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A Directed Search Model of Inequality with Heterogeneous Skills and Skill-Biased Technology

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In this paper I analyse the directed search/matching problem in an economy with heterogeneous skills and skill-biased technology. A unique symmetric equilibrium exists and is socially efficient. Matching is partially mixed in the equilibrium. A high-tech firm receives both skilled and unskilled applicants with positive probability, and favours skilled workers, while a low-tech firm receives only unskilled applicants. The model generates wage inequality among identical unskilled workers, as well as between-skill inequality, despite the fact that all unskilled workers perform the same task and have the same productivity in the two types of firms. Inequality has interesting responses to skill-biased technological progress, a general productivity slowdown, and an exogenous increase in the skill supply elasticity.

1. INTRODUCTION

Wage inequality among U.S. workers has grown significantly in the last few decades. The log weekly wage differential between the 90th and the 10th percentile of male workers increased from 1·19 in 1963 to 1·54 in 1995.[1] About only a third of this increase can be explained by changes in workers' observed skills including education, age and experience. The rest is attributed to changes in "within-group" wage inequality. Moreover, within-group wage inequality evolved differently from the college education premium—it kept rising in the 1970s, 1980s and 1990s, while the education premium fell sharply in the 1970s, rose sharply in the 1980s and continued to rise at a slower pace in the 1990s. In this paper I construct a directed search model to generate within-group inequality and analyse its joint behaviour with the skill premium.

The existing literature explains within-group inequality by exogenous differences between workers' innate ability or unobserved skills. This innate ability can affect workers' capacity to produce (Acemoglu, 1999), to adapt to new technologies (Galor and Moav, 2000), or to absorb new technology-specific skills (Lloyd-Ellis, 1999).[2] As criticized by Aghion *et al*. (1999) and Violante (2000), such models cannot account for the wage volatility along individual workers' employment histories, because the innate ability difference is fixed. As remedies, Violante (2000) assumes partially transferrable skills, and Aghion *et al*. (1999) assume a two-step adaptation process, to ensure that technological acceleration increases the variations in workers' adaptation to new machines, thus increasing the cross-sectional variance of skills and wages. However, these results rely heavily on auxiliary assumptions like vintage- or match-specific productivity.

1. All numbers in this paragraph come from Katz and Autor (1999). See also Juhn *et al*. (1993) and Levy and Murnane (1992) for the facts and the literature on wage inequality.

2. These models implicitly assume that unobserved skills are complementary with observed skills, because the innate ability allows workers to use new technologies that increase the return to observed skills. In this way, within-group inequality and the skill premium can rise simultaneously with technological progress.

Eliminating such auxiliary assumptions, I show in this paper how within-group inequality arises naturally in a directed search model and how it is related to the skill-biased technology.

The model constructed here has two types of firms, high-tech and low-tech, and two types of workers, skilled and unskilled. Skills are observable, *e.g.* education attainment. A skilled worker produces θy amount of output in a high-tech job and y in a low-tech job, while an unskilled worker performs the same task and produces the same amount y of output in either job. With $\theta > 1$, the high technology is skill-biased and, controlling for skills and job types, productivity is not match-specific. There is free entry of firms that determines both the total number and the composition of firms. After firms' entry, agents play a two-stage, directed search game. First, all firms simultaneously announce the wages and selection criteria of workers. Then, workers decide where to apply and each worker can apply to only one job in a period, which captures a search friction in the market. After receiving the applicants, a firm chooses one for the job according to the posted selection criterion. The search process is directed by firms, rather than random, because a firm deliberately chooses the posted wage and the selection criterion with an intention to affect the number and composition of applicants for the job.

I focus on symmetric mixed-strategy equilibria, where identical workers (or firms) use the same strategy and workers randomize their applications over a large number of firms. The directed search game has a unique equilibrium of this type, under the maintained assumption that the skill-biased productivity θ is larger than the differential between the entry costs of the two types of firms. In this equilibrium, matching is partially mixed. A high-tech firm receives both skilled and unskilled applicants with positive probability, and favours skilled workers, while a low-tech firm receives only unskilled applicants. The equilibrium attains the constrained social optimum.

Wage inequality among unskilled workers, as well as a positive skill premium, arises in the equilibrium. With the skill-biased productivity θ, it is optimal, both *ex ante* and *ex post*, for high-tech firms to select skilled applicants first before considering unskilled applicants. This priority implies that an unskilled worker has a lower employment probability in a high-tech firm than in a low-tech firm. However, the two jobs must provide the same expected wage to an unskilled applicant. So, high-tech firms must compensate unskilled applicants for the low selection priority by offering a higher wage to them than do low-tech firms. Therefore, wage inequality arises among unskilled workers, despite the fact that all unskilled workers perform the same task and have the same productivity. Clearly, this within-group inequality does not rely on the traditional assumptions of workers' innate ability differences or match-specific productivity.

The model provides suggestive answers to what shocks might have generated the patterns of wage inequality in the U.S. First, when there is a skill-biased technological progress, the within-group wage differential and the skill premium both rise, with the skill premium rising more precipitously. This resembles the pattern in the 1980s and 1990s. Second, when there is a general productivity slowdown, the within-group wage differential rises but the skill premium falls. This resembles the pattern in the 1970s. Third, the effects of skill supply changes on inequality are inconclusive. Within-group inequality rises when the skill supply elasticity increases moderately but, when the skill supply elasticity increases sufficiently, within-group inequality falls and may disappear altogether.

This paper makes important contributions to the theory of directed search and, more generally, to matching theory. Directed search models, originated in Peters (1991) and Montgomery (1991), have assumed that at least one side of the market is homogeneous.[3] When

3. See Burdett *et al*. (2001), Moen (1997), Acemoglu and Shimer (1999b), Cao and Shi (2000) and a part of Shi and Wen (1999). Carlton (1978) and Harris and Todaro (1970) also analyse the trade-off between price/wage and the probability of obtaining it, but their analyses are non-strategic with an exogenous relationship between the two elements.

there is heterogeneity on both sides, as in this paper, very little is known about the matching patterns, matching rates and wage shares that arise endogenously. By analysing the equilibrium explicitly, I obtain three new results. First, there is partially mixed matching in equilibrium, as described before, and it is optimal for high-tech firms to favour skilled workers. These features imply that skilled workers crowd out unskilled workers' matches but not the reverse. Thus, the typical matching functions in the literature are mis-specified, because they specify an agent's matching rate as a function of the relative number of agents on the two sides of the market. Second, the particular crowding-out between skills is a new source of within-group inequality, relative to previous models such as Montgomery (1991) (see Section 4.1), and it allows me to tie both within-group inequality and the skill premium to the skill-biased productivity. Third, the equilibrium is socially optimal and, in particular, partially mixed matching and the ranking of applicants are socially optimal.

Of course, none of these results can be obtained in the standard search theory of unemployment, developed by Diamond (1982), Mortensen (1982) and Pissarides (1990). In the latter theory, search is random, rather than being directed, and so the matching functions and wage shares are exogenous, inefficient, and inconsistent with agents' incentives.[4] Directed search reflects a fundamental belief that wages can play an important *ex ante* role in resource allocation, rather than being determined *ex post* as in the standard search theory. In this paper, directed search is also critical for the within-group wage differential by keeping identical applicants indifferent between different wages.

Finally, I contrast the model with the sequential search literature, surveyed by McMillan and Rothschild (1994). In this literature, firms post wages but workers do not know such wages and must search for them costly. When each worker discovers only one wage at a time after search, sequential search models produce a degenerate wage distribution among homogeneous workers. When each firm's offer can reach two or more workers with positive probability (Lang, 1991) or when workers search on the job (Burdett and Mortensen, 1998), there can be wage inequality among homogeneous workers but such inequality has no apparent link to skill-biased technology. Moreover, workers do not make the *ex ante* trade-off between wages and matching probability that is central to the within-group wage differential in this paper.

2. THE MODEL

2.1. *The labour market with skill-biased technology*

Consider a labour market with N workers, who are distinguished by an observable skill (*e.g.* education attainment). A fraction s of workers are skilled and the remaining unskilled. Use a subscript $i \in \{s, u\}$ to indicate these two types of workers. There are also M firms, where a fraction h use a high technology and the remaining fraction use a low technology. A subscript $j \in \{H, L\}$ indicates a firm's type. The numbers M and h are determined endogenously by firms' entry, but s and N are fixed (see Section 5.4 for an extension). Denote $n \equiv N/M$. I focus on the case where the market is large (*i.e.* $M, N \to \infty$) and neither side of the market is infinitely larger than the other side (*i.e.* $0 < n < \infty$).[5] Agents within each type are identical and all agents are risk neutral. Each firm has only one job opening.

4. An example is Blanchard and Diamond (1994), where firms rank job applicants and select workers of long unemployment duration with lower priority. Despite this priority, the exogenous matching function forces workers to apply to all firms randomly with the same probability. This is not optimal for workers and firms if search can be directed.

5. Burdett *et al*. (2001), Peters (2000) and Cao and Shi (2000) analyse directed search games with finite numbers of homogeneous buyers/workers. An earlier version of the current paper (Shi, 1997) provides some steps to approach the equilibrium here with heterogeneous agents as the limit of a finite economy.

Firms obtain zero net expected profit from creating either job. A high-tech job costs K_H to set up and a low-tech job costs K_L. A high-tech job is skill-biased, yielding output θy with a skilled worker and y with an unskilled worker, where $\theta > 1$. A low-tech job does not favour skilled workers by as much as a high-tech job does and, to simplify, I assume that it does not favour skilled workers at all, yielding output y with any worker. Note that a high-tech job filled by an unskilled worker is identical to a low-tech job. (I relax this assumption in Shi, 1997). Denote the productivity of a skilled worker in a type-j firm by Θ_j, where $\Theta_H = \theta$ and $\Theta_L = 1$. I term θ the *skill-biased productivity* and y the *general productivity*. For there to be any low-tech firms, it is necessary that $K_L < y$. Also, the skill-biased productivity is sufficient to cover the higher entry cost of a high-tech job, as assumed below:

Assumption 1. $\theta > K_H/K_L > 1$ *and* $K_L < y$.

To clarify the meanings of jobs and workers, consider an example where a high-tech job is software design and a low-tech job data-entry, while a skilled worker is a programmer and an unskilled worker a typist. A firm needs to purchase computers to set up the jobs but the computer for software design is more expensive than the computer for data entry (thus $K_H > K_L$). A programmer yields a higher value of product in software design than in data entry. But, when no programmer has applied to a software design job, the firm can hire a typist to work with the computer initially intended for software design. In this case the typist's task is data entry, no different from the job with a computer intended for data entry.[6]

It takes time to match workers with jobs. I capture this friction by assuming that each worker can apply to at most one job in a period. Unmatched jobs and workers produce nothing and get 0 payoff. The time horizon is one period (see Shi (1997) for a dynamic setting). After firms' entry, agents play the following two-stage game. First, all firms simultaneously post and commit to wages and selection criteria of workers, knowing that their decisions will affect workers' application decisions. Second, after observing all posted wages and selection criteria, workers choose which firm to apply to.[7] Then firms select workers according to the announced criteria and production follows immediately. Given the large numbers of workers and jobs, it is natural to focus on symmetric, mixed-strategy equilibria, where *ex ante* identical firms or workers use the same strategy and workers randomize over a set of preferable jobs. This focus is justified because it is difficult for agents to coordinate their decisions in a large market.[8]

A type-i worker's strategy is a vector of probabilities $P_i \equiv (p_{Hi}, \ldots; p_{Li}, \ldots)$, where p_{ji} is the probability with which a type-i worker applies to each type-j firm. A type-j firm's strategy consists of the wages, $(w_{ji})_{i=s,u}$, and a selection rule $\chi_j \in [0, 1]$. The selection rule applies only when the firm receives both types of applicants, in which case the firm prefers a skilled worker if $\chi_j = 1$, prefers an unskilled worker if $\chi_j = 0$, and is indifferent between the two types of workers if $\chi_j \in (0, 1)$. If the firm receives only one type of applicants, the firm randomly selects one with equal probability. These selection criteria are announced before workers apply and, like the announced wages, are committed to by the firm.

