# A dual geometry of the hadron in dense matter

JHEP 07 (2009)087 by

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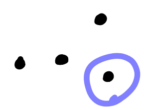
Theory of Hadronic Matter Under Extreme Conditions
5. June. 2012

# Holography as a tool for a quantum gravity

- $F \sim G\frac{mM}{r^2}$

G is small so each particle can be interpreted as a free particle $\longrightarrow$ Thermodynamics

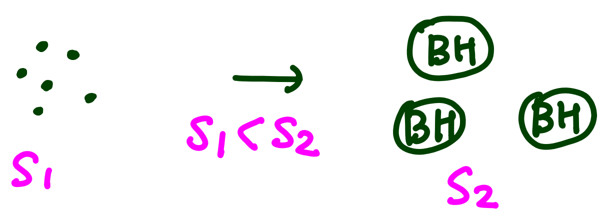


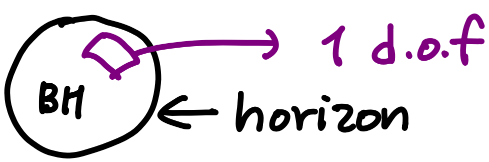
free particle

$N \to \infty$, $V \to \infty$, $\frac{N}{V} = \text{fixed}$.

Though G is small, they attract each other $\to$ Jean's instability

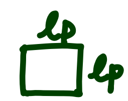
(Blackhole formulation)




In general, entropy ~ # of d.o.f ~ volume / $l_p^3$

In gravity, entropy ~ area / $l_p^2$

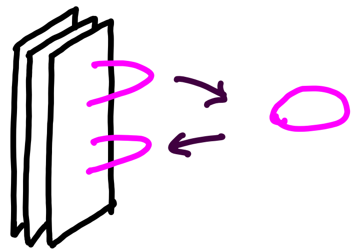


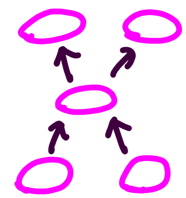
Holographic principle ['t Hooft 93, Susskind 94]

: The effective degrees of freedom in gravity are those at the boundary of the system.

# Gauge / Gravity duality

[Maldacena 97]





In the limit where the interaction between open / closed strings can be ignored

IIB closed string theory with N D3's $\xleftrightarrow[\text{closed / open string duality}]{}$ Open string theory on D3-branes

$\downarrow$ low energy, near horizon limit

Gravity theory on $\underbrace{AdS_5}_{SO(4,2)} \times \underbrace{S^5}_{SO(6)}$ $\longleftrightarrow$ $\mathcal{N}=4$ $SU(N)$ Super Yang-Mills

$\underbrace{SO(4,2)}_{\text{conformal } \mathbb{R}^{1,3}} \times \underbrace{SO(6)}_{\text{R-sym}}$

$\downarrow$ large N

Classical gravity $(g_s, R)$ $\xleftrightarrow[\text{AdS / CFT}]{}$ Strong coupling $(g_{YM}^2, N)$

$$\boxed{4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N}$$

gauge / gravity : gravity $\longleftrightarrow$ non-conformal, less SUSY

# AdS / CFT dictionary

[Witten 98][Gubser, Klebanov, Polyakov 98]

$\phi$ (p-form field in 5D) $\longleftrightarrow$ $\mathcal{O}$ in 4D

$m_5^2$ : mass squared. $\Delta$ : conformal dimension

$$\boxed{(\Delta - p)(\Delta + p + 4) = m_5^2}$$

| 4D : $\mathcal{O}(x)$ | 5D : $\phi(x, z)$ | $p$ | $\Delta$ | $m_5^2$ |
|---|---|---|---|---|
| $\bar{q}_L \gamma^\mu T^a q_L$ | $A^a_{L\mu}$ | 1 | 3 | 0 |
| $\bar{q}_R \gamma^\mu T^a q_R$ | $A^a_{R\mu}$ | 1 | 3 | 0 |
| $\bar{q}_R^\alpha q_L^\beta$ | $\frac{2}{z} X^{\alpha\beta}$ | 0 | 3 | $-3$ |
| $\left.\begin{matrix}\bar{q}_L \gamma^\mu q_L \\ \bar{q}_R \gamma^\mu q_R\end{matrix}\right\}$ baryon density | $A_\mu$ | 1 | 3 | 0 |

# Two approaches of AdS/QCD

1) Top down approach

: brane configuration for the gravity dual

ex) $N_c$ D3 and $N_f$ D7

[Kruczenski, Mateos, Myers, Winters 2004]

$N_c$ D4 and $N_f$ D8$-\overline{\text{D8}}$

[Sakai, Sugimoto 2004]

