A DISPERSION THEORY OF SYMMETRY BREAKING *)

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A B S T R A C T

A new relativistic approach to the problem of broken symmetries is proposed. This is the covariant generalization of the non-relativistic sum rules obtained in previous investigations.

The result is in the form of dispersion sum rules where the pole term represents the group theoretical answer and the cut the breaking correction. The method is applied to SU(3) and chiral $SU(3) \times SU(3)$ leading to very satisfactory results and predictions.

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I. INTRODUCTION

One of the most promising ways of treating elementary particle physics is the classification and interconnection of particles by the use of symmetry groups.

We have been accustomed to find symmetry schemes which, on the one hand, seem to be badly broken and on the other lead to amazingly good results. Therefore, one important problem is to find a method for treating broken symmetries, based more on general arguments than on specific models.

A first suggestion in this direction has been given by Gell-Mann [1] who pointed out that in a field theoretical framework, although the symmetry might be actually badly broken, the equal time commutators of charges and currents generate the algebra of the underlying symmetry group.

A method to exploit these commutation rules together with completeness has been recently developed [2),3)]. In these papers it has been shown that the one-particle contributions do correspond to the group theoretical result, whereas the many-particle terms (which are directly related to the symmetry breaking) give the corrections. The most important corrections can be evaluated by means of a dispersion analysis in terms of observable quantities like coupling constants and mass differences and have led to very reasonable results.

One difficulty found in Refs. [2),3)] comes, however, from the fact that the separation between the different contributions is not a Lorentz invariant one and, in particular, the one-particle term is multiplied by kinematical factors which tend to one in the exact symmetry limit. Although a very reasonable prescription has been given in order to properly interpret the results, a fully covariant approach does indeed represent a great advantage.

In this paper, we want to propose a relativistic generalization of the previous method which allows a clear-cut separation between one-particle and many-particle contributions.

The central idea of our approach is based on the essential use of local commutativity to obtain relativistic sum rules, analogous to the well-known dispersion relations at fixed momentum transfer. This covariant approach is especially useful in the case of dynamical groups which involve not only isospin and hypercharge but also spin and parity [4]. The well-known difficulties related to a relativistic interpretation of these groups have their counterpart, in the method of Refs. [2),3)], in a very critical dependence on the kinematical factors which makes an unambiguous interpretation rather hard.

In Section II the general dispersion approach will be discussed. Section III will be devoted to the discussion of the SU(3) results like the renormalization of vector currents, mass formulae and form factors. In Section IV we shall examine the general principles for extending the method to dynamical groups. As an application, a very beautiful sum rule for the renormalization of the axial vector current, already found by Adler and Weisberger [5], will be derived in a simple way. In addition, we shall obtain two sum rules for isovector and isoscalar magnetic moments of the nucleon, which allow to understand under a new light the electromagnetic structure of the nucleon.

II. THE GENERAL METHOD

As recalled in the previous Section, a symmetry group and its algebra can be generated by the equal time commutation relations of a set of physical charges Q_α, the generators of the group. In the case of SU(3) that we shall consider in this Section, the Q_α are simply given by the space integral of the fourth component of an octet of currents $J_\mu^{(\alpha)}(x)$. The starting point to derive sum rules for any quantity of physical interest is the consideration of the matrix element of an equal time commutator between a generator $Q_\alpha(t)$ and a local operator, having well-defined properties of transformation under the group

$$\left[Q_\alpha(0), t_\beta(\vec{x},0)\right] = f_{\alpha\beta}^{\gamma}\, t_\gamma(\vec{x},0) \tag{1}$$

The constants f are determined from the group algebra. Of course, in the particularly interesting case in which the t_β operator belongs to an octet, the f's are the structure constants of the algebra.

2.a) In order to obtain relativistically invariant sum rules, we shall derive a covariant representation for the matrix element of the commutator (1). Let us use the relation

$$Q_\alpha(t) = \int J_0^{(\alpha)}(\vec{x},t)\, d^3x = \int D_\alpha(x)\, d^4x\ \theta(t-x_0) \tag{2}$$

where

$$D_\alpha(x) = \partial_\mu J_\mu^{(\alpha)}(x) \tag{3}$$

is vanishing in the limit of exact symmetry. Thus we can write

$$\begin{aligned}&\langle a_1|[Q_\alpha(0), t_\beta(\vec{x},0)]|a_2\rangle = \\ &= \int d^4y\, \theta(-y_0)\langle a_1|[D_\alpha(y), t_\beta(\vec{x},0)]|a_2\rangle = f_{\alpha\beta}^{\gamma}\langle a_1|t_\gamma(\vec{x},0)|a_2\rangle\end{aligned} \tag{4}$$

Using invariance under translations we can extract the factor $e^{i(\bar{p}_1-\bar{p}_2)\cdot\bar{x}}$ in both sides and get

$$\int \langle a_1|[D_\alpha(z), t_\beta(0)]|a_2\rangle \,\theta(-z_0)\, d^4 z = f_{\alpha\beta}{}^{\gamma} \langle a_1|t_\gamma(0)|a_2\rangle \tag{5}$$

We now make the further assumption that the commutators between the local operators $D_\alpha(x)$ and $t_\beta(y)$ vanish for spacelike separations

$$[D_\alpha(x), t_\beta(y)] = 0 \quad \text{for} \quad (x-y)^2 < 0 \tag{6}$$

It is worth while to remark that the microcausality condition (6) gives an explicit invariant meaning to the quantity

$$M = \int \langle a_1|[D_\alpha(z), t_\beta(0)]|a_2\rangle \,\theta(-z_0)\, d^4 z \tag{7}$$

An important point to be discussed is the contribution from the integration on the surface at $t = -\infty$. The matrix elements $\langle a|D_\alpha|n\rangle$, which appear using completeness in Eq. (7), will have an oscillating behaviour in time, if the energies of the states $|a\rangle$ and $|n\rangle$ are different. This, according to the usual rules of the game, leads to a vanishing contribution. In this case, of course, the analogous formula obtained starting from $t = +\infty$

$$M = -\int \langle a_1|[D_\alpha(z), t_\beta(0)]|a_2\rangle \,\theta(z_0)\, d^4 z \tag{7'}$$

is completely equivalent to (7).

