A Differential Detection Scheme for Transmit Diversity

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Abstract—We present a transmission scheme for exploiting diversity given by two transmit antennas when neither the transmitter nor the receiver has access to channel state information. The new detection scheme can use equal energy constellations and encoding is simple. At the receiver, decoding is achieved with low decoding complexity. The transmission provides full spatial diversity and requires no channel state side information at the receiver. The scheme can be considered as the extension of differential detection schemes to two transmit antennas.

Index Terms—Antenna arrays, differential detection, space-time codes, transmitter diversity.

I. INTRODUCTION

IN THE past few years, significant progress in code design for transmitter diversity over the wireless channel has been made. The primary focus was on the case when only the receiver knows the channel, which is the case for most practical systems. For this scenario, the first bandwidth efficient transmit diversity scheme was proposed by Wittneben [17], [18], and it includes the delay diversity scheme of Seshadri and Winters [9] as a special case. Later, Foschini introduced a multilayered space-time architecture [2]. A transmit diversity scheme which achieves the same rate as that of antenna hopping diversity is constructed in [4].

More recently, space-time trellis coding has been proposed [12], which combines signal processing at the receiver with coding techniques appropriate to multiple transmit antennas, and it provides significant gain over [9] and [17]. Specific space-time trellis codes designed for 2–4 transmit antennas perform extremely well in slow fading environments (typical of indoor transmission) and come within 2–3 dB of the outage capacity computed by Telatar [16] and independently by Foschini and Gans [3]. The bandwidth efficiency is about 3–4 times that of current systems. The space-time codes presented in [12] provide the best possible tradeoff between constellation size, data rate, diversity advantage, and trellis complexity. When the number of transmit antennas is fixed, the decoding complexity of space-time trellis coding (measured by the number of trellis states at the decoder) increases exponentially as a function of both the diversity level and the transmission rate.

In addressing the issue of decoding complexity, Alamouti discovered a remarkable scheme for transmission using two transmit antennas [1]. This scheme supports a maximum likelihood detection scheme based only on linear processing at the receiver. Space-Time Block Coding introduced in [13] generalizes the transmission scheme discovered by Alamouti to an arbitrary number of transmit antennas and is able to achieve the full diversity promised by the transmit and receive antennas. These codes retain the property of having a very simple maximum likelihood decoding algorithm based only on linear processing at the receiver [13]. For real signal constellations (such as PAM), they provide the maximum possible transmission rate allowed by the theory of space-time coding [12]. For complex constellations, space-time block codes can be constructed for any number of transmit antennas, and again these codes have remarkably simple decoding algorithms based only on linear processing at the receiver. For more details on transmit diversity when the receiver knows the channel, see [12] and [13] and the references therein.

When no knowledge of the channel is available— at neither the transmitter nor at the receiver—the above schemes require the transmission of pilot symbols [11]. For one transmit antenna, differential detection schemes exist that neither require the knowledge of the channel nor employ pilot symbol transmission. These differential decoding schemes are used, for instance, in the IEEE IS-54 standard. This motivates the generalization of differential detection schemes for the case of multiple transmit antennas. A partial solution to this problem was proposed in [10], where it was assumed that the channel is not known. However, the scheme proposed in [10] requires the transmission of symbols known to the receiver at the beginning and hence is not truly differential. The scheme of [10] can be thought of as a joint channel and data estimation. In the scheme of [10], the detected sequence at time $t - 1$ is used to estimate the channel at the receiver, and these estimates are used to detect the transmitted data at time t. This is a joint channel and data estimation which can lead to error propagation. Here, we construct a truly differential detection scheme for two transmit antennas. A different nondifferential approach to transmit diversity when nobody knows the channel is reported in [5] and [6], but this approach has both exponential encoding and decoding complexities.

The outline of the paper is as follows. In Section II, the system model for transmission using two transmit antennas is considered and the transmission scheme of [1] is reviewed assuming coherent detection. In Section III, the new differential encoding algorithm is presented. The corresponding decoding algorithm is presented in Section IV. In Section V, we provide simulation results for the performance of the proposed scheme and will show that a loss of 3 dB is incurred when compared to coherent detection. Finally, some conclusions are made in Section VI.

