A Discrete Approach for the Inverse Singular Value Problem in Some Quadratic Group

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Abstract. In this paper the solution of an inverse singular value problem is considered. First the decomposition of a real square matrix $A = U\Sigma V$ is introduced, where U and V are real square matrices orthogonal with respect to a particular inner product defined through a real diagonal matrix G of order n having all the elements equal to ± 1, and Σ is a real diagonal matrix with nonnegative elements, called $G-$singular values. When G is the identity matrix this decomposition is the usual SVD and Σ is the diagonal matrix of singular values. Given a set $\{\sigma_1, \ldots, \sigma_n\}$ of n real positive numbers we consider the problem to find a real matrix A having them as $G-$singular values. Neglecting theoretical aspects of the problem, we discuss only an algorithmic issue, trying to apply a Newton type algorithm already considered for the usual inverse singular value problem.

1 Introduction

Inverse problems are an important topic in applied mathematics (statistics, data analysis, applied linear algebra), since some times it is important to recovery some general structures (for instance matrices) starting from known (e.g. experimentally) data (eigenvalues, singular values, some prescribed entries). In this work we consider a special inverse problem related to the already studied inverse singular value problem (see [2]). In particular, firstly we consider the decomposition of a real square matrix A of order n

$$A = U\Sigma V \tag{1}$$

where U and V are $G-$orthogonal (or hypernormal) matrices and Σ is a nonnegative diagonal matrix. When G is the Minkowski matrix, (1) has some interesting applications in the study of polarized light (see [9,10]). Hence, we consider the inverse singular value problem for the decomposition (1). At the present, we do not know if this problem has a practical interest, but we observe that in the last years there has been a growing interest towards other kinds of SVD (for instance the Hyperbolic Singular Value Decomposition having some applications in signal processing and other fields of the engineering [1,7,11]). The paper is organized as follows: in the Section 2 we describe some algebraic properties of a product between vectors of $\mathbb{R}^n$ and introduce the singular value decomposition (1), in

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Section 3 we present a discrete approach for the inverse singular value problem in the groups defined in Section 2 and finally in Section 4 we show a numerical example.

2 Singular Value Decomposition in Quadratic Groups

Let us denote by $\mathcal{D}_p$ the set of real diagonal matrices of dimension n, with elements equal to ± 1 and with p elements equal to $+1$. Let $G \in \mathcal{D}_p, 1 \leq p < n$, and let us define the following inner product:

$$\mathbf{x}, \mathbf{y} \in \mathbb{R}^n : \qquad (\mathbf{x}, \mathbf{y}) = \mathbf{x}^T G \mathbf{y}. \tag{2}$$

Given a vector $\mathbf{x} \in \mathbb{R}^n$, the number

$$\mathbf{x}^T G \mathbf{x} = \sum_{i=1}^{n} g_{ii} x_i^2$$

is called *Hypernormal Norm* even if it does not define a norm since it could be nonpositive. If G is the identity matrix of order n, then (2) defines the usual Euclidean scalar product. Using the hypernormal norm, the following classification among the vectors of $\mathbb{R}^n$ can be introduced.

Definition 1. *A vector* $\mathbf{x} \in \mathbb{R}^n$ *is called* Timelike *(*Strictly Timelike*) if* $(\mathbf{x}, \mathbf{x}) \geq 0$ *(if* $(\mathbf{x}, \mathbf{x}) > 0$*).*

Definition 2. *A vector* $\mathbf{x} \in \mathbb{R}^n$ *is called* Spacelike *(*Strictly Spacelike*) if* $(\mathbf{x}, \mathbf{x}) \leq 0$, *(if* $(\mathbf{x}, \mathbf{x}) < 0$*).*

Definition 3. *The set* $\{\mathbf{x} \in \mathbb{R}^n \mid (\mathbf{x}, \mathbf{x}) = 0\}$ *is called* Light Cone*.*

Definition 4. *The vectors* $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ *are* $G-$orthogonal *with respect to (2) if*

$$(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T G \mathbf{y} = 0.$$

We observe that the $G-$orthogonality does not imply the linear independence. In fact, if $\mathbf{x}$ is in the light cone, it is $G-$orthogonal to all the vectors $\mathbf{y} = \alpha \mathbf{x}$. Among the most used matrices G, we have the *Lorentz matrix* when

$$G = \begin{pmatrix} 1 & \\ & -I_{n-1} \end{pmatrix} \tag{3}$$

and the *Minkowski matrix* if

$$G = \begin{pmatrix} I_3 & \\ & -1 \end{pmatrix}$$

which has important applications in the mathematics of the relativity theory. Now we give some definition about real matrices with respect to (2).

