A discrete fracture model for two-phase flow with matrix-fracture interaction

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Outline

1. Introduction
2. Two-phase flow with a change of rock type
3. Two-phase flow with a fracture

A problem requiring multi-scale modelling

- Fractures represent heterogeneities in porous media.
  - Usually of much higher permeability than surrounding medium
  - May be of much lower permeability so that they act as a barrier
- Fracture width much smaller than any reasonable parameter of spatial discretization.

The models that we consider

- Discrete fracture models, where the fracture locations are given.
- Models that allow an interchange between the fractures and the matrix rock.
- Reduced models where fracture f is reduced to a 2-D interface γ.

Outline

1. Introduction

2. Two-phase flow with a change of rock type

3. Two-phase flow with a fracture

Saturation equation

$$\phi\frac{\partial s}{\partial t} + \operatorname{div}\mathbf{u}_w = 0, \quad \mathbf{u}_w = \mathbf{r} + \mathbf{f},$$

$$r = \underbrace{-\mathbf{K}\nabla\alpha(s)}_{\text{cap. diffusion}}, \mathbf{f} = \underbrace{b_T(s)\mathbf{u}_T + b_g(s)\mathbf{K}\mathbf{u}_G}_{\text{advection}}.$$

Pressure equation

$$\operatorname{div}\mathbf{u} = 0, \quad \mathbf{u} = -\mathbf{K}k_T(s)(\nabla p - \rho(s)\mathbf{u}_G),$$

$$\mathbf{u} = \mathbf{u}_w + \mathbf{u}_{nw}, \text{ total Darcy flow },$$

$$p = \frac{1}{2}(p_w + p_{nw}) + \beta(s), \text{ global pressure (not physical) }.$$

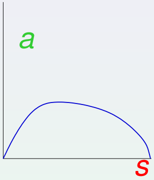
Functions α, β depend on capillary pressure and mobilities

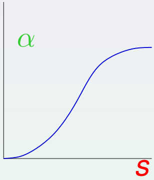
Functions k_T, b_T, b_g, ρ depend on mobilities

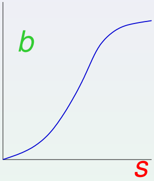
$$a(s) = \frac{k_w k_{nw}}{k_w + k_{nw}}\left(-\frac{d\pi}{ds}\right), a(0) = a(1) = 0, \quad \alpha(s) = \int_0^s a(\sigma)d\sigma,$$

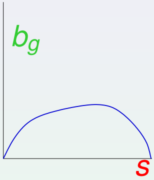
$$b(s) = \frac{k_w}{k_w + k_{nw}}, b(0) = 0, b(1) = 1, b_g(s) = \frac{k_w k_{nw}}{k_w + k_{nw}}(\rho_w - \rho_{nw})$$

$$b_g(0) \geq 0 \text{ if } \rho_w \geq \rho_{nw}, b_g(0) = b_g(1) = 0.$$



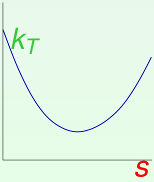


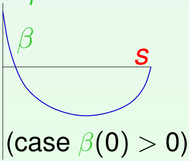


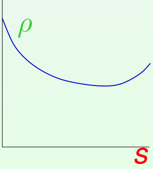


$$k_T(s) = k_w + k_{nw} \geq \underline{k}_T > 0$$

$$\rho(s) = \frac{k_{nw}\rho_{nw} + k_w\rho_w}{k_T} \qquad \beta(s) = \int_1^s (b_T - \frac{1}{2})\left(-\frac{d\pi}{ds}\right)$$







Rock type L	σ	Rock type R
$\phi^L,\ K^L,\ k_\ell^L,\ \pi^L,$		$\phi^R,\ K^R,\ k_\ell^R,\ \pi^R$
$\alpha^L,\ \beta^L,\ k_T^L,\ b_T^L,\ b_g^L,\ \rho^L$		$\alpha^R,\ \beta^R,\ k_T^R,\ b_T^R,\ b_g^R,\ \rho^R$