2) Bottom up approach

: field theory on the (asymptotic) $AdS_5$

ex) hard wall model

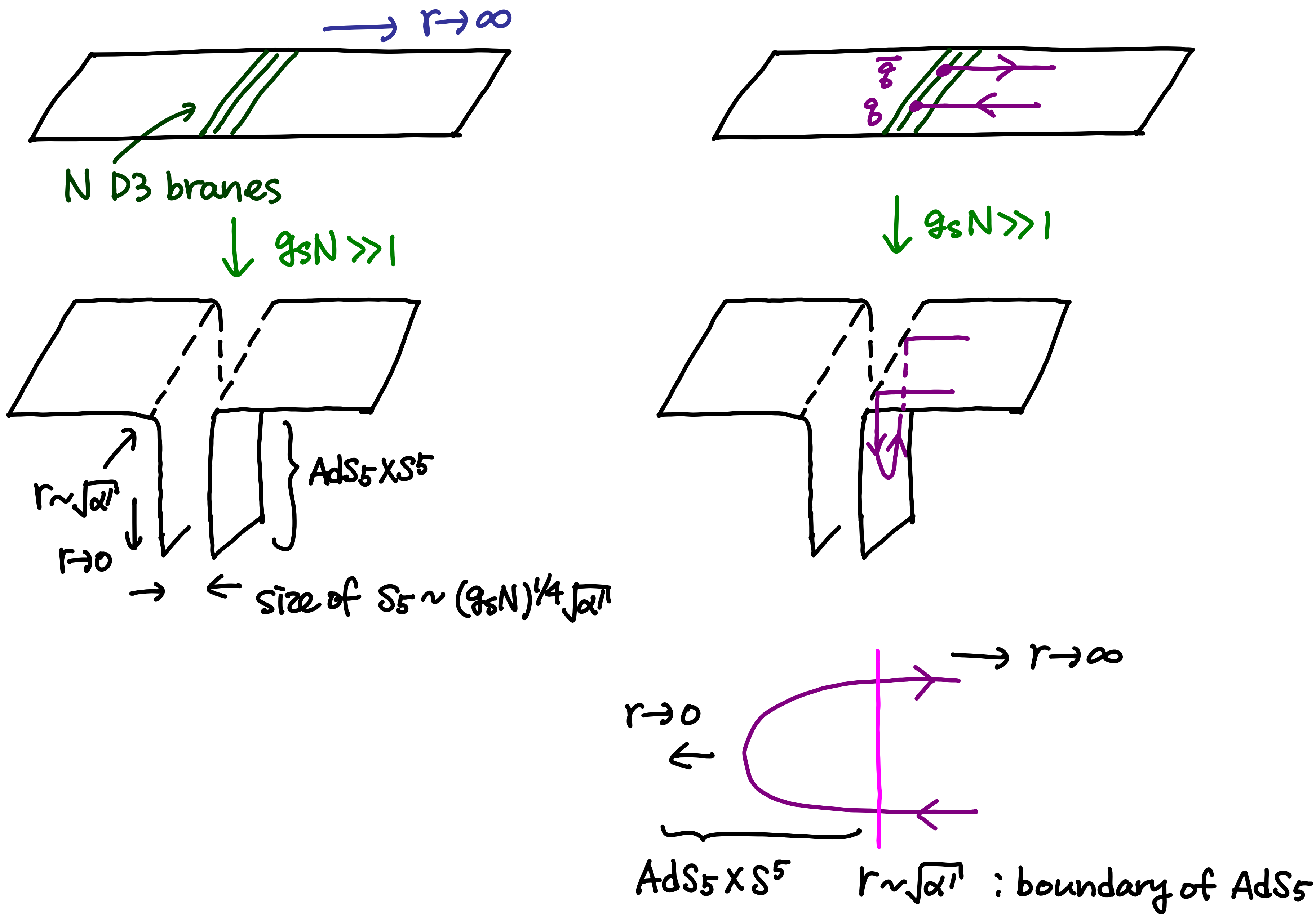
[Erlich, Katz, Son, Stephanov 2005]

[Da Rold, Pomarol 2005]

soft wall model

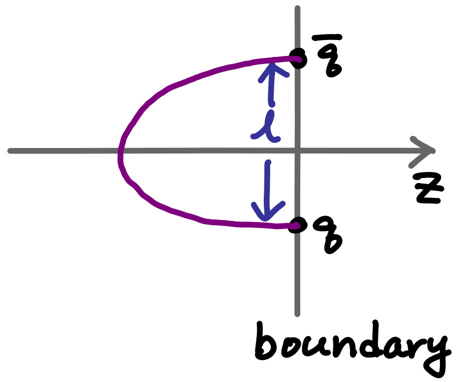
[Karch, Katz, Son, Stephanov 2006]