Definition 5. *A real square matrix A of order n is said* G-adjoint *of $B \in \mathbb{R}^{n\times n}$ if*

$$(A\mathbf{x}, \mathbf{y}) = (\mathbf{x}, B\mathbf{y})$$

for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$.

The matrix $G-$adjoint of A is usually denoted by A^+. From (2) it follows that

$$A^+ = GA^TG.$$

Definition 6. *A real square matrix A of order n is called* $G-$selfadjoint *if*

$$(A\mathbf{x}, \mathbf{y}) = (\mathbf{x}, A\mathbf{y}).$$

If A is $G-$selfadjoint then $A = A^+ = GA^TG$. The concept of $G-$orthogonality can be extended also to real matrix.

Definition 7. *A real square matrix A of order n is said* G-orthogonal *(or* hypernormal*) if*

$$A^{-1} = A^+ = GA^TG.$$

See [8] for further properties and applications of hypernormal matrices. If A is a $G-$orthogonal matrix then

$$AGA^T = G.$$

If we consider the quadratic group related to matrix G, i.e. the set

$$\mathcal{H}_G(\mathbb{R}) = \{Y \in \mathbb{R}^{n\times n} \mid \det(Y) \neq 0,\ YGY^T = G\},$$

it coincides with the set of $G-$orthogonal matrices. In [6] the conservative solution of differential systems in these groups has been considered. Related to $\mathcal{H}_G(\mathbb{R})$ it is possible to define the its algebra as the set

$$\mathtt{h}_G = \{A \in \mathbb{R}^{n\times n} \mid GA + A^TG = 0\}.$$

If $A \in \mathtt{h}_G$ then

$$A^T = -GAG.$$

If G is the identity matrix then $\mathtt{h}_G$ is the set of real skew-symmetric matrices (sometimes matrices in $\mathtt{h}_G$ are also called $G-$skew-symmetric). If

$$G = \begin{pmatrix} I_p & O \\ O & -I_{n-p} \end{pmatrix}$$

$A \in \mathtt{h}_G$ can be characterized in the following way

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{pmatrix}$$

where $A_{11} \in \mathbb{R}^{p\times p}$ and $A_{22} \in \mathbb{R}^{(n-p)\times(n-p)}$ are real skew-symmetric matrices and $A_{12} \in \mathbb{R}^{p\times(n-p)}$. Given $A \in \mathbb{R}^{n\times n}$ if there exist two real $G-$orthogonal matrices U and V, and a diagonal matrix Σ with nonnegative elements such that

$$A = U\Sigma V$$

then it is called $G-$Singular Value Decomposition of A (or $G - SVD$). In particular it is called Singular Value Decomposition in Minkowski Space if G is the matrix (3). For the existence of this decomposition the following theorem holds (see [4,9]).

Theorem 1. *The $G-SVD$ of a real matrix A exists iff*

1. *Matrix A^+A is diagonalizable and has real nonnegative eigenvalues;*
2. *$\mathcal{N}(A^+A) = \mathcal{N}(A)$*

where $\mathcal{N}(A)$ denotes the null space of A.

The elements of Σ are called $G-$singular values of A. They are the square roots of the eigenvalues of A^+A. In fact

$$A^+A = V^+\Sigma U^+U\Sigma V = V^+\Sigma^2 V = V^{-1}\Sigma^2 V.$$

3 Inverse Singular Value Problem in Quadratic Groups

Inverse problems involving eigenvalues and singular values have been extensively studied in last years (see [2,3,5]). The classical inverse singular value problem is stated as follows: given real general matrices $B_0, B_1, \ldots, B_n \in \mathbb{R}^{n\times n}$, and a set of nonnegative numbers $\sigma_1, \ldots, \sigma_n$ find the real values $c_1, \ldots, c_n$ such that the singular values of the matrix

$$B(\boldsymbol{c}) = B_0 + \sum_{i=1}^{n} c_i B_i \qquad (4)$$

are $\sigma_1, \ldots, \sigma_n$. We suppose $\sigma_i \neq \sigma_j$ if $i \neq j$. Starting from the same premise it is obvious to define an analogous inverse problem also for other kinds of singular value decomposition. It is possible to ask that matrix (4) has the $G-$singular values equal to $\sigma_1, \ldots, \sigma_n$. In [2] two different approaches are analyzed for the inverse singular value problem: the first is continuous and the other one is discrete.