The question becomes serious in the pathological situation (like for instance, $\langle N|D_I^{(-)}|P\gamma\rangle$) where the energy difference can vanish. Here we have to deal with a very unpleasant problem of asymptotic behaviour of the product of operators. We are unable up to now to give a completely satisfacory answer, however we are convinced (on the basis of some models) that the correct recipe is to treat $-\infty$ and $+\infty$ on the same ground, and to write

$$M = -\frac{1}{2}\int \langle a_1 | [D_\alpha(z), t_\beta(0)] | a_2 \rangle \, \mathcal{E}(z_0) \, d^4 z \tag{7''}$$

Let us now come back to the expression (7) and introduce the function $F(k)$ defined by

$$F(k) = \int d^4 z \, \theta(-z_0) \langle a_1 | [D_\alpha(z), t_\beta(0)] | a_2 \rangle \, e^{ikz} \tag{8}$$

and

$$M = \lim_{k \to 0} F(k) \tag{9}$$

The essential point of our method consists in the observation that $F(k)$ is the Fourier transform of an advanced causal commutator, vanishing anywhere outside the past light cone, and that one can apply to it the standard procedure to derive a (one-dimensional) dispersion relation. For simplicity, we begin by considering the case in which a_1, a_2 are both spin zero particles and t_β is a scalar operator, so that M is a scalar function of m_1^2, m_2^2 and $\Delta^2 = (p_1 - p_2)^2$. Owing to the introduction of the four vector k, which we choose of zero length $k^2 = 0$, F will depend on additional invariants. More precisely, we introduce two four vectors k_1 and k_2 such that

$$p_1 + k_1 = p_2 + k_2 , \qquad k_1 = k$$

and we shall fix

$$\begin{aligned} k_1^2 &= k^2 = 0 \\ k_2^2 &= (p_1 + k_1 - p_2)^2 = (\Delta + k_1)^2 = \Delta^2 \end{aligned} \tag{10}$$

which requires

$$\Delta k_1 = 0 \tag{11}$$

or

$$p_1 \cdot k_1 = p_2 \cdot k_2$$

Finally we define the variable

$$\nu = k_1 \cdot \frac{p_1 + p_2}{2 m_1} = \frac{k_1 \cdot p_1}{m_1} \tag{12}$$

Thus F will be a function of the "external masses" k_1^2, k_2^2 and of the two invariants Δ^2 and ν, and the quantity M under consideration will be obtained for $k_1 \to 0$, $\nu \to 0$, $k_2 \to \Delta$.

We shall study the analyticity properties of F in the variable ν, keeping Δ^2 and the external masses fixed. These properties are completely analogous to those of the scattering amplitude of particles of masses $k_1^2 = 0$, $k_2^2 = \Delta^2$ (represented by the fields D_α and t_β) on a_1, a_2. In this way we are led to a dispersion relation in the energy ν, at fixed momentum transfer Δ^2.

$$F(\nu, \Delta^2) = \frac{1}{\pi} \int_0^\infty \frac{A_I(\nu', \Delta^2)}{\nu' - \nu} d\nu' - \frac{1}{\pi} \int_{-\infty}^0 \frac{A_{II}(\nu', \Delta^2)}{\nu' - \nu} d\nu' \tag{13}$$

where the spectral functions are given

$$A_I = \frac{i}{2} \sum_n (2\pi)^4 \delta(p_1 + k_1 - p_n) \langle a_1 | D_\alpha(0) | n \rangle \langle n | t_\beta(0) | a_2 \rangle \tag{14}$$

$$A_{II} = \frac{i}{2} \sum_n (2\pi)^4 \delta(p_2 - k_1 - p_n) \langle a_1 | t_\beta(0) | n \rangle \langle n | D_\alpha(0) | a_2 \rangle \tag{14'}$$

In the expressions (14) and (14') for A_I and A_{II}, we separate the diagonal and the off-diagonal contributions. The diagonal terms, coming from particles in the same representation as a_1 and a_2, give rise to polar terms whose denominators are of the form $(\nu - (m_a^2 - m_{a'}^2)/2m_a)$ and the numerators contain "coupling constants" represented by the matrix elements $\langle a_1 | D_\alpha | a' \rangle$, $\langle a' | t_\beta | a_2 \rangle$. We see clearly here the reason for the dominance of the pole term as compared to higher contributions: in fact the denominator is of the order of the mass difference in the multiplet.

The approximation of keeping only the pole terms can be considered as a covariant definition of the group theoretical limit. We see also that the reasons for the approximate validity of the group theoretical formulae are not essentially different from those in favour of similar one-particle approximations, like for instance peripheral models.

In those cases where the kinematical situation is such that the denominators $\nu' \pm \nu$ can vanish, the preceding discussion leads to the recipe of simply using instead of $F(\nu)$:

$$F(\nu) \longrightarrow \frac{1}{2} \{ F(\nu + i\eta) + F(\nu - i\eta) \} . \tag{15}$$

2.b) Let us now discuss the relativistic generalization of the sum rules for renormalization constants discussed in Ref. [2]. We have seen there that the evaluation of the renormalization ratios is on a somewhat different footing from the usual group theoretical relations, in the sense that the higher corrections are of the second order in the breaking [6] and they are more strongly damped by a squared denominator. Therefore they need here a separate treatment. Let us consider the commutator between "opposite charges" (i.e., the generators corresponding to opposite roots) which, according to the standard formulation of the Lie algebra, equals a linear combination of the diagonal generators

$$[Q_\alpha , Q_{-\alpha}] = \alpha^i Q_i \quad {}^{*)} \tag{16}$$

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*) We denote with Q_α the generators corresponding to a (not null) root α and with Q_i the mutually commuting ones.

In particular in SU(3) $\alpha^i Q_i$ will be a linear combination of charge and hypercharge so that it has only diagonal matrix elements without renormalization. Then we take the expectation value of Eq. (16) in a well-defined state

$$\langle a|[Q_\alpha, Q_{-\alpha}]|a\rangle = \alpha^i \langle a|Q_i|a\rangle \qquad (17)$$

Using for $Q_{\pm\alpha}$ the representations

$$Q_{\pm\alpha} = \pm\int D_{\pm\alpha}(x)\ \theta(\mp x_0)\ d^4x \qquad (18)$$

we obtain

$$\langle a(p_1)|[Q_\alpha, Q_{-\alpha}]|a(p_2)\rangle =$$
$$= -\int d^4x\ d^4y \langle a(p_1)|[D_\alpha(x), D_{-\alpha}(y)]|a(p_2)\rangle\, \theta(x_0)\,\theta(-y_0) = (2\pi)^3\,\delta(\vec{p}_1 - \vec{p}_2)\, M' \qquad (19)$$

with

$$M' = \int z_0\,\theta(-z_0)\, d^4z \langle a|[D_\alpha(z), D_{-\alpha}(0)]|a\rangle \qquad (20)$$