# Geometry

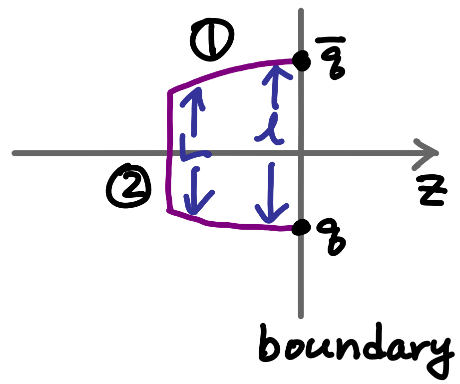


r→∞
N D3 branes
g_s N >> 1
AdS5 × S5
r ~ √α'
r→0
size of S5 ~ (g_s N)^{1/4} √α'
q̄
q
g_s N >> 1
r→∞
r→0
AdS5 × S5
r ~ √α' : boundary of AdS5

# What we want...

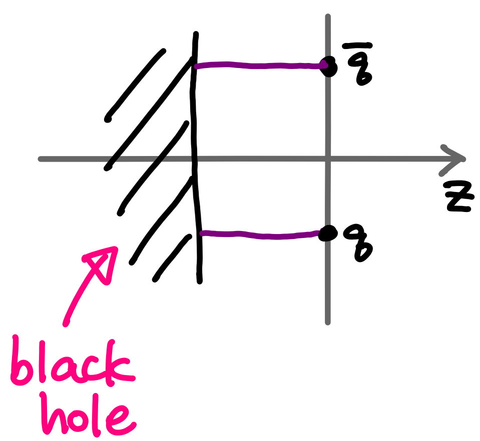



$E \sim \frac{1}{\ell}$ : Coulomb potential




① $E \sim \frac{1}{\ell}$ : Coulomb potential

② $E \sim T_s L$ : confining potential




Black hole

$\leftrightarrow$ thermal gauge theory

# $\boxed{\text{Hawking temperature}}$

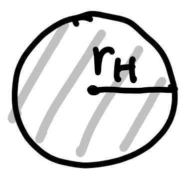
$S = \frac{A}{4\pi}$ ($S$ : entropy  $A$ : surface)

$T \sim \kappa$ ($\kappa$ : surface gravity)

Hawking temperature

$$\left( \begin{array}{l} dU = TdS + \cdots \\ dM = \kappa dA + \cdots \end{array} \right)$$

horizon




metric in Euclidean signature $ds^2 = f(r)d\tau^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2$

- $f(r_H) = 0$ : definition of the horizon

$\left( e^{-S_E} = e^{-\beta H} \quad , \beta = \frac{1}{T} \right)$

Near the horizon $\quad r \to r_H$

$$ds^2 = f(r)\,d\tau^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = \underbrace{f(r_H)}_{=0} + (r-r_H)f'(r_H) + \cdots$$

$$= (r-r_H)f'(r_H) + \cdots$$

$$\to ds^2 = (r-r_H)f'(r_H)\,d\tau^2 + \frac{d(r-r_H)^2}{(r-r_H)f'(r_H)}$$

$$\xi = 2\sqrt{\frac{r-r_H}{f'(r_H)}}$$

$$\to ds^2 = \frac{f'(r_H)^2 \xi^2}{4} d\tau^2 + d\xi^2 + \cdots$$

$$\frac{f'(r_H)}{2} d\tau = d\phi \quad \to \quad \frac{f'(r_H)}{2} \Delta\tau = 2\pi \qquad \Delta\tau = \frac{4\pi}{f'(r_H)} = \frac{1}{T}$$

$$\therefore \quad \boxed{T = \frac{f'(r_H)}{4\pi}} \quad \to \text{Hawking temperature}$$

## Black hole

: Hawking radiation

→ Hartle – Hawking state

: BH inside the thermal bath of $T = T_H$

→ Unruh state

: Hawking temperature defined
BH evaporates by Hawking radiation.

AdS Schwarzschild BH
$\xleftrightarrow[\text{dual to}]{}$ thermal SCFT on the boundary of AdS.

