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A Development of Complex Multi-Fuzzy Hypersoft Set With Application in MCDM Based on Entropy and Similarity Measure

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ABSTRACT Hypersoft set (HSS) was proposed in 2018 as a generalization of the soft set (SS). In this paper, the novelty of complex multi-fuzzy hypersoft set (CMFHSS) is discussed, which can deal with uncertainties, vagueness, and unclearness of data that lie in the information by taking into account the amplitude and phase terms (P-terms) of the complex numbers (C-numbers) at the same time. This CMFHSS establishes a hybrid framework of the multi-fuzzy set (MFS) and HSS characterized in a complex system. This framework is more flexible in two ways; firstly, it permits a wide range of values for membership function by expanding them to the unit circle in a complex frame of reference through characterization of the multi-fuzzy hypersoft set (MFHSS) involves an additional term called the P-terms to consider the periodic nature of the information. Secondly, in CMFHSS, the attributes can be further sub-partitioned into attribute values for a better understanding. We characterize its fundamental operations as a complement, union, and intersection and support them with examples. We develop the proverbial meaning of similarity measures (SM) and entropy (ENT) of CMFHSS and present the fundamental relationship. These tools can be utilized to figure out the best alternative out of a bunch that has various applications in the field of optimization. Additionally, mathematical models are given to analyze the reliability and predominance of the established methodologies. Moreover, the advantages and comparative analysis of the proposed measures with existing measures are also depicted in detail. Lastly, the mathematical models are given to represent the validity and applicability of the presented measures.

INDEX TERMS Soft set (SS), hypersoft set (HSS), multi-fuzzy set (MFS), multi-fuzzy hypersoft set (MFHSS), complex multi-fuzzy hypersoft set (CMFSS), entropy (ENT), similarity measures (SM).

I. INTRODUCTION

The significant existing theories, i.e., the theory of likelihood, the theory of fuzzy sets (f-sets) [2], [3], the theory of intuitionistic f-sets [4], the theory of vague sets [6], the theory of interval mathematics [5]. The theory of rough sets [8] can be regarded as numerical apparatuses for managing uncertainties. However, all these theories have their own troubles, as brought up in [9]. The explanation behind these difficulties is, perhaps, the deficiency of the parametrization instruments. Molodtsov [9] started the SS theory idea as another mathematical apparatus for managing uncertainties or vagueness that is liberated from the above challenges. The SS theory has rich potential for applications like economic, designing, clinical science etc. SS is called (binary, basic, rudimentary) neighborhood systems [10] and is an exceptional example of setting subordinate f-sets, as characterized by Thielle [11].

Maji *et al.* [12] conceptualized fuzzy soft sets (fs-sets) by embedding SS and fuzzy set (FS). Roy and Maji [13] introduced the application of fuzzy soft set (FSS) theory in object recognition issues. Yang *et al.* [14] presented the idea of an MFSS by consolidating the MFS and SS and practiced it to MCDM, Dey and Pal [15] generalized the idea of a

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MFSS. Zhang and Shu [16] expanded the notion of MFSS and presented the idea of possibility MFSS and applied it to a MCDM.

The fuzzy event's probability measures have played a key role in FS and their hybrid structures addressed in [17]. De Luca and Termini [18] recommended a specific arrangement of axioms for fuzzy ENT. On the opposite side, SM, which is a significant apparatus for deciding the level of SM between two articles, has gained substantially more consideration than ENT. Pappis and his teammates have given a arrangement of articles [19], [20] which took a proverbial view of the SM. The ENT and SM for some different sets, for example, interval-valued FS [21], FSS [26], and intuitionistic FSS [23] have been broadly utilized in tackling issues identified with the decision making, pattern recognition, and image processing.

Al-Qudah *et al.* [24] built up a mixed framework of complex fuzzy sets (CF-sets) and multi-fuzzy sets (MF sets) called the CMFS. This model helps take care of issues with the characteristics of multifaceted portrayal. To renovate this framework more useful, to renovate new powerful outcomes, they will form it into a CMFSS in request to consolidate the benefits of SS and apply them to the CMFS models. Their proposed model will be able to deal with uncertainties, vagueness, and unclearness of 2D multi-fuzzy data by catching the A-terms and P-terms of the C-numbers at the same time. In 2018, Al-Qudah *et al.* [25] presented the idea of CMFSS, which consolidates the benefits of both the CMFS and soft set.

In a diversity of practical applications, the attributes should be more sub-partitioned into attribute values for clearer understanding. Samarandache [27] fulfilled this need and developed the concept of the HSS as a generalization of the SS. He opened various fields in this perspective and generalized SS to the HSS by renovating it into a multi-attribute function. He also made the separation between the kinds of initial universes, crisp fuzzy (CF), intuitionistic fuzzy (IF), neutrosophic (NS), and plithogenic (PLG), respectively. Thus, he also showed that an HSS could be crisp, fuzzy, IF, NS, and PLG, respectively, and demonstrated these outcomes with models.

Saeed *et al.* [22], [28], [56] explained some basic concepts like Hypersoft (HS) subset, HS complement, not HS set, absolute set, union, intersection, AND, OR, restricted union, extended intersection, relevant complement, restricted difference, restricted symmetric difference, HS set relation, sub relation, complement relation, HS representation in matrices form, different operations on matrices and applied similarity measure technique for medical diagnosis purpose in neutrosophic environment. Saeed *et al.* [55] characterized mapping under a hypersoft set environment, then some of its essential properties like HS images, HS inverse images were also discussed. Mujahid *et al.* [52] discussed hypersoft points in different fuzzy-like environments. In 2020, Rahman *et al.* [53] defined complex HSS and developed the HS set's hybrids with a complex fuzzy set, complex intuitionistic fuzzy set, and complex neutrosophic set, respectively. They also discussed their fundamentals, i.e., subset, equal sets, null set, absolute set etc., and theoretic operations i.e. complement, union, intersection etc. In 2020, Rahman *et al.* [54] conceptualized convexity cum concavity on HSS and presented its pictorial versions with illustrative examples.

The primary commitments of our exploration are as per the following. Firstly, we present the idea of CMFHSS, which joins the benefits of both the CMFS and HSS. Secondly, we characterize a few ideal notions of CMFHSS as well as some fundamental operations, in particular the complement, union, intersection, AND, and OR. The essential properties and applicable laws relating to this idea, such as De Morgan's laws, are also verified. We present the proverbial meaning of ENT and SM of CMFHSS and study the fundamental relations between them. Additionally, mathematical models are given to analyze the reliability and the predominance of the setup methodology. Moreover, comparisons between proposed strategies and existing theories are additionally depicted in detail. Lastly, the mathematical structures are represented to justify the validity and applicability of the presented measures.

Section II focuses on some basic definitions and terminologies used in the paper. In Section III, the idea of a CMFHSS with its properties is presented. In Section IV, the set-theoretic operations of CMFHSS are conceptualized. In Section V, we present the proverbial meaning of ENT for CMFHSS, supported by an example. In Section VI, the SM between CMFHSS and the connection between the ENT and SM are examined. Section VII concludes the paper.

II. PRELIMINARIES

In this section, we discuss about some basic concepts including FS, SS, FSS, FHSS, MFS, MFSS, CMFSS, SM, ENT, CFH-set and CFH-subset.

Definition 1 [2]: The FS, $R = \{(y, I(y))|y \in Y\}$ such that

$$I : Y \to [0, 1],$$

where Y is the collection of objects and $I(y)$ represents the membership grade of $y \in Y$.

Definition 2 [9]: A pair (I, Q) is said to be SS over the universe Y, where I is a mapping given as

$$I : Q \to P(Y),$$

for $\epsilon \in Q$, $I(\epsilon)$ can be regarded as ϵ approximate elements of the SS (I, Q).

Definition 3 [12]: Let Y, Q be initial universe and set of parameters respectively. Let $P(Y)$ denote the power set of all fuzzy subsets of Y and $Q \subseteq E$. A pair (I, Q) is said to be FSS over Y, where $I : Q \to P(Y)$.

Definition 4 [28]: Suppose Y and $I(Y)$ be the universal set and all fuzzy subsets of Y respectively. Let $m_1, m_2, m_3, \cdots, m_n$ be the distinct attributes whose attribute values belongs to the sets $M_1, M_2, M_3, \ldots, M_n$ respectively, where $M_i \cap M_j = \Phi$ for $i \neq j$ and i, j belongs to

$\{1, 2, 3, \ldots, n\}$. Then the FHSS is the pair (Σ_L, L) over Y defined by a map $\Sigma_L : L \rightarrow I(Y)$, where $L = F_1 \times F_2 \times F_3 \times \ldots \times F_n$.

Definition 5 [29]: Let k be a non zero non negative integer and $Y \neq \Phi$. A MFS Q in Y is a ordered sequences $Q = \{\langle y, \lambda_1(y), \ldots, \lambda_k(y)\rangle : y \in Y\}$, where $\lambda_i : Y \rightarrow O_i = [0, 1]$, $i = 1, 2, ..., k$. A function $\lambda_Q(y) = (\lambda_1(y), \ldots, \lambda_k(y))$ is said to be multi-membership map of MF sets Q, $k =$ Dim Q and their collection in Y is $M^kFS(Y)$.

Definition 6 [14]: A pair (I, Q) is said to be MFSS with dim k if $I : Q \rightarrow M^kFS(Y)$ and $I(e), e \in Q$ is its collection of e-approximate members.

Definition 7 [25]: A pair (I, Q) is said to be a CMFSS of dim k over Y if $I : Q \rightarrow CM^k(Y)$ and be represented as $(I, Q) = \{\langle \epsilon, I(\epsilon)\rangle : \epsilon \in Q, I(\epsilon) \in CM^k(Y)\}$, where $I(\epsilon) = \{\langle y, \lambda^s_{I(\epsilon)}(y) = \rho^s_{I(\epsilon)}(y).\epsilon^{i\omega^s_{I(\epsilon)}(y)}\rangle : \epsilon \in Q, y \in Y, s = 1, 2, \ldots, k\}$, where $\lambda^s_{I(\epsilon)}(y)_{s\in k}$ is a complex multi membership function $y \in X$ with real valued functions A-part $= (\rho^s_{I(\epsilon)}(y))_{s\in k} \in [0, 1]$ and P-part $= (\omega^s_{I(\epsilon)}(y))_{s\in k}$. The collection of all such sets is represented by CM^kFSS.

Definition 8 [26]: A function S from $FS(Y, E) \times FS(Y, E)$ to $[0, 1]$ is called a SM for FSS, if it fulfills the following points.

1) $S(X_Q, \Phi_Q) = 0$, for any $Q \in E$, and $S((I, Q), (I, Q)) = 1$ for any $(I, Q) \in FS(Y, E)$,
2) $S((I, Q), (J, C)) = S((J, C), (I, Q))$, for any $(I, Q), (J, C) \in FS(Y, E)$,
3) For any $(I, Q), (J, C), (H, O) \in FS(Y, E)$ if $(I, Q) \subseteq (J, C) \subseteq (H, O)$, then $S((H, O), (I, Q)) = min(S((H, O), (J, C)), S((J, C), (I, Q)))$.

Definition 9 [26]: A real valued function E from $FS(Y, E)$ to $[0, \infty]$ for FSS is called a ENT, if E satisfies the given conditions.

1) $E(I, Q) = 0$ if (I, Q) is a SS,
2) $E(I, Q) = 1$ if $I(e) = 0.5$, for any $e \in Q$, where $[0.5]$ is the FS having membership function $[0.5](y) = 0.5$, for every $y \in Y$,
3) Suppose (I, Q) be crisp set than that of (J, C) which is, for $e \in Q$ and $y \in Y$, $I(e)(y) \leq J(e)(y)$ if $J(e)(y) \leq 0.5$ and $I(e)(y) \geq J(e)(y)$ if $J(e)(y) \geq 0.5$. Then $E(I, Q) \leq E(J, C)$,
4) $E(I, Q) = E(I^c, Q)$, where (I^c, Q) is the complement of FSS (I, Q), which can be written as $I^c(e) = (I(e))^c$, for every $e \in Q$.

Definition 10 [53]: Let $M_1, M_2, M_3, \ldots.., M_n$ be disjoint sets having attribute values of n distinct attributes $m_1, m_2, m_3, \ldots.., m_n$ respectively for $n \geq 1, G = M_1 \times M_2 \times M_3 \times \ldots.. \times M_n$ and $\xi(\underline{y})$ be a CF-set over Y for all $\underline{\epsilon} = (c_1, c_2, c_3, \ldots.., c_n) \in G$. Then, *complex fuzzy hypersoft set* (CFH-set) ϖ_G over Y is defined as:

$$\varpi_G = \{(\underline{\epsilon}, \xi(\underline{\epsilon})) : \underline{\epsilon} \in G, \xi(\underline{\epsilon}) \in C(Y)\}$$

where

$$\xi : G \rightarrow C(Y), \quad \xi(\underline{\epsilon}) = \emptyset \text{ if } \underline{\epsilon} \notin G.$$

is a CF-approximate function of ϖ_G and its value $\xi(\underline{\epsilon})$ is called $\underline{\epsilon}$-member of CFH-set $\forall \underline{\epsilon} \in G$.

Definition 11 [53]: Let $\varpi_{W_1} = (\xi_1, W_1)$ and $\varpi_{W_2} = (\xi_2, W_2)$ be two CFH-sets over the same Y. The set $\varpi_{W_1} = (\xi_1, W_1)$ is said to be the CFH-subset of $\varpi_{W_2} = (\xi_2, W_2)$, if

1) $W_1 \subseteq W_2$,
2) $\forall \underline{y} \in W_1, \xi_1(\underline{y}) \subseteq \xi_2(\underline{y})$ i.e. $r_{W_1}(\underline{y}) \leq r_{W_2}(\underline{y})$ and $\omega_{W_1}(\underline{y}) \leq \omega_{W_2}(\underline{y})$, where $r_{W_1}(\underline{y})$ and $\omega_{W_1}(\underline{y})$ are amplitude and phase terms of $\xi_1(\underline{y})$, whereas $r_{W_2}(\underline{y})$ and $\omega_{W_2}(\underline{y})$ are amplitude and phase terms of $\xi_2(\underline{y})$.

III. COMPLEX MULTI-FUZZY HYPERSOFT SET (CMFHSS)

Throughout this section, the following data is considered: $\mathcal{D} = A_1 \times A_2 \times A_3 \times \ldots \times A_n$, $\mathcal{E} = B_1 \times B_2 \times B_3 \times \ldots \times B_n$, $\mathcal{R} = C_1 \times C_2 \times C_3 \times \ldots \times C_n$, $e = (e_1, e_2, e_3, \ldots e_n)$, $\aleph = N_1 \times N_2 \times N_3 \times \ldots \times N_n$.

