A distinguisher for high-rate McEliece Cryptosystems

J.C. Faugère (INRIA, SALSA project),
Valérie Gauthier (Math. dep. Tech. Univ. of Denmark),
A. Otmani (Université Caen- INRIA, SECRET project),
L. Perret (INRIA, SALSA project),
J.-P. Tillich (INRIA, SECRET project)

May 12th, 2011

1. (Generalized) McEliece Cryptosystem McE$(\mathcal{K}_{n,k,t})$

C a $q-$ary, length n, dimension k, t-error correcting code

- Public key: $\boldsymbol{G}$ a $k \times n$ generator matrix of C in $\mathcal{K}(n,k,t)$
- Secret key: Ψ a t-error correcting procedure for C
- Encryption: $\boldsymbol{x} \to \boldsymbol{x}\boldsymbol{G} + \boldsymbol{e}$ with $\boldsymbol{e}$ of Hamming weight t
- Decryption: $\boldsymbol{y} \to \Psi(\boldsymbol{y})\boldsymbol{G}^{-1}$ with $\boldsymbol{G}^{-1}$ a right inverse of $\boldsymbol{G}$.

Alternant codes/Goppa codes

- $\boldsymbol{x} = (x_1, \dots, x_n) \in \mathbb{F}_{q^m}^n$ with $x_i \neq x_j$ if $i \neq j$
- $\boldsymbol{y} = (y_1, \dots, y_n) \in \mathbb{F}_{q^m}^n$ with $y_i \neq 0$

For any $r < n$, let $\boldsymbol{H}_r(\boldsymbol{x}, \boldsymbol{y}) \stackrel{\text{def}}{=} \begin{pmatrix} y_1 & y_2 & \cdots & y_n \\ y_1 x_1 & y_2 x_2 & \cdots & y_n x_n \\ \vdots & \vdots & & \vdots \\ y_1 x_1^{r-1} & y_2 x_2^{r-1} & \cdots & y_n x_n^{r-1} \end{pmatrix}$

Definition 1. *An alternant code is the kernel of an $\boldsymbol{H}$ of this type*

$$\mathcal{A}_r(\boldsymbol{x}, \boldsymbol{y}) = \left\{ \boldsymbol{v} \in \mathbb{F}_q^n | \boldsymbol{H}_r(\boldsymbol{x}, \boldsymbol{y}) \boldsymbol{v}^T = \boldsymbol{0}. \right\}.$$

Goppa code : $\exists \Gamma$, polynomial of degree r such that $y_i = \Gamma(x_i)^{-1}$.

Decoding Alternant and Goppa codes

Proposition 1. [decoding alternant codes] *$r/2$ errors can be decoded in polynomial time as long as $\boldsymbol{x}$ and $\boldsymbol{y}$ are known.*

Proposition 2. [The special case of binary Goppa codes] *In the case of a binary Goppa code ($q = 2$), r errors can be decoded in polynomial time, if $\boldsymbol{x}$ and Γ are known and if Γ has only simple roots.*

More generally a factor $\frac{q}{q-1}$ can be gained (exploited for instance in wild McEliece **[Bernstein-Lange-Peters 2010]**) by a suitable choice of Γ.

(public key) 2. Distinguisher problem

$\mathcal{K}^{\text{Goppa}}(n,k,t)$ the ensemble of generator matrices of t-error correcting Goppa codes of length n, dimension k
$\mathcal{K}^{\text{alt}}(n,k)$ the ensemble of generator matrices of alternant codes of length n, dimension k
$\mathcal{K}^{\text{lin}}(n,k)$ the ensemble of generator matrices of linear codes of length n and dimension k.

Can we distinguish between the cases

(i) $\boldsymbol{G} \in \mathcal{K}^{\text{Goppa}}(n,k,t)$

(ii) $\boldsymbol{G} \in \mathcal{K}^{\text{alt}}(n,k)$

(iii) $\boldsymbol{G} \in \mathcal{K}^{\text{lin}}(n,k)$?

Niederreiter Nied$(\mathcal{K}_{n,k,t})$

C a $q-$ary, length n, dimension k, $t-$error correcting code.

- Public key: $\boldsymbol{H}$ a $(n-k) \times n$ parity check matrix of C, $\boldsymbol{H} \in \mathcal{K}_{n,k,t}$
- Secret key: Ψ a t-error correcting procedure for C
- Encryption: $\boldsymbol{e} \rightarrow \boldsymbol{e}\boldsymbol{H}^T$ with $\boldsymbol{e}$ of Hamming weight t
- Decryption: To decipher $\boldsymbol{s}$, choose any $\boldsymbol{y}$ of syndrome $\boldsymbol{s}$, i.e. such that $\boldsymbol{s} = \boldsymbol{y}\boldsymbol{H}^T$, and output $\boldsymbol{y} - \Psi(\boldsymbol{y})$.