We introduce again

$$\phi(k) = \int \theta(-z_0)\, d^4z \langle a|[D_\alpha(z), D_{-\alpha}(0)]|a\rangle\, e^{ikz} \qquad (21)$$

and being now the external masses equal, we have $\Delta^2 = 0$ and we are thus led to the consideration of a forward dispersion relation. Finally

$$M' = -i \lim_{k\to 0} \frac{\partial}{\partial k_0}\phi(k) = -i\frac{E_a}{m_a} \lim_{\nu\to 0}\frac{\partial\phi}{\partial\nu} \qquad (22)$$

On the other hand, since *)

$$\langle a | Q_i | a \rangle = c^i_{aa} (2\pi)^3 \delta(\vec{p}_1 - \vec{p}_2)\, 2E_a \tag{23}$$

(c^i_{aa} being of course the appropriate Clebsch-Gordan coefficient), we get the invariant sum rule

$$\frac{-i}{\alpha_i c^i_{aa}} \frac{1}{2m_a} \lim_{\nu \to 0} \frac{\partial \phi}{\partial \nu} = 1 \tag{24}$$

Equation (24) is the relativistic equivalent of Eqs. (3.12), (3.13) of Ref. [2]. Writing now a dispersion relation for

$$\phi(\nu) = \frac{1}{\pi} \int_0^\infty \frac{A_I(\nu')}{\nu' - \nu} d\nu' - \frac{1}{\pi} \int_{-\infty}^0 \frac{A_{II}(\nu')}{\nu' - \nu} d\nu' \tag{25}$$

with

$$A_I(\nu) = \frac{i}{2} \Sigma_n (2\pi)^4 \delta(p+k-p_n) \langle a | D_\alpha(0) | n \rangle \langle n | D_{-\alpha}(0) | a \rangle \tag{26}$$

$$A_{II}(\nu) = \frac{i}{2} \Sigma_n (2\pi)^4 \delta(p-k-p_n) \langle a | D_{-\alpha}(0) | n \rangle \langle n | D_\alpha(0) | a \rangle \tag{26'}$$

We separate from the continuum the one-particle poles which have residua proportional to the square of the renormalization ratio. The contribution of the continuum could be formally expressed in terms of the cross-section of massless scalar particles whose "currents" are given by $D_{\pm\alpha}$.

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*) In order to simplify the notation, we assume here invariant normalization for the states : $\langle p_1 | p_2 \rangle = 2E\, \delta(p_1 - p_2)(2\pi)^3$.

We notice that the $\partial/\partial\nu$ derivative in Eq. (24) leads to a squared energy denominator which causes one more order of magnitude in the dominance of the pole term. This is the dispersion equivalent to the Ademollo-Gatto theorem [6].

III. APPLICATIONS

3.a) In the previous Section, in order to illustrate our method, we limited ourselves to the case of a scalar operator t_β and of spin zero particles. The generalization to higher spins does not offer any new essential difficulty, but only more complicated expressions have to be considered. As an indication we shall discuss here some explicit examples.

Let us begin by considering two spin $\frac{1}{2}$ particles, t_β still being a scalar operator. In this case the quantity $F(k)$ given by Eq. (8) is the same as the scalar meson-baryon scattering amplitude. Thus it can be written in the form *)

$$F(k) = \bar{u}_1 \left\{ \alpha(\nu, \Delta^2) + \beta(\nu, \Delta^2) \left[\gamma \cdot K, \Delta \cdot \gamma \right] \right\} u_2$$
$$\Delta = p_1 - p_2 \ , \quad K = \frac{1}{2}(k_1 + k_2) \ , \quad \nu = p_1 \cdot k_1 / m_1 \tag{27}$$

In the limit we are considering, $k_1 \to 0$, $\nu \to 0$, $k_2 \to \Delta$, only the first invariant survives. All previous considerations can be applied just by taking

$$M = \lim_{\nu \to 0} \alpha(\nu, \Delta^2) \tag{28}$$

If we now take t_β to be a four vector and a_1, a_2 two spin zero particles, the expansion of the matrix element of the commutator in invariants is

$$F_\mu(k) = G_1(\nu, \Delta^2)(p_1 + p_2)_\mu + G_2(\nu, \Delta^2)(p_1 - p_2)_\mu + k_\mu G_3(\nu, \Delta^2) \tag{29}$$

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*) This separation is more useful than the standard one $A + \gamma \cdot KB$. The relation between the two sets of invariants is : $B = -2\beta(m_1 + m_2)$; $A = \alpha + \beta \left[4m_1 \nu + (m_1^2 - m_2^2) \right]$.

Our method leads in this case to dispersion relations for both invariants G_1, G_2 (G_3 need not be considered) which correspond to the form factors appearing for instance in the $K \to \pi e \gamma$ decay.

For more complicated situations, the number of independent invariants increases. In this paper, we will not consider other cases than those discussed above.

3.b) As a first application we shall examine in detail the dispersion sum rules for the renormalization constants. As already explained, it is sufficient to take the external masses equal and, as a consequence, $\Delta^2 = 0$. We begin by recalling some useful relations. Let us consider the matrix element of a vector current between two spin zero states :

$$\langle p_1 | J_\mu^{(\alpha)} | p_2 \rangle = C_{12}^{\alpha} \left\{ (p_1 + p_2)_\mu G_1^{(\alpha)}(\Delta^2) + (p_1 - p_2)_\mu G_2^{(\alpha)}(\Delta^2) \right\} \tag{30}$$

where c_{12}^{α} is the appropriate Clebsch-Gordan coefficient and the presence of the second form factor $G_2^{(\alpha)}$ is a display of the non-conservation of $J_\mu^{(\alpha)}$. Then

$$\langle p_1 | D_\alpha(0) | p_2 \rangle = i(m_1^2 - m_2^2)\, C_{12}^{\alpha}\, G_D^{(\alpha)}(\Delta^2) \tag{31}$$

where

$$G_D^{(\alpha)}(\Delta^2) = G_1^{(\alpha)}(\Delta^2) + \frac{\Delta^2 G_2^{(\alpha)}(\Delta^2)}{m_1^2 - m_2^2} \tag{32}$$

For spin $\frac{1}{2}$ particles, the analogous formulae are :

$$\langle p_1 | J_\mu^{(\alpha)} | p_2 \rangle = C_{12}^{\alpha}\, \bar{u}_1 \{ G_1^{\alpha}(\Delta^2)\gamma_\mu + G_2^{(\alpha)}(\Delta^2)(p_1 - p_2)_\mu + G_3^{(\alpha)}(\Delta^2)\, \sigma_{\mu\nu}(p_1 - p_2)_\nu \} u_2 \qquad (30')$$

$$\langle p_1 | D_\alpha(0) | p_2 \rangle = i(m_1 - m_2) C_{12}^{\alpha}\, G_D^{(\alpha)}(\Delta^2)\, \bar{u}_1 u_2 \qquad (31')$$

$$G_D^{(\alpha)}(\Delta^2) = G_1^{(\alpha)}(\Delta^2) + \frac{\Delta^2\, G_2^{(\alpha)}(\Delta^2)}{m_1 - m_2} \qquad (32')$$