II. A SIMPLE TRANSMISSION SCHEME ASSUMING COHERENT DETECTION

A. The System Model

We consider a wireless communication system with 2 antennas at the base station and m antennas at the remote. We assume that a signal constellation is given and it is normalized such that the average energy of the constellation is 1/2. At each time slot t, signals c_t^i, $i = 1, 2$ are transmitted simultaneously from the 2 transmit antennas. The coefficient $\alpha_{i,j}$ is the path gain from transmit antenna i to receive antenna j. The path gains are modeled as samples of independent complex Gaussian random variables with variance 0.5 per real dimension. The wireless channel is assumed to be quasistatic so that the path gains are constant over a frame of length l and vary from one frame to another.

At time t, the signal r_t^j received at antenna j is given by

$$r_t^j = \sum_{i=1}^{2} \alpha_{i,j} c_t^i + \eta_t^j \tag{1}$$

where the noise samples η_t^j are independent samples of a zero-mean complex Gaussian random variable with variance 1/(2 SNR) per complex dimension. The average energy of the symbols transmitted from each antenna is normalized to be 1/2, so that the average power of the received signal at each receive antenna is 1 and the signal-to-noise ratio is SNR.

Assuming coherent detection, the receiver computes the decision metric

$$\sum_{t=1}^{2} \sum_{j=1}^{m} \left| r_t^j - \sum_{i=1}^{2} \alpha_{i,j} c_t^i \right|^2 \tag{2}$$

over all codewords

$$c_1^1 c_1^2 c_2^1 c_2^2 \cdots c_l^1 c_l^2$$

and decides in favor of the codeword that minimizes this sum.

B. Encoding Algorithm

We assume that transmission at the baseband employs a signal constellation $\mathcal{A}$ with 2^b elements. We consider the transmission matrix

$$\mathcal{G} = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \tag{3}$$

given in [1]. At time slot 1, $2b$ bits arrive at the encoder and select constellation signals s_1, s_2. Setting $x_i = s_i$ for $i = 1, 2$ in $\mathcal{G}$, at each time slot $t = 1, 2$, the entries $\mathcal{G}_{ti}$, $i = 1, 2$ are transmitted simultaneously from transmit antennas 1, 2.

C. The Coherent Detection Algorithm

Assuming coherent detection, maximum likelihood decoding can be achieved based only on linear processing at the receiver.

Maximum likelihood detection amounts to minimizing the decision statistic

$$\sum_{j=1}^{m} \left(|r_1^j - \alpha_{1,j} s_1 - \alpha_{2,j} s_2|^2 + |r_2^j + \alpha_{1,j} s_2^* - \alpha_{2,j} s_1^*|^2 \right) \tag{4}$$

over all possible values of s_1 and s_2. The minimizing values are the receiver estimates of s_1 and s_2, respectively. We expand the above metric and delete the terms that are independent of the codewords and observe that the above minimization is equivalent to minimizing

$$-\sum_{j=1}^{m} [r_1^j \alpha_{1,j}^* s_1^* + (r_1^j)^* \alpha_{1,j} s_1 + r_1^j \alpha_{2,j}^* s_2^* \\ + (r_1^j)^* \alpha_{2,j} s_2 - r_2^j \alpha_{1,j}^* s_2 - (r_2^j)^* \alpha_{1,j} s_2^* \\ + r_2^j \alpha_{2,j}^* s_1 + (r_2^j)^* \alpha_{2,j} s_1^*] + (|s_1|^2 + |s_2|^2) \\ \cdot \sum_{j=1}^{m} \sum_{i=1}^{2} |\alpha_{i,j}|^2.$$

The above metric decomposes into two parts, one of which

$$-\sum_{j=1}^{m} [r_1^j \alpha_{1,j}^* s_1^* + (r_1^j)^* \alpha_{1,j} s_1 + r_2^j \alpha_{2,j}^* s_1 + (r_2^j)^* \alpha_{2,j} s_1^*] \\ + |s_1|^2 \sum_{j=1}^{m} \sum_{i=1}^{2} |\alpha_{i,j}|^2$$

is only a function of s_1, and the other one

$$-\sum_{j=1}^{m} [r_2^j \alpha_{2,j}^* s_2^* + (r_1^j)^* \alpha_{2,j} s_2 - r_2^j \alpha_{1,j}^* s_2 - (r_2^j)^* \alpha_{1,j} s_2^*] \\ + |s_2|^2 \sum_{j=1}^{m} \sum_{i=1}^{2} |\alpha_{i,j}|^2$$

is only a function of s_2. Thus, the minimization of (4) is equivalent to minimizing these two parts separately. This in turn is equivalent to minimizing the decision statistic