Definition 12: Let $m_1, m_2, m_3, \cdots, m_n$ be the distinct attributes with the corresponding attributive values to the sets $M_1, M_2, M_3, \cdots, M_n$ respectively, where $M_i \cap M_j = \Phi$ for $i \neq j$. A pair $(\mathcal{J}, \mathcal{D})$ is called a MFHSS of dimension k over Y, where $\mathcal{J}$ is a function given as $\mathcal{J} : \mathcal{D} \rightarrow M^kFHS(Y)$. For $e \in \mathcal{D}$, $\mathcal{J}(e)$ may be regarded as the set of e-approximate elements of the MFHSS $(\mathcal{J}, \mathcal{D})$.

Definition 13: A pair $(\mathcal{J}, \mathcal{D})$ is called a CMFHSS of dimension k over Y, where $\mathcal{J}$ is a mapping given by $\mathcal{J} : \mathcal{D} \rightarrow CM^k(Y)$. A complex multi-fuzzy hypersoft set of dimension $k(CM^kFHSS(Y))$ is a mapping from parameters to $CM^k(Y)$. It is a parameterized family of complex multi-fuzzy subsets of Y, and it can be written as: as $(\mathcal{J}, \mathcal{D}) = \{\langle e, \mathcal{J}(e)\rangle : e \in \mathcal{D}, \mathcal{J}(e) \in CM^k(Y)\}$, where $\mathcal{J}(e) = \{\langle y, \lambda^s_{\mathcal{J}(e)}(y) = \rho^s_{\mathcal{J}(e)}(y).e^{i\omega^s_{\mathcal{J}(e)}(y)}\rangle : e \in \mathcal{D}, y \in Y, s = 1, 2, \ldots, k\}$, where $\mu^s_{\mathcal{J}(e)}(y)_{s\in k}$ is a complex-valued grade of multi membership function $y \in Y$. By definition, the values of $\lambda^s_{\mathcal{J}(e)}(y)_{s\in k}$ may all lie in the complex plane within the unit circle, and are thus of the form $[\lambda^s_{\mathcal{J}(e)}(y) = \rho^s_{\mathcal{J}(e)}(y).e^{i\omega^s_{\mathcal{J}(e)}(y)}]_{s\in k}$, where $(i^2 = -1)$, each of the A-terms $(\rho^s_{\mathcal{J}(e)}(y))_{s\in k}$ and the P-terms $(\omega^s_{\mathcal{J}(e)}(y))_{s\in k}$ are both real-valued, and $(\rho^s_{\mathcal{J}(e)}(y))_{s\in k} \in [0, 1]$. The set of all CM^kFHSS in Y is denoted by $CM^kFHSS(Y)$.

Example 1: Suppose a person desired to take loan from one of the bank for certain period. Suppose $Y = \{y_1 =$ JP Morgan, $y_2 =$ Wells Fargo, $y_3 =$ Goldmqan Saches$\}$ be the set of three banks in USA. It is well reputed that a year has four periods and the interest rate are different in each period. Let $a_1 =$ Repayment tenor, $a_2 =$ Interest rate, $a_3 =$ Documentation, distinct attributes whose attribute values belong to the sets E_1, E_2, E_3. Let $E_1 = \{f_1 =$ Flexible, $f_2 =$ Difficult $\}$, $E_2 = \{f_3 =$ High, $f_4 = Low\}$, $E_3 = \{f_5 =$ Easy$\}$. We construct CMFHSS having three dimension.

$$\begin{aligned}
\mathcal{J}(f_1, f_3, f_5) = \{&y_1/(0.9e^{i2\pi(2/4)}, 0.2e^{i2\pi(4/4)}, 0.9e^{i2\pi(3/4)}),\\
&y_2/(0.8e^{i2\pi(1/4)}, 0.4e^{i2\pi(3/4)}, 0.1e^{i2\pi(2/4)}),\\
&y_3/(0.4e^{i2\pi(3/4)}, 0.2e^{i2\pi(4/4)}, 0.8e^{i2\pi(1/4)})\},\\
\mathcal{J}(f_1, f_4, f_5) = \{&y_1/(0.8e^{i2\pi(2/4)}, 0.2e^{i2\pi(4/4)}, 0.3e^{i2\pi(3/4)}),\\
&y_2/(0.5e^{i2\pi(1/4)}, 0.4e^{i2\pi(2/4)}, 0.6e^{i2\pi(3/4)}),\\
&y_3/(0.1e^{i2\pi(3/4)}, 0.2e^{i2\pi(1/4)}, 0.8e^{i2\pi(4/4)})\},
\end{aligned}$$

$$\mathcal{J}(f_2, f_3, f_5) = \{y_1/(0.1e^{i2\pi(2/4)}, 0.2e^{i2\pi(4/4)}, 0.1e^{i2\pi(2/4)}), y_2/(0.8e^{i2\pi(2/4)}, 0.4e^{i2\pi(4/4)}, 0.5e^{i2\pi(2/4)}), y_3/(0.04e^{i2\pi(1/4)}, 0.2e^{i2\pi(2/4)}, 0.078e^{i2\pi(3/4)}\},$$

$$\mathcal{J}(f_2, f_4, f_5) = \{y_1/(0.2e^{i2\pi(2/4)}, 0.2e^{i2\pi(2/4)}, 0.09e^{i2\pi(3/4)}), y_2/(0.7e^{i2\pi(1/4)}, 0.4e^{i2\pi(3/4)}, 0.01e^{i2\pi(2/4)}), y_3/(0.1e^{i2\pi(3/4)}, 0.2e^{i2\pi(1/4)}, 0.8e^{i2\pi(3/4)})\},$$

In this example, the A-terms speak the belongingness degrees to the arrangement of interest rates and the P-terms speak the belongingness degrees to the period of seasons with respect to the attributes values. In the CMF value $y_1/(0.8e^{i2\pi(2=4)}, y_2/0.2e^{i2\pi(4=4)}, y_3/0.3e^{i2\pi(3=4)}$ the first value $(0.8e^{i2\pi(2=4)}$ demonstrates which is interest rate of loan is high in the late spring, since the A-term 0.8 is near to one and the P-term $(2 = 4)$ tells the year (the late spring season) which have second period w.r.t the attributes value (f_1, f_3, f_5). While the subsequent membership value $0.2e^{i2\pi(4=4)}$ demonstrates that the interest rate is low in the winter, since the P-term 0.2 which is near to zero and the P-term $(4 = 4)$ speaks to the 4th season of the year (the winter season) with respect to the attributes value (f_1, f_3, f_5). Now, we are going to describe the basic concept and operations of CMFHSS.

Definition 14: Let $(\mathcal{J}, \mathcal{D})$ and $(\varpi, \mathcal{E})$ be two CM^kFHSS over Y. Now, $(\mathcal{J}, \mathcal{D})$ is said to be a CMFHSS subset of $(\varpi, \mathcal{E})$ if,

1) $\mathcal{D} \subseteq \mathcal{E}$ and
2) $\forall e \in \mathcal{D}, \mathcal{J}(e) \sqsubseteq \varpi(e)$.

In this case, we can write $\mathcal{J}(e) \sqsubseteq \varpi(e)$.

Definition 15: $(\mathcal{J}, \mathcal{D})$ and $(\varpi, \mathcal{E})$ be two CM^kFHSS over Y. $(\mathcal{J}, \mathcal{D})$ and $(\varpi, \mathcal{E})$ are said to be a CMFHSS equal if $(\mathcal{J}, \mathcal{D})$ is a CMFHSS subset of $(\varpi, \mathcal{E})$ and $(\varpi, \mathcal{E})$ is a CMFHSS subset of $(\mathcal{J}, \mathcal{D})$.

Definition 16: A CM^kFHSS $(\mathcal{J}, \mathcal{D})$ over Y is said to be a null CMFHSS, denoted by $(\mathcal{J}, \mathcal{D})_{\phi_k}$, if $\mathcal{J}(e) = 0_k$, for all $e \in \mathcal{D}$ (i.e., $\rho^s_{\mathcal{J}(e)}(y) = 0$ and $\omega^s_{\mathcal{J}(e)}(y) = 0_\phi$, $\forall\, e \in \mathcal{D}, x \in Y, s = 1, 2, \ldots, k$).

Definition 17: A CM^kFHSS $(\mathcal{J}, \mathcal{D})$ over Y is said to be absolute CMFHSS, denoted by $(\mathcal{J}, \mathcal{D})_{U_k}$, if $\mathcal{J}(e) = 1_k$, $\forall$ $e \in \mathcal{D}$ (i.e, $\rho^s_{\mathcal{J}(e)}(y) = 1$, $\omega^s_{\mathcal{J}(e)}(y) = 2\pi$, $\forall\, e \in \mathcal{D}, y \in X, s = 1, 2, \ldots, k$).

IV. BASIC OPERATIONS ON CMFHSS-SETS

In this section, we develop some fundamental theoretic operations, laws of CMFHSS like union, intersection, complement, De Morgan's law and associative are discussed in detail.

Definition 18: Let $(\mathcal{J}, \mathcal{D})$ be a CM^kFHSS over Y and $(\mathcal{J}, \mathcal{D})^c$ denotes the complement of $(\mathcal{J}, \mathcal{D})$ which is defined as $(\mathcal{J}, \mathcal{D})^c = (\mathcal{J}^c, \rightharpoondown \mathcal{D})$, where $\mathcal{J}^c : \mathcal{D} \to CM^k(Y)$ is a mapping represented by $\mathcal{J}^c(e) = \{\langle y, \lambda^s_{\mathcal{J}^c(e)}(y) = \rho^s_{\mathcal{J}^c(e)}(y).e^{i\omega^s_{\mathcal{J}^c(e)}(y)}\rangle : e \in \rightharpoondown \mathcal{D}, y \in Y, s = 1, 2, \ldots, k\}$, where the complement of the A-term is $\rho^s_{F^c(e)}(y) = 1 - \rho^s_{\mathcal{J}(e)}(y)$ and the complement of the P-term is $\omega^s_{\mathcal{J}^c(e)}(y) = 2\pi - i\omega^s_{\mathcal{J}(e)}(y)$.

Example 2: From example 1, consider

$$\mathcal{J}(f_1, f_3, f_5) = \{y_1/(0.9e^{i2\pi(2=4)}, 0.2e^{i2\pi(4=4)}, 0.9e^{i2\pi(3=4)}), y_2/(0.8e^{i2\pi(1=4)}, 0.4e^{i2\pi(3=4)}, 0.1e^{i2\pi(2=4)}), y_3/(0.4e^{i2\pi(3=4)}, 0.2e^{i2\pi(4=4)}, 0.8e^{i2\pi(1=4)})\},$$

By applying definition 19, we get the complement

$$\mathcal{J}^c(f_1, f_3, f_5) = \{y_1/(0.1e^{i2\pi(2=4)}, 0.8e^{i2\pi(0=4)}, 0.1e^{i2\pi(1=4)}), y_2/(0.2e^{i2\pi(3=4)}, 0.6e^{i2\pi(1=4)}, 0.9e^{i2\pi(2=4)}), y_3/(0.6e^{i2\pi(1=4)}, 0.8e^{i2\pi(0=4)}, 0.2e^{i2\pi(3=4)})\},$$

Proposition 1: If $(\mathcal{J}, \mathcal{D})$ is a CM^kFHSS over Y, then

1) $((\mathcal{J}, \mathcal{D})^c)^c = (\mathcal{J}, \mathcal{D})$,
2) $((\mathcal{J}, \mathcal{D})_{\phi_k})^c = (\mathcal{J}, \mathcal{D})_{Y_k}$, where $(\mathcal{J}, \mathcal{D})_{Y_k}$ and $(\mathcal{J}, \mathcal{D})_{\phi_k}$ are the absolute and null CMFHSS, respectively.
3) $((\mathcal{J}, \mathcal{D})_{Y_k})^c = (\mathcal{J}, \mathcal{D})_{\phi_k}$

Proof: Here, we present the proof of 1 from definition 18, since the proofs of 2 and 3 are obvious from definitions 19. Suppose that $(\mathcal{J}, \mathcal{D})$ is a complex multi-fuzzy HSS having k as a dimension over Y. The complement $(\mathcal{J}, \mathcal{D})$ can be written as $(\mathcal{J}, \mathcal{D})^c = (\mathcal{J}^c, \rightharpoondown \mathcal{D})$ is defined as:

$$(\mathcal{J}, \mathcal{D})^c = \{\langle e, \rho^s_{\mathcal{J}^c(e)}(y).e^{i\omega^s_{\mathcal{J}^c(e)}}(y)\rangle : e \in \rightharpoondown \mathcal{D}, y \in Y\},$$

where $s = 1, 2, \ldots, k$

$$= \{\langle e, [1 - \rho^s_{\mathcal{J}(e)}(y)].e^{i[2\pi - \omega^s_{\mathcal{J}(e)}(y)]}\rangle : e \in \rightharpoondown \mathcal{D}, y \in Y\},$$

Now, let $(\mathcal{J}, \mathcal{D})^c = (\varpi, \mathcal{E}) = (\mathcal{J}^c, \rightharpoondown \mathcal{D})$. Then we get:

$$\begin{aligned}(\varpi, \mathcal{E})^c &= \{\langle e, [1 - \rho^s_{\mathcal{J}^c(e)}(y)].e^{i[2\pi - \omega^s_{\mathcal{J}^c(e)}(y)]}]\rangle : e \in \rightharpoondown (\rightharpoondown \mathcal{D}), y \in Y\} \\ &= \{\langle e, [1 - (1 - \rho^s)_{\mathcal{J}^c(e)}(y)].e^{i[2\pi - (2\pi - \omega^s)_{\mathcal{J}^c(e)}(y)]}]\rangle : e \in \rightharpoondown (\rightharpoondown \mathcal{D}) y \in Y\} \\ &= \{e, \rho^s_{\mathcal{J}(e)}(y).e^{i\omega^s_{\mathcal{J}(e)}(y)}\rangle : e \in \mathcal{J}, y \in Y\} = (\mathcal{J}, \mathcal{D}).\end{aligned}$$

Definition 19: The union of two CM^kFHSS $(\mathcal{J}, \mathcal{D})$ and $(\varpi, \mathcal{E})$ over Y, denoted by $(\mathcal{J}, \mathcal{D}) \cup (\varpi, \mathcal{E})$, is a CMFHSS $(\Phi, \mathcal{R})$, where $\mathcal{R} = \mathcal{D} \cup \mathcal{E}$, $\forall$ $e \in \mathcal{R}$ and $y \in Y$,

$$\Phi(e) = \begin{cases} \mathcal{J}(e) = [\rho^s_{\mathcal{J}(e)}(y).e^{i\omega^s_{\mathcal{J}(e)}(y)}]_{s \in k}, & \text{if } e \in \mathcal{D} - \mathcal{E}, \\ \varpi(e) = [\rho^s_{\varpi(e)}(y).e^{i\omega^s_{\varpi(e)}(y)}]_{s \in k}, & \text{if } e \in \mathcal{E} - \mathcal{D}, \\ \mathcal{J}(e) \cap \varpi(e) = [(\rho^s_{\mathcal{J}(e)}(y) \vee \rho^s_{\varpi(e)}(y)) \\ .e^{i[\omega^s_{\mathcal{J}(e)}(y) \cup \omega^s_{\varpi(e)}(y)]}]_{s \in k}, & \text{if } e \in \mathcal{E} \cap \mathcal{D}. \end{cases} \tag{1}$$

We write $(\Phi, \mathcal{R}) = (\mathcal{J}, \mathcal{D}) \cup (\varpi, \mathcal{E})$, where max of operator is denoted by $\cup$ and the P-term $e^{i[\omega^s_{\mathcal{J}(e)}(y) \cup \omega^s_{\varpi(e)}(y)]}]_{s \in k}$ lie in the interval $[0, 2\pi]$ and it can be evaluated by using any operators which are given below.

