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A DIRECTIVE DEVICE FOR CHARGED PARTICLES AND ITS USE IN AN ENHANCED NEUTRINO BEAM

BY

S. van der Meer

Geneva

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SUMMARY.

Divergent beams of charged particles can be made nearly parallel by a magnetic horn that is analogous to an internally reflecting conical surface in geometrical optics. Calculations show that it will accept a strongly divergent beam of charged particles over a wide momentum range. The outgoing particles will make angles with the axis of the horn of the order of a few degrees.

The current and power necessary for excitation will demand pulsed operation.

Data are presented for a horn, designed for increasing the intensity of a neutrino beam. It will concentrate π-mesons from an external target in the direction of the detectors. It is shown that this device would accept nearly all pions of one sign, produced in the target. With this horn the neutrino interaction rate could be increased by an order of magnitude over that contemplated in the present experiment.

1. PRINCIPLE OF THE MAGNETIC HORN.

In fig. 1 a conical reflecting surface is shown in cross section with a light source in point A. Light rays will be reflected several times against the inside of this "horn", as indicated. It can be shown that after each reflection the angle between the light ray and the axis has decreased by an amount φ, equal to the opening angle of the horn. Thus the rays are made more and more parallel, until their angles are small enough to permit their escape.

A similar result can be obtained with charged particles and a magnetic field, as illustrated in fig. 2. Here the horn is made of a conducting material, through which a current flows. The return path for the current is formed by a co-axial cylinder or cone further outside. Between the two surfaces a magnetic field will exist, inversely proportional to the distance from the axis. No field will exist inside the central cone. Particles leaving a source at A will traverse the inner cone, be bent back by the magnetic field, and come back to the field-free space inside. Their angle with the axis will then be smaller, although there is no simple quantitative expression as in the case of fig. 1. As shown in fig. 2, the decrease in angle is equal to the deflection between points B and C. Only one reflection is shown in this diagram.

In principle, this device will accept particles of all emission angles below a maximum value, depending on dimensions, magnetic field strength and particle momentum. Particles with larger emission angles will not be made sufficiently parallel to the axis within the length of the horn.

Particles will be lost by interactions in the inner conductor. The thickness of the metal will be determined in practice by the minimum strength, necessary for withstanding the forces exerted by the magnetic field.

The magnetic horn will concentrate charged particles without making a sharp focus. On the other hand, the accepted momentum range will be greater than that obtained with lenses of the type that produce images. The exact shape and strength of the field and the position of the source of particles are not of great importance.

2. SOME CONSIDERATIONS ON THE DIMENSIONS OF A HORN.

The trajectory of a particle in a field proportional to $^1/r$ cannot be described with simple algebraic equations and the passages between the magnetic field and the field-free interior complicate the problem of evaluating the performance of a horn. A numerical solution is necessary. In par. 5 an example will be given.

A general remark can be made concerning the energy stored in the magnetic field. This is an important parameter, since it determines the cost of the power supply.

If at equal current all dimensions of the horn are increased by a factor n, the following scaling laws apply :

	multiplied by
field in corresponding points	$^1/_n$
radius of curvature of trajectories	n
angle of outgoing particles	1
energy density in magnetic field	$^1/_{n^2}$
volume	n^3
total energy stored	n

Thus it is seen that for the same directive properties the stored energy would be increased. It is therefore advantageous to reduce the dimensions of the horn. The limit may come from one of the following considerations:

a) The mechanical pressure on the conducting surfaces due to the magnetic field is inversely proportional with the square of the linear dimensions. This pressure may become considerable and make the construction difficult.

b) For high energy particles a very high excitation current is required. The heating of the inner conductor at the narrow side of the horn may limit the reduction in size. Even with pulsed operation the instantaneous temperature rise during the pulse may be too high. If the pulses are made very short, the skin effect will become important. The decrease of the pulse duration is then compensated by the increase of the current density. The temperature increase of the outer layer during the pulse approaches a constant value for decreasing pulse length, as soon as the penetration depth for the current becomes small compared with the conductor thickness.

3. SHAPE OF THE HORN.

It is not evident that a conical horn, as shown in fig. 2, is optimum. It might be thought that e.g. a trumpet or bell-shaped cross section might have advantages. This could be examined by numerical studies. The optimum shape might depend on the relative efficiency required for particles of different momenta and different emission angles, and might therefore be a function of the particular application.

Once the current and the shape of the inner conductor have been fixed, the optimum shape of the outer conductor can be determined easily, since it does not influence the field inside. In order to reduce the stored energy as much as possible, the outer conductor should be made so that it is just cleared by the particles of highest momentum and maximum emission angle.

4. APPLICATION TO THE NEUTRINO EXPERIMENT.

A numerical example has been worked out for using a horn to intensify a neutrino beam in connection with current work in CERN (1, 2, 3). The design data given represent one of the first solutions, but not necessarily the optimum one.

For neutrino experiments it would be desirable to concentrate as many pions as possible in the direction of the detector. Precise focusing is not necessary, since in any case the neutrinos from the decaying pions will have slightly different directions. On the other hand, a wide momentum range should be accepted by the directive device, so that the properties of a horn are well suited to this application.

The source of the pions should be located inside the horn, and a short burst is desirable in order to permit short excitation pulses. Therefore it is considered that a fast ejected proton beam hits the target in the axial direction.

5. RESULTS OF TRAJECTORY CALCULATIONS.

The dimensions of the horn for which the results are given, are shown (on axes of different scale) in fig. 3.

The trajectories were determined by numerical integration with the Mercury computer. A current of 5.10^5 A (stored energy 75 kJ) was supposed to be concentrated on the outside of the inner conductor. A separate calculation confirmed that the performance of the horn will be improved slightly if the current is distributed equally over the conductor cross section.

The target should be a cylindrical rod of heavy material (e.g. tungsten) in order to obtain a good efficiency in a short length. It might be about 15 cm long (2 interaction lengths) and of diameter about 5 mm, since it is thought to be possible to concentrate most of the ejected proton beam on such a cross section. The advantage of a small diameter is that the pions emitted at an angle to the axis will have less probability of being absorbed in the target.

The first part of the horn around the target is made cylindrical, in order to have equal horn efficiency over its whole length.

Figs. 4 and 5 show the inclination to the axis of a pion leaving the horn (φ_{out}) vs emission angle from the target (φ_{in}) for various momenta. The discontinuities in the slope of these curves are caused by the sudden changes of the number of reflections, as the emission angle increases.

