

## A DIFFERENTIAL GEOMETRY-BASED NEURODYNAMICAL CLASSIFIER

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**Tijana T. Ivancevic<sup>1,2</sup>, Charles E.M. Pearce<sup>1</sup>,  
Murk Bottema<sup>3</sup>, Lakhmi C. Jain<sup>2</sup>**

<sup>1</sup>University of Adelaide,

<sup>2</sup>University of South Australia,

<sup>3</sup>Flinders University of South Australia, Australia

e-mail: tivancevic@otpusnet.com.au

**Abstract.** *A new model for a neurodynamical classifier is proposed. The classifier is viewed as a generalized bi-directional associative memory (GBAM) [11] and is described in the language of differential geometry [12-14]. GBAM is a tensor-field system resembling a two-phase biological neural oscillator in which an excitatory neural field excites an inhibitory neural field, which reciprocally inhibits the excitatory one. GBAM equations have been directly implemented in the computer algebra system 'Mathematica' and tested on two different sets of data related to detection of breast cancer. • The GBAM classifier outperformed other neural classifiers.*

**Key words:** *neurodynamics, differential geometry, classification*

### 1. INTRODUCTION

We propose a new model for a neurodynamical classifier written in the language of differential geometry. The model is tensor invariant and as a result, can be used as a classifier, controller, as well as a plausible model for the brain-like self-organizing associative-memory system. It gives a tensor-invariant model of neural networks, based on well-known differential geometry. To our knowledge this is the first direct application of differential geometry to artificial intelligence.

This approach provides a generalization of neural network models proposed by Hopfield, Cohen-Grossberg, Hecht-Nielsen, and Kosko, namely

- (i) Hopfield continuous network [1],
- (ii) Cohen-Grossberg adaptive resonance theory (ART) of asymptotically stable cognitive systems [2],
- (iii) Hecht-Nielsen counter-propagation network [3], and
- (iv) Kosko ABAM and RABAM - associative memory networks [4].

The focus in this study is on a particular classifier called a generalized bi-directional associative memory (GBAM) neurodynamical classifier. GBAM is a tensor-field system resembling a two-phase biological neural oscillator in which an excitatory neural field excites an inhibitory neural field, which reciprocally inhibits the excitatory one. This is a new generalization of Kosko ABAM and RABAM associative memory networks [4], with a new biological (oscillatory, i.e., excitatory/inhibitory) interpretation. The model includes two nonlinearly coupled (yet non-chaotic and Lyapunov stable) subsystems:

1) Activation dynamics, including two neuronal 1-dimensional tensor-fields, an excitatory vector field and an inhibitory one-form - with sigmoid (tanh) activation functions; and

2) Self-organized learning dynamics, including a symmetric synaptic 2-dimensional tensor-field, updated by differential Hebbian associative learning innovations.

Biologically, GBAM describes an oscillatory neuro-synaptic tensor-field composed of interacting excitatory and inhibitory populations of neurons as found in the cerebellum, olfactory cortex, and neocortex, all representing the basic mechanisms for the generation of oscillating (EEG-monitored) activity in the brain. The learning dynamics is itself a 2-dimensional tensor-field including four different Hebbian learning schemes (signal, differential, random and their combination).

The GBAM neuro-classifier model was implemented in computer algebra system "Mathematica" with the common output from the two activation neural fields given as a mean-field function. Three different network dimensions, with 2,3 and 4 continual neurons in each neural field, were used for simulations.

The classifier was tested on data from research projects on early detection of breast cancer. Since the late 1980's, many studies have been reported on the use of artificial intelligence (AI) systems for improving the performance of breast cancer screening programs and reducing their cost [5,6]. Classifiers based on neural networks are popular in this field because the models for both the signal and the noise in mammograms are usually poor and the relationships between various features are not well understood. The GBAM classifier was tested on two data sets associated with the detection of breast cancer. One data set pertains to the detection of cancer in fine needle aspirates (FNA) and the other pertains to the automatic detection of calcifications in screening mammograms.