$$\left| \left[\sum_{j=1}^{m} (r_1^j \alpha_{1,j}^* + (r_2^j)^* \alpha_{2,j}) \right] - s_1 \right|^2 \\ + \left(-1 + \sum_{j=1}^{m} \sum_{i=1}^{2} |\alpha_{i,j}|^2 \right) |s_1|^2$$

for detecting s_1 and the decision statistic

$$\left| \left[\sum_{j=1}^{m} (r_1^j \alpha_{2,j}^* - (r_2^j)^* \alpha_{1,j}) \right] - s_2 \right|^2 \\ + \left(-1 + \sum_{j=1}^{m} \sum_{i=1}^{2} |\alpha_{i,j}|^2 \right) |s_2|^2$$

for decoding s_2. This is the simple decoding scheme described in [1].

Similar encoding schemes with detection based only on linear processing exist for any number of transmit antennas [13], [14].

III. DIFFERENTIAL ENCODING

A. Technical Machinery

We shall restrict the constellation $\mathcal{A}$ to 2^b-PSK for some $b = 1, 2, 3, \cdots$, but in reality only BPSK, QPSK, and 8-PSK are of interest. Thus,

$$\mathcal{A} = \left\{ \frac{e^{2\pi k \boldsymbol{j}/2^b}}{\sqrt{2}} \,|\, k = 0, 1, \cdots, 2^b - 1 \right\}$$

where $\boldsymbol{j} = \sqrt{-1}$.

Given a pair of 2^b-PSK constellation symbols x_1 and x_2, we first observe that the complex vectors $(x_1\ x_2)$ and $(-x_2^*\ x_1^*)$ are orthogonal to each other and have unit lengths. Any two-dimensional vector $\mathcal{X} = (x_3\ x_4)$ can be uniquely represented in the orthonormal basis given by these vectors. In other words, there exists a unique complex vector $P_{\mathcal{X}} = (A_{\mathcal{X}}\ B_{\mathcal{X}})$ such that $A_{\mathcal{X}}$ and $B_{\mathcal{X}}$ satisfy the vector equation

$$(x_3\ x_4) = A_{\mathcal{X}}(x_1\ x_2) + B_{\mathcal{X}}(-x_2^*\ x_1^*). \tag{5}$$

The coefficients $A_{\mathcal{X}}$ and $B_{\mathcal{X}}$ are given by

$$A_{\mathcal{X}} = x_3 x_1^* + x_4 x_2^* \tag{6}$$
$$B_{\mathcal{X}} = -x_3 x_2 + x_4 x_1. \tag{7}$$

We define the set $\mathcal{V}_{\mathcal{X}}$ to consist of all the vectors $P_{\mathcal{X}}$, $\mathcal{X} \in \mathcal{A} \times \mathcal{A}$. The set $\mathcal{V}_{\mathcal{X}}$ has the following properties.

- **Property A:** It has 2^{2b} elements corresponding to the pairs $(x_3\ x_4)$ of constellation symbols.
- **Property B:** All elements of $\mathcal{V}_{\mathcal{X}}$ have unit length.
- **Property C:** For any two distinct elements $\mathcal{X} = (x_1\ x_2)$ and $\mathcal{Y} = (y_1\ y_2)$ of $\mathcal{A} \times \mathcal{A}$

$$\|P_{\mathcal{X}} - P_{\mathcal{Y}}\| = \|(x_1\ x_2) - (y_1\ y_2)\|.$$

- **Property D:** The minimum distance between any two distinct elements of $\mathcal{V}_{\mathcal{X}}$ is equal to the minimum distance of the 2^b-PSK constellation $\mathcal{A}$.

The above properties hold because the mapping $\mathcal{X} \to P_{\mathcal{X}}$ is just a change of basis from the standard basis given by vectors $\{(1\ 0),\ (0\ 1)\}$ to the orthonormal basis given by $\{(x_1\ x_2),\ (-x_2^*\ x_1^*)\}$ which preserves the distances between the points of the two-dimensional complex space.

The first ingredient of our construction is the choice of an arbitrary set $\mathcal{V}$ having Properties A and B. It is also handy if $\mathcal{V}$ has Properties C and D as well. As a natural choice for such a set $\mathcal{V}$, we may fix an arbitrary pair $\mathcal{X} \in \mathcal{A} \times \mathcal{A}$ and let $\mathcal{V} = \mathcal{V}_{\mathcal{X}}$. Because the 2^b-PSK constellation $\mathcal{A}$ always contains the signal point $1/\sqrt{2}$, we chose to fix $\mathcal{X} = ((1/\sqrt{2})\ (1/\sqrt{2}))$ in this paper.