A probabilistic model of an attacker

A (T, ϵ) adversary $\mathcal{A}$ for **Nied**$(\mathcal{K}_{n,k,t})$ is a program which runs in time at most T and is such that

$$\mathbf{Prob}_{\boldsymbol{H},\boldsymbol{e}}(\mathcal{A}(\boldsymbol{H}, \boldsymbol{e}\boldsymbol{H}^T) = \boldsymbol{e} | \boldsymbol{H} \in \mathcal{K}_{n,k,t}) \geq \epsilon$$

Most attacks actually deal with an adversary for **Nied**$(\mathcal{K}^{\mathrm{lin}}(n,k))$ instead of **Nied**$(\mathcal{K}^{\mathrm{Goppa}}(n,k,t))$.

How the distinguisher appears

$$\textbf{Adv} \stackrel{\text{def}}{=} \mathbf{Prob}(\mathcal{A}(\boldsymbol{H}, \boldsymbol{e}\boldsymbol{H}^T) = \boldsymbol{e} | \boldsymbol{H} \in \mathcal{K}^{\text{Goppa}}_{n,k,t}) - \mathbf{Prob}(\mathcal{A}(\boldsymbol{H}, \boldsymbol{e}\boldsymbol{H}^T) = \boldsymbol{e} | \boldsymbol{H} \in \mathcal{K}^{\text{lin}}_{n,k})$$

Distinguisher D:
input $\boldsymbol{H} \in \mathbb{F}_q^{(n-k)\times n}$
Step 1 : pick a random $\boldsymbol{e} \in \mathbb{F}_q^n$ of weight t
Step 2: if $\mathcal{A}(\boldsymbol{H}, \boldsymbol{e}\boldsymbol{H}^T) = \boldsymbol{e}$ then return 1, else return 0.

Advantage of $D = |\textbf{Adv}|$.

Either a decoding algorithm on linear codes or a distinguisher for Goppa codes

Proposition 3. *If* $\exists (T,\epsilon)$*-adversary against* **Nied**$(\mathcal{K}^{\textit{Goppa}}_{n,k,t})$ *, then there exists either*

(i) a $(T,\epsilon/2)$*-adversary against* **Nied**$(\mathcal{K}^{\textit{lin}}(n,k)$ *(i.e. a decoder for general linear codes working in time* T *with success probability at* $\geq \epsilon/2$*).*

(ii) A distinguisher between $\boldsymbol{H} \in \mathcal{K}^{\textit{Goppa}}_{n,k,t}$ *and* $\boldsymbol{H} \in \mathcal{K}^{\textit{lin}}_{n,k}$ *working in time* $T + O(n^2)$ *and with advantage at least* $\epsilon/2$*.*

3. Algebraic approach for attacking the McEliece cryptosystem

What is known: a basis of the code $\rightarrow$ rows of a generator matrix $\boldsymbol{G} = (g_{ij})$ of size $k \times n$.

What we also know: $\exists \boldsymbol{x}, \boldsymbol{y} \in \mathbb{F}_{q^m}^n$ s.t.

$$\boldsymbol{H}_r(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{G}^T = \boldsymbol{0}. \tag{1}$$

What we want to find: find in the case of an alternant code $\boldsymbol{x}, \boldsymbol{y}$, and in the special case of a binary Goppa code $\boldsymbol{x}$ and Γ.

The algebraic system

$\boldsymbol{H}_r(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{G}^T = \boldsymbol{0}$ translates to

$$\left\{\begin{array}{lcl} g_{1,1}Y_1 + \cdots + g_{1,n}Y_n & = & 0 \\ \vdots & & \vdots \\ g_{k,1}Y_1 + \cdots + g_{k,n}Y_n & = & 0 \\ g_{1,1}Y_1X_1 + \cdots + g_{1,n}Y_nX_n & = & 0 \\ \vdots & & \vdots \\ g_{k,1}Y_1X_1 + \cdots + g_{k,n}Y_nX_n & = & 0 \\ \vdots & & \vdots \\ g_{1,1}Y_1X_1^{r-1} + \cdots + g_{1,n}Y_nX_n^{r-1} & = & 0 \\ \vdots & & \vdots \\ g_{k,1}Y_1X_1^{r-1} + \cdots + g_{k,n}Y_nX_n^{r-1} & = & 0 \end{array}\right. \quad (2)$$

where the $g_{i,j}$'s are known coefficients in $\mathbb{F}_q$ and $k \geq n - r\,m$.