The renormalization ratio is defined as the value of $G_D(\Delta^2)$ at zero momentum transfer

$$\tau_\alpha = G_D^{(\alpha)}(0) \qquad (33)$$

and in this limit we can write

$$\langle p_1 | D_\alpha | p_2 \rangle = C_{12}^{\alpha}\, i\, \tau_\alpha\, (m_1^2 - m_2^2) \qquad \text{for bosons} \qquad (34)$$

$$\langle p_1 | D_\alpha | p_2 \rangle = C_{12}^{\alpha}\, i\, \tau_\alpha\, \bar{u}_1 u_2 \qquad \text{for fermions} \qquad (34')$$

To discuss the invariant sum rule for renormalization ratios, we can use Eq. (24)

$$\frac{1}{\alpha_i C_{aa}^{i}} \frac{1}{2m_a} \lim_{\nu \to 0} \frac{\partial F}{\partial \nu} = 1 \qquad (F = -i\,\phi) \qquad (24')$$

In the case of pseudoscalar mesons, we have an invariant amplitude only, which obeys a dispersion relation of the form

$$F(\nu) = \frac{\tau_\alpha^2 (m_a^2 - m_o^2)^2}{2 m_a (\nu_o - \nu)} + \frac{1}{\pi} \int \frac{\mathrm{Im}\, F(\nu')}{\nu' - \nu} \, d\nu' \tag{35}$$

where

$$\nu_o = \frac{m_o^2 - m_a^2}{2 m_a}$$

After derivation :

$$\tau_\alpha^2 + \frac{1}{\pi} \int \frac{\mathrm{Im}\, F(\nu')}{\nu'^2} \, d\nu' = 1 \tag{36}$$

In the fermion case, we can write the decomposition (27) and for α the dispersion relation holds *)

$$\alpha(\nu) = \frac{\tau_\alpha^2 (m_a^2 - m_o^2)^2}{2 m_a (\nu_o - \nu)} + \frac{1}{\pi} \int \frac{\mathrm{Im}\, \alpha(\nu')}{\nu' - \nu} \, d\nu' \tag{35'}$$

so that for fermions we get the completely analogous sum rule

$$\tau_\alpha^2 + \frac{1}{\pi} \int \frac{\mathrm{Im}\, \alpha(\nu')}{\nu'^2} \, d\nu' = 1 \tag{36'}$$

In connection with Eqs. (36) and (36'), we want to emphasize two points. First of all, our sum rules are defined in a unique way, without the presence of any kinematical factor. This has to be compared with the situation found in Refs. [2),3)] where p dependent sum rules were obtained.

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*) The additional factor $(m_a + m_o)^2$ in the polar term comes from the summation over the spin of the intermediate state.

Our present treatment, being a covariant one, chooses automatically what we called the best sum rule (i.e., the one we obtained elsewhere in the limit $p \to \infty$). Secondly, the deviation of r^2 from unity is represented by an integral which contains $|\langle a|D_A|n\rangle|^2$ terms. These latter can be understood as the total cross-sections for the scattering of a scalar "particle" D_A of zero mass on the target a. Thus, one might use general arguments as unitarity or Pomeranchuk theorems to get some model independent information about the size of the corrections.

3.c) In Reference [3], we have shown that the more immediate way to obtain mass formulae was through the use of the commutation relation of the type

$$[Q_A^+, N_A^+] = 0 \tag{37}$$

with

$$\begin{aligned} N_A^+ &= [Q_A^+, H] = [Q_A^+, H_B] = \\ &= i \int d^3x \, D_A^+(x) \end{aligned} \tag{38}$$

Equation (37) should be interpreted as the expression through commutation relations of the usual hypothesis that the breaking part of the Hamiltonian transforms like the eight component of an octet. As explicitly pointed out in Ref. [3], one obtains with this method energy formulae, i.e., p dependent sum rules, as a consequence of working with a non-invariant operator.

In order to obtain a covariant generalization of the method, we shall make the more restrictive hypothesis that the divergences D_A themselves belong to an octet, i.e., we admit the validity of the equal time commutation relations between the $Q_A^{(+)}$ and the local operator $D_A(\bar{x},t)$

$$[Q_A^+(t), D_A^+(\bar{x},t)] = 0 \tag{39}$$

At this point we can apply to the commutator (39) the general technique developed in the previous section, taking $t_\beta(x) = D_\beta(x)$. We begin by considering the case of pseudoscalar mesons : in particular we choose $A = K$, $a_1 = K^+$, $a_2 = K^-$. Then we introduce the quantity

$$\phi(k) = \int d^4 z \; \theta(-z_0) \langle K^+ | [D_K^+(z), D_K^+(0)] | K^- \rangle \, e^{ikz} \tag{40}$$

and Eq. (39) becomes

$$\lim_{k \to 0} \phi(k) = 0 \tag{41}$$

For $\phi(k)$ we can write a dispersion relation (at Δ^2 fixed, $\neq 0$) and, using Eqs. (31), (32) to select the polar contributions, we get :

$$\phi(\nu, \Delta^2) = \frac{1}{4m_K} \left\{ \frac{(m_K^2 - m_\pi^2)^2 \tau^{(K)} G^{(K)}(\Delta^2)}{\frac{m_\pi^2 - m_K^2}{2m_K} - \nu} + \frac{(m_K^2 - m_\pi^2)^2 \tau^{(K)} G^{(K)}(\Delta^2)}{\frac{m_\pi^2 - m_K^2}{2m_K} + \nu} \right.$$

$$\left. + \, 3\,(\pi \leftrightarrow \eta) \right\} + \frac{1}{\pi} \int \frac{\mathrm{Im}\, \phi(\nu', \Delta^2)}{\nu' - \nu} \, d\nu' \tag{42}$$

Then Eq. (41) becomes :

$$(m_K^2 - m_\pi^2)\, \tau_\pi^{(K)} G_\pi^{(K)}(\Delta^2) + 3(m_K^2 - m_\eta^2)\, \tau_\eta^{(K)} G_\eta^{(K)}(\Delta)^2 = \frac{1}{\pi} \int \frac{\mathrm{Im}\, \phi(\nu', \Delta^2)}{\nu'} \, d\nu' \tag{43}$$

In so doing we get a continuous set of sum rules, one for each value of the invariant momentum transfer Δ^2. In particular, we can choose the relation on the mass shell (considering only spurions of zero mass), that is for $\Delta^2 = 0$. We obtain

$$(m_K^2 - m_\pi^2)\, r_\pi^2 + 3\,(m_K^2 - m_\eta^2)\, r_\eta^2 = \frac{1}{\pi}\int \frac{\mathrm{Im}\, \phi(\nu', \Delta^2)}{\nu'}\, d\nu' \qquad (44)$$

If we remember that the deviation of r from unity is $O(f^2)$ (according to the Ademollo-Gatto theorem), the r presence will affect the corrections only in the $O(f^3)$ terms. Thus we can safely put $r \simeq 1$ and finally we obtain the classical SU(3) quadratic mass formula :

$$4\, m_K^2 - 3\, m_\eta^2 - m_\pi^2 = C \qquad (45)$$

and the correction C is clearly of order f^2.