We will also need an arbitrary bijective mapping $\mathcal{M}$ of blocks of $2b$ bits onto $\mathcal{V}$. Among all the possibilities for $\mathcal{M}$, we choose the following mapping. Given a block $\mathcal{B}$ of $2b$ bits, the first b bits are mapped into a constellation symbol a_3 and the second b bits are mapped into a constellation symbol a_4 using Gray mapping.
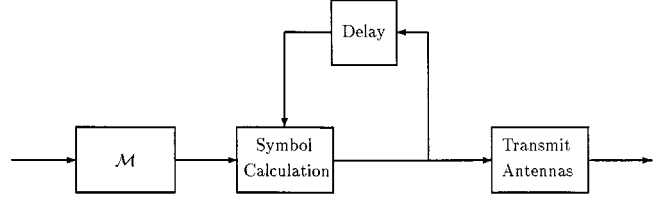

Fig. 1. Transmitter block diagram.

Let $a_1 = a_2 = 1/\sqrt{2}$, then $\mathcal{M}(\mathcal{B}) = (A(\mathcal{B})\ B(\mathcal{B}))$ is defined by

$$A(\mathcal{B}) = a_3 a_1^* + a_4 a_2^* \tag{8}$$
$$B(\mathcal{B}) = -a_3 a_2 + a_4 a_1. \tag{9}$$

Clearly, $\mathcal{M}$ maps any $2b$ bits onto $\mathcal{V}$. Conversely, given $(A(\mathcal{B})\ B(\mathcal{B}))$, the pair $(a_3\ a_4)$ is recovered by

$$(a_3\ a_4) = A(\mathcal{B})(a_1\ a_2) + B(\mathcal{B})(-a_2^*\ a_1^*). \tag{10}$$

The block $\mathcal{B}$ is then constructed by inverse Gray mapping of a_3 and a_4.

B. The Encoding Algorithm

The transmitter begins the transmission with sending arbitrary symbols s_1 and s_2 at time 1 and symbols $-s_2^*$ and s_1^* at time 2 unknown to the receiver. These two transmissions do not convey any information. The transmitter subsequently encodes the rest of the data in an inductive manner. Suppose that s_{2t-1} and s_{2t} are sent, respectively, from transmit antennas one and two at time $2t - 1$, and that $-s_{2t}^*$, s_{2t-1}^* are sent, respectively, from antennas one and two at time $2t$. At time $2t + 1$, a block of $2b$ bits $\mathcal{B}_{2t+1}$ arrives at the encoder. The transmitter uses the mapping $\mathcal{M}$ and computes $\mathcal{M}(\mathcal{B}_{2t+1}) = (A(\mathcal{B}_{2t+1})\ B(\mathcal{B}_{2t+1}))$. Then it computes

$$\begin{aligned} &(s_{2t+1}\ s_{2t+2}) \\ &= A(\mathcal{B}_{2t+1})(s_{2t-1}\ s_{2t}) + B(\mathcal{B}_{2t+1})(-s_{2t}^*\ s_{2t-1}^*). \end{aligned} \tag{11}$$

The transmitter then sends s_{2t+1} and s_{2t+2}, respectively, from transmit antennas one and two at time $2t+1$, and $-s_{2t+2}^*$, s_{2t+1}^* from antennas one and two at time $2t + 2$. This process is inductively repeated until the end of the frame (or end of the transmission).

The block diagram of the encoder is given in Fig. 1.

Example I: We demonstrate the above differential encoding scheme by an example. We assume that the constellation is BPSK consisting of the points $-1/\sqrt{2}$ and $1/\sqrt{2}$. Then the set $\mathcal{V} = \{(1\ 0), (0\ 1), (-1\ 0), (0\ -1)\}$. Recall that the Gray mapping maps a bit $i = 0, 1$ to $(-1)^i/\sqrt{2}$. We set $a_1 = a_2 = 1/\sqrt{2}$. Then the mapping $\mathcal{M}$ maps two bits onto $\mathcal{V}$ and is given by

$$\begin{aligned} \mathcal{M}(00) &= (1\ 0) \\ \mathcal{M}(10) &= (0\ 1) \\ \mathcal{M}(01) &= (0\ -1) \\ \mathcal{M}(11) &= (-1\ 0). \end{aligned}$$