$\boxed{\text{Black hole}}$ (in Euclidean signature), in Global patch.

$$ds^2 = f(r)d\tau^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{d-1}^2$$

$$\begin{cases} \omega_d = \dfrac{16\pi G}{(d-1)\,\mathrm{Vol}(S^{d-1})} \\ M : \text{ADM mass} \end{cases}$$

- Schwarzschild (flat) BH in $(d+1)$ dimensions

$$f(r) = 1 - \frac{\omega_d M}{r^{d-2}}$$

- AdS BH

$$f(r) = 1 + \frac{r^2}{R^2} - \frac{\omega_d M}{r^{d-2}}$$

$$\begin{cases} M \to 0 \ : \ f(r) \to 1 + \dfrac{r^2}{R^2} & : AdS_{d+1} \\ \Lambda \sim -\dfrac{1}{R^2} \to 0 \ : \ f(r) \to 1 - \dfrac{\omega_d M}{r^{d-2}} & : \text{flat BH} \end{cases}$$

Horizon : $f(r_H) = 0 = 1 + \dfrac{r_H^2}{R^2} - \dfrac{\omega_d M}{r_H^{d-2}}$

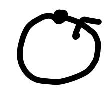
$\tau \sim \tau + \beta$ , $\beta = \dfrac{4\pi R^2 r_H}{d\cdot r_H^2 + (d-2)R^2} \sim \dfrac{4\pi R^2}{d(\omega_d R^2)^{1/d} M^{1/d}} = \dfrac{1}{T_H}$

# Confinement / deconfinement

[Witten 98]

temporal Wilson line

$$\langle W\rangle = \langle \mathrm{tr}\, e^{P\oint_\tau^{\tau+\beta} A_\tau d\tau} \rangle \sim \exp(-\beta F)$$



$\tau \sim \tau+\beta$

quark in thermal bath with temperature $T=\beta^{-1}$

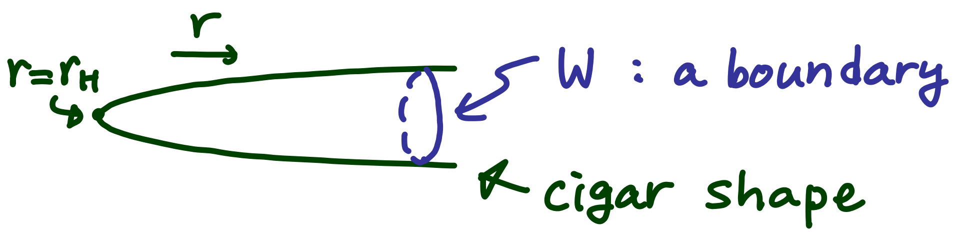
$\left[\begin{array}{ll} \langle W\rangle \neq 0 & : \text{finite free energy for external quark} \rightarrow \text{deconfinement} \\ \langle W\rangle = 0 & : \text{infinite free energy for external quark} \rightarrow \text{confinement} \end{array}\right.$

- thermal AdS : AdS in Euclidean signature


$\langle W\rangle = 0$

confinement

- thermal AdS BH : AdS BH in Euclidean signature




$\langle W\rangle \neq 0$

deconfinement

$$\boxed{\text{Hawking - Page transition}}$$

[Hawking, Page 83]

Euclidean on-shell action

$\Delta S = S_E$ (thermal AdS blackhole) $- S_E$ (thermal AdS)

$\Delta S = 0$ at $T = T_c$ : Hawking-Page transition

$T < T_c$ : thermal AdS stable
$T > T_c$ : thermal AdS BH stable

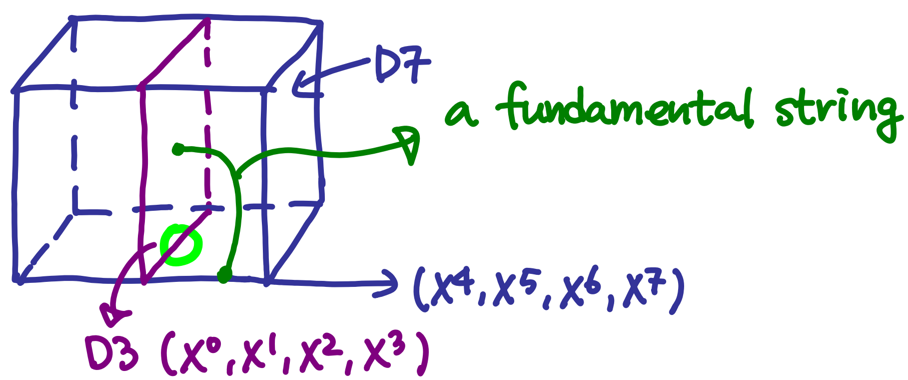
- parameter $\langle W \rangle \begin{cases} = 0 & \text{confined} \\ \neq 0 & \text{deconfined} \end{cases}$
- corresponding parameter : mass $\begin{cases} = 0 & \text{tAdS} \\ \neq 0 & \text{tAdS BH} \end{cases}$

But this argument is not always applicable.