Transmission conditions across σ

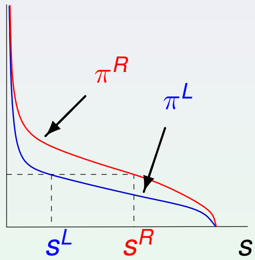
- Conservation of phases

$$\mathbf{u}_\ell^L \cdot \mathbf{n}^L + \mathbf{u}_\ell^R \cdot \mathbf{n}^R = 0,\ \ell = w, nw \quad \Longrightarrow \quad \mathbf{u}^L \cdot \mathbf{n}^L + \mathbf{u}^R \cdot \mathbf{n}^R = 0$$

- Continuity of phase pressures

- Discontinuous saturation $s^L \neq s^R$

$$\pi^L(s^L) = \pi^R(s^R)$$

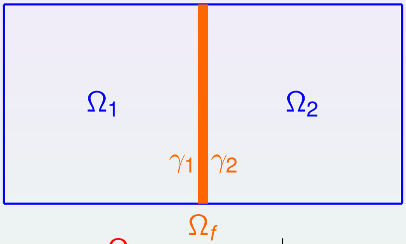


- Discontinuous global pressure $p^L \neq p^R$ $\quad (p = \frac{1}{2}(p_w + p_{nw}) + \beta(s))$

$$p^L - \beta^L(s^L) = p^R - \beta^R(s^R)$$

Three rock types for $\Omega_1, \Omega_f, \Omega_2$

$\Omega = \Omega_1 \cup \Omega_f \cup \Omega_2 \subset \mathbb{R}^n$

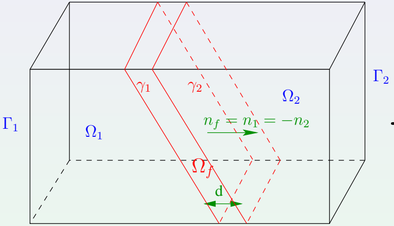


Ω_1	γ_1	Ω_f	γ_2	Ω_2
$\phi_1,\ \mathbf{K}_1,\ k_{\ell 1},\ \pi_1,$		$\phi_f,\ \mathbf{K}_f,\ k_{\ell f},\ \pi_f$		$\phi_2,\ \mathbf{K}_2,\ k_{\ell 2},\ \pi_2,$
$\alpha_1, \beta_1, k_{T1},\ b_{T1},\ b_{g1}, \rho_1$		$\alpha_f, \beta_f, k_{Tf},\ b_{Tf},\ b_{gf}, \rho_f$		$\alpha_2, \beta_2, k_{T2},\ b_{T2},\ b_{g2}, \rho_2$

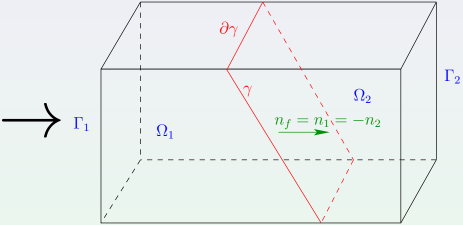
Natural idea: • Use local grid refinement

Alternative: • Model Ω_f as an interface γ

1. Shrink Ω_f to an interface hyperplane γ.



Physical model

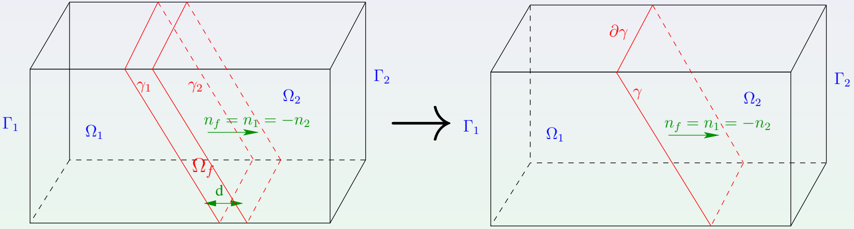


Reduced model

2. On γ introduce tangential and normal components:

$$\mathbf{K}_f = (\mathbf{K}_{f,\tau}, K_{f,n}), \quad \mathbf{u}_f = (\mathbf{u}_{f,\tau}, u_{f,n}).$$