## II. THE CLASSIFIER MODEL

### II.A. The concept of the GBAM classifier

The GBAM system introduced here resembles a two-phase biological neural oscillator in which an excitatory neural field excites an inhibitory neural field, which reciprocally inhibits the excitatory one. Mathematically, the GBAM is a tensor field system  $(\mathbf{q}, \mathbf{p}, \mathbf{W})$  defined on a manifold  $M$  called the GBAM manifold. (Definitions of tensor fields, manifolds and other constructions from differential geometry can be found in [7].) The system  $(\mathbf{q}, \mathbf{p}, \mathbf{W})$  includes two nonlinearly coupled (yet non-chaotic and stable) subsystems (see Figure 1):

- (i) Activation  $(\mathbf{q}, \mathbf{p})$  -dynamics, where  $\mathbf{q}$  and  $\mathbf{p}$  represent neuronal 1-dimensional tensor-fields, and
- (ii) Self-organized learning  $\mathbf{W}$ -dynamics, where  $\mathbf{W}$  is a symmetric synaptic 2-dimensional tensor-field.

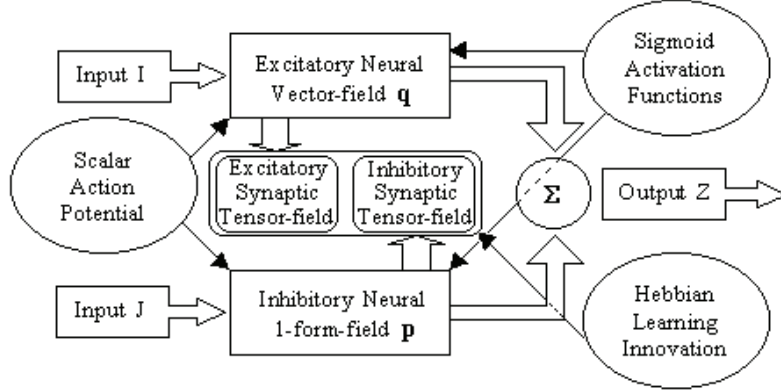


Fig. 1. Diagram of the GBAM neuro-classifier dynamics

### II.B. GBAM-activation $(\mathbf{q}, \mathbf{p})$ -dynamics

The GBAM-manifold  $M$  can be viewed as a Banach space with a  $C^\infty$ -differentiable structure on it, so that in each local chart  $U$  open in  $M$ , an  $n$ -dimensional smooth coordinate system  $U_\alpha$  exists.

GBAM-activation  $(\mathbf{q}, \mathbf{p})$ -dynamics, is defined as a system of two coupled, first-order oscillator tensor-fields, dual to each other, in a local Banach chart  $U_\alpha$ ,  $\alpha = 1, \dots, n$  on  $M$ :

- (i) An excitatory neural vector-field  $q^\alpha = q^\alpha(t) : M \rightarrow TM$ , being a cross-section of the tangent bundle  $TM$ ; and
- (ii) An inhibitory neural 1-form field  $p_\alpha = p_\alpha(t) : M \rightarrow T^*M$ , being a cross-section of the cotangent bundle  $T^*M$ .

To start with conservative linear  $(\mathbf{q}, \mathbf{p})$ -system, we postulate the GBAM scalar activation-potential  $V$  to be a negative bilinear form:

$$V = -\frac{1}{2} \sum_{\alpha=1}^n \sum_{\beta=1}^n \omega_{\alpha\beta} q^\alpha q^\beta - \frac{1}{2} \sum_{\alpha=1}^n \sum_{\beta=1}^n \omega^{\alpha\beta} p_\alpha p_\beta + \sum_{\alpha=1}^n q^\alpha p_\alpha, \quad (1)$$

where  $n$  is the number of neurons in each neural field, while  $\omega_{\alpha\beta}$  and  $\omega^{\alpha\beta}$  represent respectively inhibitory-covariant and excitatory-contravariant components of the symmetric (with zero-trace) coupling GBAM synaptic tensor  $\mathbf{W}$ .

The Lyapunov-stable, conservative, linear  $(\mathbf{q}, \mathbf{p})$ -dynamics is given as a bi-directional (excitatory-inhibitory) gradient system:

$$\begin{aligned} \dot{q}^\alpha &= -\frac{\partial V}{\partial p_\alpha} = \sum_{\beta=1}^n \omega^{\alpha\beta} p_\beta - q^\alpha \\ \dot{p}_\alpha &= -\frac{\partial V}{\partial q^\alpha} = \sum_{\beta=1}^n \omega_{\alpha\beta} q^\beta - p_\alpha \end{aligned}, \quad (2)$$

(here and in the following text, the overdot denotes time derivative).