For example arXiv : 1203.4883

D3-D7

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D3 | x | x | x | x | | | | | | |
| D7 | x | x | x | x | x | x | x | x | | |




A fundamental string

- localized on O $\mathbb{R}^{3,1}$
- fundamental for $SU(N_{D3})$ and fundamental for $SU(N_{D7})$

RNS-formalism : D3-D7 nonchiral.

($\mathcal{N}=2$ in 4D)

black D3 branes + D-instanton

A single D7 brane as a probe

↳ probe approximation / quenched approximation.

10D supergravity action [Liu, Tseytlin 99]

$$S=\frac{1}{\kappa}\int d^{10}x\sqrt{g}\left(R-\frac{1}{2}\left(\partial\Phi\right)^2+\frac{1}{2}e^{2\Phi}\left(\partial\chi\right)^2-\frac{1}{6}F_{(5)}^2\right)$$

$$\chi=-e^{-\Phi}+\chi_0$$

$$ds_{10}^2=e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2dt^2+d\vec{x}^2\right)+\frac{1}{f(r)^2}\frac{R^2}{r^2}dr^2+R^3d\Omega_5^2\right] \qquad R^4=4\pi g_sN_c\alpha'^2$$

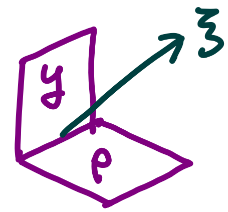
$$e^{\Phi}=1+\frac{q}{r_T^4}\log\frac{1}{f(r)^2}, \qquad \chi=-e^{-\Phi}+\chi_0, \qquad T=r_T/\pi R^2$$

$$f(r)=\sqrt{1-\left(\frac{r_T}{r}\right)^4},$$

$$ds_{10}^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f^2 dt^2 + d\vec{x}^2\right) + \frac{1}{f^2}\frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2\right]$$

$\downarrow$ introducing $\frac{d\xi^2}{\xi^2} = \frac{dr^2}{r^2 f^2}$

$$= e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f^2 dt^2 + d\vec{x}^2\right) + \frac{R^2}{\xi^2}\underbrace{\left(d\xi^2 + \xi^2 d\Omega_5^2\right)}_{\mathbb{R}^6}\right]$$




$\xi^2 = y^2 + \rho^2$

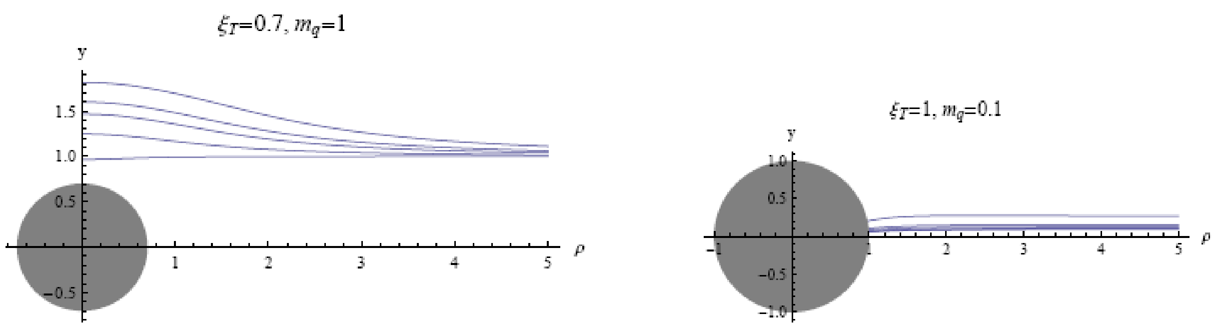
$y$: transverse

To find the induced metric on D7,

$\mathbb{R}^6 \to \mathbb{R}^4 \ \mathbb{R}^2$ : $d\rho^2 + \rho^2 d\Omega_3^2 + dy^2 + y^2 d\phi^2$

D7 { spans $(t, \vec{x}, \rho)$ and wraps $S^3$.
is orthogonal to $(y, \phi)$.

Set $\phi = 0$ using the SO(2) symmetry in $(x^8, x^9)$.