Now suppose that at time $2t - 1$, $s_{2t-1} = 1/\sqrt{2}$ and $s_{2t} = -1/\sqrt{2}$ are sent, respectively, from antennas one and two, and at time $2t$, $-s_{2t}^* = 1/\sqrt{2}$ and $s_{2t-1}^* = 1/\sqrt{2}$ are sent, respectively,

TABLE I
TRANSMITTED SYMBOLS FOR EXAMPLE I AT TIME $2t + 1$

Input bits at time $2t + 1$	Antenna 1	Antenna 2
00	$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$
10	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
01	$\frac{-1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$
11	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$

TABLE II
TRANSMITTED SYMBOLS FOR EXAMPLE I AT TIME $2t + 2$

Input bits at time $2t + 1$	Antenna 1	Antenna 2
00	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
10	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
01	$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$
11	$\frac{-1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$

from antennas one and two. Suppose that the input to the encoder at time $2t+1$ is the block of bits 10. Since $\mathcal{M}(10) = (0\ 1)$, we have $A(10) = 0$ and $B(10) = 1$. Then the values s_{2t+1} and s_{2t+2} corresponding to input bits 10 are computed as follows:

$$(s_{2t+1}\ s_{2t+2}) = 0 \cdot \left(\frac{1}{\sqrt{2}}\ \frac{-1}{\sqrt{2}} \right) + 1 \cdot \left(\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}} \right)$$
$$= \left(\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}} \right).$$

Thus, at time $2t+1$, $s_{2t+1} = 1/\sqrt{2}$ and $s_{2t+2} = 1/\sqrt{2}$ are sent, respectively, from antennas one and two, and at time $2t + 2$, $-s_{2t+2}^* = -1/\sqrt{2}$ and $s_{2t+1}^* = 1/\sqrt{2}$ are sent, respectively, from antennas one and two.

We have computed the transmitted symbols at time $2t+1$ and $2t + 2$ corresponding to the input bits 00, 10, 01 and 11 for this scenario. The results are summarized in Tables I and II.

IV. DIFFERENTIAL DECODING

For notational simplicity, we will first present the results for one receive antenna. We will write r_t for r_t^1, η_t for η_t^1 and α_1, α_2, respectively, for $\alpha_{1,1}$, $\alpha_{2,1}$ knowing that this can cause no confusion since there is only one receive antenna.

Let us assume that signals r_{2t-1}, r_{2t}, r_{2t+1}, and r_{2t+2} are received. Let

$$\Lambda(\alpha_1, \alpha_2) = \begin{pmatrix} \alpha_1 & \alpha_2^* \\ \alpha_2 & -\alpha_1^* \end{pmatrix} \qquad (12)$$

and

$$N_{2t-1} = (\eta_{2t-1}\ \eta_{2t}^*). \qquad (13)$$

The receiver recalls that

$$(r_{2t-1}\ r_{2t}^*) = (s_{2t-1}\ s_{2t})\Lambda(\alpha_1, \alpha_2) + N_{2t-1} \qquad (14)$$

and

$$(r_{2t+1}\ r_{2t+2}^*) = (s_{2t+1}\ s_{2t+2})\Lambda(\alpha_1, \alpha_2) + N_{2t+1}. \qquad (15)$$

Thus,

$$\begin{aligned}
&(r_{2t+1}\ r_{2t+2}^*) \cdot (r_{2t-1}\ r_{2t}^*) \\
&= (s_{2t+1}\ s_{2t+2})\Lambda(\alpha_1, \alpha_2)\Lambda^*(\alpha_1, \alpha_2)(s_{2t-1}^*\ s_{2t}^*) \\
&\quad + (s_{2t+1}\ s_{2t+2})\Lambda(\alpha_1, \alpha_2)N_{2t-1}^* \\
&\quad + N_{2t+1}\Lambda^*(\alpha_1, \alpha_2)(s_{2t-1}\ s_{2t})^* + N_{2t+1}N_{2t-1}^*.
\end{aligned}$$

It follows that

$$\begin{aligned}
&r_{2t+1}r_{2t-1}^* + r_{2t+2}^*r_{2t} \\
&= (|\alpha_1|^2 + |\alpha_2|^2)(s_{2t+1}s_{2t-1}^* + s_{2t+2}s_{2t}^*) \\
&\quad + (s_{2t+1}\ s_{2t+2})\Lambda(\alpha_1, \alpha_2)N_{2t-1}^* \\
&\quad + N_{2t+1}\Lambda^*(\alpha_1, \alpha_2)(s_{2t-1}\ s_{2t})^* + N_{2t+1}N_{2t-1}^*. \quad (16)
\end{aligned}$$