In figs. 4 and 5 the pion is supposed to be emitted at the end of the target (B in fig. 3). The curves for the particles starting from point A are similar.

Fig. 6 shows the position in phase space at the end of the horn for pions of 1 GeV/c, emitted at B, as a function of emission angle. From this diagram the difference with a system of lenses is apparent.

All results illustrated in figs. 4 to 6 are for particles with trajectories in a plane through the axis of the horn. Additional calculations have been made for particles whose trajectory near the target crosses the axis at a few mm distance. It was found that this does not make an important difference.

6. DISCUSSION OF THE RESULTS.

Two factors are important in estimating the performance of a horn in the present application :

a) The emission angle that should be accepted as a function of pion momentum in order to use a reasonable proportion of the pions produced in the target.

b) The maximum inclination of the outgoing pions to the axis that can be tolerated without too much loss.

ad a) In order to obtain an estimate of the required angular acceptance, some assumption had to be made about the production spectrum for pions. For this purpose, the data given in (4), and reproduced here in fig. 7 (solid curves) were used. They were redrawn for use with a system of rotational symmetry by calculating

$$\frac{\delta^2 N}{\delta p \delta \vartheta} = 2 \pi \sin \vartheta \, \frac{\delta^2 N}{\delta p \delta \omega}$$

as a function of ϑ and p. This gives the amount of pions, emitted in a "differential cone", as shown in fig. 8. Some imagination was needed in order to construct these curves from the data available and they should not be considered highly reliable. They give, however, an impression of the region of emission angles that is most important for several values of p_π . By comparing this diagram with the curves of figs. 5 and 6, it can be seen that nearly all pions are accepted and made parallel with the axis to better than 4°.

ad b) The precision with which the pion trajectories should be made parallel can be estimated in the following way. A pion emerging from the horn at an angle φ_{out} (fig. 9) should, in order to be useful, emit a neutrino at an angle

$$\varphi_\nu \approx \varphi_{out} \cdot \frac{L_2}{L_2 - 0.5 L_1}$$

if we assume that the pion decays on the average halfway between the horn and the screening wall. For practical cases $\varphi_{\nu} \approx 1.25\ \varphi_{out}$. The neutrino intensity in this direction, multiplied with the decay probability in 25 m, and with the cross section for the reaction

$$\nu + n \rightarrow p + e^{-}$$

was calculated. It is shown in fig. 10 in relative figures. The cross section for this reaction was taken from (5). It is reproduced in fig. 11, curve a.

The curves of fig. 10 give an indication of the tolerable divergence of the pion beam. They show clearly that precision of focusing is most important for high pion momentum. They also show when compared with figs. 4 and 5, that the efficiency of the horn presented here is still far from ideal.

The neutrino interaction rate was calculated for the horn described before by combining the curves of figs. 4, 5, 8 and 10 for various values of p_{π}. The result is shown in relative figures in fig. 12 (full line). The curve shown as a dotted line gives the interaction rate under the same circumstances, but without the horn.

The total interaction rate is proportional to the surface under these curves. The factor of improvement given by the presence of the horn is about 13.

The horn will concentrate either positive or negative pions and remove those of the opposite sign, except the part that passes through the central aperture. It is therefore possible to change the ratio between neutrinos and antineutrinos significantly by reversing the excitation current.

The ratio of improvement for antineutrinos was found to be about 15, using the same method of calculation as described above, and the cross section shown in fig. 11, curve b.

The same calculation was repeated for horns with semi-angles of 2.5° and 1°. It seems that these would give a somewhat lower efficiency. However, the results depend strongly on the curves of fig. 7. If at large emission angles the production of pions were higher, the optimum horn angle might be greater. If the horn angle is increased, the overall quality of focusing deteriorates, but larger emission angles are accepted.

The loss due to interactions in the material of the horn can be estimated from the results of the trajectory computation. The path length of the pions in the metal is clearly a function of the emission angle. With reasonable assumptions about the wall thickness of the inner cone, and supposing that aluminium is used, an average of about half an interaction length is found. This would result in a reduction of 40 o/o, if interacting particles would be lost. However, the pions produced in interactions would partly be focused again. The lower momentum of the secondary pions would mostly decrease the interaction rate, as can be seen from fig. 10. It is very difficult to make a good estimate of the resulting loss, but it is thought to be safe to assume that an improvement factor of at least 10 will remain.

7. ACKNOWLEDGEMENTS.

It is a pleasure to thank G. Pluym and B. de Raad for discussions and suggestions, and C.A. Ramm for stimulating the search for means of intensifying neutrino beams.

S. van der Meer

/fv

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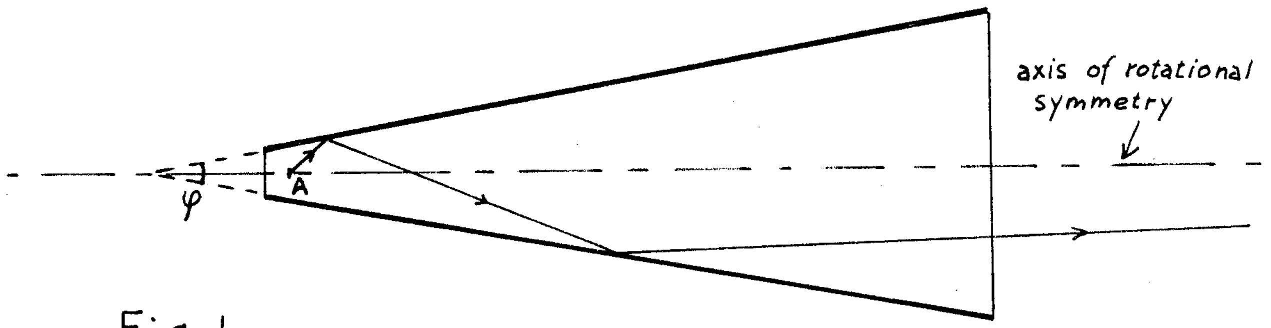