6. It should be clear from this example that the two job types and workers can be in the same industry. Similarly, by allowing each firm to have more than one job opening, one can even interpret the two job types as jobs in the same firm. Thus, the wage differential between different types of jobs in this paper can capture the differential within each industry or each firm, not just the inter-industry wage differential.

7. One can assume instead that each worker observes only two independently drawn wages (see Acemoglu and Shimer, 1999a) or that firms announce only reserve wages and hold auctions after receiving applicants (see Julien *et al.*, 2000). These formulations complicate the analysis without changing the qualitative results much.

8. In a similar game, Burdett *et al.* (2001) show that there are a continuum of asymmetric equilibria, some of which are supported by trigger strategies, but the symmetric mixed-strategy equilibrium is unique. In asymmetric equilibria, within-group inequality can even arise among homogeneous firms if the firms discriminate some subgroup of workers.

2.2. *Queue lengths and agents' strategies*

Each worker maximizes the expected wage, making a trade-off between a wage and the probability of obtaining it. When $M, N \to \infty$, the application probabilities approach zero and are inconvenient objects for the analysis. A convenient object is the *queue length*, defined as the expected number of workers applying to a firm. Let x_{ji} be the queue length of type-i workers who apply to a type-j firm, where $i \in \{s, u\}$ and $j \in \{H, L\}$. Then, $x_{js} = sNp_{js}$ and $x_{ju} = (1-s)Np_{ju}$. These queue lengths are finite in the limit $M, N \to \infty$ if $n = N/M \in (0, \infty)$. I will refer to $X_i \equiv (x_{Hi}, \ldots; x_{Li}, \ldots)$ as a type-i worker's strategy. Since each worker's application probabilities add up to one, the following restrictions must hold

$$hx_{Hs} + (1-h)x_{Ls} = ns, \tag{2.1}$$

$$hx_{Hu} + (1-h)x_{Lu} = n(1-s). \tag{2.2}$$

Let q_{ji} be the probability with which a type-i worker gets a type-j job when he/she applies to this job. When $M, N \to \infty$, these probabilities are

$$q_{js} = [\chi_j + (1-\chi_j)e^{-x_{ju}}]g(x_{js}); \tag{2.3}$$

$$q_{ju} = (1 - \chi_j + \chi_j e^{-x_{js}})g(x_{ju}), \tag{2.4}$$

where

$$g(x) \equiv \frac{1-e^{-x}}{x}. \tag{2.5}$$

Because the explanations for (2.3) and (2.4) are similar, I explain (2.3) only. For a particular skilled applicant to be selected by a type-j firm, one skilled worker must be chosen and the particular worker in discussion must be the chosen one. The firm chooses a skilled worker either when no unskilled worker has applied to the firm, or when one or more unskilled worker has applied but the firm favours a skilled applicant. The first case occurs with probability $(1-p_{ju})^{(1-s)N}$, the second with $\chi_j[1-(1-p_{ju})^{(1-s)N}]$, and the sum of these probabilities is $\chi_j + (1-\chi_j)e^{-x_{ju}}$ in the limit. Conditional on choosing a skilled applicant, the firm chooses the particular one in discussion with probability $[1-(1-p_{js})^{sN}]/(sNp_{js})$, which is the probability that the firm receives one or more skilled applicant divided by the expected number of skill applicants for that firm. The limit of this probability is $g(x_{js})$.

The function $g(\cdot)$ is continuous, strictly decreasing and strictly convex, with $g(0) = 1$ and $g(\infty) = 0$. Therefore, q_{ji} strictly decreases in x_{ji}. That is, a type-i worker who applies to a type-j firm is less likely to be chosen if more type-j workers apply to the firm. Also, q_{ji} decreases in $x_{ji'}$, where $i' \neq i$, and strictly so if type-j firms select type-i' workers with positive probability.

To describe a worker's decision, let U_i be a type-i worker's expected "market" wage in equilibrium, which is taken as given by individual agents when $M, N \to \infty$ (see Burdett *et al.*, 2001). A type-i worker applies to a type-j firm with positive probability if and only if the expected wage from that firm, $q_{ji}w_{ji}$, is equal to or greater than U_i. That is, $x_{ji} > 0$ iff $q_{ji}w_{ji} \geq U_i$. However, it can never be the case that $q_{ji}w_{ji} > U_i$. If a particular firm's offer yields $q_{ji}w_{ji} > U_i$, all type-i workers will apply to that firm with probability 1, yielding $x_{ji} = \infty$ when $N \to \infty$. Then $q_{ji} = 0$, which contradicts $q_{ji}w_{ji} > U_i$. Thus, for $i \in \{s, u\}$ and $j \in \{H, L\}$,

$$x_{ji} \begin{cases} \in (0, \infty), & \text{if } q_{ji}w_{ji} = U_i, \\ = 0, & \text{if } q_{ji}w_{ji} < U_i. \end{cases} \tag{2.6}$$

This is a type-i worker's strategy, which shows a worker's trade-off between a wage and the matching probability—a low wage job must be compensated by a high employment probability.

Now I turn to firms' decisions. A type-j firm's expected profit is[9]

$$\pi_j = [\chi_j + (1-\chi_j)e^{-x_{ju}}](1-e^{-x_{js}})(\Theta_j y - w_{js}) + (1-\chi_j+\chi_j e^{-x_{js}})(1-e^{-x_{ju}})(y - w_{ju}), \tag{2.7}$$

where $\Theta_H = \theta$ and $\Theta_L = 1$. As in (2.3), $[\chi_j + (1-\chi_j)e^{-x_{ju}}]$ is the probability with which a type-j firm selects a skilled worker. Since the firm gets one or more skilled worker with probability $1-e^{-x_{js}}$ and a skilled worker yields profit $\Theta_j y - w_{js}$, the first term on the right-hand side of (2.7) is expected profit from getting one skilled worker. Similarly, the second term is expected profit from getting an unskilled worker. A type-j firm's matching rate is $1 - e^{-(x_{js}+x_{ju})}$, the sum of the two hiring probabilities in (2.7).

A firm maximizes expected profit, taking expected market wages (U_s, U_u) and other firms' strategies as given. That is, a type-j firm chooses (w_{js}, w_{ju}, χ_j) to solve:

$$(Pj) \quad \max \pi_j \text{ s.t. (2.6).}$$

A firm does not take the queue lengths as given; rather, it takes the functional relationship (2.6) as a constraint. Given U_s and U_u, the firm effectively chooses the queue lengths by choosing the wages. In particular, a firm can offer a high wage to increase its matching probability. In contrast to Bertrand competition, an individual firm's wage offer affects the queue length smoothly rather than discontinuously, because the xs depend on the wage offer smoothly. That is, a marginal wage increase can only attract a marginal increase in the expected number of applicants.

3. SYMMETRIC EQUILIBRIUM

3.1. *Characterization, existence and uniqueness*

A *symmetric equilibrium* consists of the overall worker/job ratio (n), the fraction of high-tech firms (h), workers' expected wages (U_s, U_u), firms' strategies $(w_{ji}, \chi_j)_{i,j}$, and workers' strategies $(x_{ji})_{i,j}$, where $i \in \{s, u\}$ and $j \in \{H, L\}$, that satisfy the following requirements:

(i) Given (U_s, U_u) and other firms' strategies, each type-j firm's strategy solves (Pj);
(ii) Observing firms' decisions, each worker's decision obeys (2.6);
(iii) U_s and U_u, by affecting $(x_{ji})_{i,j}$, satisfy (2.1) and (2.2);
(iv) The numbers (n, h) are such that each firm earns zero net expected profit, *i.e.*

$$\pi_L = K_L, \qquad \pi_H = K_H. \tag{3.1}$$

The above requirements are self-explanatory. The definition requires that firms' decisions be optimal *ex ante*, *i.e.* before workers apply to firms. It is interesting to check whether a firm's selection rule is optimal *ex post*, *i.e.* after workers apply to firms. The selection rule χ_j is *ex post optimal* if it satisfies:

$$\chi_j \begin{cases} = 1, & \text{if } \Theta_j y - w_{js} > y - w_{ju}, \\ = 0, & \text{if } \Theta_j y - w_{js} < y - w_{ju}, \\ \in [0,1], & \text{if } \Theta_j y - w_{js} = y - w_{ju}. \end{cases} \tag{3.2}$$

There are nine possibilities of (x_{Hs}, x_{Hu}), *a priori*. Under Assumption 1, however, the following lemma significantly reduces the number of possibilities (see the Appendix A for a proof).

9. The term "expected profit" in this paper refers to profit before deducting the firm's entry cost. The term "net expected profit" refers to profit after deducting the entry cost.

Lemma 3.1. *In all symmetric equilibria,* $x_{Ls} = 0$ *(i.e.* $x_{Hs} = ns/h$*),* $x_{Hu} > 0$*,* $\chi_H = 1$ *and* $U_s > U_u$*. Moreover,*

$$\pi_H = (1 - e^{-x_{Hs}})\theta y + e^{-x_{Hs}}(1 - e^{-x_{Hu}})y - x_{Hs}U_s - x_{Hu}U_u, \tag{3.3}$$

$$\pi_L = (1 - e^{-x_{Lu}})y - x_{Lu}U_u. \tag{3.4}$$

This lemma states some important features of an equilibrium. First, skilled workers get a higher expected wage than unskilled workers, *i.e.* $U_s > U_u$. This is because skilled workers have a higher productivity in high-tech firms than unskilled workers. If $U_s \le U_u$ in a supposed equilibrium, the expected cost of attracting skilled workers would be equal to or lower than that of attracting unskilled workers. Given the skill-biased productivity, high-tech firms could increase expected profit in this case by attracting skilled workers through higher wages, upsetting the supposed equilibrium.

Second, skilled workers apply only to high-tech firms, *i.e.* $x_{Hs} = ns/h$ and $x_{Ls} = 0$. Because all workers have the same productivity in a low-tech firm, such a firm offers the same expected wage U_u to all workers, regardless of the worker's type. By applying only to high-tech firms, a skilled worker gets a higher expected wage U_s. Thus, there is no mixed matching of skills with low-tech firms.

Third, it is *ex ante* optimal for high-tech firms to give skilled workers the priority in selection, *i.e.* $\chi_H = 1$. If $\chi_H < 1$, a high-tech firm can deviate to a marginally higher χ_H and, at the same time, reduce w_{Hs} and increase w_{Hu} to deliver expected wages (U_s, U_u) to the applicants and to maintain the same queue lengths (x_{Hs}, x_{Hu}) as before. This deviation does not change the expected wage cost for the firm, but it increases skilled workers' utilization rate and reduces unskilled workers' utilization rate, both by $(1 - e^{-x_{Hu}})(1 - e^{-x_{Hs}})$. The net gain in expected output is $(1 - e^{-x_{Hu}})(1 - e^{-x_{Hs}})(\theta - 1)y$ (see (2.7) for $j = H$), and so expected profit increases.

Fourth, unskilled workers apply to high-tech firms with positive probability, and so there is mixed matching of skills with high-tech firms. To see why unskilled workers apply to high-tech firms, suppose that they do not. Because the skill-biased productivity is more than compensating for a high-tech firm's additional entry cost, the queue of applicants for a high-tech firm must be shorter than for a low-tech firm in order to ensure that the free-entry conditions hold for both types of firms. That is, $x_{Hs} < x_{Lu}$. In this case, however, a high-tech firm faces a higher failure rate of filling the job and so it is profitable for the firm to attract some unskilled applicants. If a high-tech firm deviates to attract an unskilled worker, the firm's expected output increases by $ye^{-x_{Hs}}$ and the expected wage cost increases by $U_u = ye^{-x_{Lu}}$, yielding a gain $y(e^{-x_{Hs}} - e^{-x_{Lu}}) > 0$.