For notational simplicity, we let

$$\mathcal{R}_1 = r_{2t+1}r_{2t-1}^* + r_{2t+2}^*r_{2t} \qquad (17)$$

$$\begin{aligned}
\mathcal{N}_1 &= (s_{2t+1}\ s_{2t+2})\Lambda(\alpha_1, \alpha_2)N_{2t-1}^* \\
&\quad + N_{2t+1}\Lambda^*(\alpha_1, \alpha_2)(s_{2t-1}\ s_{2t})^* + N_{2t+1}N_{2t-1}^* \quad (18)
\end{aligned}$$

then we have

$$\mathcal{R}_1 = (|\alpha_1|^2 + |\alpha_2|^2)A(\mathcal{B}_{2t-1}) + \mathcal{N}_1. \qquad (19)$$

Next, the receiver looks for the second vector term in the right side of (11) and recalls that

$$(r_{2t} - r_{2t-1}^*) = (-s_{2t}^*\ s_{2t-1}^*)\Lambda(\alpha_1, \alpha_2) + N_{2t} \qquad (20)$$

where

$$N_{2t} = (\eta_{2t} - \eta_{2t-1}^*). \qquad (21)$$

It follows that

$$\begin{aligned}
&(r_{2t+1}\ r_{2t+2}^*) \cdot (r_{2t} - r_{2t-1}^*) \\
&= (s_{2t+1}\ s_{2t+2})\Lambda(\alpha_1, \alpha_2)\Lambda^*(\alpha_1, \alpha_2)(-s_{2t}\ s_{2t-1}) \\
&\quad + (s_{2t+1}\ s_{2t+2})\Lambda(\alpha_1, \alpha_2)N_{2t}^* \\
&\quad + N_{2t+1}\Lambda^*(\alpha_1, \alpha_2)(-s_{2t}^*\ s_{2t-1}^*)^* + N_{2t+1}N_{2t}^*.
\end{aligned}$$

Thus,

$$\begin{aligned}
&r_{2t+1}r_{2t}^* - r_{2t+2}^*r_{2t-1} \\
&= (|\alpha_1|^2 + |\alpha_2|^2)(-s_{2t+1}s_{2t}^* + s_{2t+2}s_{2t-1}^*) \\
&\quad + (s_{2t+1}\ s_{2t+2})\Lambda(\alpha_1, \alpha_2)N_{2t}^* \\
&\quad + N_{2t+1}\Lambda^*(\alpha_1, \alpha_2)(-s_{2t}^*\ s_{2t-1}^*)^* + N_{2t+1}N_{2t}^*. \quad (22)
\end{aligned}$$

For notational simplicity, we let

$$\mathcal{R}_2 = r_{2t+1}r_{2t}^* - r_{2t+2}^*r_{2t-1} \qquad (23)$$

$$\begin{aligned}
\mathcal{N}_2 &= (s_{2t+1}\ s_{2t+2})\Lambda(\alpha_1, \alpha_2)N_{2t}^* \\
&\quad + N_{2t+1}\Lambda^*(\alpha_1, \alpha_2)(-s_{2t}^*\ s_{2t-1}^*)^* + N_{2t+1}N_{2t}^*. \quad (24)
\end{aligned}$$

Thus, we have

$$\mathcal{R}_2 = (|\alpha_1|^2 + |\alpha_2|^2)B(\mathcal{B}_{2t-1}) + \mathcal{N}_2. \qquad (25)$$

We can thus write

$$\begin{aligned}
(\mathcal{R}_1\ \mathcal{R}_2) &= (|\alpha_1|^2 + |\alpha_2|^2)(A(\mathcal{B}_{2t-1})\ B(\mathcal{B}_{2t-1})) \\
&\quad + (\mathcal{N}_1\ \mathcal{N}_2). \quad (26)
\end{aligned}$$

Because the elements of $\mathcal{V}$ have equal length, to compute $(A(\mathcal{B}_{2t-1})\ B(\mathcal{B}_{2t-1}))$, the receiver now computes the closest vector of $\mathcal{V}$ to $(\mathcal{R}_1\ \mathcal{R}_2)$. Once this vector is computed, the
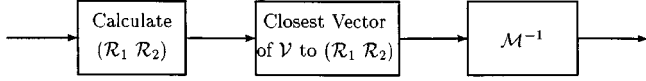
Fig. 2. Receiver block diagram.

inverse mapping of $\mathcal{M}$ is applied and the transmitted bits are recovered.