Freedom of choice in (2)

Proposition 4. *Theoretically, the system has $2n$ unknowns but we can take arbitrary values for one Y_i and for three X_i's (as long as these values are different).*

Applications

When the number of unknowns is small, ex:

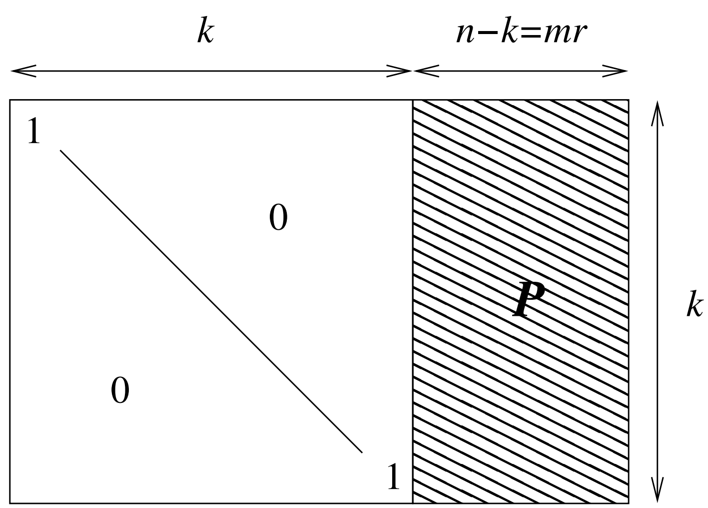
- Berger-Cayrel-Gaborit-Otmani proposal at AfricaCrypt'09 based on quasi-cyclic alternant codes
- Misoczki-Barreto at SAC'09 variant based on quasi-dyadic Goppa codes

⇒ algebraic system can be solved by (dedicated) Grobner basis techniques.

- breaks all parameters proposed in these articles ([Faugère-Otmani-Perret-Tillich;Eurocrypt 2010] with the exception of binary dyadic codes. Related to [Leander-Gauthier Umana; SCC2010]

4. A naive attack

W.l.o.g. we can assume that G is systematic in its k first positions.

$G =$ 

Step 1 – expressing the $Y_iX_i^d$'s in terms of the $Y_jX_j^d$'s for $j \in \{k+1, \ldots, n\}$.

$\boldsymbol{P} = (p_{ij})_{\substack{1 \leq i \leq k \\ k+1 \leq j \leq n}}$. We can rewrite (2) as

$$\left\{ \begin{array}{lcr} Y_i & = & \sum_{j=k+1}^{n} p_{i,j} Y_j \\ Y_i X_i & = & \sum_{j=k+1}^{n} p_{i,j} Y_j X_j \\ & \cdots & \\ Y_i X_i^{r-1} & = & \sum_{j=k+1}^{n} p_{i,j} Y_j X_j^{r-1} \end{array} \right. \tag{3}$$

for all $i \in \{1, \ldots, k\}$.

Step 2.– Exploiting $Y_i(Y_iX_i^2) = (Y_iX_i)^2$

$$\begin{cases} Y_i & = & \sum_{j=k+1}^{n} p_{i,j} Y_j \\ Y_iX_i & = & \sum_{j=k+1}^{n} p_{i,j} Y_j X_j \\ Y_iX_i^2 & = & \sum_{j=k+1}^{n} p_{i,j} Y_j X_j^2 \end{cases} \tag{4}$$

$$\Rightarrow \quad \left(\sum_{j=k+1}^{n} p_{i,j} Y_j\right)\left(\sum_{j=k+1}^{n} p_{i,j} Y_j X_j^2\right) = \left(\sum_{j=k+1}^{n} p_{i,j} Y_j X_j\right)^2$$

$$\Rightarrow \quad \sum_{j=k+1}^{n} \sum_{j'>j} p_{i,j} p_{i,j'} \left(Y_j Y_{j'} X_{j'}^2 + Y_{j'} Y_j X_j^2\right) = 0$$

Step 3. – Linearization

$$Z_{jj'} \stackrel{\text{def}}{=} Y_j Y_{j'} X_{j'}^2 + Y_{j'} Y_j X_j^2$$

$$\sum_{j=k+1}^{n} \sum_{j'>j} p_{i,j} p_{i,j'} Z_{jj'} = 0.$$

- $\binom{n-k}{2} \approx \frac{m^2r^2}{2}$ unknowns
- $k = n - mr$ equations

$\Rightarrow$ reveals $Z_{jj'}$ when $n - mr \geq \frac{m^2r^2}{2}$?