As  $\mathbf{W}$  is a symmetric and zero-trace synaptic coupling tensor, the conservative linear dynamics (2) is equivalent to the rule that the state of each neuron (in both excitatory and inhibitory neural fields) is changed in time if and only if the scalar action potential  $V$ , defined by relation (1), is lowered. Therefore, the scalar action potential  $V$  is a monotonically non-increasing Lyapunov function  $\dot{V} \leq 0$  for the conservative linear  $(\mathbf{q}, \mathbf{p})$ -dynamics (2), which converges to a local minimum or ground state of  $V$ .

Applying the inputs  $I^\alpha$  and  $J_\alpha$ , we get the non-conservative linear  $(\mathbf{q}, \mathbf{p})$ -system equations:

$$\begin{aligned} \dot{q}^\alpha &= I^\alpha + \sum_{\beta=1}^n \omega^{\alpha\beta} p_\beta - q^\alpha \\ \dot{p}_\alpha &= J_\alpha + \sum_{\beta=1}^n \omega_{\alpha\beta} q^\beta - p_\alpha \end{aligned} \quad (3)$$

Further, applying the sigmoid GBAM activation functions  $S_\alpha(\cdot)$  and  $S^\alpha(\cdot)$  to the synaptic product-terms, we get the non-conservative nonlinear  $(\mathbf{q}, \mathbf{p})$ -system equations, which generalize the transient RC-circuit neurodynamical model (see [4,8]):

$$\begin{aligned} \dot{q}^\alpha &= I^\alpha + \sum_{\beta=1}^n \omega^{\alpha\beta} S_\beta(p_\beta) - q^\alpha \\ \dot{p}_\alpha &= J_\alpha + \sum_{\beta=1}^n \omega_{\alpha\beta} S^\beta(q^\beta) - p_\alpha \end{aligned} \quad (4a)$$

The equations in (4a) apply to two-feature data. The generalization to an  $N$ -dimensional feature space is given by

$$\begin{aligned} \dot{q}_e^\alpha &= I_e^\alpha + \sum_{\beta=1}^n \omega_e^{\alpha\beta} S_\beta(p_\beta) - q_e^\alpha \\ \dot{p}_\alpha^o &= I_\alpha^o + \sum_{\beta=1}^n \omega_{\alpha\beta}^o S^\beta(q^\beta) - p_\alpha^o \end{aligned} \quad (4b)$$

where  $e = 2, 4, \dots, N$  and  $o = 1, 3, \dots, N-1$  denote respectively even and odd partitions of the total sample of  $N$  features.

In AI parlance, GBAM model (4a-4b) gives a generalization of four well-known recurrent neural-network models:

- (i) Continuous Hopfield amplifier-circuit model [1]

$$C_j \dot{v}_j = I_j - \frac{v_j}{R_j} + \sum_{i=1}^N T_{ji} u_i, \quad j = 1, \dots, N,$$

where  $v_j = v_j(t)$  represent the activation potentials in the  $j$  th processing unit,  $C_j$  and  $R_j$  denote input capacitances and leakage resistances,  $u_i = f_i(v_i(t))$  are output functions from processing elements, and  $T_{ij} = w_{ij}$  is the inverse of the resistors connection-matrix; and the functions  $f_i$  are sigmoidal.

- (ii) Cohen-Grossberg general ART-system [2],

$$\dot{v}_j = -a_j(v_j)[b_j(v_j) - \sum_{k=1}^N f_k(v_k) m_{jk}], \quad j = 1, \dots, N,$$

with proved asymptotical stability.

(iii) Hecht-Nielsen counter-propagation network [3],

$$\dot{v}_j = -A v_j + (B - v_j) I_j - v_j \sum_{k \neq j} I_k, \quad j = 1, \dots, N,$$

where  $A, B$  are positive constants and  $I_j$  are input values for each processing unit.

(iv) Kosko's BAM (ABAM and RABAM) [4]

$$\begin{aligned} \dot{v}_j &= -a_j(v_j) [b_j(v_j) - \sum_{k=1}^N f_k(v_k) m_{jk}], \quad j = 1, \dots, N \\ \dot{u}_k &= -a_k(u_k) [b_k(u_k) - \sum_{j=1}^N f_j(u_j) m_{jk}], \quad k = 1, \dots, N \end{aligned}$$

which is globally stable for the cases of signal and random-signal Hebbian learning.