1) *Sum* : $\omega^s{}_{\mathcal{J}(e) \cup \varpi(e)}(y) = \omega^s{}_{\mathcal{J}(e)}(y) + \omega^s{}_{\varpi(e)}(y)$, $\forall\, s = 1, 2, \ldots, k$.

2) Max : $\omega^s{}_{\mathcal{J}(e)\cup\varpi(e)}(y) = Max(\omega^s{}_{\mathcal{J}(e)}(y), \omega^s{}_{\varpi(e)}(y))$, $\forall$ $s = 1, 2, \ldots, k$.
3) Min : $\omega^s{}_{\mathcal{J}(e)\cup\varpi(e)}(y) = Min(\omega^s{}_{\mathcal{J}(e)}(y), \omega^s{}_{\varpi(e)}(y))$, $\forall$ $s = 1, 2, \ldots, k$.
4) Winner Takes All,

$$\omega^s{}_{\mathcal{J}(e)\cup\varpi(e)}(y) = \begin{cases} \omega^s{}_{\mathcal{J}(e)}(y), & \text{if } r^s{}_{\mathcal{J}(e)} \succ r^s{}_{\varpi(e)}, \\ \omega^s{}_{\varpi(e)}(y), & \text{if } r^s{}_{\mathcal{J}(e)} \prec r^s{}_{\varpi(e)}. \end{cases} \quad \forall\, s = 1, 2, \ldots, k.$$

Definition 20: The intersection of two CM^kFHSS $(\mathcal{J}, \mathcal{D})$ and $(\varpi, \mathcal{E})$ over Y, denoted by $(\mathcal{J}, \mathcal{D}) \cap (\varpi, \mathcal{E})$, is a CMFHSS $(\Psi, \mathcal{R})$, where $\mathcal{R} = \mathcal{D} \cup \mathcal{E}$, $\forall e \in \mathcal{R}$ and $y \in Y$,

$$\Psi(e) = \begin{cases} \mathcal{J}(e) = [\rho^s_{\mathcal{J}(e)}(y).e^{i\omega^s_{\mathcal{J}(e)}(y)}]_{s\in k}, & \text{if } e \in \mathcal{D} - \mathcal{E}, \\ \varpi(e) = [\rho^s_{\varpi(e)}(y).e^{i\omega^s_{\varpi(e)}(y)}]_{s\in k}, & \text{if } e \in \mathcal{E} - \mathcal{D}, \\ \mathcal{J}(e) \cap \varpi(e) = [(\rho^s_{\mathcal{J}(e)}(y) \wedge \rho^s_{\varpi(e)}(y)) \\ .e^{i[\omega^s_{\mathcal{J}(e)}(y) \cap \omega^s_{\varpi(e)}(y)]}]_{s\in k}, & \text{if } e \in \mathcal{E} \cap \mathcal{D}. \end{cases} \tag{2}$$

We write $(\Psi, \mathcal{R}) = (\mathcal{J}, \mathcal{D}) \cap (\varpi, \mathcal{E})$, where min of operator is denoted by $\wedge$ and the phase term $(e^{\omega^s_{\mathcal{J}(e)\cap\varpi(e)}}.(y))_{s\in k}$ of the function belong to $[0, 2\pi]$. Some theorems on the union, intersection and complement of CMFHSS will also be given. These theorems will make the connection between the set theoretic operations that have been examined previously.

Theorem 1: Suppose $(\mathcal{J}, \mathcal{D})$ and $(\varpi, \mathcal{E})$ be two CM^kFHSS over Y. Then the following conditions are satisfied.

1) $(\mathcal{J}, \mathcal{D}) \cup (\mathcal{J}, \mathcal{D})_{\phi_k} = (\mathcal{J}, \mathcal{D})$,
2) $(\mathcal{J}, \mathcal{D}) \cup (\mathcal{J}, \mathcal{D})_{\phi_k} = (\mathcal{J}, \mathcal{D})_{\phi_k}$,
3) $(\mathcal{J}, \mathcal{D}) \cup (\mathcal{J}, \mathcal{D})_{Y_k} = (\mathcal{J}, \mathcal{D})_{Y_k}$,
4) $(\mathcal{J}, \mathcal{D}) \cap (\mathcal{J}, \mathcal{D})_{Y_k} = (\mathcal{J}, \mathcal{D})$.

Proof: The proofs are obvious by applying Definitions 20 and 21.

Theorem 2: Suppose $(\mathcal{J}, \mathcal{D})$, $(\varpi, \mathcal{E})$ and $(\oplus, \mathcal{R})$ be three CM^kFHSS over Y having k as a dimension. Then the following associative laws hold true

1) $(\mathcal{J}, \mathcal{D}) \cup ((\varpi, \mathcal{E}) \cup (\oplus, \mathcal{R})) = ((\mathcal{J}, \mathcal{D}) \cup ((\varpi, \mathcal{E})) \cup (\oplus, \mathcal{R})$.
2) $(\mathcal{J}, \mathcal{D}) \cap ((\varpi, \mathcal{E}) \cap (\oplus, \mathcal{R})) = ((\mathcal{J}, \mathcal{D}) \cap ((\varpi, \mathcal{E})) \cap (\oplus, \mathcal{R})$.

Proof:

1) Suppose that $((\varpi, \mathcal{E}) \cup (\oplus, \mathcal{R})) = (M, N)$, where $N = \mathcal{E} \cup \mathcal{R}$, by definition 20, we have $((\varpi, \mathcal{E}) \cup (\oplus, \mathcal{R}))$ to be a $CM^kFHSS(M, N)$, where $N = \mathcal{E} \cup \mathcal{R}$ and $\forall e \in N$,

$$M(e) = \mathcal{E}(e) \cup \mathcal{R}(e) = \left[(\rho^s_{\varpi(e)}(y) \vee \rho^s_{\oplus(e)}(y).e^{i[\omega^s_{\varpi(e)}(y) \vee \omega^s_{\oplus(e)}(y)]}]_{s\in k}\right],$$

Suppose that $(\otimes, \aleph) = ((\mathcal{J}, \mathcal{D}) \cup (M, N))$, where $\otimes = \mathcal{J} \cup M$, By using Definition 20, we have $((\mathcal{J}, \mathcal{D}) \cup (M, N))$ to be a CM^kFHSS $(\otimes, \aleph)$, where $\aleph = \mathcal{D} \cup N$ and for all $e \in \aleph$,

$$\otimes(e) = \begin{cases} \mathcal{J}(e) = [\rho^s_{\mathcal{J}(e)}(y).e^{i\omega^s_{\mathcal{J}(e)}(y)}]_{s\in k}, & \text{if } e \in \mathcal{D} - N, \\ M(e) = [\rho^s_{\varpi(e)}(y).e^{i\omega^s_{\varpi(e)}(y)}]_{s\in k}, & \text{if } e \in N - \mathcal{D}, \\ \mathcal{J}(e) \cup M(e) = [(\rho^s_{\mathcal{J}(e)}(y) \vee \rho^s_{M(e)}(y)) \\ .e^{i[\omega^s_{\mathcal{J}(e)}(y) \cup \omega^s_{M(e)}(y)]}]_{s\in k}, & \text{if } e \in N \cap \mathcal{D}. \end{cases} \tag{3}$$

Now, let $(\mathcal{J}, \mathcal{D}) \cup ((\varpi, \mathcal{E}) \cup (\oplus, \mathcal{R})) = ((\mathcal{J}, \mathcal{D}) \cup (M, N))$. We notice the case when $e \in \mathcal{D} \cap N$ as the remainder cases which are trivial. Hence, $(\mathcal{J}, \mathcal{D}) \cup ((\varpi, \mathcal{E}) \cup (\oplus, \mathcal{R}))$

$$\begin{aligned} &= (\mathcal{J}, \mathcal{D}) \cup (M, N) \\ &= \mathcal{J}(e) \cup \varpi(e) \\ &= \mathcal{J}(e) \cup (\varpi(e) \cup \oplus(e)) \\ &= \left[(\rho^s_{\mathcal{J}(e)}(y) \vee \rho^s_{\varpi(e)\cup\oplus(e)}(y)) \; .e^{i[\omega^s_{\mathcal{J}(e)}(y) \cup \omega^s_{\varpi(e)\cup\oplus(e)}(y)]}\right]_{s\in k} \\ &= \left[(\rho^s_{\mathcal{J}(e)}(y) \vee \rho^s_{\varpi(e)\cup\oplus(e)}(y)) \; .e^{i[\omega^s_{\mathcal{J}(e)}(y) \cup \omega^s_{\varpi(e)\cup\oplus(e)}(y)]}\right]_{s\in k} \\ &= \left[(\rho^s_{\mathcal{J}(e)}(y) \vee [\rho^s_{\varpi(e)}(y) \vee \rho^s_{\oplus(e)}(y))] \; .e^{i\omega^s_{\mathcal{J}(e)}(y) \cup [\omega^s_{\varpi(e)}(y) \cup \omega^s_{\oplus(y)}}\right]_{s\in k} \\ &= \left[[\rho^s_{\mathcal{J}(e)}(y) \vee \rho^s_{\varpi(e)}(y)] \vee \rho^s_{\oplus(e)}(y)) \; .e^{i[\omega^s_{\mathcal{J}(e)}(y) \cup \omega^s_{\varpi(e)}(y)] \cup \omega^s_{\oplus(y)}}\right]_{s\in k} \\ &= \left[(\rho^s_{\mathcal{J}(e)\cup\varpi(e)}(y) \vee \rho^s_{\oplus(e)}(y)) \; .e^{i[\omega^s_{\mathcal{J}(e)\cup\varpi(e)}(y) \cup \omega^s_{\oplus(e)}(y)]}\right]_{s\in k} \end{aligned}$$

$= ((\mathcal{J}, \mathcal{D}) \cup (\varpi, \mathcal{E})) \cup (\oplus, \mathcal{R})$. Therefore, we have $(\mathcal{J}, \mathcal{D}) \cup ((\varpi, \mathcal{E}) \cup (\oplus, \mathcal{R})) = ((\mathcal{J}, \mathcal{D}) \cup (\varpi, \mathcal{E})) \cup (\oplus, \mathcal{R})$.

2) The proof resemble as that in part (1) and therefore is leave out.

Theorem 3: Suppose $(\mathcal{J}, \mathcal{D})$ and $(\varpi, \mathcal{E})$ be two CM^kFHSS over Y. Then these De Morgan's laws holds true.

1) $((\mathcal{J}, \mathcal{D}) \cup (\varpi, \mathcal{E}))^c = (\mathcal{J}, \mathcal{D})^c \cap (\varpi, \mathcal{E})^c$,
2) $((\mathcal{J}, \mathcal{D}) \cap (\varpi, \mathcal{E}))^c = (\mathcal{J}, \mathcal{D})^c \cup (\varpi, \mathcal{E})^c$.

1) *Proof:* Suppose that $(\mathcal{J}, \mathcal{D}) \cup (\varpi, \mathcal{E}) = (\oplus, \mathcal{R})$, where $\mathcal{R} = \mathcal{D} \cup \mathcal{E}$ and $\forall e \in C$

$$\oplus(e) = \begin{cases} \mathcal{J}(e) = [\rho^s_{\mathcal{J}(e)}(y).e^{i\omega^s_{\mathcal{J}(e)}(y)}]_{s\in k}, & \text{if } e \in \mathcal{D} - \mathcal{E}, \\ \varpi(e) = [\rho^s_{\varpi(e)}(y).e^{i\omega^s_{\varpi(e)}(y)}]_{s\in k}, & \text{if } e \in \mathcal{E} - \mathcal{D}, \\ \mathcal{J}(e) \cap \varpi(e) = [(\rho^s_{\mathcal{J}(e)}(y) \vee \rho^s_{\varpi(e)}(y)) \\ .e^{i[\omega^s_{\mathcal{J}(e)}(y) \cup \omega^s_{\varpi(e)}(y)]}]_{s\in k}, & \text{if } e \in \mathcal{E} \cap \mathcal{D}. \end{cases} \tag{4}$$

Since $(\mathcal{J},\mathcal{D})\cup(\varpi,\mathcal{E}) = (\oplus,\mathcal{R})$, then we have, $((\mathcal{J},\mathcal{D})\cup(\varpi,\mathcal{E}))^c = (\oplus,\mathcal{R})^c = (\oplus^c,\mathcal{R})$. Hence $\forall e\in\mathcal{R}$

$$\oplus^c(e)=\begin{cases} \mathcal{J}^c(e) \\ =[\rho^s_{\mathcal{J}^c(e)}(y).e^{i\omega^s_{\mathcal{J}^c(e)}(y)}]_{s\in k}, & \text{if } e\in\mathcal{D}-\mathcal{E},\\ \varpi^c(e) \\ =[\rho^s_{\varpi^c(e)}(y).e^{i\omega^s_{\varpi^c(e)}(y)}]_{s\in k}, & \text{if } e\in\mathcal{E}-\mathcal{D},\\ \mathcal{J}^c(e)\cap\varpi^c(e) \\ =[(\rho^s_{\mathcal{J}^c(e)}(y)\vee\rho^s_{\varpi^c(e)}(y)) \\ .e^{i[\omega^s_{\mathcal{J}^c(e)}(y)\cup\omega^s_{\varpi^c(e)}(y)]}]_{s\in k}, & \text{if } e\in\mathcal{E}\cap\mathcal{D}. \end{cases} \quad (5)$$

Since $(\mathcal{J},\mathcal{D})^c=(\mathcal{J}^c,\mathcal{D})$ and $(\varpi,\mathcal{E})^c=(\varpi^c,\mathcal{E})$ then we have, $(\mathcal{J},\mathcal{D})^c\cap(\varpi,\mathcal{E})^c = (\mathcal{J}^c,\mathcal{D})\cap(\varpi^c,\mathcal{E})$. Suppose that $(\mathcal{J}^c,\mathcal{D})\cap(\varpi^c,\mathcal{E})\cap(T,J)$, where $J = \mathcal{D}\cup\mathcal{E}$. Hence $e\in J$.

$$T(e) = \begin{cases} \mathcal{J}^c(e)=[\rho^s_{\mathcal{J}^c(e)}(y).e^{i\omega^s_{\mathcal{J}^c(e)}(y)}]_{s\in k}, & \text{if } e\in\mathcal{D}-\mathcal{E},\\ \varpi^c(e)=[\rho^s_{\varpi^c(e)}(y).e^{i\omega^s_{\varpi^c(e)}(y)}]_{s\in k}, & \text{if } e\in\mathcal{E}-\mathcal{D},\\ \mathcal{J}^c(e)\cap\varpi^c(e)=[(\rho^s_{\mathcal{J}^c(e)}(y)\vee\rho^s_{\varpi^c(e)}(y)) \\ .e^{i[\omega^s_{\mathcal{J}^c(e)}(y)\cup\omega^s_{\varpi^c(e)}(y)]}]_{s\in k}, & \text{if } e\in\mathcal{E}\cap\mathcal{D}. \end{cases} \quad (6)$$

Thus, $(\oplus^c,\mathcal{R})$ and (T,J) are the similar operators, $\forall e\in\mathcal{R}(J)$, $((\mathcal{J},\mathcal{D})\cup(\varpi,\mathcal{E}))^c = (\mathcal{J},\mathcal{D})^c\cap(\varpi,\mathcal{E})^c$ and this completes the proof.