(a) (b)

Minkowski embedding blackhole embedding

"A dual geometry of the hadron in dense matter"

B-H Lee, C Park, S-J Sin     JHEP 07 (2009) 087

## ⊙ Dual geometry for QCD with quark matters

- gravity in the Minkowskian signature

$$S_M = \int d^5x\sqrt{-G}\left[\frac{1}{2\kappa^2}\left(\mathcal{R}-2\Lambda\right)-\frac{1}{4g^2}F_{MN}F^{MN}\right]$$

$$\frac{1}{2\kappa^2} = \frac{N_c^2}{8\pi^2R^3}$$
$$\frac{1}{g^2} = \frac{N_cN_f}{4\pi^2R}$$

- equations of motion

$$\mathcal{R}_{MN}-\frac{1}{2}G_{MN}\mathcal{R}+G_{MN}\Lambda = \frac{\kappa^2}{g^2}\left(F_{MP}F_N^P-\frac{1}{4}G_{MN}F_{PQ}F^{PQ}\right),$$
$$0 = \partial_M\sqrt{-G}G^{MP}G^{NQ}F_{PQ},$$

$\begin{pmatrix} M,N=0,1,\cdots,4 \\ x^0=t\ ,\ x^4=z \end{pmatrix}$

- Ansatz

$$A_0 = A_0(z),$$
$$A_i = A_4 = 0 \quad (i=1,\ldots,3),$$
$$ds^2 = \frac{R^2}{z^2}\left(-f(z)dt^2+dx_i^2+\frac{1}{f(z)}dz^2\right)$$

$\left(z=\frac{R^2}{r}\right)$

- Solution

→ Reissner-Nördstrom AdS BH

$$f(z) = 1-mz^4+q^2z^6,$$
$$A_0 = \mu-Qz^2,$$

$$\left(q^2=\frac{2\kappa^2}{3g^2R^2}Q^2.\right)$$

## ⊙ Dual geometry of the quark-gluon plasma

In the Euclidean signature

$$S = \int d^5x\sqrt{G}\left[\frac{1}{2\kappa^2}\left(-\mathcal{R}+2\Lambda\right)+\frac{1}{4g^2}F_{MN}F^{MN}\right]$$

$$ds^2 = \frac{R^2}{z^2}\left((1-mz^4+q^2z^6)d\tau^2+d\vec{x}^2+\frac{1}{1-mz^4+q^2z^6}dz^2\right)$$

$$\underset{\hat{z}}{A}(z) = i\left(\mu - Qz^2\right)$$

horizon $z_+$ $\quad 0 = f(z_+) = 1 - mz_+^4 + q^2 z_+^6$

$$m = \frac{1}{z_+^4} + q^2 z_+^2$$ : BH mass

$$T_{RN} = \frac{1}{\pi z_+}\left(1-\frac{1}{2}q^2 z_+^6\right)$$ : Hawking temperature

Dirichlet boundary condition $A(z_+)=0$ $\quad Q^2 = \frac{\mu^2}{z_+^4}$

Horizon $z_+$ as a function of $\mu$ and $T_{RN}$

$$z_+ = \frac{3g^2R^2}{2\kappa^2\mu^2}\left(\sqrt{\pi^2T_{RN}^2 + \frac{4\kappa^2\mu^2}{3g^2R^2}} - \pi T_{RN}\right)$$

A system having the fixed chemical potential

On shell action w/ the boundary condition $A(0) = i\mu$

$$S_{RN}^D = \frac{V_3R^3}{\kappa^2}\frac{1}{T_{RN}}\left(\frac{1}{\epsilon^4} - \frac{1}{z_+^4} - \frac{2\kappa^2}{3g^2R^2}\frac{\mu^2}{z_+^2}\right),$$

→ diverges !

$V_3$ : spatial volume of the boundary.

superscript D : Dirichlet b.c.

Subscript RN : RNAdS BH

Regularize the action using the background subtraction method.

$$\bar{S}_{RN}^D = S_{RN}^D - S_{AdS}$$

- grand potential (in grand canonical ensemble)

$$\Omega_{RN} = \bar{S}^D_{RN} T_{RN} = -\frac{V_3 R^3}{\kappa^2}\left(\frac{1}{2z_+^4} + \frac{\kappa^2}{3g^2R^2}\frac{\mu^2}{z_+^2}\right)$$

- free energy (in canonical ensemble)

$$F = \Omega + \mu N \qquad N = -\frac{\partial \Omega}{\partial \mu}$$

- Imposing the Neumann B.C. at the UV cut-off

$$\bar{S}^N_{RN} = \bar{S}^D_{RN} + S_b$$

$$\begin{cases} S_b = \frac{1}{g^2}\int_{\partial M} d^4x \sqrt{G^{(4)}}\, n^M A^N F_{MN} \\ G^{(4)} = \frac{R^8}{z^8} f(z) \\ n^M = \{0,0,0,0,-\frac{z}{R}\sqrt{f(z)}\} \end{cases}$$

$$F = \bar{S}^N_{RN} T_{RN} = \Omega + \frac{2R}{g^2}\mu Q V_3$$

Q : quark # density.