Finally, a firm's expected profit is equal to the difference between expected output and expected wage cost, as in (3.3) and (3.4). Take a low-tech firm, for example. Expected output is $(1 - e^{-x_{Lu}})y$ and the expected wage cost is $(1 - e^{-x_{Lu}})w_{Lu} = x_{Lu}U_u$. The latter expression shows that the firm can calculate the expected wage cost as if it pays each expected applicant the expected wage U_u.

Lemma 3.1 indicates that an equilibrium is one of the following two types. Either unskilled workers apply to both types of firms with positive probability (*i.e.* $0 < x_{Hu} < n(1-s)/h$), which I call *equilibrium I*, or unskilled workers apply only to high-tech firms (*i.e.* $x_{Hu} = n(1 - s)/h$), which I call *equilibrium II*. Low-tech firms do not exist in equilibrium II. In both types of equilibria, skilled workers apply only to high-tech firms, *i.e.* $x_{Hs} = ns/h$. Under Assumption 1, the two types of equilibria never occur under the same parameter values. To state this result and others, define

$$\beta(n) = B^{-1}(n) \equiv 1 - (1 + n)e^{-n}. \tag{3.5}$$

Next, define n^*, $\bar{s}$ and n^{**} by the following equations:

$$n^* = B(K_L/y), \tag{3.6}$$

$$\bar{s} = \frac{B((K_H - K_L)/[(\theta - 1)y])}{B(K_L/y)}, \tag{3.7}$$

$$\beta(n^{**}) + (\theta - 1)\beta(sn^{**}) = K_H/y. \tag{3.8}$$

Under Assumption 1, $n^* \in (0, \infty)$, $\bar{s} \in (0, 1)$ and $n^{**} \in (0, \infty)$ are all well-defined. Moreover,[10]

$$d\bar{s}/d\theta < 0, \qquad \lim_{\theta \downarrow K_H/K_L} \bar{s} = 1 \qquad \text{and} \qquad \lim_{\theta \to \infty} \bar{s} = 0; \tag{3.9}$$

$$dn^{**}/ds < 0; \qquad n^{**} < n^* \text{ iff } s > \bar{s}. \tag{3.10}$$

The following proposition characterizes the equilibrium, whose proof is in Appendix B:

Proposition 3.2. *There is a unique equilibrium. The selection rule $\chi_H = 1$ is ex post (and ex ante) optimal. Equilibrium I has $h \in (0, 1)$ and exists iff $0 < s < \bar{s}$. Its other features are as follows:*

$$h = s/\bar{s}, \qquad x_{Lu} = n = n^*, \qquad x_{Hu} = n^* - n^*s/h, \qquad x_{Hs} = n^*s/h. \tag{3.11}$$

Equilibrium II has $h = 1$ and exists iff $s \geq \bar{s}$. Some other properties of equilibrium II are:

$$n = n^{**}(< n^*), \qquad x_{Lu} = 0, \qquad x_{Hu} = n^{**}(1 - s), \qquad x_{Hs} = sn^{**}. \tag{3.12}$$

With the corresponding solutions for (n, h), the two cases both have:

$$\pi_H/y = \beta(n) + (\theta - 1)\beta(ns/h), \tag{3.13}$$

$$U_u/y = e^{-n}, \qquad U_s/y = e^{-n} + (\theta - 1)e^{-ns/h}, \tag{3.14}$$

$$w_{Hs} = U_s/g(ns/h), \qquad w_{Hu} = e^{ns/h}U_u/g(n - ns/h). \tag{3.15}$$

In addition, equilibrium I has

$$\pi_L/y = \beta(n^*) \qquad \textit{and} \qquad w_{Lu} = U_u/g(n^*). \tag{3.16}$$

The equilibrium is unique for any given parameter values that satisfy Assumption 1. In equilibrium, the selection rule $\chi_H = 1$ is not only *ex ante* optimal, as explained before, but also *ex post* optimal. The reason is that, if a high-tech firm announces a selection priority for skilled workers in order to attract such workers, then the firm must have incentive to select skilled workers when these workers come to the firm.

The job composition depends on the fraction of skilled workers in the labour force, s. Both types of jobs exist if skilled workers are scarce (*i.e.* when $s < \bar{s}$), while only high-tech jobs exist if skill supply is high (*i.e.* when $s \geq \bar{s}$). The explanation is as follows. When skilled workers are scarce, a high-tech firm's matching rate is low and its expected profit is not so much higher than a low-tech firm's, despite the skill-biased productivity. Because a high-tech job is more costly to set up than a low-tech job, some firms choose to set up low-tech jobs. The situation is different when skilled workers are plenty. In this case, it is easy to fill a job with a skilled worker. Because the skilled biased productivity is high enough to cover the additional cost of a high-tech job, every firm finds it optimal to set up a high-tech job, and so low-tech jobs do not exist. Furthermore, the

10. To verify these properties, note that $\beta(n)$ defined in (3.5) is a strictly increasing function of n. Also, the function has a value 0 when $n = 0$ and a value 1 when $n \to \infty$.

critical level of skill supply, $\bar{s}$, decreases in the skill-biased productivity θ and increases in the entry cost differential of the two jobs, K_H/K_L, as shown in (3.9).[11]

Let me explain why expected wages and expected profits obey (3.14), (3.13) and (3.16). Since (3.14) implies (3.13) and (3.16) under (3.3) and (3.4), it suffices to explain (3.14). Expected wages obey (3.14) because firms equate a worker's expected wage to the worker's expected marginal output. Consider the expected wage of an unskilled worker in a high-tech firm, for example. A marginal increase in the number of unskilled workers for high-tech firms, by increasing the queue length of such workers (x_{Hu}), increases the expected wage cost in a high-tech firm by U_u and increases the firm's expected output by $ye^{-(x_{Hs}+x_{Hu})}$ (see (3.3)). Competitive entry of firms ensures that such marginal benefit and cost are equal to each other, *i.e.* $U_u = ye^{-(x_{Hu}+x_{Hs})}$. Similarly, $U_u = ye^{-x_{Lu}}$ for an unskilled worker in a low-tech firm. If both types of firms exist, then $x_{Hs} + x_{Hu} = x_{Lu} = n$. If low-tech firms do not exist, then $x_{Hu} = n(1-s)$ and again $x_{Hs} + x_{Hu} = n$. In either case, $U_u = ye^{-n}$, as in (3.14). A similar calculation yields U_s in (3.14).

3.2. *Social optimality of equilibrium*

It is interesting to consider the social optimum in this market. The social planner chooses $(M_j, \chi_j, x_{ji})_{i,j}$, where M_j is the number of type j jobs, to maximize the sum of expected net output:

$$\sum_{j=H,L} M_j \begin{Bmatrix} [\chi_j + (1-\chi_j)e^{-x_{ju}}](1-e^{-x_{js}})\Theta_j y \\ +(1-\chi_j+\chi_j e^{-x_{js}})(1-e^{-x_{ju}})y - K_j \end{Bmatrix}, \tag{3.17}$$

subject to the feasibility constraints (2.1) and (2.2), together with $M_j \geq 0$, $x_{ji} \geq 0$ and $\chi_j \in [0,1]$. Note that $M = M_H + M_L$, $h = M_H/M$ and $n = N/M$. Also note that the matching rates in (3.17) are the same functions as those in (2.7), and so the social optimum is constrained by the same search friction as is the equilibrium. The following proposition holds, whose proof is straightforward and omitted.

Proposition 3.3. *The social optimum is identical to the equilibrium allocation.*

The market internalizes the matching externalities because firms organize the market by setting wages and selection rules to "direct" workers' search decisions. Such efficiency of the equilibrium contrasts sharply with the inefficiency in the standard search theory of unemployment (*e.g.* Diamond, 1982; Mortensen, 1982; Pissarides, 1990), where search is not directed. With the skill-biased productivity, it is easy to see why giving skilled workers the priority for high-tech jobs is socially optimal. Equilibrium queue lengths are also socially optimal because firms set a worker's expected wage to the worker's expected *social* marginal contribution, which takes into account an additional worker's contribution to output and the crowding-out on existing workers.

To see this, consider a skilled worker, for example. Suppose that, after match, an additional skilled worker is made available to a high-tech firm. If the firm has already employed a skilled worker, which occurs with probability $(1-e^{-x_{Hs}})$, hiring the additional worker means replacing the existing skilled worker, in which case the additional worker's social contribution is zero. If the firm has employed an unskilled worker, which occurs with probability $e^{-x_{Hs}}(1-e^{-x_{Hu}})$, the additional skilled worker replaces the existing unskilled worker, yielding a social contribution

11. When θ falls below K_H/K_L, a case which is ruled out by Assumption 1 but which might occur in an environment where there are a continuum of skill levels, the equilibrium may have low-tech firms employ workers of all skill levels.

$(\theta - 1)y$. If the firm has failed to hire a worker, which occurs with probability $e^{-(x_{Hs}+x_{Hu})}$, the additional skilled worker's contribution is θy. Multiplying the social marginal contribution of the worker in each case by the corresponding probability, summing up and noting $x_{Hs} + x_{Hu} = n$, one obtains U_s in (3.14). Notice that a skilled worker crowds out unskilled workers' employment probability, as well as other skilled workers'.

Similarly, a firm's expected profit is equal to the firm's social marginal contribution. For example, adding a low-tech firm yields expected output $(1 - e^{-x_{Lu}})y$, but reduces the matching rate of existing firms and hence crowds out existing firms' expected output by yne^{-n}. The social marginal contribution of the additional low-tech firm is $y\beta(n)$, which is equal to π_L in (3.16).

The above discussion also helps us to see why it is socially optimal to have mixed matching of skills with high-tech firms. If there is no mixing (*i.e.* $x_{Hu} = 0$), then the social marginal contribution of an unskilled worker is $ye^{-x_{Hs}}$ in a high-tech firm and $ye^{-x_{Lu}}$ in a low-tech firm. Since $x_{Hs} < n = x_{Lu}$, expected output can be increased by allocating some unskilled workers from low-tech to high-tech firms.[12]

4. WAGE INEQUALITY AND MATCHING RATES

In the remainder of this paper I will focus on equilibrium I by assuming $s < \bar{s}$. This is because there is wage inequality among unskilled workers in equilibrium I, as I will analyse below, but not in equilibrium II. The term "equilibrium" will mean equilibrium I unless it is modified otherwise. I will abbreviate x_{Hs} as x_s, q_{Hs} as q_s, w_{Hs} as w_s, and w_{Lu} as w_L.

4.1. *Skill premium and within-group wage differential*

There is a positive skill premium in terms of expected wages, since $U_s > U_u$. However, since a skilled worker may have a sufficiently higher employment chance than an unskilled worker, the skill premium in expected wages may or may not imply a skill premium in actual wages. The latter requires θ to be large enough, as stated below (see Appendix C for a proof):

Proposition 4.1. *Skilled workers obtain a higher expected wage than unskilled workers, i.e.* $U_s > U_u$*. In high-tech firms, skilled workers obtain a higher actual wage, i.e.* $w_s > w_{Hu}$*, if and only if* $\theta > \max\{\theta_1, K_H/K_L\}$*, where* θ_1 *is defined in Appendix* C.

The model also generates a wage differential among homogeneous, unskilled workers, as stated below (see Appendix C for a proof).