The first approach exploits the property that the eigenvalues of the symmetric matrix

$$\begin{pmatrix} 0 & A \\ A^T & 0 \end{pmatrix}$$

are plus and minus of the singular values of A, then a continuous flow is derived having the diagonal matrix of the singular values as limit point. In our case this approach cannot be applied since the $G-$singular values are the eigenvalues, with signs plus and minus, of the non-symmetric matrix

$$\begin{pmatrix} 0 & A \\ GA^TG & 0 \end{pmatrix}$$

then it cannot treated as a symmetric inverse eigenvalue problem. The second approach is a Newton-type algorithm having quadratic, but not global convergence. We try to apply this second approach to our problem. Following [2] we define

$$M(\Sigma) = \{U^+ \Sigma V \in \mathrm{I\!R}^{n\times n} \mid U, V \in \mathcal{H}_G(\mathrm{I\!R})\} \tag{5}$$

the set of all real matrices in $\mathrm{I\!R}^{n\times n}$ whose $G-$singular values are $\sigma_1, \ldots, \sigma_n$. Denoting by

$$B = \{B(\boldsymbol{c}) \mid \boldsymbol{c} \in \mathrm{I\!R}^n\}$$

the set of matrices of the form (4), the inverse G-singular value problems is equivalent to find an intersection of the two sets $M(\Sigma)$ and B. If $X^{(m)} \in M(\Sigma)$ there exist two matrices $U^{(m)}, V^{(m)} \in \mathcal{H}_G(\mathrm{I\!R})$ such that

$$U^{(m)^+} X^{(m)} V^{(m)} = \Sigma. \tag{6}$$

We recall that

$$U^{(m)^{-1}} = U^{(m)^+} = GU^{(m)^T}G, \qquad V^{(m)^{-1}} = V^{(m)^+} = GV^{(m)^T}G.$$

Now we need to find an intercept of the line tangent to the manifold $M(\Sigma)$ at $X^{(m)}$ with the set B. Since a tangent vector $T(X)$ to $M(\Sigma)$ at a point $X \in M(\Sigma)$ has the form

$$T(X) = XK - HX$$

where K, H are $G-$skew-symmetric matrices and it is well-known that

$$X + T(X) = X + XK - HX$$

represents the line tangent to $M(\Sigma)$ emanating from X then we need to find two matrices $H^{(m)}, K^{(m)} \in \mathtt{h}_G$ such that

$$X^{(m)} + X^{(m)}K^{(m)} - H^{(m)}X^{(m)} = B(\boldsymbol{c}^{(m+1)}). \tag{7}$$

From (6) it is also:

$$X^{(m)} = U^{(m)} \Sigma V^{(m)^{-1}},$$

then (7) becomes:

$$U^{(m)}\Sigma V^{(m)^{-1}} + U^{(m)}\Sigma V^{(m)^{-1}}K^{(m)} - H^{(m)}U^{(m)}\Sigma V^{(m)^{-1}} = B(\boldsymbol{c}^{(m+1)})$$

$$\Sigma V^{(m)^{-1}} + \Sigma V^{(m)^{-1}}K^{(m)} - U^{(m)^{-1}}H^{(m)}U^{(m)}\Sigma V^{(m)^{-1}} = U^{(m)^{-1}}B(\boldsymbol{c}^{(m+1)})$$

$$\Sigma + \Sigma V^{(m)^{-1}} K^{(m)} V^{(m)} - U^{(m)^{-1}} H^{(m)} U^{(m)} \Sigma = U^{(m)^{-1}} B(\boldsymbol{c}^{(m+1)}) V^{(m)}$$

$$\Sigma + \Sigma G V^{(m)^T} G K^{(m)} V^{(m)} - G U^{(m)^T} G H^{(m)} U^{(m)} \Sigma = G U^{(m)^T} G B(\boldsymbol{c}^{(m+1)}) V^{(m)}.$$