2) The proof is same as that of part (1) and therefore is leave out.

V. ENTROPY (ENT) ON CMFHS-SETS

ENT is one of the principal characteristics of f-sets as its addresses the primary inquiry when managing f-sets. How fuzzy is an FS? ENT is a tool that is used to measure the fuzziness of FS. In this part, we present the idea of ENT of CMFHSS. Some related theorems and an application for a person selection decision issue who wishes to buy a car are built to utilize the recently created ENT-based CMFHSS to demonstrate its validity and importance.

Definition 21: A function $E : CM^kFHSS(Y)\to[0,1]$ is said to be ENT on CM^kFHSS, if E fulfils the given conditions.

1) $E(\mathcal{J},\mathcal{D})=0 \Leftrightarrow \rho^s_F(e)(y)=1$ and $\omega^s_{F(e)}(e)(y)=2\pi$, $\forall\, e\in\mathcal{D}, y\in Y, s=1,2,\ldots,k$.
2) $E(\mathcal{J},\mathcal{D})=1 \Leftrightarrow \rho^s_{\mathcal{J}(e)}(y)=0.5$ and $\omega^j_{\mathcal{J}}(e)(y)=\pi$, $\forall\, e\in\mathcal{D}, y\in Y, s=1,2,\ldots,k$.
3) $E(\mathcal{J},\mathcal{D})=E(\mathcal{J},\mathcal{D})^c$.
4) if $(\mathcal{J},\mathcal{D})\subseteq(\varpi,\mathcal{D})$, i.e, $\rho^s_{\mathcal{J}(e)}(y)\le\rho^s_{\varpi(e)}(y)$ and $\omega^s_{\mathcal{J}(e)}(y)\le\omega^s_{\varpi(e)}(y)$, $e\in E, y\in Y, s=1,2,\ldots,k$, then $E(\mathcal{J},\mathcal{D})\ge E(\varpi,\mathcal{D})$.

Theorem 4: Let $Y=\{y_1,y_2,\ldots,y_p\}$ be the nonempty universal set of elements and $\mathcal{D}$ be the universal set of parameters. Hence $(\mathcal{J},\mathcal{D})=\{\mathcal{D}(e)=r^s_{\mathcal{J}(e)}(y).e^{i\omega^s_{\mathcal{J}(e)}}(y)|l=1,2,3,\ldots,m\}$, where $e\in\mathcal{D}$, is a family of CM^kFHSS. Define $E(\mathcal{J},\mathcal{D})$ as follows: $E(\mathcal{J},\mathcal{D})=\frac{1}{2m}\Sigma^m_{l=1}[E^r_l(\mathcal{J},\mathcal{D})+E^\omega_l\frac{(\mathcal{J},\mathcal{D})}{2\pi}]$,

where,

$$E^r_l(\mathcal{J},\mathcal{D})=\frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}[1-|\rho^s_{\mathcal{J}(e_l)}(y_p)-\rho^s_{\mathcal{J}^c(e_l)}(y_p)|],$$

and

$$E^\omega_l(\mathcal{J},\mathcal{D})=\frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}[1-|\omega^s_{\mathcal{J}(e_l)}(y_p)-\omega^s_{\mathcal{J}^c(e_l)}(y_p)|],$$

then $E(\mathcal{J},\mathcal{D})$ is an ENT of CM^kFHSS.

Proof: We prove that the $E(\mathcal{J},\mathcal{D})$ fulfils the all requirements given in Definition 22.

1) $E(\mathcal{J},\mathcal{D})=0, \Leftrightarrow \frac{1}{2m}\Sigma^m_{l=1}[E^r_l(\mathcal{J},\mathcal{D})+E^\omega_l\frac{(\mathcal{J},\mathcal{D})}{2\pi}]=0$, $\Leftrightarrow E^r_l(\mathcal{J},\mathcal{D})=0$ and $E^\omega_l(\mathcal{J},\mathcal{D})=0$
$$\Leftrightarrow \forall e_l\in\mathcal{D},\quad y_p\in Y,\ s=1,2,3\ldots,n$$
$\Sigma^n_{p=1}\Sigma^k_{s=1}[1-|\rho^s_{\mathcal{J}(e_l)}(y_p)-\rho^s_{\mathcal{J}^c(e_l)}(y_p)|]=0$, and $\Sigma^n_{p=1}\Sigma^k_{s=1}[1-|\omega^s_{\mathcal{J}(e_l)}(y_p)-\omega^s_{\mathcal{J}^c(e_l)}(y_p)|]=0$,
$$\Leftrightarrow \forall e_l\in\mathcal{D},\quad y_p\in Y,\ s=1,2,3\ldots,n,$$
$|\rho^s_{\mathcal{J}(e_l)}(y_p)-\rho^s_{\mathcal{J}^c(e_l)}(y_p)=1$, $|\omega^s_{\mathcal{J}(e_l)}(y_p)-\omega^s_{\mathcal{J}^c(e_l)}(y_p)|=2\pi$, $\Leftrightarrow\forall\ e_l\in\mathcal{D}, y_p\in Y, s=1,2,3\ldots,n$, $\rho^s_{\mathcal{J}(e_l)}(y_p)=1$, $\omega^s_{\mathcal{J}(e_l)}(y_p)=2\pi$,
2) For $(\mathcal{J},\mathcal{D})\in CM^kFSS(Y)$, we have $E(\mathcal{J},\mathcal{D})=1$, $\Sigma^m_{l=1}[E^r_l(\mathcal{J},\mathcal{D})+E^\omega_l\frac{(\mathcal{J},\mathcal{D})}{2\pi}]=2m$, $\Leftrightarrow E^r_l(\mathcal{J},\mathcal{D})=1$, and $E^\omega_l(\mathcal{J},\mathcal{D})]=2\pi$, $\Leftrightarrow\forall e_l\in\mathcal{D}, y_p\in Y, s=1,2,3\ldots,n$, $\frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}[1-|\rho^s_{\mathcal{J}(e_l)}(y_p)-\rho^s_{\mathcal{J}^c(e_l)}(y_p)|]=1$, and $\frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}[1-|\omega^s_{\mathcal{J}(e_l)}(y_p)-\omega^s_{\mathcal{J}^c(e_l)}(y_p)|]=2\pi$, $\Leftrightarrow\forall e_l\in\mathcal{D}, y_p\in Y, s=1,2,3\ldots,n$, $\Sigma^n_{p=1}\Sigma^k_{s=1}[1-|\rho^s_{\mathcal{J}(e_l)}(y_p)-\rho^s_{\mathcal{J}^c(e_l)}(y_p)|]=nk$, and $\Sigma^n_{p=1}\Sigma^k_{s=1}[2\pi-|\omega^s_{\mathcal{J}(e_l)}(y_p)-\omega^s_{\mathcal{J}^c(e_l)}(y_p)|]=2\pi(nk)$, $\Leftrightarrow\forall\ e_l\in\mathcal{D}, y_p\in Y, s=1,2,3\ldots,n$, $[1-|\rho^s_{\mathcal{J}(e_l)}(y_p)-\rho^s_{\mathcal{J}^c(e_l)}(y_p)|]=1$, and $[2\pi-|\omega^s_{\mathcal{J}(e_l)}(y_p)-\omega^s_{\mathcal{J}^c(e_l)}(y_p)|]=2\pi$, $\Leftrightarrow\forall e_l\in\mathcal{D}, y_p\in Y, s=1,2,3\ldots,n$, $|\rho^s_{\mathcal{J}^c(e_l)}(y_p)-\rho^s_{\mathcal{J}(e_l)}(y_p)|=0$, and $|\omega^s_{\mathcal{J}^c(e_l)}(y_p)-\omega^s_{\mathcal{J}(e_l)}(y_p)|=0$, $\Leftrightarrow\forall e_l\in\mathcal{D}, y_p\in Y, s=1,2,3\ldots,n$, $\rho^s_{\mathcal{J}(e_l)}(y_p)=\frac{1}{2}$ and $\omega^s_{\mathcal{J}(e_l)}(y_p)=\pi$,
3) For $E(\mathcal{J},\mathcal{D})\in CM^kFSS(Y)$, we have, $E^r_l(\mathcal{J},\mathcal{D})=\frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}[1-|\rho^s_{\mathcal{J}(e_l)}(y_p)-\rho^s_{\mathcal{J}^c(e_l)}(y_p)|]$, $\frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}[1-|\rho^s_{\mathcal{J}^c(e_l)}(y_p)-\rho^s_{\mathcal{J}(e_l)}(y_p)|], = E^r_l(\mathcal{J},\mathcal{D})^c$, Similarly, we can prove $E^r_l(\mathcal{J},\mathcal{D})=E^r_l(\mathcal{J},\mathcal{D})^c$ it is obvious that $E(\mathcal{J},\mathcal{D})=E(\mathcal{J},\mathcal{D})^c$.
4) Suppose $(\mathcal{J},\mathcal{D})$ and $(\varpi,\mathcal{D})\in CM^kFSS(Y)$. If $(\mathcal{J},\mathcal{D})\subseteq(\varpi,\mathcal{D})$,
$$\Rightarrow\forall e_l\in\mathcal{D},\quad y\in Y,\ s=1,2,3\ldots,k,$$
$\rho^s_{\mathcal{J}(e_l)}(y_p)\le\rho^s_{\varpi(e_l)}(y_p)$ and $\omega^s_{\mathcal{J}(e_l)}(y_p)\le\omega^s_{\varpi(e_l)}(y_p)$ $\Rightarrow\forall e_l\in\mathcal{D}, y\in Y, s=1,2,3\ldots,k$,
$$|\rho^s_{\mathcal{J}(e_l)}(y_p)-\rho^s_{\mathcal{J}^c(e_l)}(y_p)|\le|\rho^s_{\varpi(e_l)}(y_p)-\rho^s_{\varpi^c(e_l)}(y_p)|,$$

and $|\omega^s_{\mathcal{J}(e_l)}(y_p) - \omega^s_{\mathcal{J}^c(e_l)}(y_p)| \leq |\omega^s_{\varpi(e_l)}(y_p) - \omega^s_{\varpi^c(e_l)}(y_p)|$,

$$\Rightarrow \forall e_l \in \mathcal{D}, \quad y \in Y, \ s = 1, 2, 3 \ldots, k,$$

$1 - |\rho^s_{\mathcal{J}(e_l)}(y_p) - \rho^s_{\mathcal{J}^c(e_l)}(y_p)| \geq 1 - |\rho^s_{\varpi(e_l)}(y_p) - \rho^s_{\varpi^c(e_l)}(y_p)|$,
and

$$2\pi - |\omega^s_{\mathcal{J}(e_l)}(y_p) - \omega^s_{\mathcal{J}^c(e_l)}(y_p)| \geq 2\pi - |\omega^s_{\varpi(e_l)}(y_p) - \omega^s_{\varpi^c(e_l)}(y_p)|,$$

$$\Rightarrow \frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}([1 - |r^s_{\mathcal{J}(e_l)}(y_p) - \rho^s_{\mathcal{J}^c(e_l)}(y_p)|]), \geq \frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}([1 - |\rho^s_{\varpi(e_l)}(y_p) - \rho^s_{\varpi^c(e_l)}(y_p)|]),$$

and

$$\frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}([2\pi - |\omega^s_{\mathcal{J}(e_l)}(y_p) - \omega^s_{\mathcal{J}^c(e_l)}(y_p)|]) \geq \frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}([2\pi - |\omega^s_{\varpi(e_l)}(y_p) - \omega^s_{\varpi^c(e_l)}(y_p)|]),$$

$$\Rightarrow E^r_l(\mathcal{J}, \mathcal{D}) \geq E^r_l(\varpi, \mathcal{D}),$$

and

$$\Rightarrow E^\omega_l(\mathcal{J}, \mathcal{D}) \geq E^\omega_l(\varpi, \mathcal{D}),$$
$$\Rightarrow E^r_l(\mathcal{J}, \mathcal{D}) + E^\omega_l(\mathcal{J}, \mathcal{D}) \geq E^r_l(\varpi, \mathcal{D}) + E^\omega_l(\varpi, \mathcal{D}),$$
$$\Rightarrow \frac{1}{2m}\Sigma^m_{l=1}[E^r_l(\mathcal{J}, \mathcal{D}) + E^\omega_l\frac{(\mathcal{J}, \mathcal{D})}{2\pi}] \geq \frac{1}{2m}\Sigma^m_{l=1}[E^r_l(\varpi, \mathcal{D}) + E^\omega_l\frac{(\varpi, \mathcal{D})}{2\pi}],$$
$$\Rightarrow E(\mathcal{J}, \mathcal{D}) \geq E(\varpi, \mathcal{D}).$$

A. THE PROPOSED ENT-BASED CMFHSS WITH APPLICATION

In this section, we utilize the idea of CMFHSS to build a novel algorithm and strategy, called ENT-based CMFHSS, in which we expand ENT depending on CMFHSS under a fuzzy environment. Also, a person selection decision issue who wish to buy a car is built to utilize the recently created ENT-based CMFHSS to demonstrate its validity and importance.