1. Shrink Ω_f to an interface hyperplane γ.



Physical model   Reduced model

2. On γ introduce tangential and normal components:

$$\mathbf{K}_f = (\mathbf{K}_{f,\tau}, K_{f,n}), \quad \mathbf{u}_f = (\mathbf{u}_{f,\tau}, u_{f,n}).$$

3. Average conservation equation and tangential component of Darcy's law across the fracture:

$$\mathrm{div}_\tau \mathbf{u}_\gamma = Q_\gamma + \mathbf{u}_1 \cdot \mathbf{n}_1 + \mathbf{u}_2 \cdot \mathbf{n}_2, \quad \mathbf{u}_\gamma = -\mathbf{K}_{f,\tau} d \nabla_\tau P_\gamma$$

with $\mathbf{u}_\gamma = \int_0^d \mathbf{u}_{f,\tau}\, d\sigma, \quad Q_\gamma = \int_0^d q_f\, d\sigma, \quad P_\gamma = \frac{1}{d}\int_0^d p_f d\sigma.$

4. Integrate normal component of Darcy's law across the fracture:

$\rightarrow$ Darcy's law across the interface γ:

$$-\frac{1}{\delta(s_\gamma)}\mathbf{u}_i \cdot \mathbf{n} + (p_i - \beta_i(s_i)) = (P_\gamma - \beta_f(S_\gamma)), \quad i = 1, 2.$$

- $\delta(s_\gamma) = \dfrac{2\mathbf{K}_{f\mathbf{n}}\, k_f(s_\gamma)}{d}$.
- Note that when $\delta(s_\gamma) \rightarrow \infty$, then $(p_i - \beta_i(s_i)) = (P_\gamma - \beta_f(S_\gamma))$.

5. Average conservation equation and tangential component of Darcy's law for the wetting phase across the fracture:

$$\phi_\gamma \frac{\partial s_\gamma}{\partial t} + \mathrm{div}_\tau \mathbf{u}_{w\gamma} = Q_{w\gamma} + \mathbf{u}_{w1} \cdot \mathbf{n}_1 + \mathbf{u}_{w2} \cdot \mathbf{n}_2,$$
$$\mathbf{u}_{w\gamma} = -\mathbf{K}_\gamma \nabla_\tau \alpha_f(s_\gamma) + \mathbf{f}(s_\gamma),$$

with $\mathbf{u}_{w\gamma} = \int_0^d \mathbf{u}_{wf,\tau}\, d\sigma, \quad Q_{w\gamma} = \int_0^d q_{wf}\, d\sigma, \quad s_\gamma = \frac{1}{d}\int_0^d s_f d\sigma.$

6. Integrate normal component of Darcy's law (phase w) across the fracture:

$\rightarrow$ Darcy's law for the wetting phase across the interface γ:

$$\frac{1}{\delta_w(s_\gamma)} \mathbf{u}_{wi} \cdot \mathbf{n}_i + \pi_i(s_i) = \pi_\gamma(s_\gamma) + \frac{f_\gamma(s_\gamma)}{\delta_w(s_\gamma)} \mathbf{u}_i \cdot \mathbf{n}_i, \quad i = 1, 2.$$

- $\delta_w(s_\gamma) = \mathbf{K}_{f\mathbf{n}}\, k_{wf\mathbf{n}}(s_f)/d,$
- Note that if $\delta_w(s_\gamma) \rightarrow \infty$, then $\pi_i(s_i) - \pi_\gamma(s_\gamma) \rightarrow 0$.