Setting

$$\tilde{K}^{(m)} = G V^{(m)^T} G K^{(m)} V^{(m)}$$

$$\tilde{H}^{(m)} = G U^{(m)^T} G H^{(m)} U^{(m)}$$

$$\tilde{B}^{(m)} = G U^{(m)^T} G B(\boldsymbol{c}^{(m+1)}) V^{(m)}.$$

the equation becomes

$$\Sigma + \Sigma \tilde{K}^{(m)} - \tilde{H}^{(m)} \Sigma = \tilde{B}^{(m)}. \tag{8}$$

The matrices $\tilde{K}^{(m)}$ and $\tilde{H}^{(m)}$ are in the tangent space $\mathtt{h}_G$, in fact

$$G\tilde{K}^{(m)} + \tilde{K}^{(m)^T} G = G\left[G V^{(m)^T} G K^{(m)} V^{(m)}\right] + \left[G V^{(m)^T} G K^{(m)} V^{(m)}\right]^T G =$$

$$= V^{(m)^T} G K^{(m)} V^{(m)} + V^{(m)^T} K^{(m)^T} G V^{(m)} =$$

$$= V^{(m)^T} G K^{(m)} V^{(m)} - V^{(m)^T} G K^{(m)} V^{(m)} = 0.$$

The proof for $\tilde{H}^{(m)}$ is similar. We can exploit the n^2 scalar equations (8) to evaluate the elements of unknown matrices $\tilde{K}^{(m)}$ and $\tilde{H}^{(m)}$ and the unknown vector $\boldsymbol{c}^{(m+1)}$. Let us consider if they are enough to compute the elements unknown. Matrices $\tilde{K}^{(m)}$ and $\tilde{H}^{(m)}$ are in $\mathtt{h}_G$, whose dimension is $[n(n-1)]/2$ hence they are characterized by $n^2 - n$ parameters plus n elements of vector $\boldsymbol{c}^{(m+1)}$ we have exactly n^2 unknowns, which can be computed from the equations. For $i \neq j$ (8) gives

$$\tilde{b}_{ij}^{(m)} = \sigma_i \tilde{k}_{ij}^{(m)} - \tilde{h}_{ij}^{(m)} \sigma_j \tag{9}$$

while the equations for diagonal elements are

$$\tilde{b}_{ii}^{(m)} = \sigma_i, \qquad i = 1, \ldots, n. \tag{10}$$

Using (10) it is possible to compute the elements of the vector $\boldsymbol{c}^{(m+1)}$. In fact

$$\sigma_i = \tilde{b}_{ii}^{(m)} = \boldsymbol{e}_i^T \tilde{B}^{(m)} \boldsymbol{e}_i$$

where $\boldsymbol{e}_i$ is the $i-$th unit vector of $\mathbb{R}^n$, and from (4)

$$\tilde{B}^{(m)} = G U^{(m)^T} G B(\boldsymbol{c}^{(m+1)}) V^{(m)} =$$

$$= G U^{(m)^T} G \left[B_0 + \sum_{j=1}^{n} c_j B_j\right] V^{(m)} =$$

$$= G U^{(m)^T} G B_0 V^{(m)} + \sum_{j=1}^{n} c_j G U^{(m)^T} G B_j V^{(m)}.$$

Hence

$$\tilde{b}_{ii}^{(m)} = \boldsymbol{e}_i^T \left[GU^{(m)^T} GB_0 V^{(m)} + \sum_{j=1}^{n} c_j GU^{(m)^T} GB_j V^{(m)} \right] \boldsymbol{e}_i =$$

$$= \boldsymbol{e}_i^T GU^{(m)^T} GB_0 V^{(m)} \boldsymbol{e}_i + \sum_{j=1}^{n} c_j \boldsymbol{e}_i^T GU^{(m)^T} GB_j V^{(m)} \boldsymbol{e}_i.$$