Fig. 1

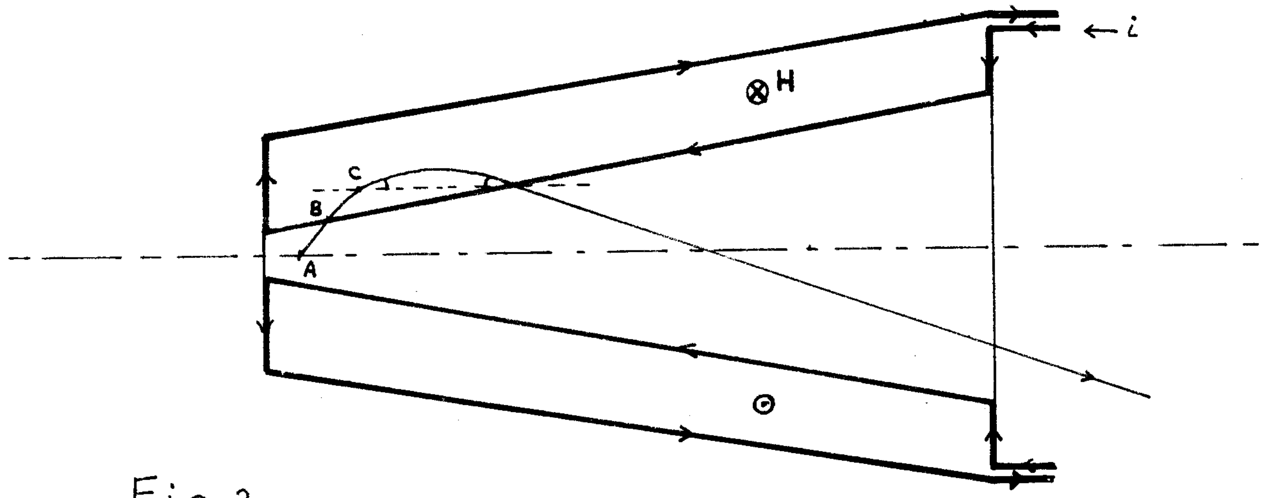
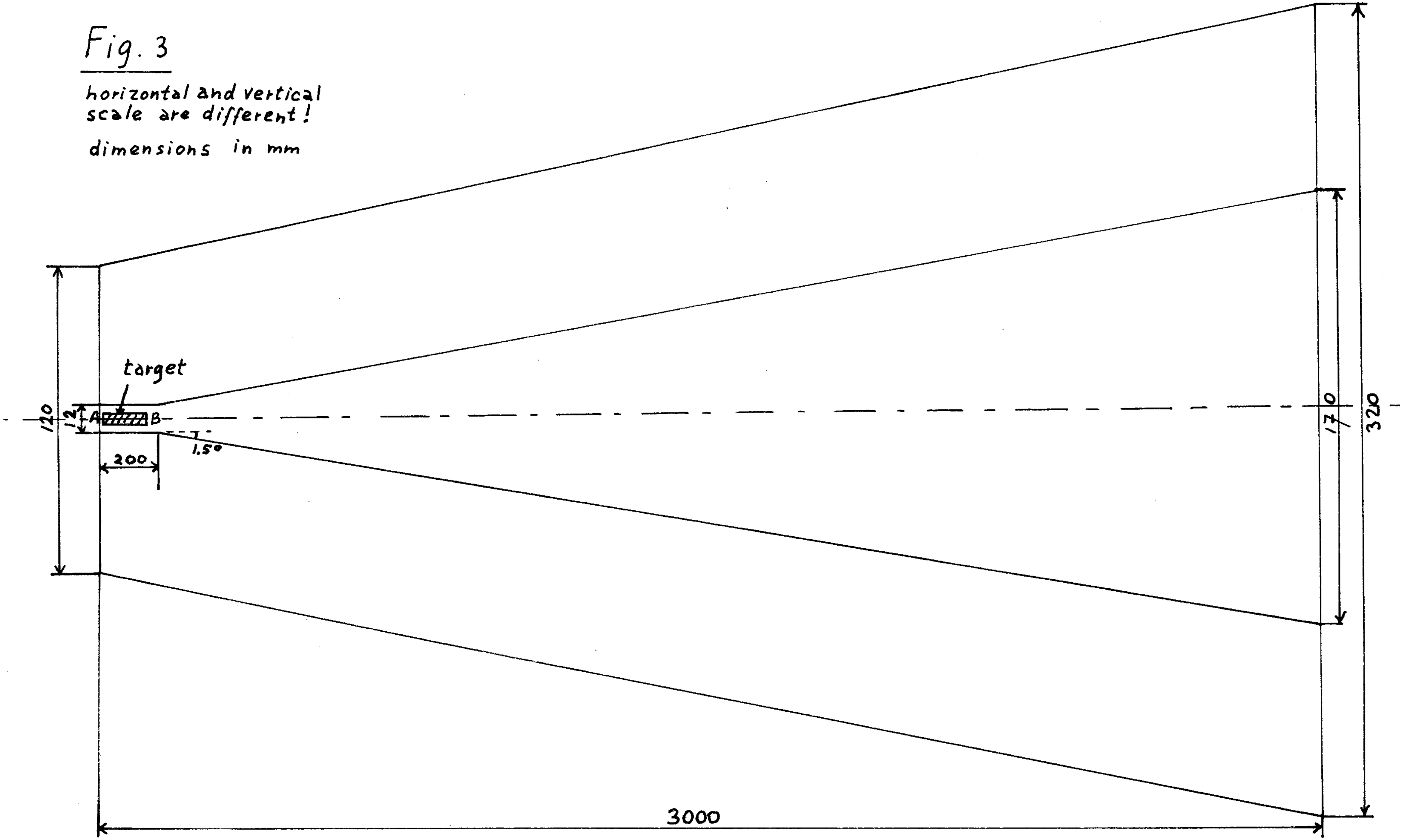


Fig. 2

Fig. 3

horizontal and vertical scale are different!