Model in Ω_i

$$\phi_i \frac{\partial s_i}{\partial t} + \operatorname{div} \mathbf{u}_{wi} = q_{wi}, \quad \mathbf{u}_{wi} = -\mathbf{K}_i \nabla \alpha(s_i) + \mathbf{f}_i(s_i),$$
$$\operatorname{div} \mathbf{u}_i = q_i, \quad \mathbf{u}_i = -\mathbf{K}_i k_i(s_i)(\nabla p_i - \rho_i(s_i)\mathbf{u}_{Gi}),$$

Model on γ

$$\phi_\gamma \frac{\partial s_\gamma}{\partial t} + \operatorname{div} \mathbf{u}_{w\gamma} = Q_{w\gamma} + \sum_{i=1}^{2} \mathbf{u}_{wi} \cdot \mathbf{n}_i, \ \mathbf{u}_{w\gamma} = -\mathbf{K}_\gamma \nabla \alpha(s_\gamma) + \mathbf{f}_\gamma(s_\gamma),$$
$$\operatorname{div} \mathbf{u}_\gamma = Q_\gamma + \sum_{i=1}^{2} \mathbf{u}_i \cdot \mathbf{n}_i, \ \mathbf{u}_\gamma = -\mathbf{K}_\gamma k_\gamma(s_\gamma)(\nabla p_\gamma - \rho_\gamma(s_\gamma)\mathbf{u}_{G\gamma}),$$

Transmission conditions

$$\frac{1}{\delta_w(s_\gamma)} \mathbf{u}_{wi} \cdot \mathbf{n}_i + \pi_i(s_i) = \pi_\gamma(s_\gamma) + \frac{f_\gamma(s_\gamma)}{\delta_w(s_\gamma)} \mathbf{u}_i \cdot \mathbf{n}_i,$$
$$-\frac{1}{\delta(s_\gamma)} \mathbf{u}_i \cdot \mathbf{n} + p_i = P_\gamma - [\beta(s)]_{\gamma,i}.$$

Robin-to-Neumann operator

$$\mathcal{L}_i^{RtN}(p_\gamma, \mathbf{u}_G, q_i) = -(\mathbf{u}_i \cdot \mathbf{n}_i)_{|\gamma},$$

where $(p_i, \mathbf{u}_i)$ is the solution of

$$\begin{aligned}
\operatorname{div} \mathbf{u}_i &= q_i && \text{in } \Omega_i \times (0,T),\\
\mathbf{u}_i &= -\mathbf{K}_i(\nabla p_i - \rho_i(s_i)\,\mathbf{u}_G) && \text{in } \Omega_i \times (0,T),\\
p_i &= 0 && \text{on } \partial\Omega_i \cap \partial\Omega \times (0,T),\\
-\frac{1}{\delta(s_\gamma)}\mathbf{u}_i \cdot \mathbf{n}_i + p_i &= p_\gamma - [\beta(s)]_{\gamma,i} && \text{on } \gamma \times (0,T).
\end{aligned}$$

Interface problem

$$\begin{aligned}
\textstyle\sum_{i=1}^{2} \mathcal{L}_i^{RtN}(p_\gamma, \mathbf{u}_G, q) - \mathbf{u}_\gamma &= Q_\gamma && \text{in } \gamma \times (0,T),\\
\mathbf{u}_\gamma &= -\mathbf{K}_\gamma\, k_\gamma(s_\gamma)\,\nabla p_\gamma && \text{in } \gamma \times (0,T),\\
\operatorname{div} \mathbf{u}_\gamma &= 0 && \text{in } \gamma \times (0,T),\\
p_\gamma &= 0 && \text{on } \partial\gamma \times (0,T),
\end{aligned}$$

Schur method: Pressure equation

Robin-to-Neumann operator

$$\mathcal{L}_i^{RtN}(p_\gamma, \mathbf{u}_G, q_i) = -(\mathbf{u}_i \cdot \mathbf{n}_i)_{|\gamma},$$

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p_i &= 0 && \text{on } \partial\Omega_i \cap \partial\Omega \times (0,T), \\
-\frac{1}{\delta(s_\gamma)}\mathbf{u}_i \cdot \mathbf{n}_i + p_i &= p_\gamma - [\beta(s)]_{\gamma,i} && \text{on } \gamma \times (0,T).
\end{aligned}$$