We can proceed analogously in the fermion case and get, using Eq. (31')

$$\phi(\nu, \Delta^2) = \Bigg\{ \frac{1}{4m_p} \frac{(m_p^2 - m_\Sigma^2)(m_\Xi - m_\Sigma)\, r_{p\Sigma}\, G_{\Sigma\Xi}(\Delta^2)}{\frac{m_\Sigma^2 - m_p^2}{2m_p} - \nu}$$

$$+ \frac{1}{4m_\Xi} \frac{(m_\Xi^2 - m_\Sigma^2)(m_p - m_\Sigma)\, r_{\Sigma\Xi}\, G_{p\Sigma}(\Delta^2)}{\frac{m_\Sigma^2 - m_\Xi^2}{2m_\Xi} + \nu} + 3\,(\Sigma \to \Lambda)\Bigg\}$$

$$+ \frac{1}{\pi}\int \frac{\mathrm{Im}\, \phi(\nu', \Delta^2)}{\nu' - \nu}\, d\nu' = 0$$

Fixing $\Delta^2 = 0$ *) and neglecting the deviation of r from unity, we have the linear mass formula for baryons :

$$2\, m_\Xi + 2\, m_p - 3\, m_\Lambda - m_\Sigma = C \qquad (45')$$

$$C = O(f^2)$$

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*) We can notice that in this case - where the external masses are different - the value $\Delta^2 = 0$ can be reached either through analytic continuation in the unphysical region, or by keeping $\vec{p}_1 = \vec{p}_2 = \vec{p}$ and then taking the limit $p \to \infty$. This corresponds to the procedure we used in Ref. [3)] (Section 6.3) to derive mass formulae and to write corrections in a covariant way.

As far as the corrections are concerned, they can be evaluated through the continuum contribution, i.e., by means of the dispersion integral of $\mathrm{Im}\,\emptyset$. Formally, it can be related to the imaginary part of a spurion baryon scattering amplitude

$$(D_K^+) + \Xi^- \longrightarrow (D_K^-) + P \tag{46}$$

In this connection, one might get arguments that such an amplitude should be strongly damped at high energies, because the process goes through the exchange of large quantum numbers $(\Delta S = 2)$. We know that, as a consequence of general theorems [7], the favoured channel at high energy is for no quantum number exchange. This can suggest, in the framework of dispersion theory, an explanation of the smallness of the corrections to the mass formulae.

3.d) As a final example, we shall treat the case in which t_β is a current density. In particular, we want to consider the strangeness changing vector current $(\Delta Q = \Delta S)$ which describes, for instance, the $K \to \pi e \nu$ decay. In this way, we shall be able to connect the form factors for this process with the electromagnetic ones.

We start by considering the commutator between the strange charge Q_K^+ and the opposite current $J^{(K^-)}$:

$$\left[Q_K^+ , J_\mu^{(K^-)} \right] = J_\mu^{(Q)} + \frac{3}{2} J_\mu^{(Y)} \tag{47}$$

We recall now the definition of the matrix element of a current between pseudoscalar particles.

$$\langle p_1 | J_\mu^{(\alpha)} | p_2 \rangle = C_{12}^{\alpha} \left\{ (p_1 + p_2)_\mu G_1^{(\alpha)}(\Delta^2) + (p_1 - p_2)_\mu G_2^{(\alpha)}(\Delta^2) \right\}$$

where

$$G_1^{(\alpha)}(0) = z_\alpha$$

If $J_\mu^{(\alpha)}$ is a conserved current [this is the case of $J^{(3)}$, $J^{(y)}$ if we consider semi-strong interactions breaking SU(3)], then

$$\partial_\mu J_\mu^{(\alpha)} = 0 \tag{48}$$

and thus

$$G_2^{(\alpha)}(\Delta^2) = 0 \tag{49}$$

and

$$Z_\alpha = 1 \tag{50}$$

We take now the matrix element of (47) between two π^+ states :

$$\langle\pi^+|[Q_K^+, J_\mu^{(K^-)}]|\pi^+\rangle = \langle\pi^+|\, J_\mu^{(Q)}\,|\pi^+\rangle \tag{51}$$

To evaluate the left-hand side commutator, we first write it in the covariant form :

$$\int\langle\pi^+(p_1)|\,[D_K^+(z), J_\mu^{(K^-)}(0)]\,|\pi^+(p_2)\rangle\ \theta(-z_0)\ d^4z \tag{52}$$

and then we introduce

$$\begin{aligned} \phi_\mu(k) &= \int d^4z\ \theta(-z_0)\langle\pi^+|[D_K^+(z), J_\mu^{(K^-)}(0)]|\pi^+\rangle\ e^{ikz} \\ \langle\pi^+|[Q_K^+, J_\mu^{(K^-)}]|\pi^+\rangle &= \lim_{k\to 0}\ \phi_\mu(k) \end{aligned} \tag{53}$$

On invariance grounds, we put

$$\phi_\mu(k) = (p_1+p_2)_\mu\,\phi_1(\nu,\Delta^2) + (p_1-p_2)_\mu\,\phi_2(\nu,\Delta^2) + k_\mu\,\phi_3(\nu,\Delta^2) \tag{54}$$

so that we can identify

$$\phi_1(0,\Delta^2) = G^{(Q)}_{1\pi\pi}(\Delta^2) \tag{55}$$

$$\phi_2(0,\Delta^2) = 0 \tag{56}$$

With the standard procedure we get for ϕ_1 and ϕ_2 the following dispersive representations (we separate explicitly the $\bar{K}_o$ polar contribution) :

$$\phi_1(\nu,\Delta^2) = \frac{(m^2_K - m^2_\pi)\, r_{\pi K}}{m^2_K - m^2_\pi - 2m_n \nu}\, G^{(-)}_{1\pi K}(\Delta^2) + \frac{1}{\pi}\int \frac{\operatorname{Im}\phi_1(\nu',\Delta^2)}{\nu'-\nu}\, d\nu' \tag{57}$$

$$\phi_2(\nu,\Delta^2) = \frac{(m^2_K - m^2_\pi)\, r_{\pi K}}{m^2_K - m^2_\pi - 2m_n \nu}\, G^{(-)}_{2\pi K}(\Delta^2) + \frac{1}{\pi}\int \frac{\operatorname{Im}\phi_2(\nu',\Delta^2)}{\nu'-\nu}\, d\nu' \tag{58}$$