## ⊙ Dual geometry of the hadronic phase

$$ds^2 = \frac{R^2}{z^2}\left(f(z)\,d\tau^2 + d\vec{x}^2 + \frac{1}{f(z)}\,dz^2\right)$$

In the absence of quark matters

confinement

thermal AdS

$f(z) = 1$

deconfinement

Schwarzschild AdS BH

$f(z) = 1 - mz^4$

In the presence of quark matters

confinement

thermal charged AdS

$f(z) = 1 + q^2 z^6$

deconfinement

Reisner-Nordstrom AdS BH

$f(z) = 1 - mz^4 + q^2 z^6$

satisfies the Einstein and Maxwell eq.

asymptotically AdS.

∴ Dual geometry of the hadronic phase

$$\boxed{ds^2 = \frac{R^2}{z^2}\left((1+q^2z^6)d\tau^2 + d\vec{x}^2 + \frac{1}{1+q^2z^6}dz^2\right)}$$

- Grand canonical ensemble → Dirichlet boundary condition

$A(z_{IR}) = i\alpha\mu$ ($\alpha$ : constant)

$A(z) = i(\mu - Qz^2)$ → $Q = \frac{(1-\alpha)}{z_{IR}^2}\mu$

Regularized on-shell action of the tcAdS

$$\bar{S}^D_{tc} = S^D_{tc} - S^D_{tAdS} = -\frac{V_3R^3}{\kappa^2}\frac{1}{T_{tc}}\left(\frac{1}{z_{IR}^4} + \frac{2\kappa^2}{3g^2R^2}\frac{(1-\alpha)^2\mu^2}{z_{IR}^2}\right)$$

grand potential $\Omega = T_{tc}\bar{S}^D_{tc}$

$$N = -\frac{\partial\Omega}{\partial\mu} = \frac{2}{3}(1-\alpha)\frac{2R}{g^2}QV_3$$

- Canonical ensemble → Neumann boundary condition

$\mu N = S_b T_{tc}$

$S_b = \frac{\mu}{T_{tc}} \frac{2R}{g^2} Q V_3$

"$\alpha = -\frac{1}{2}$"

Renormalized action

$$\bar{S}^D_{tc} = -\frac{V_3 R^3}{\kappa^2}\frac{1}{T_{tc}}\left(\frac{1}{z_{IR}^4} + \frac{3\kappa^2}{2g^2R^2}\frac{\mu^2}{z_{IR}^2}\right)$$

$$\mu = \frac{2}{3} Q z_{IR}^2.$$

$\mu$ : chemical potential

$Q$ : quark # density

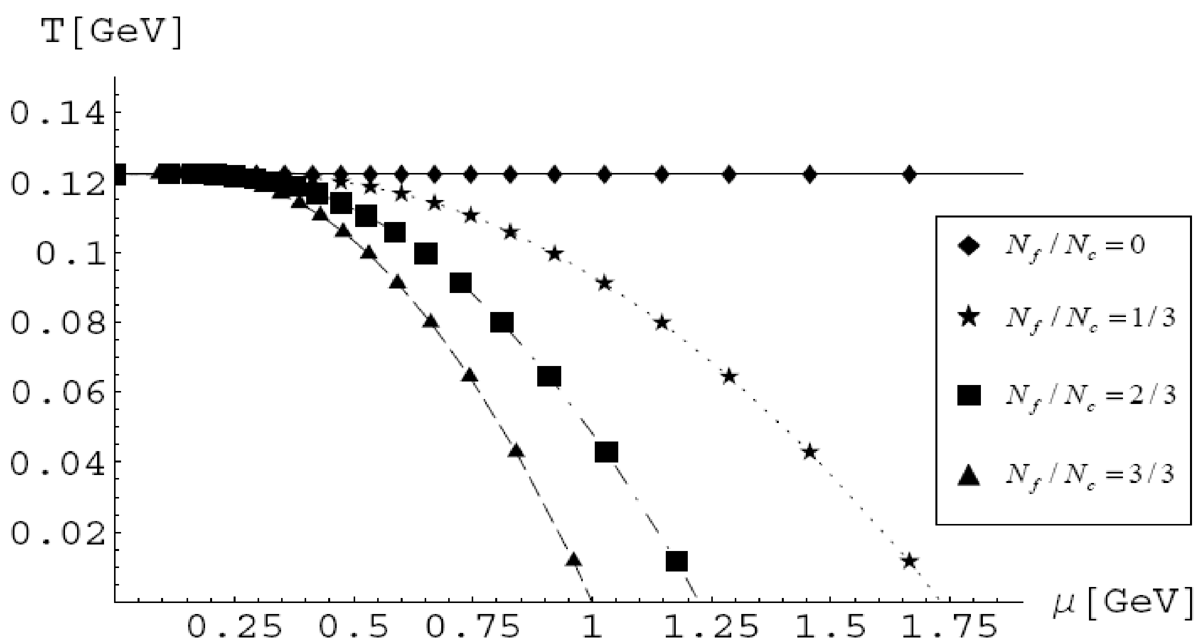
## ⊙ Confinement / Deconfinement phase transition