dimensions in mm



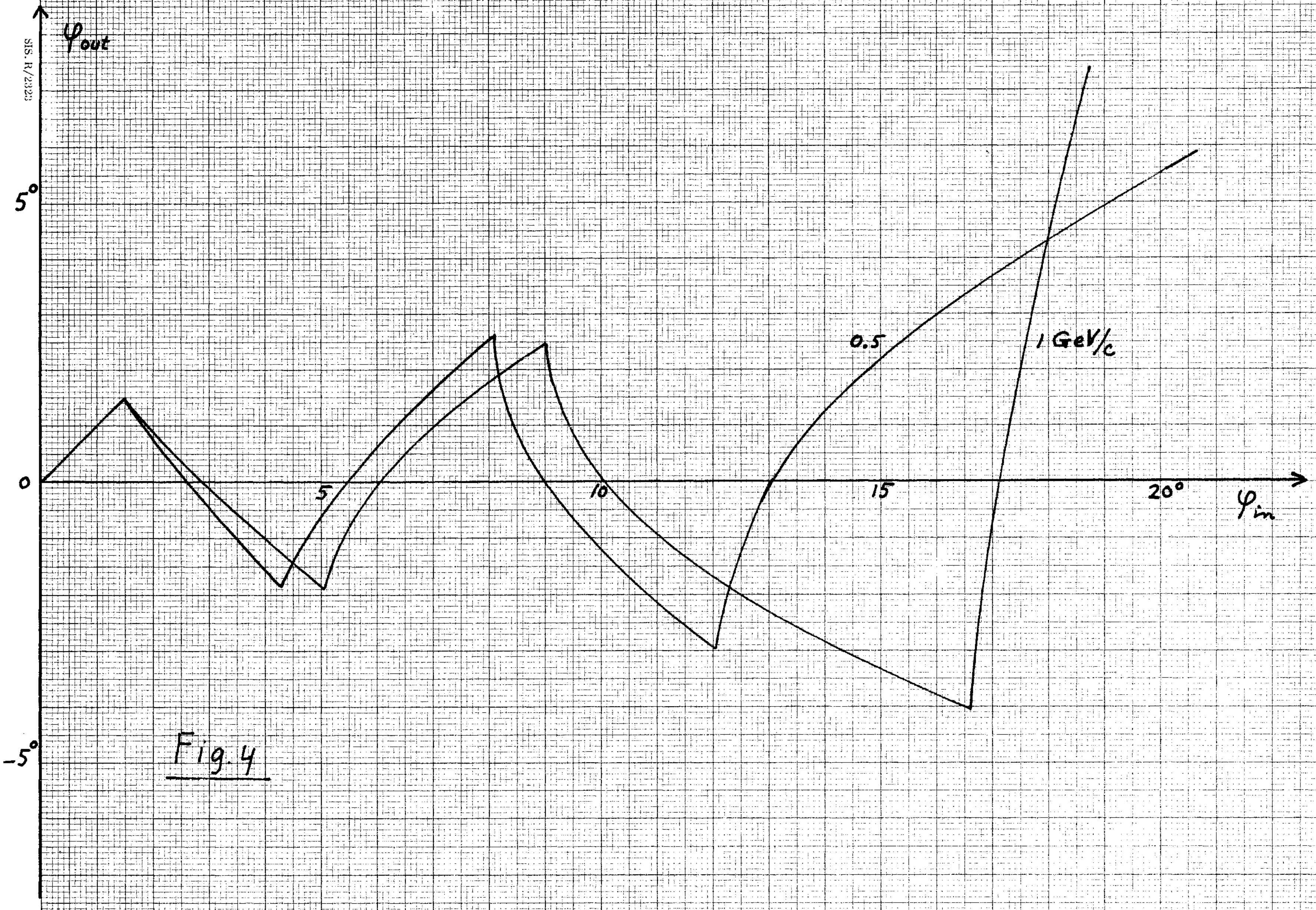


Fig. 4

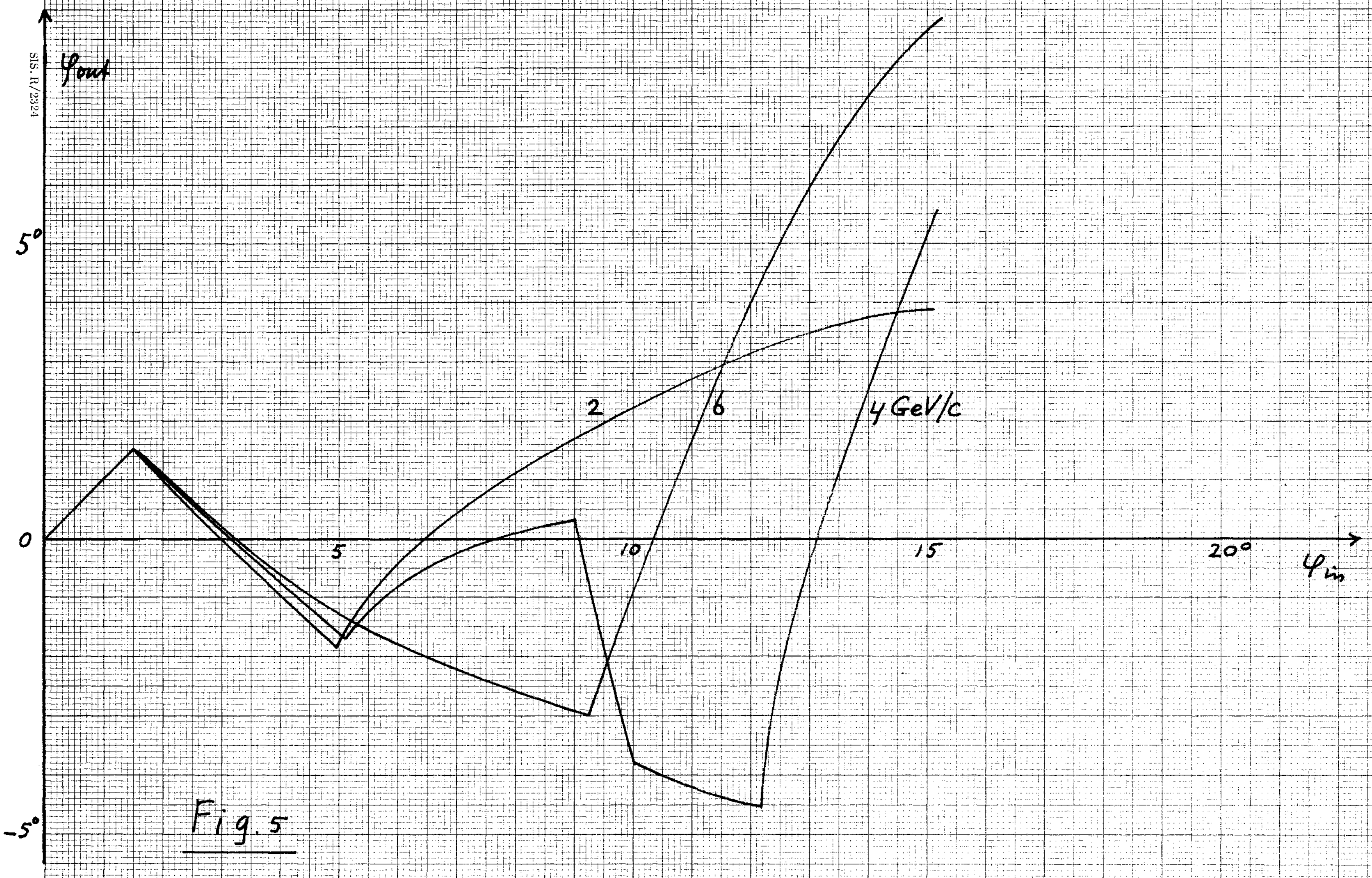


Fig. 5

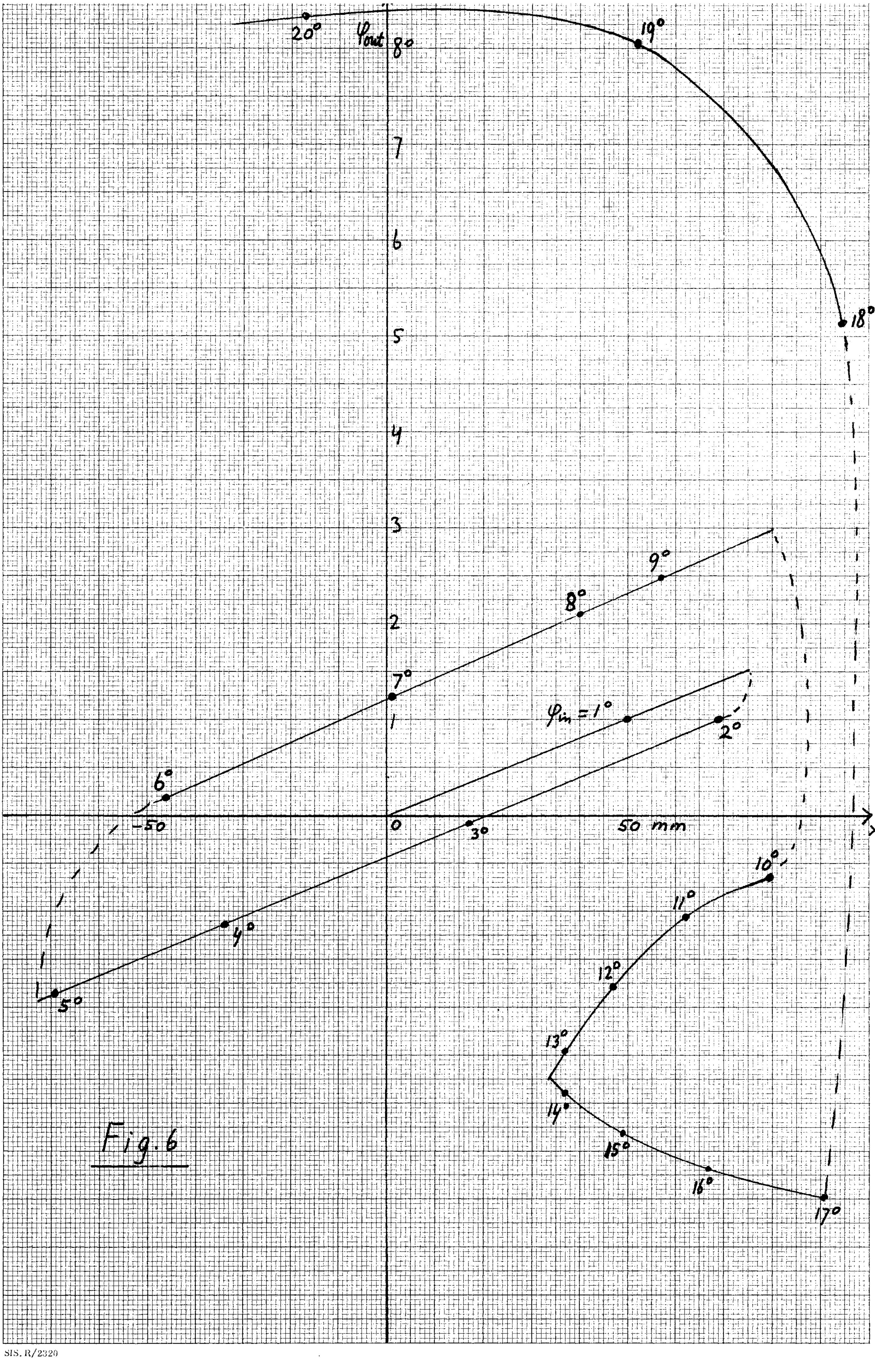

20°
φout
80
19°
7
6
5
18°
4
3
9°
8°
2
7°
1
φin = 1°
2°
6°
-50
0
3°
50 mm
10°
11°
4°
12°
5°
13°
14°
15°
16°
17°

Fig. 6

Number of π^- GeV/c^{-1} sterad^{-1} produced by 10^{11} protons of 25 GeV interacting in Al target