(We do not write an analogous expression for ϕ_3 because it does not contribute as $k \to 0$.) Then, from (55) and (56), we get [using the fact that $r_{\pi K} = 1 + O(f^2)$]

$$G^{(-)}_{1\pi K}(\Delta^2) + \frac{1}{\pi}\int \frac{\operatorname{Im}\phi_1(\nu',\Delta^2)}{\nu'}\, d\nu' = G^{(Q)}_{1\pi\pi}(\Delta^2) \tag{59}$$

$$G^{(-)}_{2\pi K}(\Delta^2) + \frac{1}{\pi}\int \frac{\operatorname{Im}\phi_2(\nu',\Delta^2)}{\nu'}\, d\nu' = 0 \tag{60}$$

One sees that the deviation of $G_{1\,\pi K}^{(K^-)}$ from $G_{1\,\pi\pi}^{(Q)}$ (or $G_{2\,\pi K}^{(K^-)}$ from zero), represented by the continuum contribution, is $O(f)$. Of course, in the lowest order approximation of Eqs. (59) and (60) we recover the usual SU(3) limits :

$$G_{1\,\pi K}^{(-)}(\Delta^2) = G_{1\,\pi\pi}^{(Q)}(\Delta^2) \quad , \quad G_{2\,\pi K}^{(-)}(\Delta^2) = O(f)$$

We remark, however, once again, that in our method we can obtain an unambiguous identification of what has to be taken as the correct limit of SU(3). In other words, the problem of the kinematical factors and of the p dependence of the sum rule does not exist.

Concluding this Section, we have seen that the dispersion approach introduces a unique separation between diagonal and not diagonal contributions.

The relations one obtains, taking only the pole contribution, are very reasonable indeed and coincide with what many people had already guessed on the basis of intuitive considerations. In this context, it is amusing to notice that this method leads automatically to mass formulae which are linear for fermions and quadratic for bosons.

One can remark that from purely group theoretical standpoints, all choices of the kinematical factors are on the same footing. However, we want to stress that if the sum rules are cast in the form (5), the corrections can be simply expressed as dispersion integrals analogous to those appearing in fixed Δ^2 dispersion relations.

Thus, one might take advantage of arguments as unitarity or Pomeranchuk theorems for an estimate of these corrections or for obtaining some upper bounds.

IV. DYNAMICAL SYMMETRIES

Our aim is now to discuss the application of the previous method to dynamical groups, which involve also space-time quantum numbers like spin and parity.

4.a) In this case the main problem still open is to understand the correct physical meaning of the group theoretical results, especially in relation to Lorentz invariance and unitarity. On the other hand, we have already seen that the method of Refs. [2),3)], based on completeness, leads to very serious questions of interpretation, since the form of the sum rules depends critically on the choice of the reference frame. Thus, we have in this case to rely almost completely on the dispersion approach which, as we shall show in some examples, is of great use as an unambiguous means of investigating those controversial questions.

Let us start with the $SU(3) \times SU(3)$ group of Gell-Mann [4)] which connects states of opposite parity. If we indicate :

$$J_\mu^{(\alpha)} \sim \bar{\psi} \lambda_\alpha \gamma_\mu \psi \tag{61}$$

$$\bar{J}_\mu^{(\alpha)} \sim \bar{\psi} \lambda_\alpha \gamma_5 \gamma_\mu \psi \tag{62}$$

the generators of the group are

$$Q_\alpha = \int J_0^{(\alpha)} d^3x \tag{63}$$

$$\bar{Q}_\alpha = \int \bar{J}_0^{(\alpha)} d^3x \tag{64}$$

The commutation rules between the operators in Eqs. (61), (62), (63) and (64) can be easily obtained from those of $SU(3)$ by using the following rule

$$[\text{even}, \text{even}] \to \text{even}$$
$$[\text{even}, \text{odd}] \to \text{odd} \qquad (65)$$
$$[\text{odd}, \text{odd}] \to \text{even} .$$

The dispersion treatment can be easily extended to this case by writing

$$Q_\alpha = \int D_\alpha(x)\, \Theta(-x_0)\, d^4x$$
$$\bar{Q}_\alpha = \int \bar{D}_\alpha(x)\, \Theta(-x_0)\, d^4x \qquad (66)$$

$$D_\alpha = \partial_\mu j_\mu^{(\alpha)}$$
$$\bar{D}_\alpha = \partial_\mu \bar{j}_\mu^{(\alpha)} \qquad (67)$$

Therefore, as in the previous Sections, the equal time commutators between charges and currents can be reduced to the investigation of the expressions

$$\int d^4x\, \Theta(-x_0) \langle a | [D_\alpha(x), D_\beta(0)] | a \rangle\, e^{ikx} \qquad (68)$$

$$\int d^4x\, \Theta(-x_0) \langle a | [D_\alpha(x), j_\mu^{(\beta)}(0)] | a \rangle\, e^{ikx} \qquad (69)$$

in which both even and odd charges and currents appear.

If we now make the assumption that the commutators between all D and j vanish for spacelike separations, the expressions in (68) and (69) are fully covariant and can be taken as the basis to derive relativistic dispersion relations.

The contribution to the dispersion integrals will come, first from the one-particle poles, then from the many-particle cut. Retaining only the polar contributions, one gets a set of relations involving observable quantities and renormalization ratios which can be considered as the simplest physical consequence of the dynamical group. Of course, there is also the contribution from higher terms and there is hope that at least some of these effects could be reasonably estimated as in standard dispersion theory.

We want to stress that all identities obtained on this basis have an unambiguous physical meaning and this shows, at least in our opinion, that the causality condition is necessary to give a clear interpretation of the dynamical groups.

We shall illustrate the previous consideration by means of two examples. The first one refers to the renormalization ratio of the axial vector current. We shall present a simple derivation of a beautiful sum rule obtained by Adler and Weisberger in a more involved manner, using the method of Refs. [2] and [3]. A second example will be based on the use of commutation rules between charges and currents. We shall get two sum rules for the anomalous magnetic moments of the nucleons whose physical meaning is rather interesting.