1) THE PROPOSED ENT BASED CMFHSS WITH APPLICATION

Let Y be a non-empty universal set, and suppose $Y \subset A$ be the set of alternatives under discussion, presented by $Y = \{x_1, x_2, ..., x_m\}$. Let $\mathcal{D} = A_1 \times A_2 \times ... \times A_n$, where $n \geq 1$ and A_i is the set of all attribute values of the attribute a_i, $i = 1, 2, 3, ..., n$. The construction steps for the proposed CMFHSS-based ENT are as per the following or see fig 5

1) Input each of the CMFHSS.
2) Calculate ENT for each CMFHSS using the formula $E(\mathcal{J}, \mathcal{D}) = \frac{1}{2m}\Sigma^m_{l=1}[E^r_l(\mathcal{J}, \mathcal{D}) + E^\omega_l\frac{(\mathcal{J}, \mathcal{D})}{2\pi}]$, where $E^r_l(\mathcal{J}, \mathcal{D}) = \frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}[1 - |\rho^s_{\mathcal{J}(e_l)}(y_p) - \rho^s_{\mathcal{J}^c(e_l)}(y_p)|]$, and $E^\omega_l(\mathcal{J}, \mathcal{D}) = \frac{1}{nk}\Sigma^n_{p=1}\Sigma^k_{s=1}[1 - |\omega^s_{\mathcal{J}(e_l)}(y_p) - \omega^s_{\mathcal{J}^c(e_l)}(y_p)|]$.
3) Find such CMFHSS which has minimum ENT and chose it for best optimal.
4) Choose any one, if it received more than one optimal.

Example 3: Assume that a car company CEO who has three clients needs to know which client would perhaps to purchase a car from him. Assume that he has clients Harry, Jonas, and William looking for a car. Let $X = \{a = \text{Acura}, b = \text{Bentley}, c = \text{BMW}\}$ be collection of cars, let a_1 = Feature, a_2 = Cost, a_3 = Colour, be distinct attributes whose corresponding attribute values belong to the sets F_1, F_2, F_3. Let $F_1 = \{f_1 = \text{Safety feature}, f_2 = \text{Convenience feature}\}$, $F_2 = \{f_3 = \text{High}\}$, $F_3 = \{f_4 = \text{Pearl}, f_5 = \text{metallic}\}$. The attractiveness of the set the car to Harry, Jonas, and William encoded into CMFHSS $(\mathcal{J}, \mathcal{D})$, $(\varpi, \mathcal{D})$ and $(\oplus, \mathcal{D})$ respectively. Construct the CM^kFHSS $(\mathcal{J}, \mathcal{D})$, $(\varpi, \mathcal{D})$ and $(\oplus, \mathcal{D})$ presenting attractiveness of the car of Harry, Jonas, and William respectively.

1) This can be done with the support of customers.

$$(\mathcal{J}, \mathcal{D}) = \left\{\mathcal{J}(f_1, f_3, f_4) = \left\{\frac{(0.3e^{i0.4\pi}, 0.4e^{i0.3\pi}, 0.7e^{i0.1\pi})}{a}, \frac{(0.8e^{i0.2\pi}, 0.4e^{i0.8\pi}, 0.1e^{i0.6\pi})}{b}, \frac{(0.8e^{i0.2\pi}, 0.7e^{i0.2\pi}, 0.8e^{i0.5\pi})}{c}\right\},\right.$$
$$\left\{\mathcal{J}(f_1, f_3, f_5) = \left\{\frac{(0.3e^{i0.3\pi}, 0.3e^{i0.4\pi}, 0.6e^{i0.4\pi})}{a}, \frac{(0.8e^{i0.2\pi}, 0.6e^{i0.1\pi}, 0.7e^{i0.9\pi})}{b}, \frac{(0.2e^{i0.9\pi}, 0.2e^{i0.4\pi}, 0.8e^{i0.3\pi})}{c}\right\},\right.$$
$$\left\{\mathcal{J}(f_2, f_3, f_4) = \left\{\frac{(0.3e^{i0.9\pi}, 0.2e^{i0.7\pi}, 0.3e^{i0.8\pi})}{a}, \frac{(0.2e^{i0.4\pi}, 0.2e^{i0.6\pi}, 0.2e^{i0.3\pi})}{b}, \frac{(0.9e^{i0.3\pi}, 0.3e^{i0.6\pi}, 0.3e^{i0.2\pi})}{c}\right\}\right.$$
$$\left.\left\{\mathcal{J}(f_2, f_3, f_5) = \left\{\frac{(0.3e^{i0.5\pi}, 0.7e^{i0.3\pi}, 0.3e^{i0.8\pi})}{a}, \frac{(0.3e^{i0.4\pi}, 0.9e^{i0.3\pi}, 0.3e^{i0.9\pi})}{b}, \frac{(0.4e^{i0.7\pi}, 0.2e^{i0.1\pi}, 0.7e^{i0.4\pi})}{c}\right\}\right\},$$

$$(\varpi, \mathcal{D}) = \left\{\mathcal{J}(f_1, f_3, f_4) = \left\{\frac{(0.2e^{i0.7\pi}, 0.2e^{i0.8\pi}, 0.9e^{i0.2\pi})}{a},\right.\right.$$

$$\frac{(0.7e^{i0.5\pi},0.2e^{i0.6\pi},0.3e^{i0.8\pi})}{b},\frac{(0.4e^{i0.3\pi},0.8e^{i0.9\pi},0.5e^{i0.3\pi})}{c}\Bigg\},$$

$$\Bigg\{\mathcal{J}(f_1,f_3,f_5)=\Bigg\{\frac{(0.6e^{i0.9\pi},0.2e^{i0.8\pi},0.2e^{i0.9\pi})}{a},\frac{(0.2e^{i0.8\pi},0.6e^{i0.1\pi},0.7e^{i0.3\pi})}{b},\frac{(0.3e^{i0.6\pi},0.2e^{i0.7\pi},0.7e^{i0.3\pi})}{c}\Bigg\},$$

$$\Bigg\{\mathcal{J}(f_2,f_3,f_4)=\Bigg\{\frac{(0.3e^{i0.9\pi},0.8e^{i0.9\pi},0.5e^{i0.9\pi})}{a},\frac{(0.8e^{i0.5\pi},0.6e^{i0.7\pi},0.9e^{i0.2\pi})}{b},\frac{(0.6e^{i0.9\pi},0.2e^{i0.9\pi},0.2e^{i0.6\pi})}{c}\Bigg\}$$

$$\Bigg\{\mathcal{J}(f_2,f_3,f_5)=\Bigg\{\frac{(0.6e^{i0.5\pi},0.7e^{i0.4\pi},0.8e^{i0.8\pi})}{a},\frac{(0.3e^{i0.4\pi},0.9e^{i0.3\pi},0.3e^{i0.2\pi})}{b},\frac{(0.4e^{i0.7\pi},0.1e^{i0.3\pi},0.9e^{i0.5\pi})}{c}\Bigg\}\Bigg\},$$

$(\oplus,\mathcal{D})$

$$=\Bigg\{\mathcal{J}(f_1,f_3,f_4)=\Bigg\{\frac{(0.4e^{i0.8\pi},0.6e^{i0.1\pi},0.8e^{i0.8\pi})}{a},\frac{(0.2e^{i0.9\pi},0.8e^{i0.4\pi},0.2e^{i0.6\pi})}{b},\frac{(0.3e^{i0.2\pi},0.8e^{i0.2\pi},0.8e^{i0.1\pi})}{c}\Bigg\},$$

$$\Bigg\{\mathcal{J}(f_1,f_3,f_5)=\Bigg\{\frac{(0.7e^{i0.2\pi},0.9e^{i0.1\pi},0.3e^{i0.6\pi})}{a},\frac{(0.5e^{i0.2\pi},0.2e^{i0.8\pi},0.4e^{i0.6\pi})}{b},\frac{(0.2e^{i0.6\pi},0.1e^{i0.5\pi},0.8e^{i0.4\pi})}{c}\Bigg\},$$

$$\Bigg\{\mathcal{J}(f_2,f_3,f_4)=\Bigg\{\frac{(0.5e^{i0.9\pi},0.2e^{i0.6\pi},0.4e^{i0.7\pi})}{a},\frac{(0.8e^{i0.6\pi},0.6e^{i0.7\pi},0.9e^{i0.2\pi})}{b},\frac{(0.2e^{i0.9\pi},0.3e^{i0.8\pi},0.6e^{i0.5\pi})}{c}\Bigg\}$$

$$\Bigg\{\mathcal{J}(f_2,f_3,f_5)=\Bigg\{\frac{(0.2e^{i0.5\pi},0.2e^{i0.5\pi},0.1e^{i0.4\pi})}{a},\frac{(0.5e^{i0.4\pi},0.3e^{i0.4\pi},0.5e^{i0.2\pi})}{b},\frac{(0.2e^{i0.5\pi},0.9e^{i0.2\pi},0.4e^{i0.5\pi})}{c}\Bigg\}\Bigg\}.$$

2) Calculate the Entropies of $(\mathcal{J},\mathcal{D})$, $(\varpi,\mathcal{D})$ and $(\oplus,\mathcal{D})$ using the formula mention in algorithm, see Table 1. Hence the Entropies of the CM^kFSSs $(\mathcal{J},\mathcal{D})$, $(\varpi,\mathcal{D})$ and $(\oplus,\mathcal{D})$ are as given below $E(\mathcal{J},\mathcal{D})=0.569$, $E(\varpi,\mathcal{D})=0.553$, $E(\oplus,\mathcal{D})=0.51$ respectively.
3) Optimal solution is to choose $(\oplus,\mathcal{D})$ as it hs minimum value of ENT.
4) William has higher chance to purchase car.

All alternatives are ranked by ENT based CMFHSS depicted in the following clustered cone 1.

B. COMPARATIVE STUDIES

A few comparisons of the initiated techniques with shortcomings are discussed to analyze the proposed technique's validity and predominance. Additionally, we will compare

TABLE 1. Entropies.

$E_1^r(\mathcal{J},\mathcal{D})$	0.53
$E_2^r(\mathcal{J},\mathcal{D})$	0.5
$E_3^r(\mathcal{J},\mathcal{D})$	0.467
$E_4^r(\mathcal{J},\mathcal{D})$	0.65
$E_1^\omega(\mathcal{J},\mathcal{D})$	1.2711
$E_2^\omega(\mathcal{J},\mathcal{D})$	2.724
$E_3^\omega(\mathcal{J},\mathcal{D})$	3.351
$E_4^\omega(\mathcal{J},\mathcal{D})$	0.533
$E_1^r(\varpi,\mathcal{D})$	3.0722
$E_2^r(\varpi,\mathcal{D})$	0.511
$E_3^r(\varpi,\mathcal{D})$	0.6
$E_4^r(\varpi,\mathcal{D})$	0.449
$E_1^\omega(\varpi,\mathcal{D})$	2.7121
$E_2^\omega(\varpi,\mathcal{D})$	3.77
$E_3^\omega(\varpi,\mathcal{D})$	4.118
$E_4^\omega(\varpi,\mathcal{D})$	2.621
$E_1^r(\bigoplus,\mathcal{D})$	0.51
$E_2^r(\bigoplus,\mathcal{D})$	0.511
$E_3^r(\bigoplus,\mathcal{D})$	0.6
$E_4^r(\bigoplus,\mathcal{D})$	0.556
$E_1^\omega(\bigoplus,\mathcal{D})$	2.86
$E_2^\omega(\bigoplus,\mathcal{D})$	2.79
$E_3^\omega(\bigoplus,\mathcal{D})$	4.12
$E_4^\omega(\bigoplus,\mathcal{D})$	2.51

TABLE 2. Comparison of the proposed ENT based CMFHSS with existing entropies.

SN	References	Entropies	Ranking
1	[42]	Not valid	×
2	[21]	Not valid	×
3	[43]	Not valid	×
4	[44]	Not valid	×
5	[45]	Not valid	×
6	[46]	Not valid	×
7	[47]	Not valid	×
8	[48]	Not valid	×
9	[49]	Not valid	×
10	Proposed Method in this paper	$E(\mathcal{J},\mathcal{D}) = 0.569$, $E(\varpi,\mathcal{D}) = 0.553$, $E(\bigoplus,\mathcal{D}) = 0.51$	$E(\bigoplus,\mathcal{D}) \geq E(\varpi,\mathcal{D}) \geq E(\mathcal{J},\mathcal{D})$

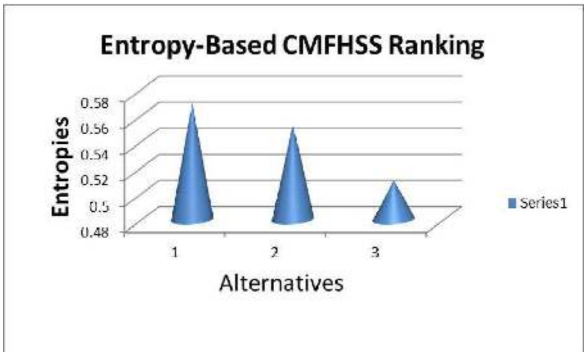

FIGURE 1. Ranking of alternative by ENT based CMFHSS.

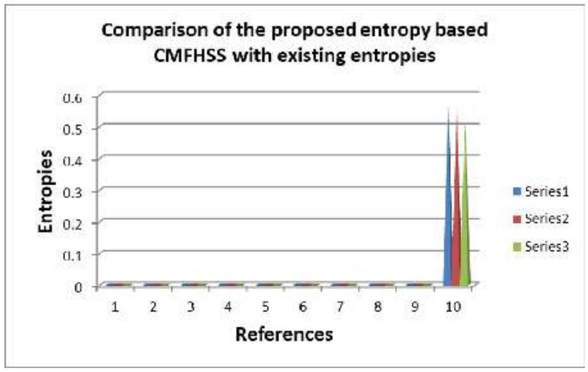

FIGURE 2. Comparison of the proposed ENT based CMFHSS with existing entropies.

our proposed ENT based CMFHSS with nine other existing entropies, including the idea presented by Szmidt *et al.* [42] based on non-probabilistic-type ENT measure for intuitionistic f-sets, the notion initiated by Zhang *et al.* [21] an axiomatic definition of ENT for IVFS and relationship between ENT and similarity measure of IVFSS set, Majumdar *et al.* [43] based on ENT of a single-valued neutrosophic set, and the idea founded by Ye *et al.* [44] based on the ENT measures of interval-valued NSS, and the idea founded by Aydodu *et al.* [45] based on ENT and similarity measure of interval-valued neutrosophic sets, and the idea established by Athira *et al.* [46] based on ENT and distance measures of Pythagorean fs-sets and their applications, and the idea presented by Lvqing *et al.* [47] based on two classes of ENT measures for complex f-sets, and the idea built by Kumar *et al.* [48] based on complex intuitionistic fs-sets with distance Measures and Entropies, and the idea endowed by Selvachandran *et al.* [49] based on vague ENT measure for complex, vague soft sets. However, when the attributes are further sub-divided into attribute values and the issues that include complex (two-dimensional) information/date (the degree of the influence and the total time of the influence) then all current disadvantages are failed to manage. This need is fulfilled in the proposed ENT-based CMFHSS.