For the multiplicative coefficient $|\alpha_1|^2 + |\alpha_2|^2$ in (26) to be small, both $|\alpha_1|$ and $|\alpha_2|$ have to be small. This means that both subchannels from transmit antennas 1 and 2 to the receive antenna must undergo fading. In this light, the decoder suffers from the detrimental effect of fading only if both subchannels from transmit antennas 1 and 2 to the receive antenna have small path gains. This means that the decoder enjoys a two-level transmit diversity gain. This physical argument can be rigorized, albeit by employing tedious mathematical techniques. Indeed, from the resemblance of (26) to an analogous formula for maximum ratio combining, it can be proved that the above detection method provides two-level diversity assuming 2 transmit and one receive antennas.

The same procedure can be used for more than one receive antenna. For each receive antenna j, we compute $\mathcal{R}_1^j$ and $\mathcal{R}_2^j$, using the same method for $\mathcal{R}_1$ and $\mathcal{R}_2$ given above, assuming only receiver antenna j exists. Then the closest vector of $\mathcal{V}$ to $\left(\sum_{j=1}^m \mathcal{R}_1^j \sum_{j=1}^m \mathcal{R}_2^j\right)$ is computed. Subsequently, the transmitted bits are computed by applying the inverse mapping of $\mathcal{M}$. It must be again clear that $2m$-level diversity is achieved.

The block diagram of the receiver is given in Fig. 2.

Remark: For the differential encoding and decoding methods presented above to work, it was required that the set $\mathcal{V}$ have Properties A and B. It is possible to relax Property A even more and work with a set $\mathcal{V}$ with an arbitrary number, say L, of vectors having unit lengths. In such a scenario, the mapping $\mathcal{M}$ is required to be a bijective mapping of blocks of $\log_2 L$ bits to $\mathcal{V}$. Applying this minor change, the methods of encoding and decoding presented in this paper will continue to function effectively.

V. PERFORMANCE ANALYSIS

Equation (26) is the vector analog of [7, equation (4.2.113), p. 268] for DPSK, assuming one transmit antenna. The same analysis can be modified to prove that the performance of the proposed differential detection scheme is 3 dB worse than that of the transmit diversity scheme of [1] (which employs coherent detection) at high signal-to-noise power ratios. Similarly, the analysis in [8] can be refined to compute the bit error rate of this system. This 3 dB penalty can also be physically justified from (26) by noticing that the multiplicative term $N_{2t+1}N_{2t-1}^*$ in (18) can be ignored at high signal-to-noise power ratios, as it is much smaller compared to the other terms. Similarly, the term $N_{2t+1}N_{2t}^*$ in (24) can be ignored at high signal-to-noise power ratios. The two remaining terms in (18) and (24) double the power of noise in (26) as compared to the coherent detection. This doubling of the noise power is equivalent to the aforementioned 3 dB loss.

In Figs. 3–5, we present simulation results for both the performance of the coherent detection two-level transmit diversity
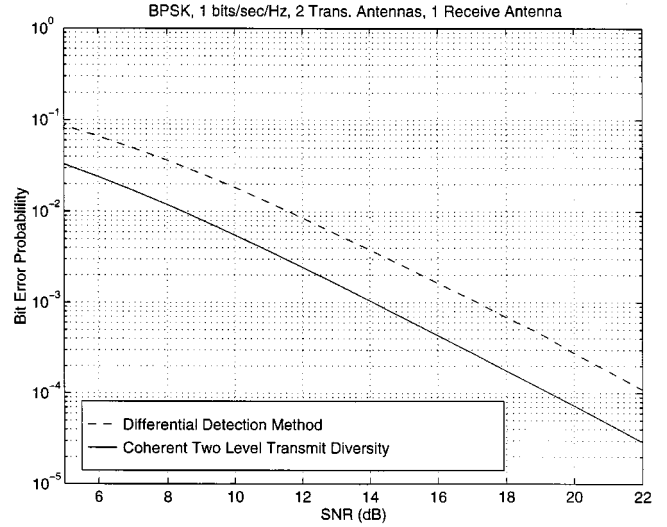