The vector $\boldsymbol{c}^{(m+1)}$ is the solution of the linear system

$$A^{(m)} \boldsymbol{c}^{(m+1)} = \boldsymbol{\sigma} - \boldsymbol{b}, \tag{11}$$

where $A \in \mathbb{R}^{n \times n}$ is the matrix with elements

$$a_{ij}^{(m)} = \boldsymbol{e}_i^T GU^{(m)^T} GB_j V^{(m)} \boldsymbol{e}_i$$

and

$$b_i = \boldsymbol{e}_i^T GU^{(m)^T} GB_0 V^{(m)} \boldsymbol{e}_i$$

and $\boldsymbol{\sigma}$ denotes the vector $(\sigma_1, \ldots, \sigma_n)$. If (11) has a unique solution then vector $\boldsymbol{c}^{(m+1)}$ is known. To compute matrices $\tilde{K}^{(m)}$ and $\tilde{H}^{(m)}$ we exploit (9). We recall the structure of matrices in $\mathtt{h}_G$

$$\tilde{K}^{(m)} = \begin{pmatrix} K_{11}^{(m)} & K_{12}^{(m)} \\ K_{12}^{(m)^T} & K_{22}^{(m)} \end{pmatrix}$$

where $K_{11}^{(m)} \in \mathbb{R}^{p \times p}$ and $K_{22}^{(m)} \in \mathbb{R}^{(n-p)\times(n-p)}$ are real skew-symmetric matrices. First we consider the equations (i, j), with $1 \leq i < j \leq p$ and $p + 1 \leq i < j \leq n$:

$$\tilde{b}_{ij}^{(m)} = \sigma_i \tilde{k}_{ij}^{(m)} - \tilde{h}_{ij}^{(m)} \sigma_j \tag{12}$$

and

$$\tilde{b}_{ji}^{(m)} = \sigma_j \tilde{k}_{ji}^{(m)} - \tilde{h}_{ji}^{(m)} \sigma_i = -\sigma_j \tilde{k}_{ij}^{(m)} + \sigma_i \tilde{h}_{ij}^{(m)}. \tag{13}$$

Solving (12) and (13) we obtain:

$$\tilde{k}_{ij}^{(m)} = \frac{\sigma_i \tilde{b}_{ij}^{(m)} - \sigma_j \tilde{b}_{ji}^{(m)}}{\sigma_i^2 - \sigma_j^2}$$

$$\tilde{h}_{ij}^{(m)} = \frac{\sigma_i \tilde{b}_{ji}^{(m)} + \sigma_j \tilde{b}_{ij}^{(m)}}{\sigma_i^2 - \sigma_j^2}.$$

From the equations (i, j), $1 \leq i \leq p$ and $p + 1 \leq j \leq n$, it is:

$$\tilde{b}_{ij}^{(m)} = \sigma_i \tilde{k}_{ij}^{(m)} - \tilde{h}_{ij}^{(m)} \sigma_j \tag{14}$$

and

$$\tilde{b}_{ji}^{(m)} = \sigma_j \tilde{k}_{ji}^{(m)} - \tilde{h}_{ji}^{(m)} \sigma_i = \sigma_j \tilde{k}_{ij}^{(m)} - \sigma_i \tilde{h}_{ij}^{(m)}. \tag{15}$$

Solving (14) and (15) we obtain:

$$\tilde{k}_{ij}^{(m)} = \frac{\sigma_j \tilde{b}_{ji}^{(m)} - \sigma_i \tilde{b}_{ij}^{(m)}}{\sigma_j^2 - \sigma_i^2}$$

$$\tilde{h}_{ij}^{(m)} = \frac{\sigma_i \tilde{b}_{ji}^{(m)} - \sigma_j \tilde{b}_{ij}^{(m)}}{\sigma_j^2 - \sigma_i^2}.$$

We use $\tilde{K}^{(m)}$ and $\tilde{H}^{(m)}$ to compute $K^{(m)}$ and $H^{(m)}$, in fact

$$K^{(m)} = G(V^{(m)^{-1}})^T G \tilde{K}^{(m)} V^{(m)^{-1}} = V^{(m)} \tilde{K}^{(m)} G V^{(m)^T} G$$

$$H^{(m)} = G(U^{(m)^{-1}})^T G \tilde{H}^{(m)} U^{(m)^{-1}} = U^{(m)} \tilde{K}^{(m)} G U^{(m)^T} G.$$

The next step is to project $B(\boldsymbol{c}^{(m+1)})$ to $M(\Sigma)$. It is possible to use the exponential map, since, given a matrix H in $\mathtt{h}_G$, then e^H belongs to the quadratic group. The computation of the exponential of a matrix is too expensive, then the Cayley transform can be used, if

$$D = \left(I + \frac{H^{(m)}}{2}\right)\left(I - \frac{H^{(m)}}{2}\right)^{-1}, \qquad F = \left(I + \frac{K^{(m)}}{2}\right)\left(I - \frac{K^{(m)}}{2}\right)^{-1},$$

then

$$X^{(m+1)} = D^+ X^{(m)} F.$$

The computation of the matrix can be avoided since only $G-$orthogonal matrices $U^{(m)}$ and $V^{(m)}$ are needed:

$$U^{(m+1)} = D^+ U^{(m)}, \qquad V^{(m+1)} = F V^{(m)}.$$

The convergence of the method will not be considered in this work.