Proposition 4.2. $w_{Hu} > w_L$*. That is, an unskilled worker in a high-tech firm is paid a higher wage than an identical unskilled worker in a low-tech firm.*

Unskilled workers in high-tech firms earn higher wages than their peers in low-tech firms not because they have match-specific productivity with high-tech firms, nor because they are complementary with skilled workers in production, but because they bear a higher risk of failing to get the job. That is, because high-tech firms give skilled workers the selection priority, an

12. This result contrasts with that in Shi (2001), where I show that the socially efficient assignment of machines qualities to skills exhibits complete separation even in the presence of the search friction. The latter result arises from an assumption on the form of mixed matches. There, the social planner is able to assign different subgroups of machines of the same quality to different skills, and different subgroups of workers of the same skill to different machine qualities, but is restricted to not assign each individual machine to different skills or each individual worker to different machine qualities. That is, mixed matching is allowed at the aggregate level in Shi (2001) but not at the individual level. In contrast, the current paper allows for both levels of mixing.

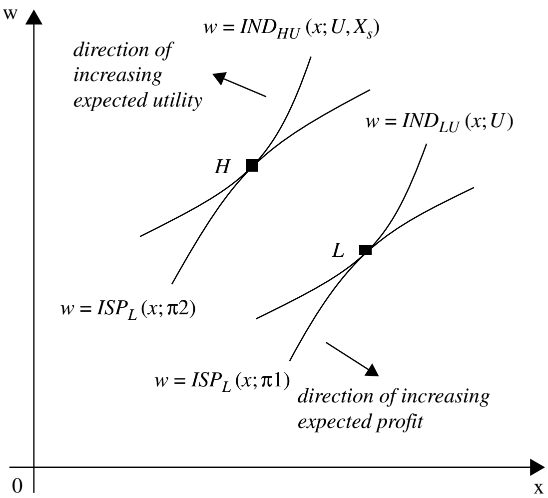

FIGURE 1
Unskilled workers' trade-off between wage and queue length

unskilled worker who applies to a high-tech firm has a lower probability of getting the job than does an identical unskilled worker who applies to a low-tech firm. To compensate for this lower probability, high-tech firms must offer a higher wage to unskilled applicants than do low-tech firms. Because these workers and jobs may belong to the same industry or even the same firm, the within-group wage differential in my model is not necessarily an inter-industry wage differential documented by Katz and Summers (1989).

Figure 1 illustrates the wage differential among unskilled workers. The iso-profit curve $w = ISP_L(x, \pi_1)$ represents $\pi_L = \pi_1$, where $\pi_L = (1 - e^{-x_{Lu}})(y - w_L)$ is expected profit of a low-tech firm. The curve $w = IND_{Lu}(x; U)$ represents $g(x_{Lu})w_L = U$, which is the indifference curve of an unskilled worker applying for a low-tech job. The point L is a low-tech firm's optimal offer to an unskilled worker. Similarly, a high-tech firm's expected profit is $(1 - e^{-x_{Hu}})(y - w_{Hu})$, conditional on receiving only unskilled applicants. Such expected profit is a constant π_2 along the iso-profit curve $w = ISP_L(x; \pi_2)$.[13] The indifference curve of an unskilled worker who applies to a high-tech job is $w = IND_{Hu}(x; U, x_s)$, which comes from the constraint $e^{-x_s} g(x_{Hu})w_{Hu} = U$. Point H is a high-tech firm's optimal offer to an unskilled worker. For the same queue length x of unskilled workers, an unskilled worker is chosen with probability $g(x)$ if the job is a low-tech job and with $e^{-x_s} g(x)$ if the job is a high-tech job. Thus, the indifference curve IND_{Hu} lies above the indifference curve IND_{Lu} for any $x_s > 0$. As a result, a high-tech job must offer a higher wage than a low-tech job in order to deliver the same expected wage to an unskilled applicant.

The within-group wage differential here is different from that in other directed search models. In particular, Montgomery (1991) shows that identical workers can get different wages from firms that differ in the worker's value of marginal product (due to different product demand, etc.). In my model unskilled workers in high-tech and low-tech firms have the same value of marginal product; yet, they obtain different wages from these two types of firms. The skill-biased

13. Because a high-tech firm may receive skilled applicants, its expected profit also consists of a part generated by successfully hiring a skilled worker. With the selection priority, however, the offers to unskilled workers do not affect skilled workers' application decisions, and so the offers to unskilled workers can be depicted under given offers to skilled workers.

technology is important for this wage differential, because it gives high-tech firms incentive to rank the applicants. If $\theta = 1$, there would not be a wage differential among unskilled workers.[14]

4.2. *Matching rates and unemployment rates*

The two types of workers experience different matching rates and unemployment rates. Let N_s be the number of employed skilled workers, N_{Hu} the number of unskilled workers employed in high-tech firms, and N_L the number of unskilled workers employed in low-tech firms. Then,

$$N_s = Mh(1 - e^{-n^*s/h}); \qquad N_{Hu} = Mh(e^{-n^*s/h} - e^{-n^*}); \qquad N_L = M(1-h)(1 - e^{-n^*}). \tag{4.1}$$

Let the average matching rate be α_s for a skilled worker and α_u for an unskilled worker. Then,

$$\alpha_s \equiv \frac{N_s}{sN} = \frac{1 - e^{-n^*s/h}}{n^*s/h}; \tag{4.2}$$

$$\alpha_u \equiv \frac{N_{Hu} + N_L}{(1-s)N} = \frac{1 - e^{-n^*} - h(1 - e^{-n^*s/h})}{n^*(1-s)}. \tag{4.3}$$

The unemployment rate is $1 - \alpha_i$ for type i workers. Using the equilibrium conditions in Proposition 3.2, one can establish the following proposition (the proof is omitted):

Proposition 4.3. *Skilled workers have a higher matching rate than unskilled workers, i.e. $\alpha_s > \alpha_u$, and a lower unemployment rate. For given n^*, α_u is a decreasing function of h.*

The matching rates for the two types of workers differ in magnitudes and functional forms. Skilled workers' matching rate is a nice decreasing function of the number of skilled workers per high-tech firm (n^*s/h). In contrast, unskilled workers' matching rate depends separately on the skill composition s, the firm composition h and the overall worker/firm ratio n^*. In particular, an increase in the fraction of high-tech firms in the economy reduces α_u, because it induces some unskilled workers to switch to high-tech firms in which they are selected with low priority. Since the matching rate for each worker depends on the compositions of skills and jobs, as well as the aggregate worker/job ratio, the matching functions in standard search models are mis-specified.

Nevertheless, a standard search model captures well the overall matching rate per worker, which is:

$$\alpha \equiv s\alpha_s + (1-s)\alpha_u = \frac{1 - e^{-n^*}}{n^*}. \tag{4.4}$$

This average matching rate depends only on the overall worker/firm ratio and not on the job composition or the skill composition. On the firms' side, the matching rate is $N_L/[M(1-h)]$ for a low-tech firm and $(N_s + N_{Hu})/(Mh)$ for a high-tech firm, both being equal to $1 - e^{-n^*}$. Thus, each firm's matching rate depends only on the overall worker/firm ratio.

14. Note that unskilled workers face the same queue length in the two types of firms, because $x_s + x_{Hu} = x_{Lu} = n$. Thus, identical workers receive different wages here not because a high-wage job has a strictly longer queue than a low-wage job, but rather because a high-wage job has a less favourable queue for unskilled workers. This might explain the paradoxical finding in Holrtzer *et al.* (1991) that jobs paying more than the minimum wage attract fewer applicants than do minimum wage jobs, if jobs paying more than the minimum wage are intended for better workers.

5. EQUILIBRIUM RESPONSES TO SHOCKS

5.1. *Measures of relative wages and wage inequality*

Use the letter R to indicate log relative wages, H high-tech jobs, and U unskilled workers. Define

$$RE = \ln\left(\frac{U_s}{U_u}\right); \qquad RU = \ln\left(\frac{w_{Hu}}{w_L}\right); \qquad RH = \ln\left(\frac{w_s}{w_{Hu}}\right); \qquad RB = \ln\left(\frac{w_s}{AU}\right), \quad (5.1)$$

where $\ln(AU)$ is the following weighted average of log wages of unskilled workers:

$$\ln(AU) = \frac{N_{Hu}}{N_{Hu} + N_L} \ln w_{Hu} + \frac{N_L}{N_{Hu} + N_L} \ln w_L.$$

Following the common practice I define a wage differential as the standard deviation of log wages of the corresponding group of employed workers, which takes into account of both the relative wage and the employment distribution. The letter D indicates such wage differentials. Define

$$DU = \frac{(N_{Hu} N_L)^{1/2}}{N_{Hu} + N_L} RU; \tag{5.2}$$

$$DH = \frac{(N_s N_{Hu})^{1/2}}{N_s + N_{Hu}} RH; \qquad DB = \frac{[N_s(N_{Hu} + N_L)]^{1/2}}{N_s + N_{Hu} + N_L} RB; \tag{5.3}$$

$$DT = [a_s(1 - a_s)(RH)^2 + 2a_s(1 - h)RH \cdot RU + h(1 - h)(RU)^2]^{1/2}, \tag{5.4}$$

where $a_s = N_s/(N_s + N_{Hu} + N_L)$. Here, DU is the wage differential among unskilled workers, DH the wage differential in high-tech firms, DB the wage differential in terms of average log wages of the two worker types, and DT the overall wage differential. DU is within-group inequality, while DH and DB are between-skill inequality. DH is a narrower measure of the skill premium than DB.

To illustrate the quantitative responses of wage differentials to shocks in later exercises, I use the following example. Normalize $y = 10$. To circumvent the difficulty of precisely defining skill categories, I choose $s = 0{\cdot}2$, match RU with the 50-10 percentile log relative wage and match RH with the 90-50 percentile log relative wage in the U.S. data (see Juhn *et al.*, 1993, Table 2). The 50-10 percentile log relative wage is 0·50 in 1964 and 0·64 in 1988, with an average value 0·57. The 90-50 percentile log relative wage is 0·44 in 1964 and 0·54 in 1988, with an average value 0·49. According to the decomposition in Juhn *et al*. (1993, Table 4), about a third of the changes in the 50-10 percentile log relative wage is due to skill changes, which the measure RU does not capture. Thus, I match RU with the remainder, *i.e.* $RU = 0{\cdot}57 \times 2/3 \approx 0{\cdot}38$. Also, about 42% of the changes in the 90-50 percentile log relative wage is due to factors other than skills. Since RH in the current model is generated solely by the skill difference, I set $RH = 0{\cdot}49 \times 58\% \approx 0{\cdot}285$. Finally, the average wage/output ratio is set to the realistic value 0·64. The procedure yields: $K_L = 2{\cdot}15$, $K_H = 3{\cdot}51$, and $\theta = 1{\cdot}91$. These parameter values satisfy Assumption 1.

5.2. *Skill-biased technological progress*

Consider an increase in θ as a skill-biased technological improvement.[15] The following proposition summarizes the effects (the proof is straightforward and omitted):

15. Another way to model skill-biased technological progress is through a reduction in the relative cost K_H/K_L. According to Greenwood and Yorukoglu (1997), the cost of skill-biased equipment like computers fell significantly around 1974 before the rise of the skill premium. In this paper, a decrease in K_H/K_L has similar effects to those of an increase in θ.