For more detailed see table 2, fig 2.

VI. SIMILARITY MEASURE BETWEEN CMFHS-SETS

SM evaluates the degree in which various patterns, images, or sets are alike. Such kind of measures are utilized broadly in the use of fs-sets. We present a definition of a SM for *CMFHSS* as following.

Definition 22: A function $S : CM^kFHSS(Y) \times CM^kFHSS(Y) \rightarrow [0, 1]$ is said to be SM between two CM^kFHSS $(\mathcal{J},\mathcal{D})$ and $(\varpi,\mathcal{D})$, if S satisfies the following axiomatic requirements

1) $S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = S((\varpi,\mathcal{D}),(\mathcal{J},\mathcal{D}))$,
2) $S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 1 \Leftrightarrow (\mathcal{J},\mathcal{D}) = (\varpi,\mathcal{D})$,
3) $S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 0 \Leftrightarrow \forall\, e \in \mathcal{D}, x \in Y, s = 1, 2, 3 \ldots, K$, the following restrictions are fulfilled $\rho^s_{\mathcal{J}(e)} = 1, \rho^s_{\varpi(e)} = 0$ or $\rho^s_{\mathcal{J}(e)} = 0, \rho^s_{\varpi(e)} = 1$ and $\omega^s_{\mathcal{J}(e)} = 2\pi, \omega^s_{\varpi(e)} = 0$ or $\omega^s_{\mathcal{J}(e)} = 0, \omega^s_{\varpi(e)} = 2\pi$,
4) $\forall\ (\mathcal{J},\mathcal{D}), (\varpi,\mathcal{D})$ and $(\oplus,\mathcal{D}) \in CM^kFHSS$, if $(\mathcal{J},\mathcal{D}) \subseteq (\varpi,\mathcal{D}) \subseteq (\oplus,\mathcal{D})$, then $S((\mathcal{J},\mathcal{D}), (\oplus,\mathcal{D})) \leq S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$ and $S((\mathcal{J},\mathcal{D}),(\oplus,\mathcal{D})) \leq S((\varpi,\mathcal{D}), (\oplus,\mathcal{D}))$. Now, we develop the formula to find the SM between two CM^kFHSS as follows.

Theorem 5: Let $Y = \{y_1, y_2, \ldots, y_p\}$ be the universal set of elements and $\mathcal{D}$ be the universal set of parameters.

$(\mathcal{J},\mathcal{D}) = \{\mathcal{D}(e) = \rho^s_{\mathcal{J}(e)}(y).e^{i\omega^s_{\mathcal{J}(e)}}(y)|l = 1,2,3,\ldots,m\}$, and $(\varpi,\mathcal{D}) = \{\mathcal{D}(e) = \rho^s_{\varpi(e)}(y).e^{i\omega^s_{\varpi(e)}}(y)|l = 1,2,3,\ldots,m\}$, are two families of CM^kFHSS.

Define $S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$ as follows, $S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = \frac{1}{2m}\Sigma^m_{l=1}[S^r_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) + \frac{S^\omega_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))}{2\pi}]$,
where,

$$S^r_{l=1}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 1 - \frac{1}{n}\Sigma^n_{l=1}max\{(|\rho^s_{\mathcal{J}(e)}(y_p) - r^s_{\varpi(e)}(y_p)|)_{s\in k}\},$$

and

$$S^\omega_{l=1}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 2\pi - \frac{1}{n}\Sigma^n_{l=1}max\{(|\omega^s_{\mathcal{J}(e)}(y_p) - \omega^s_{\varpi(e)}(y_p)|)_{s\in k}\},$$

then $S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$ is a SM between two CM^kFHSS $(\mathcal{J},\mathcal{D})$ and $(\varpi,\mathcal{D})$.

Proof: It is sufficient to prove that $S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$ fulfill the properties listed in Definition 23.

1) For $S^r_{l=1}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 1 - \frac{1}{n}\Sigma^n_{p=1}max \{(|\rho^s_{\mathcal{J}(e_l)}(y_p) - \rho^s_{\varpi(e_l)}(y_p)|)_{s\in k}\}$,

$$= 1 - \frac{1}{n}\Sigma^n_{p=1}max\{(|\rho^s_{\varpi(e_l)}(y_p) - \rho^s_{\mathcal{J}(e_l)}(y_p)|)_{s\in k}\}$$
$$= S^r_{l=1}((\varpi,\mathcal{D}),(\mathcal{J},\mathcal{D})),$$

and

$$S^\omega_{l=1}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$$
$$= 2\pi - \frac{1}{n}\Sigma^n_{p=1}max\{(|\omega^s_{\mathcal{J}(e_l)}(y_p) - \omega^s_{\varpi(e_l)}(y_p)|)_{s\in k}\},$$
$$= 2\pi - \frac{1}{n}\Sigma^n_{p=1}max\{(|\omega^s_{\varpi(e_l)}(y_p) - \omega^s_{\mathcal{J}(e_l)}(y_p)|)_{s\in k}\}$$
$$= S^\omega_{l=1}((\varpi,\mathcal{D}),(\mathcal{J},\mathcal{D})),$$

So we have

$$S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = \frac{1}{2m}\Sigma^m_{l=1}[S^r_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) + \frac{S^\omega_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))}{2\pi}],$$
$$= \frac{1}{2m}\Sigma^m_{l=1}[S^r_l((\varpi,\mathcal{D}),(\mathcal{J},\mathcal{D})) + \frac{S^\omega_l((\varpi,\mathcal{D}),(\mathcal{J},\mathcal{D}))}{2\pi}]$$
$$= S((\varpi,\mathcal{D}),(\mathcal{J},\mathcal{D})).$$

2) $S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 1$

$$\Leftrightarrow \frac{1}{2m}\Sigma^m_{l=1}[S^r_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) + \frac{S^\omega_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))}{2\pi}] = 1,$$
$$\Leftrightarrow S^r_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 1,$$
$$\Leftrightarrow S^\omega_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 2\pi,$$

$\Leftrightarrow S^r_{l=1}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 1 - \frac{1}{n}\Sigma^n_{p=1}max\{(|\rho^s_{\mathcal{J}(e_l)}(y_p) - \rho^s_{\varpi(e_l)}(y_p)|)_{s\in k}\}, 2\pi - \frac{1}{n}\Sigma^n_{p=1}max\{(|\omega^s_{\mathcal{J}(e_l)}(y_p) - \omega^s_{\varpi(e_l)}(y_p)|)_{s\in k}\} = 2\pi, \forall e_l \in \mathcal{D}, y \in Y, s = 1,2,\ldots k,$

$\Leftrightarrow \frac{1}{n}\Sigma^n_{p=1}max\{(|\rho^s_{\mathcal{J}(e_l)}(y_p) - \rho^s_{\varpi(e_l)}(y_p)|)_{s\in k} = 0,$

and

$$\Leftrightarrow \frac{1}{n}\Sigma^n_{p=1}max\{(|\omega^s_{\mathcal{J}(e_l)}(y_p) - \omega^s_{\varpi(e_l)}(y_p)|)_{s\in k} = 0,$$
$$\forall e_l \in \mathcal{D}, \quad y \in Y, \ s = 1,2,\ldots k,$$
$$\Leftrightarrow \Sigma^n_{p=1}max\{(|\rho^s_{\mathcal{J}(e_l)}(y_p) - \rho^s_{\varpi(e_l)}(y_p)|)_{s\in k} = 0,$$
$$\Leftrightarrow \Sigma^n_{p=1}max\{(|\omega^s_{\mathcal{J}(e_l)}(y_p) - \omega^s_{\varpi(e_l)}(y_p)|)_{s\in k} = 0,$$
$$\forall e_l \in \mathcal{D}, \quad y \in Y, \ s = 1,2,\ldots k,$$
$$\Leftrightarrow \rho^s_{\mathcal{J}(e_l)}(y_p) = \rho^s_{\varpi(e_l)}(y_p),$$
$$\Leftrightarrow \omega^s_{\mathcal{J}(e_l)}(y_p) = \omega^s_{\varpi(e_l)}(y_p),$$
$$\forall e_l \in \mathcal{D}, \quad y \in Y, \ s = 1,2,\ldots k,$$
$$\Leftrightarrow (\mathcal{J},\mathcal{D}) = (\varpi,\mathcal{D}).$$

3) $S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 0,$

$$\frac{1}{2m}\Sigma^m_{l=1}[S^r_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) + \frac{S^\omega_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))}{2\pi}] = 0,$$
$$\Leftrightarrow S^r_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 0,$$

and

$$\Leftrightarrow S^\omega_l((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 0,$$
$$\Leftrightarrow 1-\frac{1}{n}\Sigma^n_{p=1}max\{(|\rho^s_{\mathcal{J}(e_l)}(y_p)-\rho^s_{\varpi(e_l)}(y_p)|)_{s\in k}\}=0,$$

and

$$\Leftrightarrow 2\pi - \frac{1}{n}\Sigma^n_{p=1}max\{(|\omega^s_{\mathcal{J}(e_l)}(y_p) - \omega^s_{\varpi(e_l)}(y_p)|)_{s\in k}\}$$
$$= 0,$$
$$\forall \ e_l \in \mathcal{D}, \quad y \in Y, \ s = 1,2,\ldots k,$$
$$\Leftrightarrow \frac{1}{n}\Sigma^n_{p=1}max\{(|\rho^s_{\mathcal{J}(e_l)}(y_p) - \rho^s_{\varpi(e_l)}(y_p)|)_{s\in k}\} = 1,$$
$$\Leftrightarrow \frac{1}{n}\Sigma^n_{p=1}max\{(|\omega^s_{\mathcal{J}(e_l)}(y_p) - \omega^s_{\varpi(e_l)}(y_p)|)_{s\in k}\} = 2\pi,$$
$$\forall e_l \in \mathcal{D}, \quad y \in Y, \ s = 1,2,\ldots k,$$
$$\Leftrightarrow max\{(|\rho^s_{\mathcal{J}(e_l)}(y_p) - \rho^s_{\varpi(e_l)}(y_p)|)_{s\in k}\} = 1,$$

and

$$\Leftrightarrow max\{(|\omega^s_{\mathcal{J}(e_l)}(y_p) - \omega^s_{\varpi(e_l)}(y_p)|)_{s\in k}\} = 2\pi,$$
$$\forall e_l \in \mathcal{D}, \quad y \in Y, \ s = 1,2,\ldots k,$$

$\Leftrightarrow \rho^s_{\mathcal{J}(e_l)} = 0, \rho^s_{\varpi(e_l)} = 1, \rho^s_{\mathcal{J}(e_l)} = 1, \rho^s_{\varpi(e_l)} = 0$ and $\omega^s_{\mathcal{J}(e_l)} = 0, \omega^s_{\varpi(e_l)} = 2\pi$ or $\omega^s_{\mathcal{J}(e_l)} = 2\pi, \omega^s_{\varpi(e_l)} = 0.$

4) $(\mathcal{J},\mathcal{D}) \subseteq (\varpi,\mathcal{D}) \subseteq (\oplus,\mathcal{D}),$

$$\Rightarrow \rho^s_{\mathcal{J}(e_l)}(y_p) \le \rho^s_{\varpi(e_l)}(y_p) \le \rho^s_{\oplus(e_l)}(y_p)$$

and

$$\omega^s_{\mathcal{J}(e_l)}(y_p) \le \omega^s_{\varpi(e_l)}(y_p) \le \omega^s_{\oplus(e_l)}(y_p),$$
$$\forall e_l \in \mathcal{D}, \quad y \in Y, \ s = 1,2,\ldots k,$$
$$\Rightarrow |\rho^s_{\mathcal{J}(e_l)}(y_p) - \rho^s_{\oplus(e_l)}(y_p)|$$
$$\le |\rho^s_{\mathcal{J}(e_l)}(y_p) - r^s_{\varpi(e_l)}(y_p)|,$$

and

$$\Rightarrow |\omega^{s}_{\mathcal{J}(e_l)}(y_p) - \omega^{s}_{\oplus(e_l)}(y_p)| \leq |\omega^{s}_{\mathcal{J}(e_l)}(y_p) - \omega^{s}_{\varpi(e_l)}(y_p)|, \quad \forall e_l \in \mathcal{D}, \quad y \in Y, \ s = 1, 2, \ldots k,$$

$$\Leftrightarrow 1 - \frac{1}{n}\Sigma^{n}_{p=1} max\{(|\rho^{s}_{\mathcal{J}(e_l)}(y_p) - \rho^{s}_{\oplus(e_l)}(y_p)|)_{s\in k}\} \leq 1 - \frac{1}{n}\Sigma^{n}_{p=1} max\{(|\rho^{s}_{\mathcal{J}(e_l)}(y_p) - \rho^{s}_{\varpi(e_l)}(y_p)|)_{s\in k}\},$$

$$\Leftrightarrow 2\pi - \frac{1}{n}\Sigma^{n}_{p=1} max\{(|\omega^{s}_{\mathcal{J}(e_l)}(y_p) - \omega^{s}_{\oplus(e_l)}(y_p)|)_{s\in k}\} \leq 2\pi - \frac{1}{n}\Sigma^{n}_{p=1} max\{(|\omega^{s}_{\mathcal{J}(e_l)}(y_p) - \omega^{s}_{\varpi(e_l)}(y_p)|)_{s\in k}\},$$

$$\Rightarrow S^{r}_{l=1}((\mathcal{J}, \mathcal{D}), (\oplus, \mathcal{D})) \leq S^{r}_{l=1}((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D})),$$

and

$$\Rightarrow S^{\omega}_{l=1}((\mathcal{J}, \mathcal{D}), (\oplus, \mathcal{D})) \leq S^{\omega}_{l=1}((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D})),$$

$$\Rightarrow S^{r}_{l=1}((\mathcal{J}, \mathcal{D}), (\oplus, \mathcal{D})) + S^{\omega}_{l=1}((\mathcal{J}, \mathcal{D}), (\oplus, \mathcal{D})) \leq S^{r}_{l=1}((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D})) + S^{\omega}_{l=1}((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D})),$$

$$\Rightarrow \frac{1}{2m}\Sigma^{m}_{l=1}[S^{r}_{l}((\mathcal{J}, \mathcal{D}), (\oplus, \mathcal{D})) + \frac{S^{\omega}_{l}((\mathcal{J}, \mathcal{D}), (\oplus, \mathcal{D}))}{2\pi}] \leq \frac{1}{2m}\Sigma^{m}_{l=1}[S^{r}_{l}((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D})) + \frac{S^{\omega}_{l}((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D}))}{2\pi}],$$

$$S((\mathcal{J}, \mathcal{D}), (\oplus, \mathcal{D})) \leq S((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D})).$$

A. THE PROPOSED SM-BASED CMFHSS WITH APPLICATION

In this section, we utilize the idea of CMFHSS to build a novel algorithm and strategy, called SM-based CMFHSS, in which we expand SM depending on CMFHSS under a fuzzy environment. Besides, a car selection decision issue is built to utilize the recently created ENT-based CMFHSS to demonstrate its validity and importance.