- This happens for the Courtois-Finiasz-Sendrier scheme, ex: $n = 2^{21}$, $r = 10$, $m = 21$ which has to choose small values of r.

Linearized System

Definition 2. *Assume that the public key $\boldsymbol{G}$ of the McEliece cryptosystem is in systematic form* $(\boldsymbol{I}_k \mid \boldsymbol{P})$

The linearized system associated to $\boldsymbol{G}$ is

$$\left\{\begin{array}{rcl} \displaystyle\sum_{j=k+1}^{n} \sum_{j'>j} p_{1,j} p_{1,j'} Z_{jj'} & = & 0 \\ \displaystyle\sum_{j=k+1}^{n} \sum_{j'>j} p_{2,j} p_{2,j'} Z_{jj'} & = & 0 \\ & \vdots & \\ \displaystyle\sum_{j=k+1}^{n} \sum_{j'>j} p_{k,j} p_{k,j'} Z_{jj'} & = & 0 \end{array}\right.$$

The dimension of the solution space is denoted by D.

Algebraic Distinguisher

Solving this system requires that

- Number of equations k is greater than the number of unknowns $\binom{n-k}{2}$
- rank is (almost) equal to the number of unknowns

If $\boldsymbol{G}$ is random then one would expect that the rank is $\min\left\{k, \binom{n-k}{2}\right\}$

$$\Longrightarrow D = \max\left\{0, \binom{n-k}{2} - k\right\}$$

But for several structured (Goppa, alternant) codes rank $< \min\left\{k, \binom{n-k}{2}\right\}$ and this defect can be quantified

Example $q = 2$ and $m = 14$

r	3	4	5	6	7	8	9	10	11	12	13	14
$\binom{n-k}{2}$	861	1540	2415	3486	4753	6216	7875	9730	11781	14028	16471	19110
k	16342	16328	16314	16300	16286	16272	16258	16244	16230	16216	16202	16188
D_{rand}	0	0	0	0	0	0	0	0	0	0	269	2922
$D_{\text{alternant}}$	42	126	308	560	882	1274	1848	2520	3290	4158	5124	6188
D_{Goppa}	252	532	980	1554	2254	3080	4158	5390	6776	8316	10010	11858

Example $q = 2$ and $m = 14$

r	15	16	17	18	19	20	21	22	23	24	25	26	27
$\binom{n-k}{2}$	21945	24976	28203	31626	35245	39060	43071	47278	51681	56280	61075	66066	71253
k	16174	16160	16146	16132	16118	16104	16090	16076	16062	16048	16034	16020	16006
D_{rand}	5771	8816	12057	15494	19127	22956	26981	31202	35619	40232	45041	50046	55247
$D_{\text{alternant}}$	7350	8816	12057	15494	19127	22956	26981	31202	35619	40232	45041	50046	55247
D_{Goppa}	13860	16016	18564	21294	24206	27300	30576	34034	37674	41496	45500	50046	55247

Alternant Case

Let $\ell \stackrel{\text{def}}{=} \lfloor \log_q(r-1) \rfloor$.

$$D_{\text{alternant}} = \frac{1}{2} m(r-1) \left((2\ell+1)r - 2\frac{q^{\ell+1}-1}{q-1} \right)$$

as long as $\binom{n-k}{2} - D_{\text{alternant}} < k$.

Goppa Case

Let ℓ the unique integer such that $q^{\ell} - 2q^{\ell-1} + q^{\ell-2} < r \leqslant q^{\ell+1} - 2q^{\ell} + q^{\ell-1}$

$$D_{\mathsf{Goppa}} = \begin{cases} \frac{1}{2}m(r-1)(r-2) = D_{\mathsf{alternant}} & \text{for} \quad r < q-1 \\ \frac{1}{2}mr\Big((2\ell+1)r - 2q^{\ell} + 2q^{\ell-1} - 1\Big) & \text{for} \quad r \geqslant q-1 \end{cases}$$

as long as $\binom{n-k}{2} - D_{\mathsf{Goppa}} < k$.