Interface problem

$$\begin{aligned}
\textstyle\sum_{i=1}^2 \mathcal{L}_i^{RtN}(p_\gamma, \mathbf{u}_G, q) - \mathbf{u}_\gamma &= Q_\gamma && \text{in } \gamma \times (0,T), \\
\mathbf{u}_\gamma &= -\mathbf{K}_\gamma\, k_\gamma(s_\gamma)\, \nabla p_\gamma && \text{in } \gamma \times (0,T), \\
\operatorname{div} \mathbf{u}_\gamma &= 0 && \text{in } \gamma \times (0,T), \\
p_\gamma &= 0 && \text{on } \partial\gamma \times (0,T),
\end{aligned}$$

Schur method: Saturation problem

Robin-to-Neumann operator

$$\mathcal{L}_{wi}^{RtN}(s_\gamma, s_0, q_{wi}) = -\mathbf{u}_{wi} \cdot \mathbf{n}_i|_\gamma,$$

where $(s_i, \mathbf{u}_{wi})$ is the solution

$$\begin{aligned}
\phi_i \frac{\partial s_i}{\partial t} + \operatorname{div} \mathbf{u}_{wi} &= q_{wi} && \text{in } \Omega_i \times (0,T),\\
\mathbf{u}_{wi} &= -\mathbf{K}_i \nabla \alpha_i(s_i) + \mathbf{f}_i(s_i) && \text{in } \Omega_i \times (0,T),\\
s_i &= 0 && \text{in } \partial\Omega_i \cap \partial\Omega \times (0,T),\\
\frac{1}{\delta_w(s_\gamma)} \mathbf{u}_{wi} \cdot \mathbf{n}_i + \pi_i(s_i) &= \pi_\gamma(s_\gamma) + \frac{f_\gamma(s_\gamma)}{\delta_w(s_\gamma)} \mathbf{u}_i \cdot \mathbf{n}_i && \text{on } \gamma \times (0,T).
\end{aligned}$$

Interface problem

$$\begin{aligned}
\Phi_\gamma \frac{\partial s_\gamma}{\partial t} + \mathbf{u}_{w\gamma} &= Q_{w\gamma} + \sum_{i=1}^{2} \mathcal{L}_{w}^{RtN}(s_\gamma, q_w) && \text{in } \gamma \times (0,T),\\
\mathbf{u}_{w\gamma} &= -\mathbf{K}_\gamma \nabla \alpha_\gamma(s_\gamma) + \mathbf{f}_\gamma(s_\gamma) && \text{in } \gamma \times (0,T),\\
s_\gamma &= 0 && \text{in } \partial\gamma \times (0,T),
\end{aligned}$$

Robin-to-Neumann operator

$$\mathcal{L}^{RtN}_{wi}(s_\gamma, s_0, q_{wi}) = -\mathbf{u}_{wi} \cdot \mathbf{n}_i|_\gamma,$$

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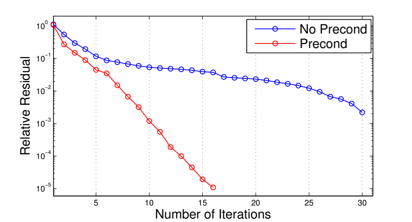
\mathbf{u}_{wi} &= -\mathbf{K}_i \nabla \alpha_i(s_i) + \mathbf{f}_i(s_i) && \text{in } \Omega_i \times (0, T),\\
s_i &= 0 && \text{in } \partial\Omega_i \cap \partial\Omega \times (0, T),\\
\frac{1}{\delta_w(s_\gamma)} \mathbf{u}_{wi} \cdot \mathbf{n}_i + \pi_i(s_i) &= \pi_\gamma(s_\gamma) + \frac{f_\gamma(s_\gamma)}{\delta_w(s_\gamma)} \mathbf{u}_i \cdot \mathbf{n}_i && \text{on } \gamma \times (0, T).
\end{aligned}$$

