r.
4. Use Table 2 to £nd $\Gamma(1+\theta)$.
5. Set $\beta = -m_1/\Gamma(1+\theta)$.
6. Return 50% delay value of $\beta(\ln 2)^{\theta}$.

Figure 4: Algorithm for computing the WED metric.

Compared to the multi-dimensional look-up tables in PRIMO and H-Gamma, only two 1-D tables are needed. This not only reduces the memory requirement, but also makes WED easy to implement.

3.4 Stability of the WED Algorithm

Some delay metrics are not guaranteed to return nonnegative real values. By de£nition, a Weibull distribution WEI(α, β) is stable when both α and β are positive. In Equation (24), the WED metric will return a positive delay value as long as θ and β are positive. Clearly from Fig.3, θ is nonnegative as long as r is positive. From [4], we know that for an RC circuit m_2 is always positive. Therefore, θ is guaranteed to be positive for any node in an RC circuit. Again from [4], m_1 is negative for an RC circuit. Furthermore, $\Gamma(1+\theta)$ is positive as long as θ is positive. Consequently, β is always positive. Combining the above arguments, we conclude that WED will always return a nonnegative value for delays at any nodes in any RC circuit.

4 Experiments

In this section, we present some experimental results of WED.

4.1 A Simple Test Case

The £rst experiment is a simple RC chain as shown in Fig. 5. As a comparison, we implemented D2M and PRIMO. We also implemented the metric in [14], which we call TDP, as well as the metric in [6], which we call KM. The input signal is a step signal. We use SPICE for the golden result. Results are presented in Table 3. Note PRIMO returns a negative delay at node n_1. As seen in the table, WED produced the most accurate delays for n_1, n_3 and n_4. PRIMO and D2M are slightly more accurate at the far nodes n_5 and n_6. What is striking about the WED results is that the magnitude of the deviations from SPICE are less than for any of the other metrics.

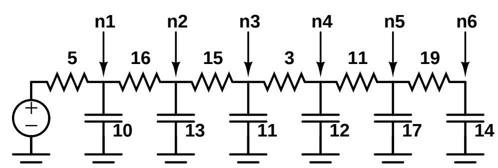

Figure 5: A simple RC circuit to demonstrate the accuracy of WED.

node	KM	TDP	D2M	PRIMO	WED	SPICE
n1	850	0	113	-39	45	40
n2	1500	401	797	648	661	467
n3	1689	1494	1514	1486	1474	1437
n4	1703	1631	1640	1621	1620	1589
n5	1731	1961	1982	1974	2025	1975
n6	1741	2191	2258	2245	2348	2257

Table 3: Comparison between different delay metrics for the simple test case in Fig. 5.

4.2 Comparison with H-gamma

We did not implement H-gamma as its complexity and look-up table construction make it dif£cult, if not prohibitive. However, we can still make a quick comparison of WED to H-gamma by running WED on the same RC circuit used in [1] and [8], as shown in Fig. 6. The delays of D2M, PRIMO, H-gamma, WED, and SPICE for this circuit are listed in Table 4, assuming a step input signal. As one can see that for this test example, WED is about as accurate as PRIMO but slightly worse than H-gamma. This is not necessarily surprising given H-gamma's complexity. WED's simplicity and ease of implementation may outweigh H-gamma's superior accuracy for several physical design applications.

4.3 Results on Industrial Nets

Finally, we perform tests on several nets from an industrial ASIC part in a 0.20um technology. We selected 432 high capacitance nets, and extracted SPICE netlists from the actual Steiner routes. The nets were identi£ed as those with a large Elmore error in order to provide a realistic, yet challenging set to the delay metrics. Each net has at least two sinks with an average of 5.4 sinks per net. The nets typically are