Proposition 5.1. *An increase in the skill-biased productivity has the following effects:*

$$\frac{dn^*}{d\theta} = 0, \ \frac{dh}{d\theta} > 0; \quad \frac{dx_s}{d\theta} < 0, \ \frac{dx_{Hu}}{d\theta} > 0, \frac{dx_{Lu}}{d\theta} = 0; \quad \frac{d\alpha_s}{d\theta} > 0, \ \frac{d\alpha_u}{d\theta} < 0;$$
$$\frac{dU_s}{d\theta} > 0, \ \frac{dU_u}{d\theta} = 0; \quad \frac{dw_L}{d\theta} = 0, \ \frac{dw_{Hu}}{d\theta} < 0, \ \frac{dw_s}{d\theta} > 0.$$

Let me explain these effects one at a time. For any given number of high-tech firms, the skill-biased technological progress increases a high-tech firm's profit. So, the number of high-tech firms is higher in the new equilibrium and each skilled worker's matching rate (α_s) increases. However, the overall worker-job ratio n^* is equal to the queue length of workers for a low-tech firm, as explained in Section 3.1. Because a low-tech firm's expected profit does not depend on θ or s, the free-entry condition for such a firm does not change, and so the overall worker-job ratio does not change. The increase in the number of high-tech firms is matched one for one by the decrease in the number of low-tech firms.

Because there are now more high-tech firms, each gets a smaller expected number of skilled applicants (x_s). So, an unskilled worker's employment chance improves in high-tech firms, which induces unskilled workers to increase their application to high-tech firms (*i.e.* x_{Hu} increases). This shift in the application reduces the average matching rate of unskilled workers and increases their unemployment rate, because the shift puts more unskilled applicants in firms that give them a low priority. The shift in application also keeps each low-tech firm's matching probability unchanged.

Wages respond as follows. First, the increased demand for skilled workers increases their expected wage. The higher expected wage comes from increased matching rates and increased actual wages for skilled workers. Actual wages rise for skilled workers, despite the higher matching rate, because firms smooth the higher expected wage cost using both the actual wage and the matching probability. Second, actual and expected wages do not change for unskilled workers in low-tech firms (see (3.14)). This is because the queue length of workers for each low-tech firm is unchanged, implying that the trade-off between the wage and the matching probability is the same as before for a low-tech firm and an unskilled worker applying to such a firm. Third, because the employment probability increases for unskilled workers applying to a high-tech firm, such workers' actual wage (w_{Hu}) must decrease in order to keep the expected wage U_u unchanged. The fall in w_{Hu} can be illustrated in Figure 1 as a result of a fall in x_s, which shifts southeast the indifference curve of an unskilled worker who applies to a high-tech firm, IND_{Hu}.

Because w_s rises and w_{Hu} falls, the relative wage between skills in high-tech firms increases, which increases the between-skill wage differential DH. Since more unskilled workers are employed in high-tech firms now than before, the lower tail of the wage distribution in high-tech firms fattens. This further increases DH, provided that there are more skilled workers than unskilled workers employed in high-tech firms. Thus, the skill-biased technological progress raises the between-skill wage differential sharply.

The wage differential among unskilled workers, DU, responds to θ ambiguously. On the one hand, the relative wage among unskilled workers, w_{Hu}/w_L, falls, which leads to a lower wage differential among unskilled workers. On the other hand, some unskilled workers shift from low-tech firms to high-tech firms, *i.e.* from low wages to high wages. This shift increases the wage differential among unskilled workers, provided that there are more unskilled workers employed in low-tech firms than in high-tech firms. Analytically it is not clear which of these two effects dominates. The response of the average log wage among unskilled workers, AU, is also ambiguous analytically; for the fall in w_{Hu} reduces AU but the shift in employment of unskilled workers to high-wage jobs increases AU.

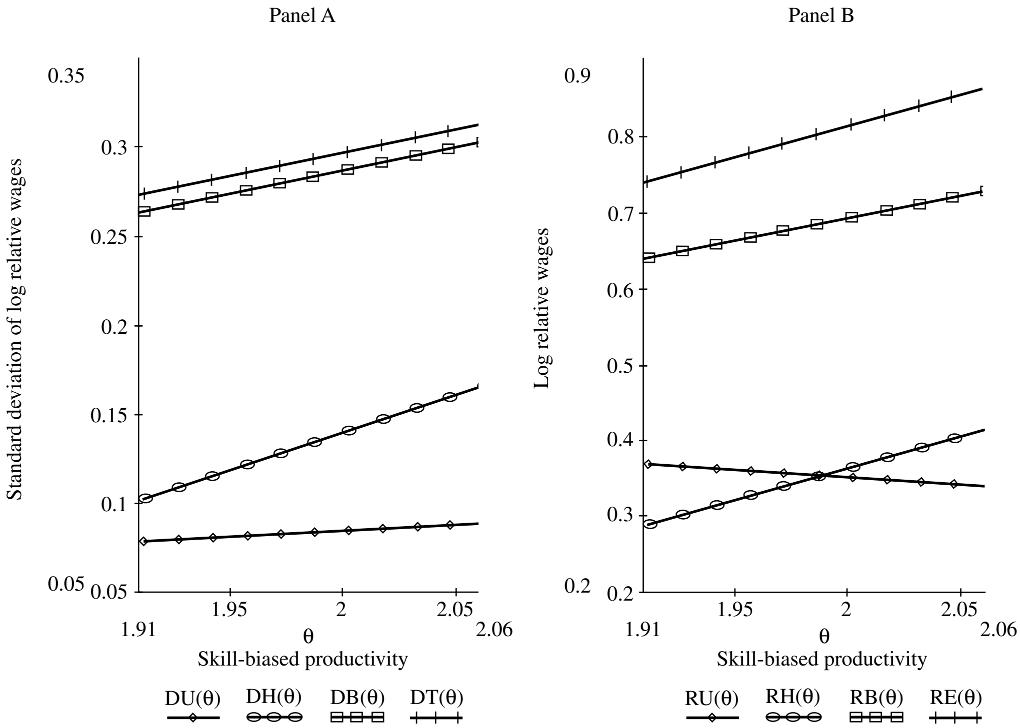

FIGURE 2

Effects of an increase in skill-biased productivity

To illustrate the responses of wage inequality to skill-biased progress, consider the numerical example in Section 5.1 and increase θ from its base value 1·91 to 2·06, with a step 0·015, and compute the equilibrium for each step. Figure 2 depicts the responses of wage inequality (panel A) and log relative wages (panel B). First, the skill-biased productivity progress increases log relative wages between skills, RH and RB, and widens between-skill wage differentials, DH and DB, as analysed above. Second, the mean of the log wage level of unskilled workers (AU, not shown) and the standard deviation (DU) both increase. Thus, the effect of the shift in unskilled workers' employment to high-tech firms dominates the fall in the relative wage (RU). Third, between-skill wage differentials increase by much more than does the within-skill wage differential. Finally, the overall wage differential increases. These results indicate that a skill-biased technological progress can generate simultaneously the sharply rising skill premium and the moderately rising within-group wage differential in the 1980s. The two wage differentials rise concurrently despite the absence of match-specific productivity and the complementarity between skilled and unskilled workers.

However, skill-biased technological progress alone fails to generate the opposite movements between the skill premium and within-group inequality in the 1970s. In addition, the positive response of unskilled workers' average wage in the above numerical example does not accord well with the U.S. data, although the response is ambiguous analytically; for example, Juhn *et al.* (1993, Figure 2) shows that the 10th percentile real wage fell steadily between 1974 and 1988. This is a failure shared by most models cited in the introduction that rely on skill-biased technological progress as the driving force of inequality. Acemoglu (1999) shows that skill supply shocks are important for explaining the behaviour of unskilled workers' average wage. For these reasons, I examine shocks other than the skill-biased progress below.

5.3. *A general productivity slowdown*

Productivity growth in the U.S. economy slowed down significantly in the 1970s relative to the 1960s. For example, the annual growth rate of GDP per worker was 2·2% in the 1960s but 0·4% in the 1970s; the annual growth rate of total factor productivity was 1·9% in the 1960s but 0·2% in the 1970s (see Jones, 1998, p. 42). Because the current model does not have a growth trend, I capture such productivity slowdown by a decrease in the general productivity, y. The following proposition summarizes the effects of y (see Appendix C for a proof).[16]

Proposition 5.2. *An increase in y has the following effects:*

$$\frac{dn^*}{dy} < 0,\ \frac{dh}{dy} < 0; \qquad \frac{dx_s}{dy} < 0,\ \frac{dx_{Hu}}{dy} < 0,\ \frac{dx_{Lu}}{dy} < 0; \qquad \frac{d\alpha_s}{dy} > 0,\ \frac{d\alpha_u}{dy} > 0;$$
$$\frac{dU_s}{dy} > 0,\ \frac{d(U_s/U_u)}{dy} < 0; \qquad \frac{dw_s}{dy} > 0,\ \frac{dw_{Hu}}{dy} > 0; \qquad \frac{d(w_{Hu}/w_L)}{dy} < 0.$$

A general productivity slowdown reduces entry of both types of firms by making them less profitable than before. Since the supply of workers is fixed, the overall worker/job ratio (n^*) increases. So does the ratio of skilled workers to high-tech firms (n^*s/h). All workers have higher unemployment rates and lower expected wages than before. As indicated by (3.14), the lower expected wages come from both the decrease in y and the increase in queue lengths $(n^*, n^*s/h)$. Actual wages also fall for both types of workers.

The contraction is not uniform between the two types of firms. Low-tech firms contract by more than high-tech firms do, and so the fraction of high-tech firms in the economy (h) increases. This is because, for the same decrease in the general productivity, a high-tech firm's expected profit relative to the entry cost falls by less than a low-tech firm's, due to the entry-cost differential. As net expected profit falls by a smaller proportion for a high-tech firm than for a low-tech firm, the proportion of firms exiting from high-tech jobs is smaller than from low-tech jobs.

The non-uniform contraction affects matching rates and wages as follows. First, an unskilled worker's average matching rate and expected wage decrease by more than a skilled worker's, because unskilled workers apply mostly to low-tech firms which contract more severely. Second, the relative wage among unskilled workers, w_{Hu}/w_L, increases because high-tech firms' demand for labour falls by less than low-tech firms'. Third, unskilled workers increase their application to high-tech firms, and so the queue length of unskilled workers for each high-tech firm, n^*-n^*s/h, increases more precipitously than that for each low-tech firm, n^*. Finally, the relative wage of skilled to unskilled workers in high-tech firms, w_s/w_{Hu}, may fall. As unskilled workers increase application to high-tech firms, their congestion level increases relative to that of skilled workers. To ensure the same expected wage for unskilled applicants as low-tech firms do, high-tech firms may reduce unskilled workers' wage w_{Hu} by less than skilled workers' wage w_s.

The wage differential among unskilled workers (DU) rises unambiguously, because the relative wage w_{Hu}/w_L increases and the shift of some unskilled workers to high-tech firms fattens the upper tail of the wage distribution of unskilled workers. In contrast, the skill premium DH may fall or rise. The shift of unskilled workers' application toward high-tech firms fattens the lower tail of the wage distribution in high-tech firms and increases the standard deviation DH, but the relative wage w_s/w_{Hu} may fall and reduce the differential DH. Similarly, the analytical

16. The effects here should be more generally interpreted as those of uneven increases between productivity and capital costs (K_L, K_H). This interpretation is useful because capital costs in the 1970s might have increased more rapidly than productivity, due to energy shocks and high inflation.