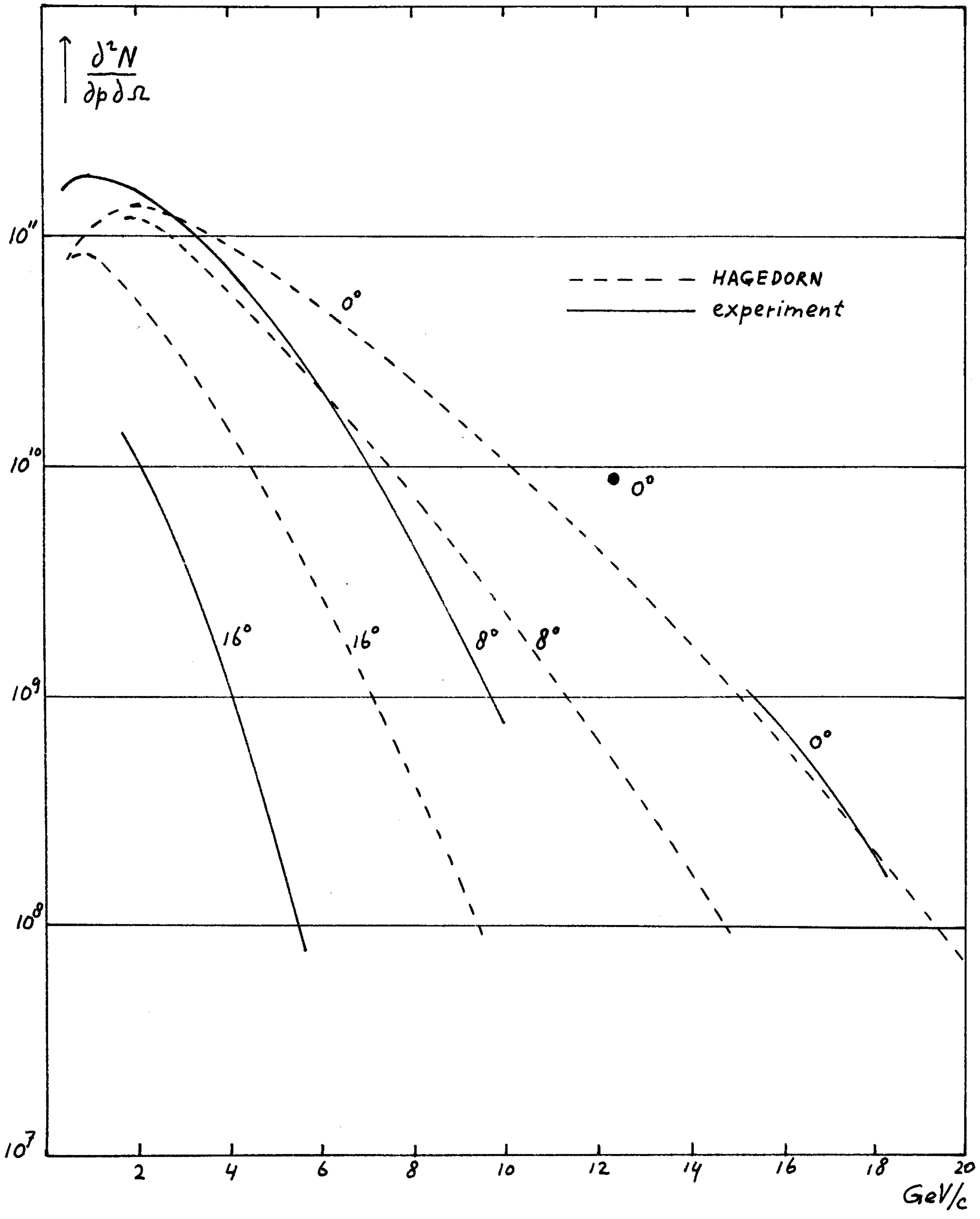


Fig. 7

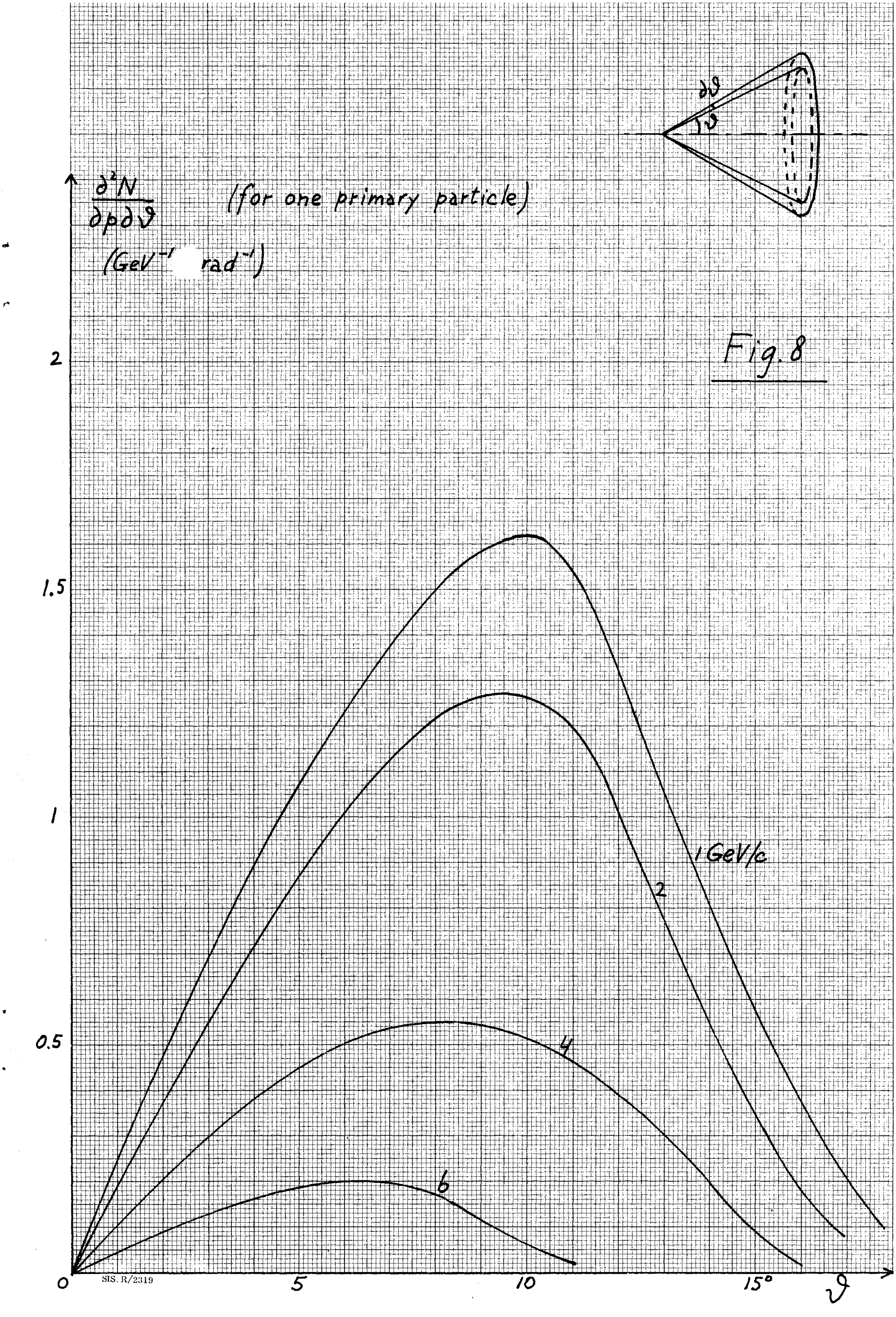


Fig. 8

SIS. R/2319

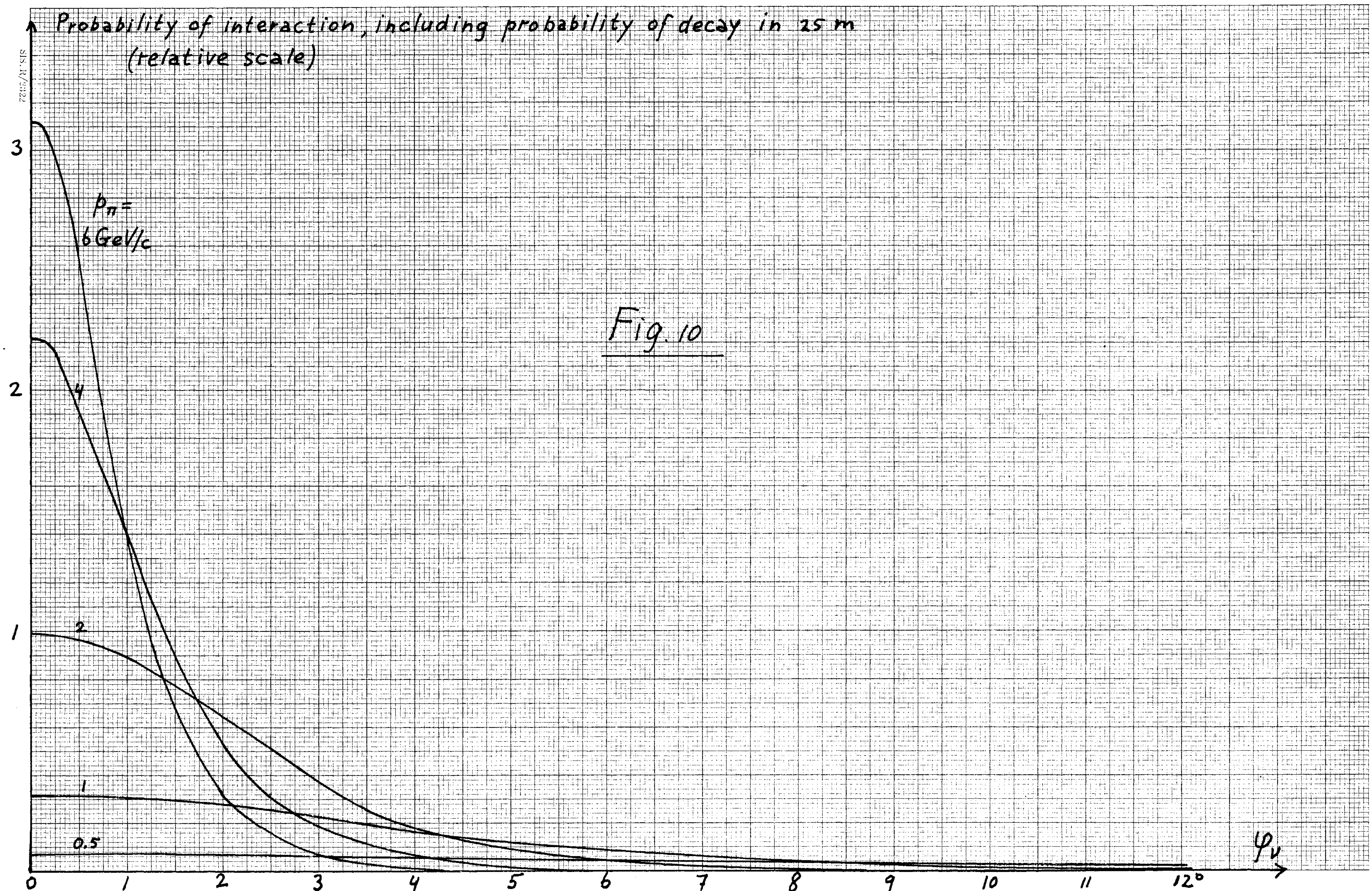

Probability of interaction, including probability of decay in 25 m
(relative scale)
$p_\pi =$
6 GeV/c
4
2
1
0.5
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
12°
φ_ν