4 A Numerical Example and Conclusions

In this last section we present a numerical example concerning the discrete method introduced in the previous section.

Example 1. In this case we have considered a problem of dimension 4, the matrix G is

$$G = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}$$

and we have chosen the following random symmetric matrices B_i:

$$B_0 = \begin{pmatrix} 1.4353 & 1.1345 & 0.7833 & 0.5754 \\ 1.1345 & 0.7065 & 0.8811 & 1.1264 \\ 0.7833 & 0.8811 & 0.9568 & 1.2707 \\ 0.5754 & 1.1264 & 1.2707 & 1.7857 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0.5462 & 1.0596 & 0.9154 & 1.0762 \\ 1.0596 & 1.8168 & 0.3103 & 0.4132 \\ 0.9154 & 0.3103 & 1.2816 & 0.3617 \\ 1.0762 & 0.4132 & 0.3617 & 1.9886 \end{pmatrix}$$

$$B_2 = \begin{pmatrix} 0.8796 & 0.7333 & 0.7728 & 0.5254 \\ 0.7333 & 1.1831 & 0.9896 & 0.9110 \\ 0.7728 & 0.9896 & 1.1885 & 0.5023 \\ 0.5254 & 0.9110 & 0.5023 & 1.2917 \end{pmatrix}$$

$$B_3 = \begin{pmatrix} 1.9338 & 0.8019 & 1.6053 & 0.1655 \\ 0.8019 & 1.6375 & 1.1775 & 1.0814 \\ 1.6053 & 1.1775 & 0.6922 & 0.5885 \\ 0.1655 & 1.0814 & 0.5885 & 1.7120 \end{pmatrix}$$

$$B_4 = \begin{pmatrix} 0.9805 & 1.2666 & 0.7582 & 1.4403 \\ 1.2666 & 0.8244 & 0.9508 & 0.5583 \\ 0.7582 & 0.9508 & 1.3864 & 1.0502 \\ 1.4403 & 0.5583 & 1.0502 & 0.3976 \end{pmatrix}.$$

Then we have taken the following vector $\boldsymbol{c}$:

$$\overline{\boldsymbol{c}} = (6.2520E-01, 7.3336E-01, 3.7589E-01, 9.8765E-03)^T$$

and have considered as $G-$singular values those of matrix $B(\overline{\boldsymbol{c}})$:

$$\boldsymbol{\sigma} = (0.39364, 1.1338, 2.5057, 10.4847)^T.$$

Hence we have perturbed each entry of the vector $\overline{\boldsymbol{c}}$ with random quantities between 0 and 0.5, and have considered this one as the initial guess for the iterations. In fact the algorithm, when it converges, has only a local convergence (see [2]) then it is necessary to start from a point near the solution. In Table 1 the error, measured in the 2-norm, on the $G-$singular values at each iteration are shown.

Table 1. Example 1

Iteration	**Error**
0	$2.5289e+0$
1	$1.1410e+0$
2	$7.2304e-1$
3	$3.2589e-2$
4	$8.9911e-3$
5	$3.0011e-4$
6	$9.8812e-5$

In the paper we have presented a discrete approach to the inverse singular value problem in quadratic groups. The algorithm is similar to the one already introduced for the usual inverse singular value problem. The algorithm described needs to be investigated more carefully, in fact it breaks down very often (the matrix $A^{(m)}$ is some times singular or the matrix $B(\boldsymbol{c}^{(m)})$ does not satisfy the hypotheses on the existence of the G-singular value decomposition. A possible solution to these troubles could be a different parameterization of the manifold (5). This features will be analyzed in future together with the solvability of the problem. Moreover it should be considered the analogous problem for the Hyperbolic Singular Value Decomposition (see [1,7,11]) which has some applications in signal processing and other fields of engineering. Some other aspects of the problem need further investigations, for instance the application of continuous techniques (see [2]).

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