Interface problem

$$\begin{aligned}
\Phi_\gamma \frac{\partial s_\gamma}{\partial t} + \mathbf{u}_{w\gamma} &= Q_{w\gamma} + \sum_{i=1}^{2} \mathcal{L}^{RtN}_{w}(s_\gamma, q_w) && \text{in } \gamma \times (0, T),\\
\mathbf{u}_{w\gamma} &= -\mathbf{K}_\gamma \nabla \alpha_\gamma(s_\gamma) + \mathbf{f}_\gamma(s_\gamma) && \text{in } \gamma \times (0, T),\\
s_\gamma &= 0 && \text{in } \partial\gamma \times (0, T),
\end{aligned}$$

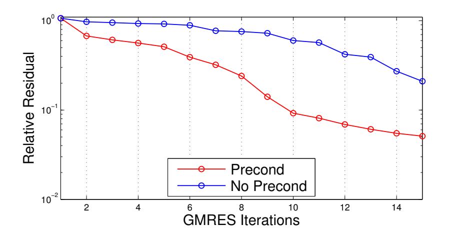
- Splitting diffusion and advection
- Mixed finite elements for 2nd order terms
- Cell-centered finite volumes and Godunov's method for advection
- Implicit Euler for diffusion, explicit Euler for advection
- Different time steps for diffusion and advection
- Domain decomposition solution

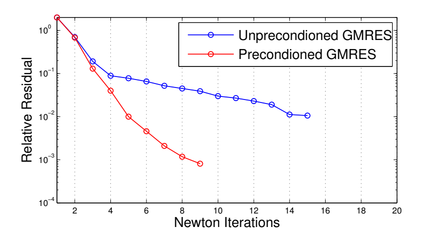
- Local preconditioner for the pressure: $(\text{div}_\tau)^{-1}$



Efficiency of the CG preconditioner

Interface GMRES-Newton convergence



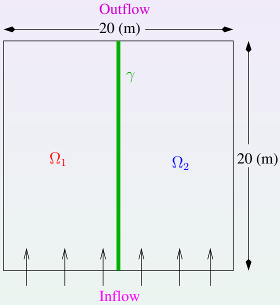


GMRES- Relatif residual

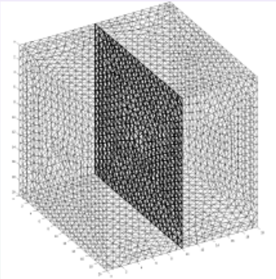
- Unpreconditioned GMRES-Newton
- Preconditioned GMRES-Newton

Inexact Newton- Relatif Residual

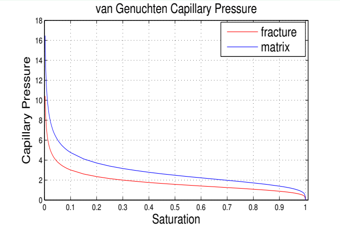
Effect of preconditioner on interface Newton convergence,
15 GMRES iterations