Fig. 10

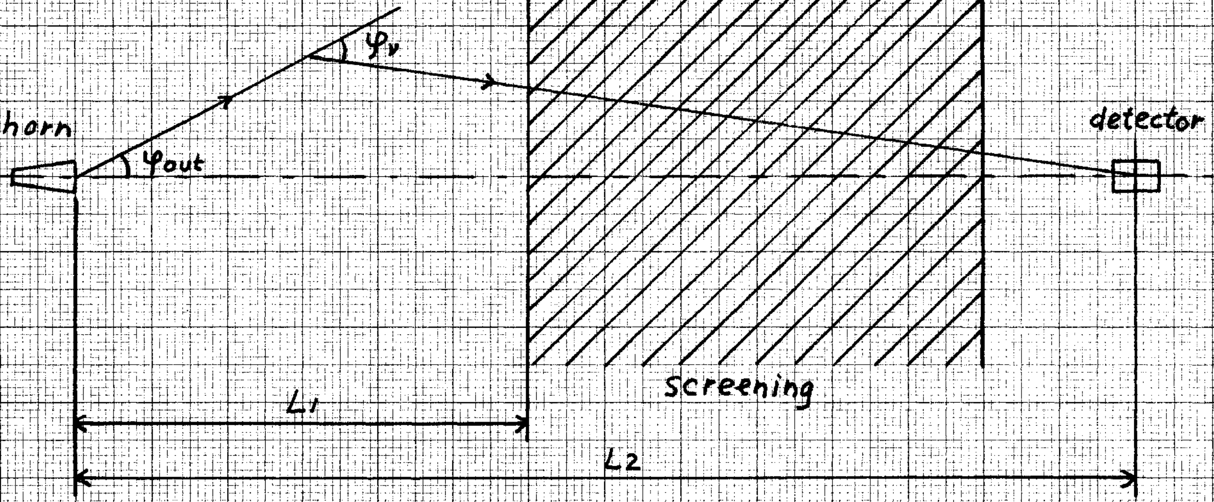

horn
φ_{out}
φ_ν
detector
screening
L_1
L_2

Fig. 9

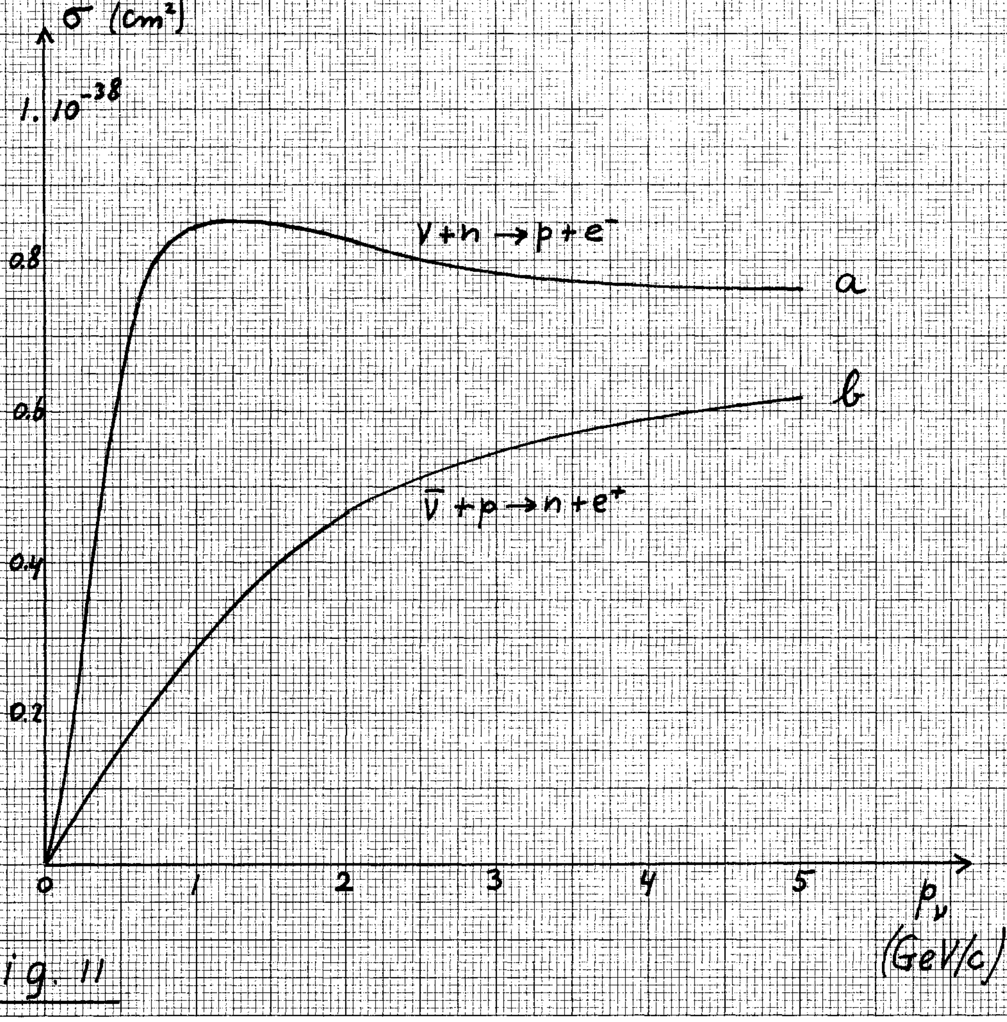
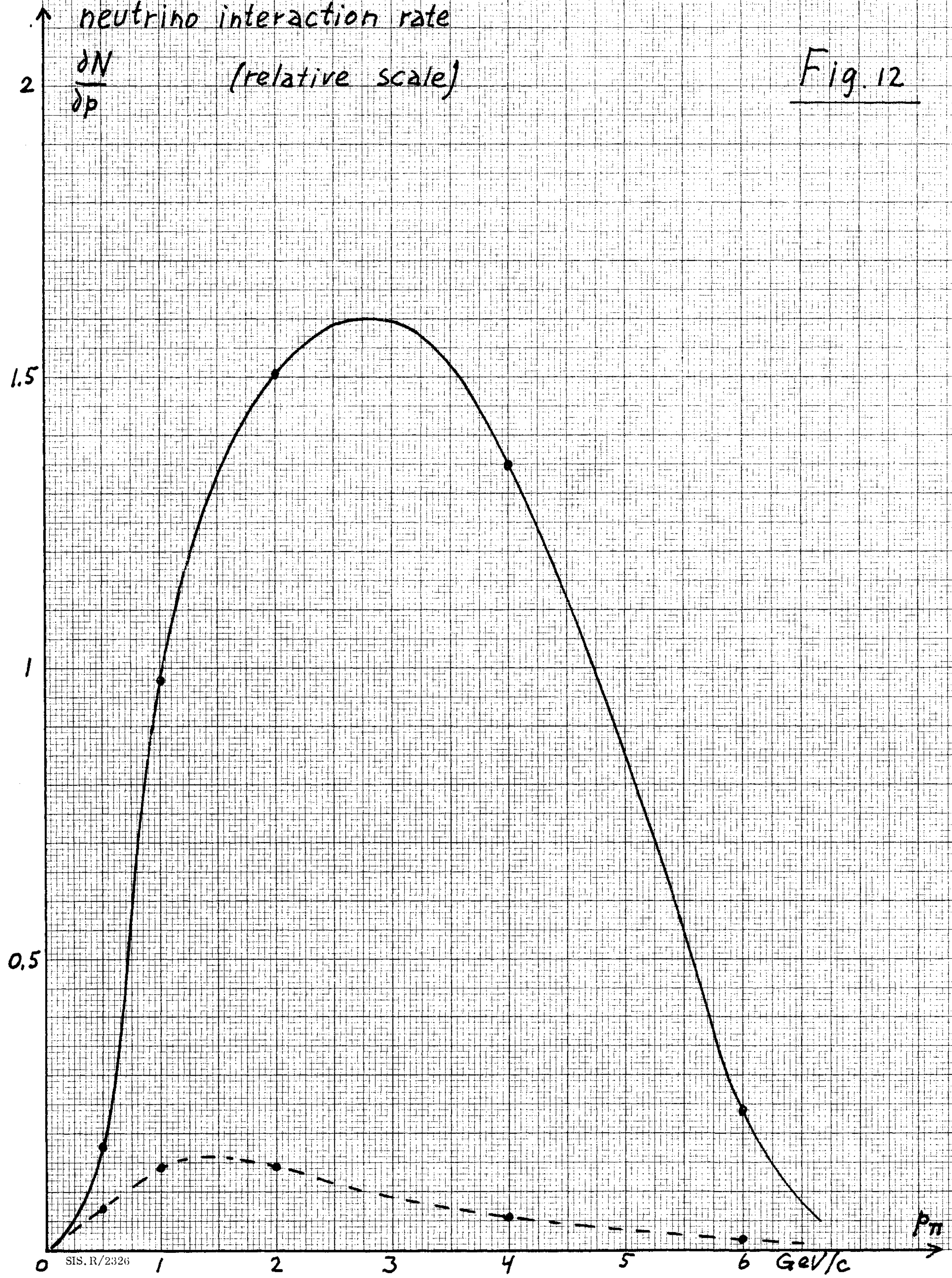

σ (cm^2)
1.10^{-38}
0.8
0.6
0.4
0.2
0
1
2
3
4
5
p_ν (GeV/c)
$\nu + n \rightarrow p + e^-$
a
$\bar{\nu} + p \rightarrow n + e^+$
b

Fig. 11

Fig. 12



SIS. R/2326