4.b) Let us now consider the commutator

$$[\bar{Q}^{(+)}, \bar{Q}^{(-)}] = 2Q^{(3)} \tag{70}$$

where $\bar{Q}^{\pm}$ are the "axial" charges transforming like $\tau^{\pm}$ and Q^{3} is the isovector charge. We write *) :

$$F(k) = \int \langle P_1 | [\bar{D}^{(+)}(x), \bar{D}^{(-)}(0)] | P_2 \rangle e^{ikx} \theta(-x_0) d^4x \tag{71}$$

and, as in the previous Section, $k^2 = 0$. Equation (24') tells us that

*) More precisely, $F(k)$ is the no spin flip part of the amplitude [see Eq. (27)].

$$\frac{1}{g\,\omega_a} \lim_{\nu \to 0} \frac{\partial \bar{F}(\nu)}{\partial \nu} = 1 \tag{72}$$

The pole term can be evaluated by recalling the definition (for $k^2 = 0$)

$$\langle P | \bar{D}^{(+)} | N \rangle = i\,(\omega_P + \omega_N)\, \bar{u}_P \gamma_5 u_N\, r_A \tag{73}$$

where the renormalization ratio r_A is, according to universality, equal to the ratio g_A/g_V

$$r_A = \frac{g_A}{g_{A0}} = \frac{g_A}{g_V}$$

The dispersion relation for $F(\nu)$ has the form

$$\begin{aligned} F(\nu) = {} & \frac{(\omega^2_N - \omega^2_P)^2}{2\omega_P} \, \frac{1}{\frac{\omega^2_N - \omega^2_P}{2\omega_P} - \nu} + \\ & + \frac{1}{\pi} \int_0^{\infty} \frac{A_I(\nu')}{\nu' - \nu}\, d\nu' + \frac{1}{\pi} \int_{-\infty}^{0} \frac{A_{II}(\nu')}{\nu' - \nu}\, d\nu' \end{aligned} \tag{74}$$

where

$$\begin{aligned} A_I &= \frac{1}{2} (2\pi)^4 \sum_{n \neq N} \langle P | \bar{D}^{(+)} | n \rangle \langle n | \bar{D}^{(-)} | P \rangle\, \delta(p + k - p_n) \\ A_{II} &= \frac{1}{2} (2\pi)^4 \sum_{n \neq N} \langle P | \bar{D}^{(-)} | n \rangle \langle n | \bar{D}^{(+)} | P \rangle\, \delta(p - k - p_n) \end{aligned} \tag{75}$$

The continuum distribution can be reasonably estimated by using the approximate relation

$$\langle P | \bar{D}^{(+)} | n \rangle = \frac{\alpha_n}{\omega^2_\pi}\, T_{\pi P, n} \tag{76}$$

where

$$\alpha_\pi = \langle 0 | \bar{D}^{(+)} | \pi \rangle \tag{77}$$

and $T_{\pi^- p, n}$ is the scattering amplitude $\pi^- p \to n$. Equation (76) is the generalization of the well-known Goldberger-Treiman relation [12] [obtained by taking for $|n\rangle$ the one-neutron state *)]

$$\begin{aligned} \langle P | \bar{D}^{(+)} | N \rangle &= 2m\, i\, \bar{u}_P \gamma_5 u_N r_A \\ &= i\, \bar{u}_P \gamma_5 u_N\, g_{\pi N} \frac{\alpha_\pi}{m^2_\pi} \end{aligned} \tag{78}$$

or

$$\alpha_\pi = \frac{r_A\, 2m\, m^2_\pi}{g_{\pi N}} \tag{79}$$

Equation (76) can be obtained in two ways. The most intuitive derivation makes use of the Gell-Mann - Lévy [11] proportionality relation

$$\bar{D}^{(+)} = C\, \Phi_\pi^{(+)} \tag{80}$$

and assumes that the continuation from mass zero virtual pion to real pion is harmless.

A less restrictive proof, analogous to the dispersion analysis of the Goldberger-Treiman relation, can be obtained by continuing $\langle P | \bar{D}^+ | N \rangle$ in the square momentum $k^2 = (p-p_n)^2$ associated with $\bar{D}$ and by retaining only the lowest contribution given by the pion pole :

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*) The standard form of the Goldberger-Treiman formula is obtained by multiplying both sides of Eqs. (78) and (79) by g_A^o and recalling that $g_A = g_A^o\, r_A$ and a_π (related to the π life-time) is equal to $g_A^o\, \alpha_\pi$.

$$\langle P | \bar{D}^{(+)} | N \rangle = \frac{\alpha_\pi \, \Gamma_{\pi P, m}}{\omega^2_\pi - k^2} \tag{81}$$

Inserting now Eq. (76) into (74) and (75), and using Eq. (71), we obtain

$$1 = R^2_A + \frac{\alpha^2_\pi}{2\omega^4_\pi} \, \frac{1}{\pi} \int_{\omega_\pi + \frac{\omega^2_\pi}{2m}}^{\infty} \frac{d\nu'}{\nu'} \left\{ \sigma^{tot}_{\pi^- p}(\nu') - \sigma^{tot}_{\pi^+ p}(\nu') \right\} \tag{82}$$

and finally, using Eq. (79), we arrive at the final form of the Adler-Weisberger relation

$$R^2_A = \left\{ 1 + \frac{2m^2}{g^2_{\pi N}} \, \frac{1}{\pi} \int \frac{d\nu'}{\nu'} \left[\sigma^{tot}_{\pi^- p}(\nu') - \sigma^{tot}_{\pi^+ p}(\nu') \right] \right\}^{-1} \tag{83}$$

The result that the higher corrections to axial vector renormalization can be expressed in terms of physical cross-sections, depends of course on the fact that $\bar{D}^{\mp}$ has the same quantum numbers as the pion. It is clear that analogous results could be obtained by exploiting other particles of the octet like K meson.

4.c) Let us now consider the commutator

$$\left[\bar{Q}_3 , j_\mu^{(V)} \right] = 0$$
$$\left[\bar{Q}_3 , j_\mu^{(S)} \right] = 0 \tag{84}$$

where $j_\mu^{(v)}$, $j_\mu^{(s)}$ are the isovector and isoscalar electromagnetic currents *). Taking the matrix elements between nucleon states, we get

$$\lim_{q \to 0} F^{(v,s)}(q) = 0 \tag{85}$$

where

$$F^{(v,s)}(q) = \varepsilon_\mu \int d^4x \langle N(p_2)| [\bar{D}_3(x), j_\mu^{(v,s)}(0)] |N(p_1)\rangle e^{iqx} \theta(-x_0) \tag{86}$$

The amplitude can be treated in close analogy with the Chew, Goldberger, Low, Nambu [13] theory of photoproduction with the following identification of their variables

$$P = \frac{p_1 + p_2}{2} \qquad \nu = \frac{P \cdot q}{m}$$
$$k = p_1 + q - p_2 \qquad \Delta^2 = (p_1 - p_2)^2 . \tag{87}$$