1) THE PROPOSED SIMILARITY BASED CMFHSS WITH APPLICATION

Suppose $Y \neq \Phi$ universal set, and let $Y \subset A$ alternatives under discussion, represented by $X = \{y_1, y_2, ..., y_m\}$. Let $\mathcal{D} = A_1 \times A_2 \times ... \times A_n$, where $n \geq 1$ and A_i is the set of all attribute values of the attribute a_i, $i = 1, 2, 3, ..., n$. The construction steps for the proposed CMFHSS-based Similarity or see fig 6 are as per the following:

1) Input each of the CMFHSS.
2) Calculate similarity measure for each CMFHSS using the formula

$$S((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D})) = \frac{1}{2m}\Sigma^{m}_{l=1}[S^{r}_{l}((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D})) + \frac{S^{\omega}_{l}((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D}))}{2\pi}],$$

where $S^{r}_{l=1}((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D}))$

$$= 1 - \frac{1}{n}\Sigma^{n}_{l=1} max\{(|\rho^{s}_{\mathcal{J}(e)}(y_p) - r^{s}_{\varpi(e)}(y_p)|)_{s\in k}\},$$

and

$$S^{\omega}_{l=1}((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D})) = 2\pi - \frac{1}{n}\Sigma^{n}_{l=1} max\{(|\omega^{s}_{\mathcal{J}(e)}(y_p) - \omega^{s}_{\varpi(e)}(y_p)|)_{s\in k}\}.$$

3) Find such CMFHSS which has maximum similarity and chose it for best optimal.
4) Choose any one, if it received more than one optimal.

Example 4: A country with an administration that shows the falling tendency. To overcome this issue, the administration authorities need to put a rescue package into action. Four panels which are free of one another and an assessment board are set up by the government. Every one of these panels has arranged four unique ventures and submitted them to the administration.

Let $X = \{a = $ 1st package, $b =$ 2nd package, $c =$ third package$\}$ be the set of levels. Let $a_1 =$ tax collection, $a_2 =$ Higher global growth, $a_3 =$ Lower interest rates, be distinct attributes whose corresponding attribute values belong to the sets F_1, F_2, F_3. Let $F_1 = \{f_1 =$ fairness, $f_2 =$ Convenience of payment$\}$, $F_2 = \{f_3 =$ increased export spending. $\}$, $F_3 = \{f_4 =$ reduce the cost of borrowing, $f_5 =$ increase consumer spending and investment$\}$.

1) Here, our point is to choose the ideal rescue package according to parameters given
Model for CMFHSS are encoded in the followings tables

$$(\varpi, \mathcal{D}) = \Big\{\varpi(f_1, f_3, f_4) = \Big\{\frac{(0.2e^{i0.7\pi}, 0.2e^{i0.8\pi}, 0.9e^{i0.2\pi})}{a}, \frac{(0.7e^{i0.5\pi}, 0.2e^{i0.6\pi}, 0.3e^{i0.8\pi})}{b}, \frac{(0.4e^{i0.3\pi}, 0.8e^{i0.9\pi}, 0.5e^{i0.3\pi})}{c}\Big\},$$

$$\varpi(f_1, f_3, f_5) = \Big\{\frac{(0.6e^{i0.9\pi}, 0.2e^{i0.8\pi}, 0.2e^{i0.9\pi})}{a}, \frac{(0.2e^{i0.8\pi}, 0.6e^{i0.1\pi}, 0.7e^{i0.3\pi})}{b}, \frac{(0.3e^{i0.6\pi}, 0.2e^{i0.7\pi}, 0.7e^{i0.3\pi})}{c}\Big\},$$

$$\varpi(f_2, f_3, f_4) = \Big\{\frac{(0.3e^{i0.7\pi}, 0.5e^{i0.0.5\pi}, 0.7e^{i0.1\pi})}{a}, \frac{(0.9e^{i0.2\pi}, 0.6e^{i0.1\pi}, 0.7e^{i0.3\pi})}{b}, \frac{(0.3e^{i0.6\pi}, 0.3e^{i0.4\pi}, 0.2e^{i0.4\pi})}{c}\Big\},$$

$$\varpi(f_2,f_3,f_5) = \left\{\frac{(0.6e^{i0.9\pi},0.8e^{i0.6\pi},0.9e^{i0.2\pi})}{a}, \frac{(0.6e^{i0.8\pi},0.2e^{i0.3\pi},0.6e^{i0.3\pi})}{b}, \frac{(0.4e^{i0.7\pi},0.1e^{i0.9\pi},0.3e^{i0.2\pi})}{c}\right\}, \Big\},$$

$$(\oplus,\mathcal{D}) = \Big\{\oplus(f_1,f_3,f_4) = \left\{\frac{(0.2e^{i0.7\pi},0.6e^{i0.1\pi},0.7e^{i0.4\pi})}{a}, \frac{(0.3e^{i0.9\pi},0.4e^{i0.8\pi},0.1e^{i0.9\pi})}{b}, \frac{(0.2e^{i0.5\pi},0.3e^{i0.8\pi},0.9e^{i0.4\pi})}{c}\right\},$$

$$\oplus(f_1,f_3,f_5) = \left\{\frac{(0.7e^{i0.9\pi},0.7e^{i0.2\pi},0.8e^{i0.6\pi})}{a}, \frac{(0.1e^{i0.7\pi},0.6e^{i0.3\pi},0.6e^{i0.3\pi})}{b}, \frac{(0.1e^{i0.6\pi},0.2e^{i0.7\pi},0.7e^{i0.3\pi})}{c}\right\},$$

$$\oplus(f_2,f_3,f_4) = \left\{\frac{(0.8e^{i0.9\pi},0.9e^{i0.4\pi},0.1e^{i0.7\pi})}{a}, \frac{(0.8e^{i0.8\pi},0.6e^{i0.1\pi},0.5e^{i0.3\pi})}{b}, \frac{(0.2e^{i0.6\pi},0.5e^{i0.9\pi},0.6e^{i0.3\pi})}{c}\right\},$$

$$\oplus(f_2,f_3,f_5) = \left\{\frac{(0.6e^{i0.9\pi},0.2e^{i0.8\pi},0.2e^{i0.9\pi})}{a}, \frac{(0.2e^{i0.8\pi},0.2e^{i0.1\pi},0.7e^{i0.7\pi})}{b}, \frac{(0.3e^{i0.6\pi},0.2e^{i0.7\pi},0.7e^{i0.9\pi})}{c}\right\}, \Big\},$$

$$(\otimes,\mathcal{D}) = \Big\{\otimes(f_1,f_3,f_4) = \left\{\frac{(0.2e^{i0.7\pi},0.5e^{i0.7\pi},0.2e^{i0.4\pi})}{a}, \frac{(0.2e^{i0.6\pi},0.5e^{i0.8\pi},0.3e^{i0.6\pi})}{b}, \frac{(0.7e^{i0.6\pi},0.3e^{i0.7\pi},0.2e^{i0.9\pi})}{c}\right\},$$

$$\otimes(f_1,f_3,f_5) = \left\{\frac{(0.6e^{i0.9\pi},0.2e^{i0.8\pi},0.2e^{i0.9\pi})}{a}, \frac{(0.2e^{i0.8\pi},0.6e^{i0.1\pi},0.6e^{i0.9\pi})}{b}, \frac{(0.2e^{i0.6\pi},0.2e^{i0.7\pi},0.9e^{i0.7\pi})}{c}\right\},$$

$$\otimes(f_2,f_3,f_4) = \left\{\frac{(0.6e^{i0.9\pi},0.2e^{i0.8\pi},0.3e^{i0.3\pi})}{a}, \frac{(0.2e^{i0.8\pi},0.9e^{i0.1\pi},0.7e^{i0.8\pi})}{b}, \frac{(0.3e^{i0.6\pi},0.2e^{i0.7\pi},0.6e^{i0.3\pi})}{c}\right\},$$

$$\otimes(f_2,f_3,f_5) = \left\{\frac{(0.3e^{i0.4\pi},0.2e^{i0.8\pi},0.2e^{i0.9\pi})}{a}, \frac{(0.2e^{i0.8\pi},0.6e^{i0.1\pi},0.7e^{i0.2\pi})}{b}, \frac{(0.3e^{i0.6\pi},0.1e^{i0.9\pi},0.5e^{i0.6\pi})}{c}\right\}, \Big\},$$

and ideal CMFHSS are

$$(\mathcal{J},\mathcal{D}) = \Big\{\mathcal{J}(f_1,f_3,f_4) = \left\{\frac{(0.3e^{i0.4\pi},0.4e^{i0.3\pi},0.7e^{i0.1\pi})}{a}, \frac{(0.8e^{i0.2\pi},0.4e^{i0.8\pi},0.1e^{i0.6\pi})}{b}, \frac{(0.8e^{i0.2\pi},0.7e^{i0.2\pi},0.8e^{i0.5\pi})}{c}\right\},$$

$$\mathcal{J}(f_1,f_3,f_5) = \left\{\frac{(0.3e^{i0.3\pi},0.1e^{i0.2\pi},0.7e^{i0.4\pi})}{a}, \frac{(0.8e^{i0.2\pi},0.6e^{i0.8\pi},0.7e^{i0.9\pi})}{b}, \frac{(0.2e^{i0.9\pi},0.2e^{i0.4\pi},0.4e^{i0.3\pi})}{c}\right\},$$

$$\mathcal{J}(f_2,f_3,f_4) = \left\{\frac{(0.4e^{i0.8\pi},0.3e^{i0.4\pi},0.6e^{i0.4\pi})}{a}, \frac{(0.2e^{i0.4\pi},0.6e^{i0.1\pi},0.7e^{i0.9\pi})}{b}, \frac{(0.6e^{i0.9\pi},0.2e^{i0.4\pi},0.8e^{i0.3\pi})}{c}\right\},$$

$$\mathcal{J}(f_2,f_3,f_5) = \left\{\frac{(0.3e^{i0.3\pi},0.3e^{i0.4\pi},0.6e^{i0.4\pi})}{a}, \frac{(0.8e^{i0.2\pi},0.6e^{i0.1\pi},0.7e^{i0.9\pi})}{b}, \frac{(0.7e^{i0.3\pi},0.8e^{i0.3\pi},0.6e^{i0.3\pi})}{c}\right\}, \Big\},$$

2) Calculate the SM of $(\mathcal{J},\mathcal{D})$, $(\varpi,\mathcal{D})$ and $(\oplus,\mathcal{D})$ using the formula mention in algorithm in Step (2), see Table 3.
Hence the degree of similarity between $(\mathcal{J},\mathcal{D})$ and $(\varpi,\mathcal{D})$, $(\oplus,\mathcal{D})$, $(\otimes,\mathcal{D})$ respectively is given by $S_1 = S((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D})) = 0.6519$, $S_2 = S((\mathcal{J},\mathcal{D}),(\oplus,\mathcal{D})) = 0.6143$, $S_3 = S((\mathcal{J},\mathcal{D}),(\otimes,\mathcal{D})) = 0.6260$.
3) Thus, the government officials should select the rescue package $(\varpi,\mathcal{D})$ with highest score. Hence, they will select $(\varpi,\mathcal{D})$.

The table 4, fig 3 and 4 depict the correlation of the proposed measures with existing measures given by Li *et al.* [30],

TABLE 3. Similarity measures.

$S^{\tau}_{l=1}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$	0.733
$S^{\tau}_{l=2}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$	0.533
$S^{\tau}_{l=3}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$	0.5
$S^{\tau}_{l=4}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$	0.466
$S^{\omega}_{l=1}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$	4.7125
$S^{\omega}_{l=2}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$	4.6075
$S^{\omega}_{l=3}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$	5.0264
$S^{\omega}_{l=4}((\mathcal{J},\mathcal{D}),(\varpi,\mathcal{D}))$	4.3981
$S^{\tau}_{l=1}((\mathcal{J},\mathcal{D}),(\bigoplus,\mathcal{D}))$	0.5667
$S^{\tau}_{l=2}((\mathcal{J},\mathcal{D}),(\bigoplus,\mathcal{D}))$	0.466
$S^{\tau}_{l=3}((\mathcal{J},\mathcal{D}),(\bigoplus,\mathcal{D}))$	0.466
$S^{\tau}_{l=4}((\mathcal{J},\mathcal{D}),(\bigoplus,\mathcal{D}))$	0.466
$S^{\omega}_{l=1}((\mathcal{J},\mathcal{D}),(\bigoplus,\mathcal{D}))$	4.607
$S^{\omega}_{l=2}((\mathcal{J},\mathcal{D}),(\bigoplus,\mathcal{D}))$	4.7122
$S^{\omega}_{l=3}((\mathcal{J},\mathcal{D}),(\bigoplus,\mathcal{D}))$	4.816
$S^{\omega}_{l=4}((\mathcal{J},\mathcal{D}),(\bigoplus,\mathcal{D}))$	4.3981
$S^{\tau}_{l=1}((\mathcal{J},\mathcal{D}),(\bigotimes,\mathcal{D}))$	0.443
$S^{\tau}_{l=2}((\mathcal{J},\mathcal{D}),(\bigotimes,\mathcal{D}))$	0.466
$S^{\tau}_{l=3}((\mathcal{J},\mathcal{D}),(\bigotimes,\mathcal{D}))$	0.7
$S^{\tau}_{l=4}((\mathcal{J},\mathcal{D}),(\bigotimes,\mathcal{D}))$	0.433
$S^{\omega}_{l=1}((\mathcal{J},\mathcal{D}),(\bigotimes,\mathcal{D}))$	4.921
$S^{\omega}_{l=2}((\mathcal{J},\mathcal{D}),(\bigotimes,\mathcal{D}))$	4.188
$S^{\omega}_{l=3}((\mathcal{J},\mathcal{D}),(\bigotimes,\mathcal{D}))$	5.131
$S^{\omega}_{l=4}((\mathcal{J},\mathcal{D}),(\bigotimes,\mathcal{D}))$	4.3981

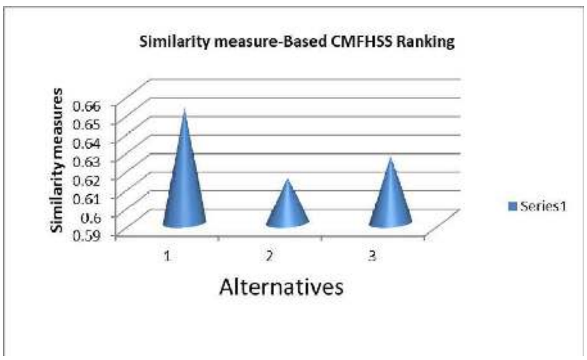

FIGURE 3. Ranking of alternative by SM based CMFHSS.