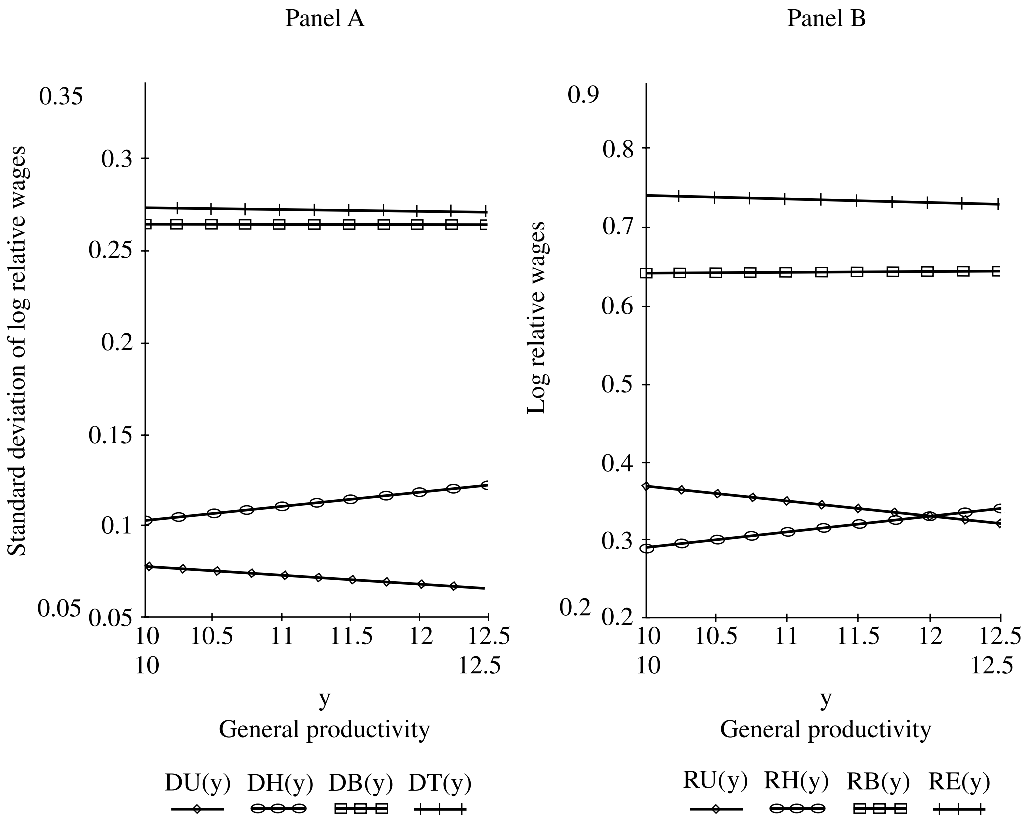

FIGURE 3

Effects of an increase in general productivity

responses of the overall between-skill wage differential (DB) and the overall wage differential among all workers (DT) are ambiguous.

To see the quantitative responses, consider the numerical example in Section 5.1. Fix $\theta = 1{\cdot}91$ and reduce y from 12·5 to 10, with a step 0·25. Figure 3 illustrates the responses of wage inequality (panel A) and log relative wages (panel B) (read the figures backward). First, the log relative wage RU and the wage differential DU among unskilled workers both rise when y falls, as discussed above. Second, the log relative wage in high-tech jobs RH falls. This exerts a negative effect on the wage differential DH that dominates the effect of the change in the skill employment distribution in high-tech firms, and so the wage differential DH falls. Third, the overall log relative wage between skills RB and the corresponding wage differential DB both rise, but the magnitudes are very small. Finally, the overall wage differential DT rises slightly. These results suggest that a general productivity slowdown might be useful for explaining the fall in the education premium and the concurrent rise in within-group wage inequality in the 1970s.

5.4. *Response of skill supply: an extension*

Exogenous changes in skill supply are considered to be an important cause for the changes in wage inequality in the 1970s (see Katz and Murphy, 1992). In particular, the proportion of educated workers increased sharply in the 1970s in the U.S. A part of this increase was exogenous, caused by the baby-boom generation entering the labour force and the compulsory Vietnam draft. To examine the effects of skill supply changes, I now allow skill supply to respond to wage differentials in the following way:

$$s = S\left(\frac{U_s}{U_u}\right) \equiv b \cdot \ln\left(\frac{U_s}{U_u}\right), \qquad b > 0. \tag{5.5}$$

This specification captures the following features: (i) $s > 0$ only if $U_s > U_u$; (ii) a higher relative expected wage for skilled workers attracts more workers to upgrade their skills ($S' > 0$); and (iii) this attraction diminishes as the relative wage increases ($S'' < 0$). An exogenous increase in skill supply can be modelled by an increase in the skill supply elasticity, b. The following proposition summarizes the effects:[17]

Proposition 5.3. *Under the specification* (5.5)*, a small or moderate increase in b has no effect on* (x_s, x_{Hu}, x_{Lu}), $(n^*, n^*s/h)$, (U_s, U_u), (w_s, w_{Hu}, w_L), *or* DH, *but it increases s and h, and reduces* α_u. *If there are more unskilled workers employed in low-tech firms than in high-tech firms, then* DU, DB *and* DT *increase. A sufficiently large increase in b reduces* DU *and may eliminate* DU *altogether.*

When the increase in b is small or moderate, the economy remains in equilibrium I. The worker/job ratios in the two types of firms, n^* and n^*s/h, do not change, because the entry conditions for these firms do not change (see (3.6) and (3.7)). Thus, the queue length, the expected wage and the actual wages are all the same as before. However, skill supply increases, as expected, which induces more firms to set up high-tech jobs. As the fraction of high-tech jobs increases, unskilled workers increase their application to these jobs. This shift of unskilled workers balances the increased skill supply and keeps the skill distribution in high-tech firms unchanged. Since the relative wage of skilled workers in high-tech firms does not change either, the wage differential DH does not change.

In contrast, the wage differential among unskilled workers (DU) changes. As unskilled workers increase application to high-tech firms, the wage distribution of unskilled workers shifts toward high wages. This increases DU if more unskilled workers are employed in low-tech firms than in high-tech firms, which occurs when skill supply is initially low. The increase in within-group inequality relative to the skill premium is consistent with the evidence in the 1970s in the U.S. When the skill supply elasticity increases sufficiently, however, more unskilled workers are employed in high-tech firms than in low-tech firms, in which case the wage differential among unskilled workers falls when skill supply increases further.

Sufficiently large increases of the skill supply elasticity can even switch the economy from equilibrium I into equilibrium II by increasing skill supply beyond the critical level $\bar{s}$ defined in (3.7). In this case low-tech firms and the wage differential among unskilled workers vanish, and the skill premium starts to change with skill supply. The switch increases skilled workers' unemployment rate; meanwhile, the skill premium may rise as unskilled workers move completely to high-tech firms and the lower tail of the wage distribution fattens.[18] As Acemoglu (1999) argues, such concurrent increases in skilled workers' unemployment rate and the skill premium are an important aspect of the U.S. data in the 1970s.

17. To prove the proposition, one substitutes (5.5) into (3.6)–(3.16) and differentiates the equations with respect to b. This exercise is straightforward and hence omitted. The equilibrium responses to shocks in θ and y are similar to those obtained previously, with only slight changes in the magnitudes. The parameter b can be identified by setting the initial value of s to the number 0·2 used in previous calculation, which results in $b = 0{\cdot}27$.

18. Skilled workers' unemployment rate rises iff the ratio of skilled workers to high-tech firms, ns/h, increases. To verify that ns/h increases, set $\pi_H = K_H$ in (3.13), differentiate the equation with respect to s, and note that n falls from n^* to n^{**} when the economy switches from equilibrium I to equilibrium II.

Here the equilibrium switches in a direction opposite to that in Acemoglu (1999), who employs a standard search model. In my model, the increase in skill supply switches the equilibrium from one in which skills are partially separated by jobs into one without separation. In Acemoglu's model, in contrast, the increase in skill supply switches the equilibrium from one without separation into one with complete separation. The root of Acemoglu's result is the exogenous matching function and exogenous wage shares. When skill supply is low, low-tech firms would like to attract unskilled workers by offering them a better employment chance than high-tech firms do, but the exogenous matching function prevents this from occurring, thus resulting in the pooling equilibrium. When skill supply is high, it is inefficient to create low-tech firms, but the exogenous matching function assigns workers to low-tech firms nevertheless, thus enabling those firms to survive by employing unskilled workers.

Because of the above fundamental difference, the current model also has the following results that contrast with Acemoglu's model. First, when the equilibrium switches from I to II, unskilled workers' expected utility (wage) increases from ye^{-n^*} to $ye^{-n^{**}}$ (see Proposition 3.2). Second, workers' overall matching rate, $(1-e^{-n})/n$, increases when the economy switches into equilibrium II, and so the overall unemployment rate falls. In contrast, unskilled workers are worse off in Acemoglu's model and the overall unemployment rate increases when the economy switches into equilibrium II. Finally, there is wage inequality among unskilled workers in this paper, even after controlling for all skills. In Acemoglu's model, there is wage inequality among workers with the same observable skill only if these workers differ in unobserved skills.

6. CONCLUSION

In this paper I analyse the directed search/matching problem in an economy with heterogeneous skills and skill-biased technology. I show that a unique symmetric equilibrium exists and is socially efficient. Matching is partially mixed in the equilibrium. A high-tech firm receives both skilled and unskilled applicants with positive probability, and favours skilled workers, while a low-tech firm receives only unskilled applicants. The model generates wage inequality among unskilled workers, as well as between-skill inequality. Since high-tech firms favour skilled applicants, they must compensate unskilled applicants for the low employment probability by offering them a higher wage than low-tech firms do. This within-group inequality does not rely on the traditional assumptions of innate ability differences and match- or vintage-specific productivity, because unskilled workers in this paper perform the same task and have the same productivity in the two types of firms.

The model produces interesting responses of wage inequality to three key shocks occurred in the last three decades—a skill biased technological progress since 1974, a general productivity slowdown in the 1970s, and an exogenous increase in skill supply in the 1970s. First, skill-biased technological progress generates concurrent increases in within-group inequality and the skill premium, with the latter rising more sharply. Within-group wage inequality among unskilled workers rises because unskilled workers increase their application to expanding high-tech firms, which fattens the upper tail of the wage distribution among unskilled workers. The skill premium rises more sharply than within-group wage inequality because skill-biased progress increases skill wages and also shifts employment of unskilled workers to high-tech firms, the latter of which fattens the lower tail of the wage distribution across skills. Second, a general productivity slowdown increases within-group wage inequality and reduces the skill premium. The wage differential among unskilled workers increases because a general productivity slowdown makes low-tech firms contract by more than high-tech firms. The relative contraction of low-tech firms induces unskilled workers to increase application to high-tech firms, which fattens the upper tail of the wage distribution among unskilled workers and increases wage inequality among these workers. In addition, the increase in unskilled workers' application to high-tech firms increases the relative congestion of unskilled workers to skilled workers in high-tech firms and calls for a decrease in the relative wage of skilled workers to unskilled workers in these firms. The skill premium thus falls. Third, an exogenous increase in the elasticity of skill supply generates an expansion of high-tech firms, increases unskilled workers' application to high-tech firms, and flattens the upper tail of the wage distribution among unskilled workers. This increases wage inequality among unskilled workers when the skill supply elasticity increases slightly or moderately. When the supply elasticity increases sufficiently, however, low-tech firms can disappear altogether, in which case wage inequality among unskilled workers vanishes. The above responses of wage inequality suggest that a skill-biased technological progress is important for the concurrent rise of the skill premium and within-group wage inequality in the 1980s and 1990s, while a general productivity slowdown or a moderate increase in skill supply is important for their opposite movements in the 1970s.

Since the trade-off between wages and matching probabilities is critical to the within-group wage differential in this paper, one might want to check whether it exists in the data. To do so, one can think that the observed density of the wage distribution is proportional to the matching probability in this model. Then the model predicts that such a density is a decreasing function of wages among workers entering the market, controlling for skills.[19] Note, however, that the within-group wage differential does not imply a positive relationship between workers' unemployment duration and observed wages. Rather, a dynamic extension of the current model may imply a positive relationship between workers' unemployment duration and the wages they *applied* to but failed to get.