Example $q = 2$ and $m = 14$

r	3	4	5	6	7	8	9	10	11	12	13	14
$\binom{n-k}{2}$	861	1540	2415	3486	4753	6216	7875	9730	11781	14028	16471	19110
k	16342	16328	16314	16300	16286	16272	16258	16244	16230	16216	16202	16188
D_{rand}	0	0	0	0	0	0	0	0	0	0	269	2922
$D_{\text{alternant}}$	42	126	308	560	882	1274	1848	2520	3290	4158	5124	6188
$T_{\text{alternant}}$	42	126	308	560	882	1274	1848	2520	3290	4158	5124	6188
D_{Goppa}	252	532	980	1554	2254	3080	4158	5390	6776	8316	10010	11858
T_{Goppa}	252	532	980	1554	2254	3080	4158	5390	6776	8316	10010	11858

Example $q = 2$ and $m = 14$

r	15	16	17	18	19	20	21	22	23	24	25	26	27
$\binom{n-k}{2}$	21945	24976	28203	31626	35245	39060	43071	47278	51681	56280	61075	66066	71253
k	16174	16160	16146	16132	16118	16104	16090	16076	16062	16048	16034	16020	16006
D_{rand}	5771	8816	12057	15494	19127	22956	26981	31202	35619	40232	45041	50046	55247
$D_{\text{alternant}}$	7350	8816	12057	15494	19127	22956	26981	31202	35619	40232	45041	50046	55247
$T_{\text{alternant}}$	7350	8610	10192	11900	13734	15694	17780	19992	22330	24794	27384	30100	32942
D_{Goppa}	13860	16016	18564	21294	24206	27300	30576	34034	37674	41496	45500	50046	55247
T_{Goppa}	13860	16016	18564	21294	24206	27300	30576	34034	37674	41496	45500	49686	54054

Simplified Formulas for binary Goppa Codes

- Let $\ell \stackrel{\mathrm{def}}{=} \lceil \log_2 r \rceil + 1$.

$$D_{\mathsf{Goppa}} = \frac{1}{2} mr \left((2\ell + 1)r - 2^{\ell} - 1 \right)$$

as long as $\binom{mr}{2} - D_{\mathsf{Goppa}} < n - mr$.

Binary Goppa Codes

In particular, assuming that $n = 2^m$, the binary Goppa code distinguishing problem is solved for any $r < r_{\max}$

m	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
$r_{\max}$	5	8	8	11	16	20	26	34	47	62	85	114	157	213	290	400

- $m = 13$ and $r = 19$ corresponds to a 90-bit security McEliece public key.
- All CFS parameters fits in the range of validity of the algebraic distinguisher.

5. Explanation

- Formulas obtained through experimentations for random codes, alternant codes and irreducible Goppa codes over fields of size $q \in \{2, 4, 8, 16\}$.
- We have an explanation for alternant codes and binary Goppa codes by guessing a basis of the solution vector space over $\mathbb{F}_q$.
- It does not provide a proof.

Explanation for Alternant Codes – Step I

- Note that the entries of the system are in $\mathbb{F}_q$ and solutions are sought in $\mathbb{F}_{q^m}$.
- Let us view $\mathbb{F}_{q^m}$ as a $\mathbb{F}_q$-vector space of dimension m, and let $\pi_i : \mathbb{F}_{q^m} \to \mathbb{F}_q$ be the function giving the i-th coordinate.
- Hence, if a vector $\boldsymbol{v}$ with $v_j \in \mathbb{F}_{q^m}$ is a solution then $\pi_i(\boldsymbol{v}) = \Big(\pi_i(v_j)\Big)_j$ whose entries are in $\mathbb{F}_q$ is also a solution.

$\implies$ Any solution with entries over $\mathbb{F}_{q^m}$ would potentially provide a basis of m solutions with entries over $\mathbb{F}_q$

Explanation for Alternant Codes – Step II

- We have used $Y_iY_iX_i^2 = (Y_iX_i)^2$ which leads to:

$$\forall i \in \{1,\ldots,k\}, \qquad \sum_{j=k+1}^{n} \sum_{j'>j} p_{i,j}p_{i,j'}Y_jY_{j'}\left(X_j^2 + X_{j'}^2\right) = 0$$

- But we can use any relation $Y_iX_i^aY_iX_i^b = Y_iX_i^cY_iX_i^d$ with a, b, c, d in $\{0,\ldots,r-1\}$ such that $a+b=c+d$

$$\sum_{j=k+1}^{n} \sum_{j'>j} p_{i,j}p_{i,j'}Y_jY_{j'}(X_j^aX_{j'}^b + X_j^bX_{j'}^a + X_j^cX_{j'}^d + X_j^dX_{j'}^c) = 0$$