### II.C. Self-organized learning $\mathbf{W}$ -dynamics

The continuous and smooth (at least  $C^1$ -differentiable) unsupervised update law for the coupling synaptic GBAM tensor-field  $\mathbf{W}$  can be viewed both as an inhibitory-covariant Hebbian learning scheme, generalized from [4]:

$$\dot{\omega}_{\alpha\beta} = -\omega_{\alpha\beta} + \Phi_{\alpha\beta}(q^\alpha, p_\alpha), \quad (5a)$$

and, as an excitatory-contravariant Hebbian learning scheme:

$$\dot{\omega}^{\alpha\beta} = -\omega^{\alpha\beta} + \Phi^{\alpha\beta}(q^\alpha, p_\alpha) \quad (5b)$$

where the three terms from the left to the right denote respectively the new-update value, the old value and the innovation of the synaptic tensor  $\mathbf{W}$ . In this case the nonlinear (usually sigmoid) innovation functions  $\Phi_{\alpha\beta}$  and  $\Phi^{\alpha\beta}$  are defined by one of following four Hebbian models:

(i) Signal Hebbian learning, with innovation in both variance-forms:

$$\begin{aligned} \Phi_{\alpha\beta} &= S_\alpha(q^\alpha) S_\beta(p_\beta), \\ \Phi^{\alpha\beta} &= S^\alpha(q^\alpha) S^\beta(p_\beta) \end{aligned} \quad (6)$$

(ii) Differential Hebbian learning, with innovation in both variance-forms:

$$\begin{aligned} \Phi_{\alpha\beta} &= S_\alpha(q^\alpha) S_\beta(p_\beta) + \dot{S}_\alpha(q^\alpha) \dot{S}_\beta(p_\beta), \\ \Phi^{\alpha\beta} &= S^\alpha(q^\alpha) S^\beta(p_\beta) + \dot{S}^\alpha(q^\alpha) \dot{S}^\beta(p_\beta) \end{aligned} \quad (7)$$

(terms with overdots are called "signal velocities").

(iii) Random signal Hebbian learning, with innovation in both variance-forms:

$$\begin{aligned} \Phi_{\alpha\beta} &= S_\alpha(q^\alpha) S_\beta(p_\beta) + n_{\alpha\beta}, \\ \Phi^{\alpha\beta} &= S^\alpha(q^\alpha) S^\beta(p_\beta) + n^{\alpha\beta} \end{aligned} \quad (8)$$

where  $n_{\alpha\beta} = \{n_{\alpha\beta}(t)\}$ ,  $n^{\alpha\beta} = \{n^{\alpha\beta}(t)\}$  respectively denote covariant and contravariant additive, zero-mean, Gaussian white-noise processes independent of the main innovation signal; and

- (iv) Random differential signal Hebbian learning, with innovation in both variance-forms:

$$\begin{aligned}\Phi_{\alpha\beta} &= S_{\alpha}(q^{\alpha})S_{\beta}(p_{\beta}) + \dot{S}_{\alpha}(q^{\alpha})\dot{S}_{\beta}(p_{\beta}) + n_{\alpha\beta}, \\ \Phi^{\alpha\beta} &= S^{\alpha}(q^{\alpha})S^{\beta}(p_{\beta}) + \dot{S}^{\alpha}(q^{\alpha})\dot{S}^{\beta}(p_{\beta}) + n^{\alpha\beta}\end{aligned}\quad (9)$$

#### II.D. Total GBAM ( $\mathbf{q}, \mathbf{p}, \mathbf{W}$ ) -neurodynamics and biological interpretation

Total GBAM tensorial neurodynamics is defined as a union of neural oscillatory activation ( $\mathbf{q}, \mathbf{p}$ ) -dynamics (4a-4b) and synaptic learning  $\mathbf{W}$ -dynamics (5a-5b):

$$\begin{aligned}\dot{q}_e^{\alpha} &= I_e^{\alpha} + \sum_{\beta=1}^n \omega_e^{\alpha\beta} S_{\beta}(p_{\beta}) - q_e^{\alpha} \\ \dot{p}_{\alpha}^o &= I_{\alpha}^o + \sum_{\beta=1}^n \omega_{\alpha\beta}^o S^{\beta}(q^{\beta}) - p_{\alpha}^o \\ \dot{\omega}_e^{\alpha\beta} &= -\omega_e^{\alpha\beta} + \Phi_e^{\alpha\beta}(q^{\alpha}, p_{\alpha}), \\ \dot{\omega}_{\alpha\beta}^o &= -\omega_{\alpha\beta}^o + \Phi_{\alpha\beta}^o(q^{\alpha}, p_{\alpha}),\end{aligned}\quad (10)$$

where the tensorial innovation  $\Phi$ -functions are given by one of Hebbian models (6-9),  $\alpha = 1, \dots, n$  is the number of continual-time neurons in each neural-activation field,  $e = 2, 4, \dots, N$  and  $o = 1, 3, \dots, N-1$  denote respectively even and odd partitions of the total sample of  $N$  features.