Chen [31], Chen *et al.* [32], Hung *et al.* [50], Hong *et al.* [33], Dengfeng [34], Li *et al.* [35], Liang *et al.* [36], Mitchell [37], Ye [44], Wei [51], Zhang [38], Peng *et al.* [39], Boran *et al.* [40] and Begam *et al.* [41].

B. ADVANTAGES AND COMPARATIVE ANALYSIS OF THE CMFHSS

In the following, few comparisons of the initiated techniques with shortcomings are presented to inspect the proposed strategies' validity and prevalence. Moreover, the proposed SM is compared with other existing measures and gets the following disadvantages to comprehend with an example, including the idea presented in the previous section. However, all existing shortcomings fail to manage issues that involve

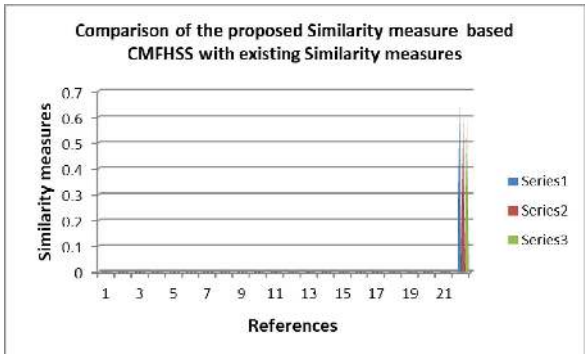

FIGURE 4. Comparison of the proposed SM based CMFHSS with existing similarity measure.

2D information/date i.e., two unique sorts of data/information relating to the problem parameters. The comparability of proposed techniques is shown with the help of example 4, see the outcomes in table 5 and fig 7.

Example 5: For example 4, if we have one-dimensional information likewise

$$
\begin{aligned}
(\varpi,\mathcal{D}) = \Bigg\{ & \varpi(f_1,f_3,f_4) \\
= & \Bigg\{ \frac{(0.2e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.9e^{i2\pi(0.0)}}{a}, \frac{(0.7e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)})}{b}, \frac{(0.4e^{i2\pi(0.0)}, 0.8e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)})}{c} \Bigg\}, \\
\varpi(f_1,f_3,f_5) = & \Bigg\{ \frac{(0.6e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)})}{a}, \frac{(0.2e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{b}, \frac{(0.3e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{c} \Bigg\}, \\
\varpi(f_2,f_3,f_4) = & \Bigg\{ \frac{(0.3e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{a}, \frac{(0.9e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{b}, \frac{(0.3e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)})}{c} \Bigg\}, \\
\varpi(f_2,f_3,f_5) = & \Bigg\{ \frac{(0.6e^{i2\pi(0.0)}, 0.8e^{i2\pi(0.0)}, 0.9e^{i2\pi(0.0)})}{a}, \frac{(0.6e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.6^{i2\pi(0.0)})}{b}, \frac{(0.4e^{i2\pi(0.0)}, 0.1e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)})}{c} \Bigg\}, \Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
(\oplus,\mathcal{D}) = \Bigg\{ & \oplus(f_1,f_3,f_4) \\
= & \Bigg\{ \frac{(0.2e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{a},
\end{aligned}
$$

TABLE 4. Comparison of the proposed similarity measure based CMFHSS with existing SM.

SN	References	SM	Ranking
1	[30]	Not valid	×
2	[31]	Not valid	×
3	[32]	Not valid	×
4	[50]	Not valid	×
5	[50]	Not valid	×
6	[50]	Not valid	×
7	[33]	Not valid	×
8	[34]	Not valid	×
9	[35]	Not valid	×
10	[36]	Not valid	×
11	[36]	Not valid	×
12	[36]	Not valid	×
13	[37]	Not valid	×
14	[44]	Not valid	×
15	[51]	Not valid	×
16	[38]	Not valid	×
17	[39]	Not valid	×
18	[39]	Not valid	×
19	[39]	Not valid	×
20	[40]	Not valid	×
21	[41]	Not valid	×
22	Proposed Method in this paper	$S_1 = 0.6519, S_2 = 0.6143, S_3 = 0.6260.$	$S_1 \geq S_3 \geq S_2$

$$\frac{(0.3e^{i2\pi(0.0)}, 0.4e^{i2\pi(0.0)}, 0.1e^{i2\pi(0.0)})}{b}, \frac{(0.2e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)}, 0.9e^{i2\pi(0.0)})}{c}\Bigg\},$$

$$\oplus(f_1, f_3, f_5) = \Bigg\{\frac{(0.7e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)}, 0.8e^{i2\pi(0.0)})}{a}, \frac{(0.1e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)})}{b}, \frac{(0.1e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{c}\Bigg\},$$

$$\oplus(f_2, f_3, f_4) = \Bigg\{\frac{(0.8e^{i2\pi(0.0)}, 0.9e^{i2\pi(0.0)}, 0.1e^{i2\pi(0.0)})}{a}, \frac{(0.8e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)})}{b}, \frac{(0.2e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)})}{c}\Bigg\},$$

$$\oplus(f_2, f_3, f_5) = \Bigg\{\frac{(0.6e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)})}{a}, \frac{(0.2e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{b}, \frac{(0.3e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{c}\Bigg\},\Bigg\},$$

$$(\otimes, \mathcal{D}) = \Bigg\{\otimes(f_1, f_3, f_4) = \Bigg\{\frac{(0.2e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)})}{a}, \frac{(0.2e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)})}{b}, \frac{(0.7e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)})}{c}\Bigg\},$$

$$\otimes(f_1, f_3, f_5) = \Bigg\{\frac{(0.6e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)})}{a}, \frac{(0.2e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)})}{b}, \frac{(0.2e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.9e^{i2\pi(0.0)})}{c}\Bigg\},$$

$$\otimes(f_2, f_3, f_4) = \Bigg\{\frac{(0.6e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)})}{a}, \frac{(0.2e^{i2\pi(0.0)}, 0.9e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{b}, \frac{(0.3e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)})}{c}\Bigg\},$$

$$\otimes(f_2, f_3, f_5) = \Bigg\{\frac{(0.3e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)})}{a}, \frac{(0.2e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{b}, \frac{(0.3e^{i2\pi(0.0)}, 0.1e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)})}{c}\Bigg\},\Bigg\},$$

and ideal CMFHSS are

$$(\mathcal{J}, \mathcal{D}) = \Bigg\{\mathcal{J}(f_1, f_3, f_4)$$

TABLE 5. Comparison of the proposed similarity measure based CMFHSS with existing SM.

SN	References	SM	Ranking
1	[30]	Not valid	×
2	[31]	Not valid	×
3	[32]	Not valid	×
4	[50]	Not valid	×
5	[50]	Not valid	×
6	[50]	Not valid	×
7	[33]	Not valid	×
8	[34]	Not valid	×
9	[35]	Not valid	×
10	[36]	Not valid	×
11	[36]	Not valid	×
12	[36]	Not valid	×
13	[37]	Not valid	×
14	[44]	Not valid	×
15	[51]	Not valid	×
16	[38]	Not valid	×
17	[39]	Not valid	×
18	[39]	Not valid	×
19	[39]	Not valid	×
20	[40]	Not valid	×
21	[41]	$S_1 = 0.34$, $S_2 = 0.18$, $S_3 = 0.24$,	$S_1 \geq S_3 \geq S_2$.
22	Proposed Method in this paper	$S_1 = 0.279$, $S_2 = 0.245$, $S_3 = 0.255$,	$S_1 \geq S_3 \geq S_2$.

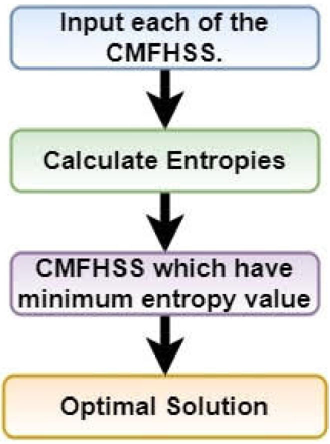

FIGURE 5. Construction steps for the proposed CMFHSS-based ENT.

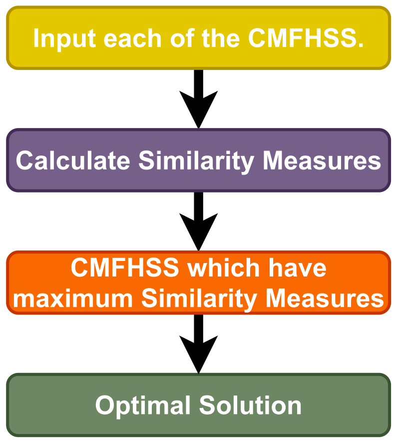

FIGURE 6. Construction steps for the proposed CMFHSS-based SM.

$$= \left\{ \frac{(0.3e^{i2\pi(0.0)}, 0.4e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{a}, \frac{(0.8e^{i2\pi(0.0)}, 0.4e^{i2\pi(0.0)}, 0.1e^{i2\pi(0.0)})}{b}, \frac{(0.8e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)}, 0.8e^{i2\pi(0.0)})}{c} \right\},$$

$$\mathcal{J}(f_1, f_3, f_5) = \left\{ \frac{(0.3e^{i2\pi(0.0)}, 0.1e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{a}, \frac{(0.8e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{b}, \frac{(0.2e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.4e^{i2\pi(0.0)})}{c} \right\},$$

$$\mathcal{J}(f_2, f_3, f_4) = \left\{ \frac{(0.4e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)})}{a}, \frac{(0.2e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)})}{b}, \frac{(0.6e^{i2\pi(0.0)}, 0.2e^{i2\pi(0.0)}, 0.8e^{i2\pi(0.0)})}{c} \right\},$$

$$\mathcal{J}(f_2, f_3, f_5) = \left\{ \frac{(0.3e^{i2\pi(0.0)}, 0.3e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)})}{a}, \frac{(0.8e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)})}{b}, \frac{(0.7e^{i2\pi(0.0)}, 0.8e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)})}{c} \right\}, \right\},$$

$$S_1 = S((\mathcal{J}, \mathcal{D}), (\varpi, \mathcal{D})) = 0.279,$$
$$S_2 = S((\mathcal{J}, \mathcal{D}), (\oplus, \mathcal{D})) = 0.245,$$
$$S_3 = S((\mathcal{J}, \mathcal{D}), (\otimes, \mathcal{D})) = 0.255.$$

C. SENSITIVITY ANALYSIS

1) By ignoring the imaginary parts and $n = 1$ s.t $A_1 = A_2 = A_3 \ldots = A_n$, then the proposed CMFHSS reduced to Multi fuzzy soft set [14].
2) $k = 1$ and $n = 1$ s.t $A_1 = A_2 = A_3 \ldots = A_n$, then the proposed CMFHSS reduced to CMFSS [25].

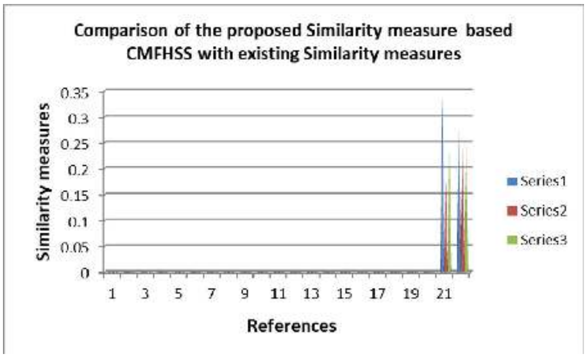

FIGURE 7. Comparison of the proposed SM based CMFHSS with existing SM.

The proposed measures dependent on CMFHSS are more remarkable and more general than existing strategies are examined in [14], [25]. We are presently dealing with building up a more top to bottom theoretical structure concerning the comparability gauges and have plans to broaden this to different sorts of SM in the future. We are inspired by [25], and anticipate broadening our work to other generalizations of CMFHSS, for example, Intuitionistic CMFHSS, Neutrosophic CMFHSS, Plithogenic CMFHSS, Plithogenic Intuitionistic CMFHSS, Plithogenic Intuitionistic CMFHSS, and Plithogenic Neutrosophic CMFHSS and apply the work in clinical imaging issues, pattern recognition, recommender frameworks, social, the economic system, approximate reasoning, image processing and game theory.

VII. CONCLUSION

A new scientific device to demonstrate the data or information seen repeatedly throughout some time is established. The CMFHSS set is developed by combining a MFS and HSS characterized in a complex system. This framework is more flexible in two ways; firstly, it broadens the membership function through their translation in a unit circle with phase and amplitude parts. Secondly, in CMFHSS the attributes can be further sub-partitioned into attribute values for a better understanding. We characterized its fundamental operations as a complement, union, and intersection and supported them with examples. This study will dispense a theoretical basis to address vagueness and periodicity in designing, clinical, material science, autos, and many others. This new comprehension of the P-terms opens new zones for some applications in the field of physical science and other natural sciences, where P-terms may also present the temperature, pressure, distance, or any factor that affects and cooperates with its corresponding A-terms in the choice cycle. Moreover, we presented the proverbial meaning of ENT and SM of CMFHSS and studied the fundamental relations. Additionally, mathematical models are given to analyze the reliability and predominance of the setup methodologies. Furthermore, the advantages and comparative analysis of the proposed measures with existing measures are also depicted in detail. Lastly, the mathematical models are given to represent the validity and applicability of the proposed methodologies.

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