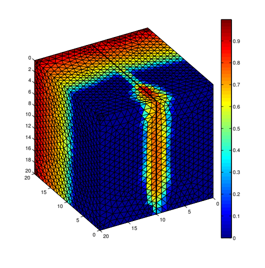
Domain of calculation

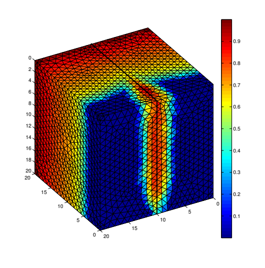


A conforming mesh with 72088 tetrahedra

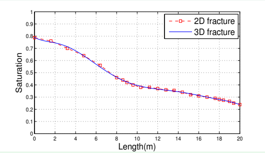


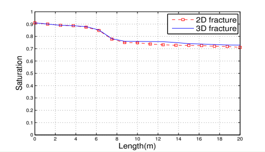
Capillary pressure curves



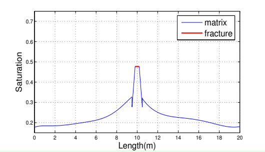


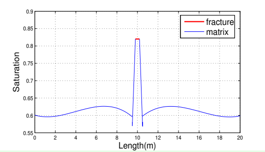
Saturation at two different times



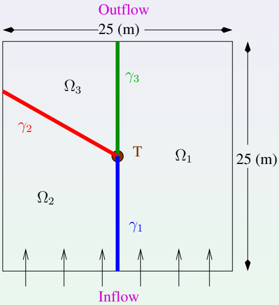


Along the fracture

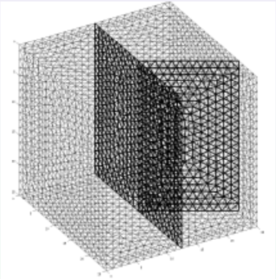




Along a line orthogonal to the fracture

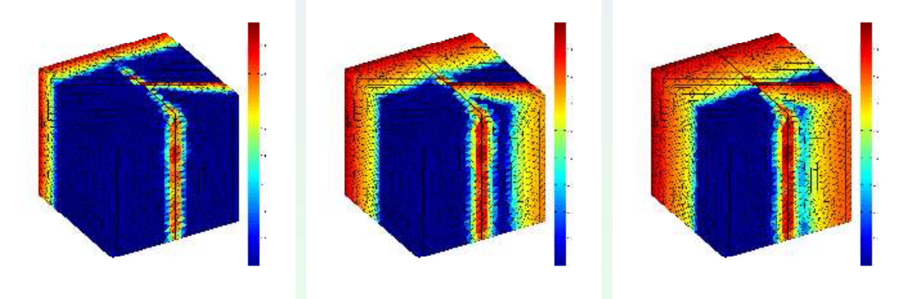


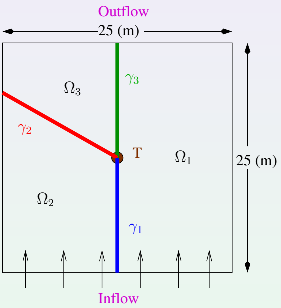
Domain of calculation


A conforming mesh with 65104 tetrahedra

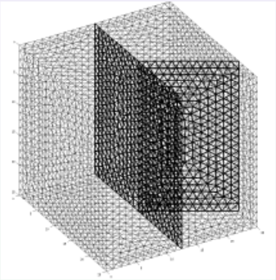
Domain Ω_3 has a larger permeability.

Saturation at three different times





Domain of calculation

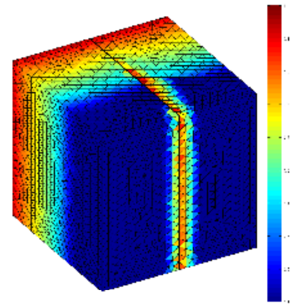


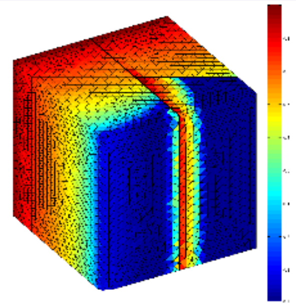
A conforming mesh with 65104 tetrahedra

γ_2 is now a barrier.

Numerical experiment with a fracture intersecting a barrier

Saturation at two different times:





1. Mesh generation
3-D and surface meshing by P. Laug, Inria, Gamma3.
    - **BLSURF**: mesh generator for composite parametric surfaces
    - **GHS3D**: 3-D mesh generator for tetrahedral elements
2. Modeling software
MATLAB Reservoir Simulation Toolbox (**MRST**) developed by SINTEF Applied Mathemathics, K.-A. Lie et al.

3. Articles

• J. Jaffré, M. Mnejja, and J. E. Roberts, A discrete fracture model for two-phase flow with matrix-fracture interaction (2011).
• V. Reichenberger, H. Jakobs, P. Bastian, and R. Helmig, A mixed-dimensional finite volume method for two-phase flow in fractured porous media (2006).
• J. E. P Monteagudo and A. Firoozabadi, Control-volume model for simulation of water injection in fractured media: incorporating matrix heterogeneity and reservoir wettability effects (2006).

Things to do

- Simulate actual 3-D examples
- Consider fractures which do not extend up to the boundary
- Use nonconforming meshes
- Use different time steps in fractures and in matrix rock