Fig. 3. Performance of the differential detection and coherent detection: two-level transmit diversity scheme for BPSK constellation.
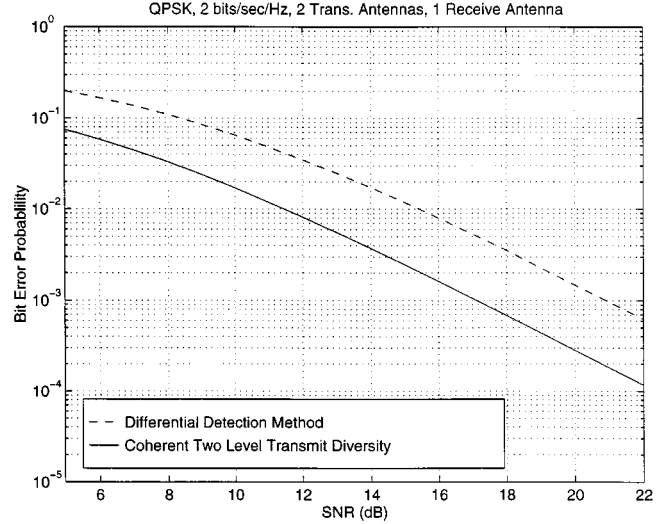

Fig. 4. Performance of the differential detection and coherent detection: two-level transmit diversity scheme for QPSK constellation.
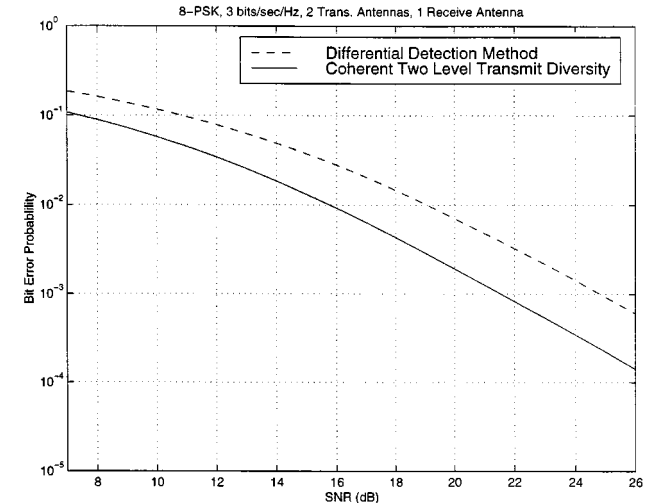

Fig. 5. Performance of the differential detection and coherent detection: two-level transmit diversity scheme for 8-PSK constellation.

scheme and the proposed differential detection method. The results are presented for BPSK, QPSK, and 8-PSK constellations. The framization is that of IS-54 standard. The average receive signal-to-noise power ratio per transmission is denoted by SNR. The fading is assumed to be constant over each frame and vary from one frame to another. The 3 dB loss due to noncoherent detection can also be observed from these simulation results.

VI. Conclusion and Final Remarks

We presented a differential detection transmit diversity method where neither the receiver nor the transmitter has access to channel state information. We provided evidence that the proposed method is 3 dB worse than two-level transmit diversity combining [1]. The underlying constellation may be assumed to be a 2^b-PSK constellation for $b = 1, 2, 3, \cdots$. This differential detection transmit diversity method was presented for the case that the channel is quasistatic and nonfrequency selective. Naturally, it is expected that the performance of this scheme degrades when the channel is frequency selective.

There are two main reasons that the differential detection transmit diversity method presented here works for 2 transmit antennas, as follows.

1) A full rate 2×2 complex orthogonal design exists [13]. Thus, the signals to be transmitted at times $2t + 1$ and $2t + 2$ can be expressed in terms of the linear combination of those of $2t - 1$ and $2t$. The coefficients of this linear expansion are determined by the transmitted data.
2) The elements of the set $\mathcal{V}$ have equal lengths. In this light, after an appropriate combining, minimum distance decoding can be applied.

It is a nontrivial task to extend the differential detection transmit diversity method described in this paper to $n > 2$ transmit antennas. In doing so, we have employed the theory of generalized orthogonal designs. The presentation of such differential detection methods is the topic of a forthcoming paper [15].

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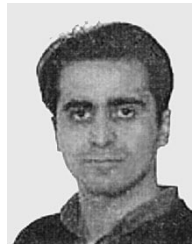
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