Equation (86) is valid for any choice of the "polarization vector" but we shall choose

$$\varepsilon \cdot k = 0 \tag{88}$$

We expand F in the form

$$F = \alpha M_A + \beta M_B + \gamma M_C + \delta M_D \tag{89}$$

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*) The sum rules we are going to obtain, as well as Eq. (83), can be derived by just using $SU(2) \times SU(2)$.

where $\quad (F_{\mu\nu} = k_\mu \varepsilon_\nu - k_\nu \varepsilon_\mu)$

$$M_A = -\frac{1}{2} \gamma_5 \gamma_\mu \gamma_\nu F_{\mu\nu}$$

$$M_B = -2i \gamma_5 P_\mu q_\nu F_{\mu\nu} \tag{90}$$

$$M_C = -\gamma_5 \gamma_\mu q_\nu F_{\mu\nu}$$

$$M_D = -2\gamma_5 \gamma_\mu P_\nu F_{\mu\nu} + i\omega \gamma_5 \gamma_\mu \gamma_\nu F_{\mu\nu}$$

and $\alpha, \beta, \gamma, \delta$ are invariant functions of ν, Δ^2, q^2.

In the limit $q_\mu \to 0$ only M_A survives, since :

$$M_D = \gamma q \gamma_\mu \gamma_\nu F_{\mu\nu} + M_C \tag{90'}$$

Equation (85) will thus involve $\alpha(\nu, \Delta^2, q^2)$ only :

$$\lim_{\substack{\Delta^2, q^2 \to 0 \\ \nu \to 0}} \alpha^{(\nu,s)}(\nu, \Delta^2, q^2) = 0 \tag{91}$$

In complete analogy to the Weisberger-Adler case, the invariant function α can be analyzed through a dispersion relation in which $\Delta^2 = q^2 = 0$ is taken.

In the limit $\nu \to 0$ we get the two sum rules *)

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*) The evaluation of the nucleon polar term requires a little care because we have to deal with a ratio of a vanishing numerator with a vanishing dispersion denominator. This can be easily handled by the device of introducing a fictitious mass difference between the nucleons appearing in the matrix element $\langle N_1 | \overline{D} | N_2 \rangle$. Putting this difference to zero leads to a completely unambiguous determination of the polar term. A physical realization of this procedure can be found by considering the commutators involving $\overline{Q}^{(+)}$ instead of $\overline{Q}_3$.

$$R_A \cdot \mu'^{(v,s)} + \frac{1}{\pi}\int_0^{\infty} \frac{d\nu'}{\nu'} \, \mathrm{Im}\, \alpha^{(v,s)}(\nu') = 0 \tag{92}$$

where $\mu'^{(v,s)}$ are the anomalous isovector and isoscalar magnetic moments of the nucleon

$$\mu'^{(v)} = \frac{\mu'_p - \mu_N}{2} \; ; \quad \mu'^{(s)} = \frac{\mu'_p + \mu_N}{2}$$

Im $\alpha^{(v,s)}$ can be related to the sum

$$\sum_n \langle N(p_1)|\bar{D}|n\rangle\langle n| j_\mu |N(p_2)\rangle \, \delta(p_1 + q - p_n) \tag{93}$$

As in the previous case, using Eq. (76), Im α can be related to the physical photoproduction amplitude A

$$\mathrm{Im}\, \alpha = \frac{\alpha_0}{\omega_\pi^2} \, \mathrm{Im}\, A = R_A \frac{2\omega}{g_{\pi N}} \, \mathrm{Im}\, A \tag{94}$$

We are finally led to the two sum rules

$$\frac{\mu'_p + \mu_N}{2} + \frac{2\omega}{g_{\pi N}} \frac{1}{\pi}\int \mathrm{Im}\, A^{(s)}(\nu') \frac{d\nu'}{\nu'} = 0 \tag{95}$$

$$\frac{\mu'_p - \mu_N}{2} + \frac{2\omega}{g_{\pi N}} \frac{1}{\pi}\int \mathrm{Im}\, A^{(v)}(\nu') \frac{d\nu'}{\nu'} = 0 \tag{96}$$

Let us now discuss the physical meaning of Eqs. (95) and (96). We note first of all that the "group theoretical result", in which the dispersion integrals are neglected, is that both anomalous magnetic moments of the nucleon vanish. This has already been pointed out by

Gell-Mann, and follows from the form (61) of the electromagnetic currents, i.e., from the absence of an "elementary" Pauli electromagnetic coupling. If we want, however, to consider the dispersion corrections, we see that Eqs. (95) and (96) are on an entirely different footing, since the strong contribution of the 33 isobar is present in Eq. (96) only. A preliminary estimate of this term, based on the Gourdin-Salin [14] model, shows an almost complete cancellation between the N and N^*_{33} contributions, leaving a very small difference to be explained in terms of higher resonances.

We obtain, in this way, a simple explanation of the fact that the anomalous magnetic moment of the nucleon is essentially isovector. Equations (95) and (96) can also be viewed from a different standpoint.

Equation (84), from which our sum rules have been deduced, can be considered as coming from the more general $(SU_6)\times(SU_6)$ group where both N and N_{33} are members of the same multiplet. In this more general framework, the group theoretical result is obtained by neglecting everything but N and N_{33} terms, so that Eq. (95) leads to

$$\frac{\mu'_p+\mu_N}{2}=0 \tag{97}$$

whereas Eq. (96) gives a relation between $\mu'_p-\mu_N$ and the transition N,N_{33} magnetic moment.

If we supplement Eq. (97) with the well-known $SU(6)$ relation [15]

$$2\mu_p+3\mu_N=0 \tag{98}$$

where

$$\mu_p=\mu'_p+1 \tag{99}$$

is relating the <u>total</u> magnetic moments of the nucleons, we finally get

$$\begin{aligned} \mu_P &= 3 \\ \mu_N &= -2 \end{aligned} \tag{100}$$

The gyromagnetic ratios of P and N are given by pure Clebsch-Gordan coefficients !

Of course, in order to accept Eq. (100) with full confidence, one needs a completely covariant and consistent treatment of all commutation rules appearing in $SU(6) \times SU(6)$ symmetry. Work in this direction is in progress *).

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*) S. Fubini and G. Segrè. It is to be noted that, as pointed out by Dashen and Gell-Mann [16], the new commutation rules do not only involve operators like $\int j_\mu \, d^3x$ and $\int \bar{j}_\mu \, d^3x$, but also $\int \bar{\psi} \gamma_0 \gamma_\mu \psi \, d^3x$ and $\int \bar{\psi} \gamma_0 \gamma_5 \gamma_\mu \psi \, d^3x$.

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