Explanation for Alternant Codes – Step III

- For $r \geqslant q$, the automorphism $x \longmapsto x^{q^\ell}$ for any $0 \leqslant \ell \leqslant m-1$ can be used.

$$\forall e \in \{0, \ldots, r-1\}, \quad Y_i X_i^e = \sum_{j=k+1}^{n} p_{ij} Y_j X_j^e \implies Y_i^q X_i^{eq} = \sum_{j=k+1}^{n} p_{ij} Y_j^q X_j^{eq}$$

- We therefore can use the same trick, for instance $Y_i (Y_i X_i)^q = Y_i^q Y_i X_i^q$,

$$\sum_{j=k+1}^{n} \sum_{j'>j} p_{i,j} p_{i,j'} \left(Y_j Y_{j'}^q X_{j'}^q + Y_{j'} Y_j^q X_j^q + Y_j^q Y_{j'} X_{j'}^q + Y_{j'}^q Y_j X_j^q \right) = 0.$$

Explanation for Alternant Codes

- However the equations obtained $\left(Y_i X_i^a Y_i X_i^b\right)^q = \left(Y_i X_i^c Y_i X_i^d\right)^q$ do not provide new solutions after decomposition over $\mathbb{F}_q$ since they are linearly dependent of those obtained from $Y_i X_i^a Y_i X_i^b = Y_i X_i^c Y_i X_i^d$.
- Hence, we only consider equations obtained from integers a, b, c, d, ℓ such that $a + bq^\ell = c + dq^\ell$

$$Y_i X_i^a \left(Y_i X_i^b\right)^{q^\ell} = Y_i X_i^c \left(Y_i X_i^d\right)^{q^\ell}$$

$$\boldsymbol{Z}_{a,b,c,d,\ell} \stackrel{\text{def}}{=} \left(Y_j X_j^a Y_{j'}^{q^\ell} X_{j'}^{bq^\ell} + Y_{j'} X_{j'}^a Y_j^{q^\ell} X_j^{bq^\ell} + Y_j X_j^c Y_{j'}^{q^\ell} X_{j'}^{dq^\ell} + Y_{j'} X_{j'}^c Y_j^{q^\ell} X_j^{dq^\ell}\right)_{1 \leqslant j < j' \leqslant n-k}$$

Explanation for Alternant Codes

- Let us assume that $d > b$ and set $\delta \stackrel{\text{def}}{=} d - b$ and then $a = c + q^\ell \delta$

$$\Longrightarrow \quad \boldsymbol{Z}_{a,b,c,d,\ell} = \boldsymbol{Z}_{c+q^\ell\delta,b,c,b+\delta,\ell}$$

- Let $\mathcal{B}_r$ be the set $\boldsymbol{Z}_{c+q^\ell\delta,b,c,b+\delta,\ell}$ obtained with $\delta = 1$ and satisfying:

$$\begin{cases} 0 \leqslant b \leqslant r-2 \text{ and } 0 \leqslant c \leqslant r-1-q^\ell & \text{if} \quad 1 \leqslant \ell \leqslant \lfloor \log_q(r-1) \rfloor \\ 0 \leqslant b < c \leqslant r-2 & \text{if} \quad \ell = 0. \end{cases}$$

Proposition 5.

- *Any $\boldsymbol{Z}_{c+q^\ell\delta,b,c,b+\delta,\ell}$ belongs to the $\mathbb{F}_{q^m}$-vector space generated by $\mathcal{B}_r$*
- *The cardinality of $\mathcal{B}_r$ with $r \geqslant 3$ is equal to D/m.*

Heuristic

For random choices of x_i's and y_i's defining the alternant code, the set $\left\{\pi_i(\boldsymbol{Z}) \mid \boldsymbol{Z} \in \mathcal{B}_r \text{ and } 1 \leq i \leq m\right\}$ forms a basis of the vector space that is solution to the linearized system.

Conclusion

- Large dimension comes from the many different ways of combining the equations together yielding the same linearized system
- What happens for random generator is proven now.
- Binary Goppa codes can also be explained but no explanation for non-binary Goppa codes.
- The most difficult task is identifying a basis of the vector space of solutions.
- A slightly better distinguisher can be obtained by taking the subcode of codewords of even weights.
- Distinguisher $\Rightarrow$ attack ?
- Approach requires $\frac{k}{n}$ very close to 1. Should very high rates be avoided in a McEliece like scheme ?