I have abstracted from dynamics and wage inequality among skilled workers in order to emphasize the result that homogeneous, unskilled workers can rationally choose to work for different wages. It is feasible but more involved to incorporate dynamics (see Shi, 1997) and to generate wage inequality among skilled workers by allowing the skill-biased productivity θ to have different realizations across matches. The model has also abstracted from other important sources of the within-group wage differential, such as the employer size. In a separate paper (Shi, 2001) I show that the size-wage differential can arise among homogeneous workers when there is directed search in both the goods market and the labour market. It remains to check how the size-wage differential interacts with a skill-biased technology.

APPENDIX

A. *Proof of Lemma* 3.1

First I establish the following lemma.

Lemma A.1. (i) *If a type-j firm attracts only unskilled workers, i.e. if $x_{js}=0$, then*

$$y/U_u = e^{x_{ju}} \qquad \text{and} \qquad \pi_j/y = \beta(x_{ju}), \tag{A.1}$$

*where $\beta(\cdot)$ is defined in (*3.5*).*

(ii) *If a type-j firm attracts only skilled workers in an equilibrium, i.e. if $x_{ju}=0$, then*

$$\Theta_j y/U_s = e^{x_{js}} \qquad \text{and} \qquad \pi_j/(\Theta_j y) = \beta(x_{js}). \tag{A.2}$$

19. It is important to confine this test to market entrants. If all employed workers of the same skill are considered, the actual wage density function is likely to be hump-shaped, because workers at the lower end of the wage distribution are more likely to separate from the current jobs and look for higher wages.

(iii) *If* $x_{Ls} > 0$ *and* $x_{Lu} > 0$, *then*

$$U_s = U_u = ye^{-(x_{Ls}+x_{Lu})}, \qquad \pi_L/y = \beta(x_{Ls} + x_{Lu}). \tag{A.3}$$

(iv) *If* $x_{Hs} > 0$ *and* $x_{Hu} > 0$, *then* $U_s > U_u$, $\chi_H = 1$, *and*

$$\pi_H = (1 - e^{-x_{Hs}})\theta y + e^{-x_{Hs}}(1 - e^{-x_{Hu}})y - x_{Hs}U_s - x_{Hu}U_u. \tag{A.4}$$

Proof. Part (i) and Part (ii) are analogous to each other; so I prove Part (i) only. If a type-j firm attracts only unskilled workers in an equilibrium, the expected profit function of this firm is $\pi_j = (1-e^{-x_{ju}})(y - w_{ju})$ and the constraint is $g(x_{ju})w_{ju} = U_u$, where $g(\cdot)$ is defined in (2.5). Substituting w_{ju} from the constraint, I have $\pi_j = (1 - e^{-x_{ju}})y - x_{ju}U_u$. To maximize π_j, the first-order condition for x_{ju} is $y/U_u = e^{x_{ju}}$. Substituting back into the expected profit function, one gets π_j as in (A.1).

Part (iii). If $x_{Ls} > 0$ and $x_{Lu} > 0$, then $w_{Ls} = U_s/q_{Ls}$ and $w_{Lu} = U_u/q_{Lu}$ by (2.6). Substitute (w_{Ls}, w_{Lu}) in (2.7) with $j = L$, I have $\pi_L = [1 - e^{-(x_{Ls}+x_{Lu})}]y - x_{Ls}U_s - x_{Lu}U_u$. The first-order conditions for x_{Ls} and x_{Lu} immediately lead to the expressions for (U_s, U_u) in (A.3). Substituting (U_s, U_u), the above expression for π_L becomes that in (A.3).

Part (iv). If $x_{Hs} > 0$ and $x_{Hu} > 0$, then $w_{Hs} = U_s/q_{Hs}$ and $w_{Hu} = U_u/q_{Hu}$ by (2.6). Substituting (w_{Hs}, w_{Hu}) in (2.7) with $j = H$, I have

$$\begin{aligned}\pi_H = {} & [\chi_H + (1 - \chi_H)e^{-x_{Hu}}](1 - e^{-x_{Hs}})\theta y \\ & + (1 - \chi_H + \chi_H e^{-x_{Hs}})(1 - e^{-x_{Hu}})y - x_{Hs}U_s - x_{Hu}U_u.\end{aligned}$$

Then, $\partial\pi_H/\partial\chi_H > 0$ and so $\chi_H = 1$. Substituting $\chi_H = 1$ into the above expression of π_H, one obtains (A.4). To maximize π_H, the first order conditions for x_{Hs} and x_{Hu} then imply $U_s/y = (\theta - 1)e^{-x_{Hs}} + e^{-(x_{Hs}+x_{Hu})} > e^{-(x_{Hs}+x_{Hu})} = U_u/y$. Thus, $U_s > U_u$. ‖

To establish Lemma 3.1, it suffices to show that case (iv) of Lemma A.1 applies in all symmetric equilibria and that case (i) applies for $j = L$ (*i.e.* $x_{Ls} = 0$ and $x_{Hs} = ns/h$). Lemmas A.2 through A.4 below establish these desired results. To economize on space, I only sketch the proofs of these lemmas. Detailed proofs are available upon request.

Lemma A.2. $x_{Hs} > 0$ *in equilibrium.*

Proof. Suppose that $x_{Hs} = 0$ in an equilibrium. I derive a contradiction.

Case (i): $0 < x_{Hu} < n(1-s)/h$. Consider a deviation by a high-tech firm, $(w^d_{Hs}, w^d_{Hu}, \chi^d_H)$, that satisfies the following conditions: $\chi^d_H = 1$, $w^d_{Hs} = U_s + \varepsilon$, and $e^{-x^d_{Hs}}w^d_{Hu} = w_{Hu}$, where $\varepsilon > 0$ is an arbitrarily small number and x^d_{Hs} is the queue length of skilled workers that the deviator attracts. The queue length x^d_{Hs} satisfies $g(x^d_{Hs})w^d_{Hs} = U_s$. The deviator's expected profit, denoted π^d_H, is given by (2.7) with $(w^d_{Hs}, w^d_{Hu}, x^d_{Hs})$ replacing (w_{Hs}, w_{Hu}, x_{Hs}). Note that $\pi^d_H = \pi_H$ when $\varepsilon = 0$. The deviation is profitable if $d\pi^d_H/d\varepsilon > 0$ when $\varepsilon \to 0$. Computation yields

$$\left.\frac{d\pi^d_H}{d\varepsilon}\right|_{\varepsilon=0} = 2\left[(\theta - 1 + e^{-x_{Hu}})\frac{y}{U_s} - 1\right].$$

Because Part (i) of Lemma A.1 applies to high-tech firms in the supposed equilibrium, $y/U_u = e^{x_{Hu}}$. Because Part (iii) of Lemma A.1 applies in the supposed equilibrium, $y/U_s = y/U_u$. Thus, $y/U_s = e^{x_{Hu}}$. Substituting this result, it is evident that $d\pi^d_H/d\varepsilon > 0$ when $\varepsilon \to 0$.

Case (ii): $x_{Hu} = n(1 - s)/h$. Consider the same deviation as in Case (i). Since Part (ii) of Lemma A.1 applies to low-tech firms, $y/U_s = e^{x_{Ls}}$, and so the deviation is profitable iff $\theta - 1 +$

$e^{-x_{Hu}} - e^{-x_{Ls}} > 0$ (see Case (i) for $d\pi_H^d/d\varepsilon|_{\varepsilon=0}$). Since $\pi_L/y = \beta(x_{Ls})$ and $\pi_H/y = \beta(x_{Hu})$ in the supposed equilibrium, the two free-entry conditions then yield $x_{Ls} = B(K_L/y)$ and $x_{Hu} = B(\theta K_H/y)$. Because $K_H < \theta K_L$ by Assumption 1, $x_{Hu} < B(\theta K_L/y)$. Thus, the following condition is sufficient for the deviation to be profitable: $\theta - 1 + e^{-B(\theta K_L/y)} - e^{-B(K_L/y)} > 0$. This condition is satisfied for all $\theta > 1$, because its left-hand side is an increasing function of θ and has a value 0 when $\theta = 1$.

Case (iii): $x_{Hu} = 0$. There is no high-tech firm in this case. Moreover, Part (iii) of Lemma A.1 applies, yielding $y/U_s = e^n$ and $K_L/y = \beta(n)$. Consider a firm that deviates by setting up a high-tech job and announcing the following offer: $w_{Hs}^d > U_s$, $\chi_H^d = 1$, and $w_{Hu}^d = 0$. This offer attracts only skilled applicants and the queue length of such applicants is x_{Hs}^d that satisfies $g(x_{Hs}^d)w_{Hs}^d = U_s$. Part (ii) of Lemma A.1 applies to the deviator and so the deviator's best decisions obey (A.2) for $j = H$, with x_{Hs}^d replacing x_{Hs} and π_H^d replacing π_H. Since $U_s/y = e^{-n}$, (A.2) implies $x_{Hs}^d = n + \ln\theta$ and $\pi_H^d/y = \theta - (1 + n + \ln\theta)e^{-n}$. Then, $\pi_H^d > \theta y\beta(n)$, *i.e.* $(\theta - 1)(1 + n) - \ln\theta > 0$. The latter condition holds, because its left-hand side is an increasing function of θ for all $\theta > 1$, and is equal to 0 when $\theta = 1$. Because $K_L/y = \beta(n)$, the firm gets $\pi_H^d > \theta K_L > K_H$. That is, the deviation is profitable. ‖

Lemma A.3. *$x_{Hs} = ns/h$ (i.e. $x_{Ls} = 0$) in equilibrium.*

Proof. By Lemma A.2, $x_{Hs} > 0$. To establish the current Lemma, suppose that $0 < x_{Hs} < ns/h$. I derive a contradiction. Note that the case with both $0 < x_{Hs} < ns/h$ and $0 < x_{Hu} < n(1-s)/h$ cannot occur, because in such a case Parts (iii) and (iv) of Lemma A.1 apply to both types of firms which yield contradicting results on the difference $U_s - U_u$. So, $x_{Hu} = n(1-s)/h$ or $x_{Hu} = 0$, if $0 < x_{Hs} < ns/h$.

Case (i): $x_{Hu} = n(1-s)/h$. In this case, part (ii) of Lemma A.1 applies to low-tech firms and Part (iv) applies. Consider that a low-tech firm deviates to wages (w_{Ls}^d, w_{Lu}^d) that satisfy

$$(1 - \chi_L + \chi_L e^{-x_{Ls}})w_{Lu}^d = U_u + \varepsilon;$$
$$(1 - \chi_L + \chi_L e^{-x_{Ls}})g(x_{Lu}^d)w_{Lu}^d = U_u;$$
$$[\chi_L + (1 - \chi_L)e^{-x_{Lu}^d}]g(x_{Ls})w_{Ls}^d = U_s;$$

where $\varepsilon > 0$ is an arbitrarily small number. Computing the deviator's expected profit, π_L^d, one can show that $d\pi_L^d/d\varepsilon|_{\varepsilon=0} > 0$ iff $e^{-x_{Ls}}y/U_u > 1$. The latter condition holds in the supposed equilibrium because $y/U_u > y/U_s = e^{x_{Ls}}$.