Artificial neural networks are generally inspired by biological neural systems, but in fact, some important features of biological systems are not present in most artificial neural networks. In particular, uni-directional neural networks, which include all associative neural networks except the BAM model introduced by Kosko [4], do not resemble oscillatory biological neural systems. GBAM is a generalization of Kosko's ABAM and RABAM neural systems and inherits their oscillatory (excitatory/inhibitory) neuro-synaptic behavior. Such oscillatory behavior is a basic characteristic of a number of biological systems. Examples oscillatory neural ensembles: in the human nervous system are:

- (i) Motoneurons and Renshaw interneurons in the spinal cord;
- (ii) Pyramidal and basket cells in the hippocampus;
- (iii) Mitral and granule cells in the olfactory bulb;
- (iv) Pyramidal cells and thalamic inter-neurons in cortico-thalamic system;
- (v) Interacting excitatory and inhibitory populations of neurons found in the cerebellum, olfactory cortex, and neocortex, all representing the basic mechanisms for the generation of oscillating (EEG-monitored) activity in the brain.
- (vi) So, GBAM can be considered as a model for any of above-mentioned oscillatory biological neural systems.

## III. CLASSIFICATION RESULTS AND DISCUSSION

## III.A. GBAM classification

The full GBAM neuro-classifier as described by equations (10), was tested on two sets of data related to the diagnosis of breast cancer. The first dataset, from the University of Wisconsin, comprises ten features (radius, perimeter, fractal dimension, etc.) measured on the nuclei of cells in fine needle aspiration (FNA) slides [9]. The shape and texture of cell nuclei in FNA samples are used to determine the disease state of suspicious tissue. The second dataset comprises four features measured at locations of suspected microcalcifications in digital mammograms [10]. To test the simplified and computationally significantly cheaper version of GBAM as given by equations (4a), the two features were selected (volume and radius) of the candidate calcifications. In this case the objective was to classify each candidate location as a true microcalcifications or as noise.

In each of the tests, the features were entered without scaling or other pre-processing. The classification was performed with three different network dimensions:  $n = 2, 3, 4$ , where  $n$  denotes number of neurons in each  $(\mathbf{q}, \mathbf{p})$  -field. The derivation of the activation and learning equations, as well as their digital simulation was performed in the computer algebra system “Mathematica” (version 4). The numerical solution of the corresponding initial-value problems were performed using the Mathematica function NDSolve, one of the most powerful ODE-integrators for both non-stiff and stiff high-dimensional systems. The duration of the simulation for each individual case was 0.04 seconds. The starting step-size was  $10^{-7}$  seconds. As output  $Z$  the mean-field of outputs of the two activation vector fields,

$$Z_{AM} = \frac{1}{2n} \left[ \sum_{\alpha=1}^n q^{\alpha}(t_{FIN}) + \sum_{\beta=1}^n p_{\beta}(t_{FIN}) \right], \quad (11)$$

was used, where  $t_{FIN} = 0.04$  denotes the final time for each simulation.

Agreement matrices for the classification results are given in Tables 1-2.

Table 1 Statistics (contingency table) for GBAM neuro-classifier from the University of Wisconsin

Classifier result	Actual benign	Actual malignant	Total
Classified as benign	32	11	43
Classified as malignant	3	54	57
Total	35	65	100
Sensitivity		83.07%	
Specificity	91.42%		

Of 100 cases, where 35 were benign and 65 were malignant, the GBAM neuro-classifier classified 32 as benign and 54 as malignant (Table I). The reliability of classifiers were compared by the following numbers: *Sensitivity = correct positive (malignant) / total positive and Specificity = correct negative (benign) / total negative*. Therefore, the sensitivity of GBAM neuro-classifier is 83.07%, and the specificity is 91.42%.

Table 2 Statistics (contingency table) for GBAM neuro-classifier: mammography database

Classifier result	Actual false calcification	Actual true calcification	Total
Classified as false calc.	9303	24	9327
Classified as true calc.	697	95	792
Total	10000	119	10119
Sensitivity		79.83%	
Specificity	93.03%		

Of 10119 pixels from the mammography dataset, where 10000 were false calcification and 119 were true calcification, the GBAM neuro-classifier classified 9303 as false calcification and 95 as true calcification (Table II). The sensitivity of GBAM neuro-classifier on mammography database is 79.83%, and the specificity is 93.03%.