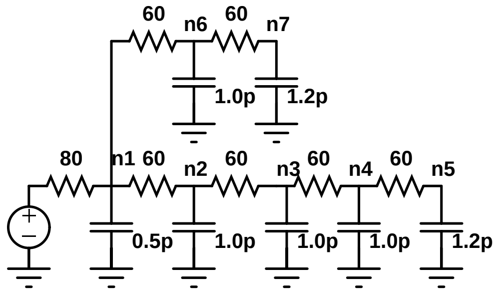

Figure 6: A simple RC circuit to compare the accuracy of WED, D2M, PRIMO and H-gamma.

node	D2M	PRIMO	H-γ	WED	SPICE
n1	299	241	194	246	196
n2	514	498	486	485	476
n3	696	699	701	698	700
n4	830	836	840	855	844
n5	905	909	912	943	919
n6	420	376	355	386	374
n7	492	450	431	470	453

Table 4: Comparison between D2M, PRIMO, H-gamma and WED for a small test circuit shown in Fig 6.

extremely "stiff" in the sense that the delays at different sinks may be orders of magnitude apart. As an example, Table 5 lists the delays for three sinks in one net "test_17" as well as the relative error compared to SPICE. The SPICE delays range from 80ps to 1170ps. Observe that the near-end node n1 causes the biggest problems for the delay metrics, resulting in the largest margins of error, while almost all the delay metrics do much better for the easier far-end node n3.

	D2M	Err	PRIMO	Err	WED	Err	SPICE
n1	138	74%	32	-60%	81	2%	80
n2	722	8%	701	5%	684	2%	669
n3	1164	-1%	1159	-1%	1187	2%	1170

Table 5: Delay values of three fan-out nodes in net "test_17". All units in pico-second.

The total number of nets considered prohibits presenting net-by-net results for all the nets in the design. Instead, we report the accuracy of different delay metrics by calculating the statistical characteristics of the error distributions. First, for each net, we separate all sink nodes into three categories: near-end, middle and far-end. If the SPICE delay for a given sink is less than 25% of the delay to the furthest sink in the net, it is classi£ed as a near-end sink. If lies between 25% than 75%, then it is a middle sink. Finally, larger than 75%, then it's a far-end sink. For our example net "test_17", n1 is a near-end, n2 is middle, and n3 is a far-end sink. Out of 2313 sinks considered, 507 are near-end, 636 are middle, and 1170 are far-end. The breakdown is also listed in the £rst column of Table 6.

We ran scaled Elmore, D2M, PRIMO, and WED for every sink and computed the delay and error relative to SPICE. Table 6 reports the mean and the standard derivation of the error distribution. A small mean error indicates that on average the delay metric performs well, while a small standard distribution indicates stability from net to net. Note that among 507 near-end sinks, PRIMO generates negative delays for 281 sink nodes. Since negative delay is unrealistic, we rounded them to zero. It is clearly shown that WED performs better than the other metrics at both the near-end and middle nodes, but marginally worse than D2M at the far-end.

nodes		S.Elmore	D2M	PRIMO	WED
near end (507)	mean	205.1%	56.4%	73.0%	49.3%
	std	139.1%	39.6%	36.3%	31.1%
middle (636)	mean	28.0%	15.9%	8.4%	6.3%
	std	20.7%	11.8%	5.9%	5.9%
far end (1170)	mean	6.7%	0.7%	0.9%	0.8%
	std	2.6%	0.5%	0.4%	0.6%
overall (2313)	mean	55.8%	17.0%	18.7%	12.9%
	std	102.9%	29.2%	33.6%	24.4%

Table 6: Statistical characteristics of the error distribution in an industrial ASIC design. The numbers in the £rst column indicates the total number of nodes in the particular category.

5 Conclusions

This paper describes WED a new delay metric for RC circuits. Two circuit moments are used to match the circuit's impulse response to a Weibull distribution. WED can be easily implemented as two one-dimensional table look-up operations and the procedure can be run in constant time. Experimental results on industrial design demonstrate that WED outperforms existing delay metrics. WED has a wide spectrum of potential applications such as interconnect synthesis, placement and routing. In the future work, we seek to apply WED to those applications in the physical design ¤ow.

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