Case (ii): $x_{Hu} = 0$. In this case, only skilled workers apply to high-tech firms. So, Part (ii) of Lemma A.1 applies to high-tech firms, yielding $\theta y/U_s = e^{x_{Hs}}$. Also, Part (iii) of that Lemma applies to low-tech firms, so $y/U_s = e^{(x_{Ls}+x_{Lu})}$. Then, $x_{Hs} = x_{Ls} + x_{Lu} + \ln\theta > x_{Ls} + x_{Lu}$. Using Parts (ii) and (iii) of Lemma A.1 to obtain π_H and π_L, the free-entry conditions and Assumption 1 imply $\beta(x_{Ls} + x_{Lu}) = K_L/y > K_H/(\theta y) = \beta(x_{Hs})$. Since $\beta'(x) > 0$, this result implies $x_{Ls} + x_{Lu} > x_{Hs}$, contradicting the earlier result. ‖

Lemma A.4. *$x_{Hu} > 0$ in equilibrium.*

Proof. Suppose that $x_{Hu} = 0$. I derive a contradiction. Consider that a high-tech firm maintains $\chi_H = 1$ and the wage w_{Hs} for skilled workers but deviates to offer w_{Hu}^d to unskilled workers, where w_{Hu}^d satisfies $e^{-x_{Hs}}w_{Hu}^d = U_u + \varepsilon$ ($\varepsilon > 0$). For $\varepsilon > 0$, the offer will attract some unskilled workers and the queue length of unskilled workers attracted, denoted x_{Hu}^d, satisfies $e^{-x_{Hs}}g(x_{Hu}^d)w_{Hu}^d = U_u$. Note that the deviation does not affect the queue length of skilled applicants. The deviator's expected profit is

$$\pi_H^d = (1 - e^{-x_{Hs}})\theta y + e^{-x_{Hs}}(1 - e^{-x_{Hu}^d})y - x_{Hs}U_s - x_{Hu}^d U_u.$$

Then, $d\pi_H^d/d\varepsilon > 0$ for sufficiently small ε iff $e^{-x_{Hs}}y/U_u - 1 > 0$, in which case the deviation is profitable. To show that the latter condition holds in the supposed equilibrium, note that $x_{Hu} = 0$ and $x_{Ls} = 0$ (by Lemma A.3). Part (i) of Lemma A.1 applies to low-tech firms and Part (ii) to high-tech firms. So, $y/U_u = e^{x_{Lu}}$. Moreover, the free-entry conditions imply $\beta(x_{Hs}) = K_H/(\theta y) < K_L/y = \beta(x_{Lu})$. Since $\beta'(x) > 0$, this result implies $x_{Hs} < x_{Lu}$. Thus, $e^{-x_{Hs}}y/U_u - 1 = e^{x_{Lu}-x_{Hs}} - 1 > 0$, and so the deviation is profitable. ‖

B. *Proof of Proposition* 3.2

Consider equilibrium I first, where $x_{Lu} > 0$. Under Lemma 3.1, $x_{Lu} > 0$ if and only if $0 < h < 1$. Maximizing π_H in (A.4), one can derive the first-order conditions of x_{Hs} and x_{Hu} as follows:

$$U_u/y = e^{-(x_{Hs}+x_{Hu})}; \tag{B.1}$$

$$U_s/y = (\theta - 1)e^{-x_{Hs}} + e^{-(x_{Hs}+x_{Hu})}. \tag{B.2}$$

Since only unskilled workers apply to low-tech firms, Part (i) of Lemma A.1 applies to low-tech firms, which yields $U_u/y = e^{-x_{Lu}}$. Combining this with (B.1), I have $x_{Lu} = x_{Hs} + x_{Hu}$. With this result and $x_{Hs} = ns/h$, the adding up condition (2.2) implies $x_{Lu} = n$ and $x_{Hu} = n - ns/h$, as (3.11) indicates. Substituting these expressions for the xs into (B.1) and (B.2), I get (3.14). The equations in (3.15) come from (2.6), applied to a high-tech firm for the two types of workers, together with $q_{Hs} = g(x_{Hs})$ and $q_{Hu} = e^{-x_{Hs}}g(x_{Hu})$. The wage w_{Lu} in (3.16) comes from (2.6), applied to a low-tech firm for unskilled workers, with $q_{Lu} = g(x_{Lu})$. The expression for π_L in Part (i) of Lemma A.1 yields $\pi_L = y\beta(n)$, as in (3.16). Then, the free-entry condition $\pi_L = K_L$ yields $n = n^*$, where n^* is defined in (3.6). Substituting $x_{Hs} = ns/h$ and $x_{Hu} = n - ns/h$ into (A.4) in Part (iv) of Lemma A.1, I have π_H as in (3.13). Since $\beta(n) = K_L/y$, the free-entry condition $\pi_H = K_H$ and the definition of n^* in (3.6) imply $h = s/\bar{s}$. The requirement for the current equilibrium, $0 < h < 1$, is then equivalent to $s < \bar{s}$.

To establish the existence of equilibrium I under $s < \bar{s}$, it now suffices to show that there is no incentive for a low-tech firm to attract skilled workers. Consider a deviation by a low-tech firm, $(w_{Ls}^d, w_{Lu}^d, \chi_L^d)$, that attracts a positive queue length, x_{Ls}^d, of skilled workers. Given $U_s > U_u$, this deviation is not profitable if it also attracts some unskilled applicants (see the proof of Part (iii) in Lemma A.1). Suppose that the deviation drives away all unskilled workers. Then, Part (ii) of Lemma A.1 applies to the deviator, which yields $U_s/y = e^{-x_{Ls}^d}$ and $\pi_L^d/y = \beta(x_{Ls}^d)$. Because $U_s > U_u = ye^{-n^*}$ in the current case, $x_{Ls}^d < n^*$ and so $\pi_L^d/y < \beta(n^*) = \pi_L/y$, *i.e.* the deviation is not profitable. This shows that equilibrium I exists iff $s < \bar{s}$.

Now consider equilibrium II, where $x_{Lu} = 0$. In this case, all workers apply to high-tech firms and so low-tech firms do not exist, *i.e.* $h = 1$. Then, $x_{Hs} = ns$ and $x_{Hu} = n(1 - s)$. Again, (A.4) applies. To maximize π_H, the first-order conditions for (x_{Hs}, x_{Hu}) yield (3.14), with which (A.4) yields (3.13). The wages in (3.15) come from (2.6). With (3.13) and $h = 1$, the free-entry condition $\pi_H = K_H$ yields $\beta(n) + (\theta - 1)\beta(ns) = K_H/y$. By (3.8), this solves $n = n^{**}$.

For the outcome $h = 1$ to be consistent with equilibrium, there should be no incentive for a firm to set up a low-tech job. Consider a possible entrant in the low-tech job. Because $U_s > U_u$, the best decision of the entrant should attract only unskilled workers (see the proof of Part (iii) of Lemma A.1). Let w_{Lu}^d be the entrant's offer wage to unskilled workers and x_{Lu}^d the queue length of such applicants to the entrant. Then, Part (i) of Lemma A.1 implies that the entrant's maximum expected profit is $\pi_L^d = y\beta(x_{Lu}^d)$, where $e^{x_{Lu}^d} = y/U_u$. For the entry to be not profitable, it is necessary and sufficient that $K_L \geq \pi_L^d = y\beta(x_{Lu}^d)$. Because $x_{Lu}^d = \ln(y/U_u) = n^{**}$ in the current equilibrium and $K_L/y = \beta(n^*)$, the entry is not profitable iff $n^* \geq n^{**}$. Since $n^* \geq n^{**}$ iff $s \geq \bar{s}$, a type II equilibrium exists iff $s \geq \bar{s}$.

Finally, the choice $\chi_H = 1$ is *ex post* optimal if $\theta y - w_{Hs} > y - w_{Hu}$. Abbreviating ns/h as x_s, I can use (3.14) and (3.15) to show that $\theta y - w_{Hs} > y - w_{Hu}$ iff

$$0 < (\theta - 1)\left(1 - \frac{x_s}{e^{x_s} - 1}\right) + \left(\frac{n - x_s}{e^{n - x_s} - 1} - \frac{x_s}{e^{x_s} - 1} e^{-(n - x_s)}\right). \tag{B.3}$$

Note that $x/(e^x - 1) < 1$ and $x - 1 + e^{-x} > 0$ for all $x > 0$. Since $n - x_s = x_{Hu} > 0$, then

$$\frac{n - x_s}{e^{n - x_s} - 1} - \frac{x_s}{e^{x_s} - 1} e^{-(n - x_s)} > \frac{n - x_s}{e^{n - x_s} - 1} - e^{-(n - x_s)} > 0.$$

Indeed, (B.3) holds and $\chi_H = 1$ is *ex post* optimal. ‖

C. *Proofs of Propositions* 4.1, 4.2 *and* 5.2

For Proposition 4.1, substitute (3.14) into (3.15). Then, $w_s > w_{Hu} \iff$

$$\theta > 1 + \frac{e^{n^* s/h} - 1}{n^* s/h} \cdot \frac{n^* - n^* s/h}{e^{n^* - n^* s/h} - 1} - e^{-n^* + n^* s/h}. \tag{C.1}$$

The right-hand side of (C.1) is an increasing function of s/h and hence a decreasing function of θ (note that n^* is independent of θ and s/h $(= \bar{s})$ is a decreasing function of θ, see (3.9)). Denote this function by $RHS(\theta)$ temporarily. When $\theta \to \infty$, $\bar{s} \to 0$ and $\theta > RHS(\theta)$. When $\theta \to K_H/K_L$, $\bar{s} \to 1$ and $RHS(\theta) \to (e^{n^*} - 1)/n^*$. If $(e^{n^*} - 1)/n^* > K_H/K_L$, there is a number θ_1 $(> K_H/K_L)$ such that $\theta > RHS(\theta)$ iff $\theta > \theta_1$. If $(e^{n^*} - 1)/n^* \leq K_H/K_L$, then $\theta > RHS(\theta)$ for all $\theta > K_H/K_L$. Then, $w_s > w_{Hu}$ if $\theta > \max\{\theta_1, K_H/K_L\}$.

For Proposition 4.2, compare w_L in (3.16) with w_{Hu} in (3.15). Then, $w_{Hu} > w_L \iff (n^* - x_s)(1 - e^{-n^*}) - n^*(e^{-x_s} - e^{-n^*}) > 0$. Temporarily denote the left-hand side of the inequality by $LHS(x_s)$ for given n^*. Note that $x_s \in (0, n^*)$. Since $LHS(0) = LHS(n) = 0$ and $LHS(\cdot)$ is concave for $x_s \in (0, n^*)$, $LHS(x_s) > 0$ for all $x_s \in (0, n^*)$.

For Proposition 5.2, temporarily drop the subscript s on x and denote $n^* s/h$ by x. Differentiating (3.6), (3.7) and the equation for x_s in (3.11) with respect to y yields

$$\frac{dn^*}{dy} = -\frac{\beta(n^*)}{y\beta'(n^*)}; \qquad \frac{dx}{dy} = -\frac{\beta(x)}{y\beta'(x)}; \qquad \frac{dh}{dy} = \frac{n\beta(x)\beta'(n^*) - x\beta'(x)\beta(n^*)}{n^*\beta'(n^*)x\beta'(x)y/h}.$$

Since $\beta' > 0$, then $dn^*/dy < 0$ and $dx/dy < 0$, implying $dx_{Lu}/dy < 0$, $dU_u/dy > 0$ and $d\alpha_s/dy > 0$. To show $dh/dy < 0$, temporarily denote the numerator of the expression for dh/dy by $RHS(n^*)$ for fixed $x < n^*$. Then $dh/dy < 0$ if and only if $RHS(n^*) < 0$. Using $n^* > x$, I can show that $RHS'(n^*) < (2 - x)\beta(x) - x^2 e^{-x} = 2 - x - (2 + x)e^{-x}$. The function $2 - x - (2 + x)e^{-x}$ has a value zero when $x = 0$, a derivative $-\beta(x) < 0$, and hence is negative for all $x > 0$. Thus, $RHS'(n^*) < 0$ for all $n^* > x$. Because $RHS(x) = 0$, $RHS(n^*) < 0$ for $n^* > x$.

The matching rate for an unskilled worker, α_u, can be shown to be a decreasing function of (n^*, h). Since (n^*, h) both fall with y, $d\alpha_u/dy > 0$. The responses of wages stated in the proposition can be verified directly. ‖

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