### III.B. Data Test on the standard MLP and comparison with GBAM

To compare the performance of the GBAM classifier with a well-known neural classifier, the experiments described in the previous section were repeated using a multilayer perceptron (MLP) trained with backpropagation algorithm (see, for instance, [3]). The feedforward neural network was made of three layers (input layer with two neurons, hidden layer and output layer). The output layer contained one neuron for classification (with output 0 or 1; 0 = benign, 1 = malignant - for the Wisconsin database, and 0=false calcification and 1=true calcification for mammography database). Different backpropagation learning algorithms were used: On-Line BACKPROP, RPROP and Quick-Prop. Different activation functions were used: logistic, tanh(.), linear and Gaussian. The total number of completed epochs was 100,000. One epoch was defined as one presentation of the entire training data set to the network. The learning rate was 0.01.

A linear transfer function was used for the input layer. For the hidden layer, several transfer function were used: linear, hyperbolic tangent, Gaussian and logistic function. The logistic function was used for the output layer. All features values were recorded with four significant digits.

Table 3 Experimental evaluation and comparison for the dataset from the University of Wisconsin

Training algorithm	Activation function in the hidden layer	MAX RMS error	Percent correct
On-line backpropagation	logistic	0.9607	79%
On-line backpropagation	tanh	0.9686	80%
On-line backpropagation	linear	1.0294	20%
On-line backpropagation	Gaussian	1	35%
RPROP	logistic	0.9884	81%
RPROP	tanh	0.9587	78%
GBAM	*****	*****	87%

In all cases results with GBAM classifier are better than with different types of MLP.



Table 4 Experimental evaluation and comparison for the mamography dataset

Training algorithm	Activation function in the hidden layer	MAX RMS error	Percent correct
On-line backpropagation	logistic	1	84.80%
On-line backpropagation	tanh	0.9998	84.37%
On-line backpropagation	linear	1	86.08%
On-line backpropagation	Gaussian	0.9965	79.87%
RPROP	logistic	0.9997	81.8
RPROP	tanh	1	83.73
GBAM	*****	*****	92.87

As shown in Tables 3 and 4, the classification results obtained with the GBAM neuro-classifier, were better than all types of BACKPROP-MLP classifier for both datasets.

#### IV. CONCLUSION

In this study we have developed a new neurodynamical classifier, using the tensor-invariant formalism from differential geometry. This new theoretical model of neurodynamics is based on bio-physically realistic excitatory/inhibitory neural activation and self-organized learning dynamics. The model generalizes standard associative neural models of Hopfield, Cohen-Grossberg, Hecht-Nielsen, and Kosko. The GBAM model was tested on two breast-cancer datasets and compared with MLP classification.

The GBAM classifier was implemented in the computer algebra system “Mathematica” and on datasets for classifying cell nuclei in FNA and slides for detecting microcalcifications in digital mammograms. The key results are:

- (1) A low number of benign examples were classified as malignant
- (2) GBAM classifiers outperformed other neural classifiers, and
- (3) Classification results are invariant of the number dimension and the output-form of the neural activation field. The last result is a natural characteristic of the tensor-invariant form of the structure of the classifier.

These results indicate the potential for developing new classifiers from a relatively abstract view which are, none the less, easy to implement by direct use of a computer algebra system.

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## NEURODINAMIČKI KLASIFIKATOR BAZIRAN NA DIFERENCIJALNOJ GEOMETRIJI

**Tijana T. Ivancevic, Charles E.M. Pearce,  
Murk Bottema, Lakhmi C. Jain**

*Predložen je novi model neurodinamičkog klasifikatora. Ovaj klasifikator predstavlja generalizaciju asocijativne memorije u dva pravca (GBAM) [11] i opisan je jezikom diferencijalne geometrije. GBAM sistem je formalno definisan tenzorskim poljem koje modelira dvo-fazni biološki nervni oscilator, u kojem ekscitatorno nervno polje pobugjuje inhibitorno nervno polje, koje povratno inhibira ekscitatorno polje. Jednačine GBAM-a su direktno implementirane u sistemu kompjuterske algebre "Mathematica" i testirane na dva različita skupa podataka za detekciju karcinoma dojke. GBAM klasifikator se pokazao superiornijim u odnosu na druge neuralne klasifikatore.*

**Ključne reči:** *Neurodinamika, diferencijalna geometrija, klasifikacija*