In grand canonical ensemble

$$\Delta S = S_{RN}^{D} - S_{tc}^{D}$$

$$S_{RN}^{D} = \frac{V_3 R^3}{\kappa^2}\frac{1}{T_{RN}}\left(\frac{1}{\epsilon^4} - \frac{1}{z_+^4} - \frac{2\kappa^2}{3g^2R^2}\frac{\mu^2}{z_+^2}\right)$$

$$S_{tc}^{D} = \frac{V_3 R^3}{\kappa^2}\frac{1}{T_{tc}}\left(\frac{1}{\epsilon^4} - \frac{1}{z_{IR}^4} - \frac{3\kappa^2}{2g^2R^2}\frac{\mu^2}{z_{IR}^2}\right)$$

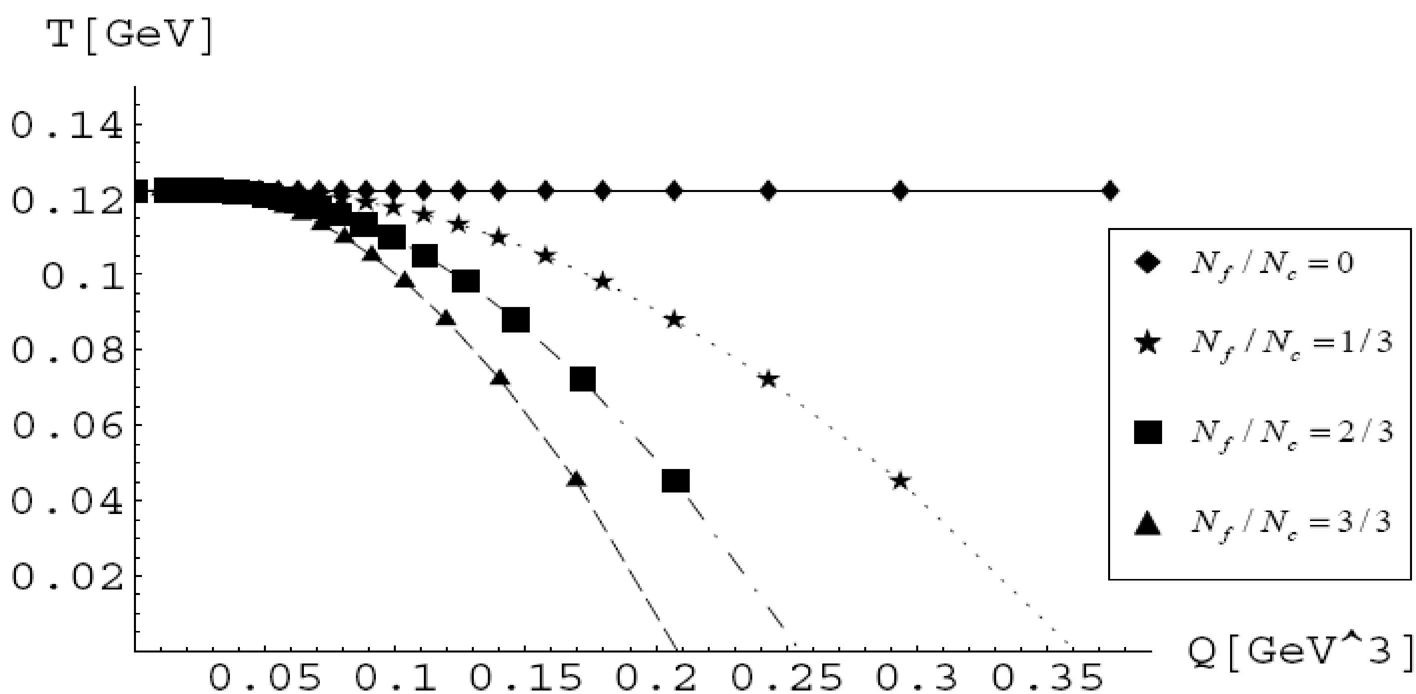


In canonical ensemble

$$\Delta S \equiv S_{RN}^N - S_{tc}^N$$

$$S_{RN}^N = \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{RN}} \left( \frac{1}{\epsilon^4} - \frac{1}{z_+^4} + \frac{4\kappa^2 Q^2}{3g^2 R^2} z_+^2 \right)$$

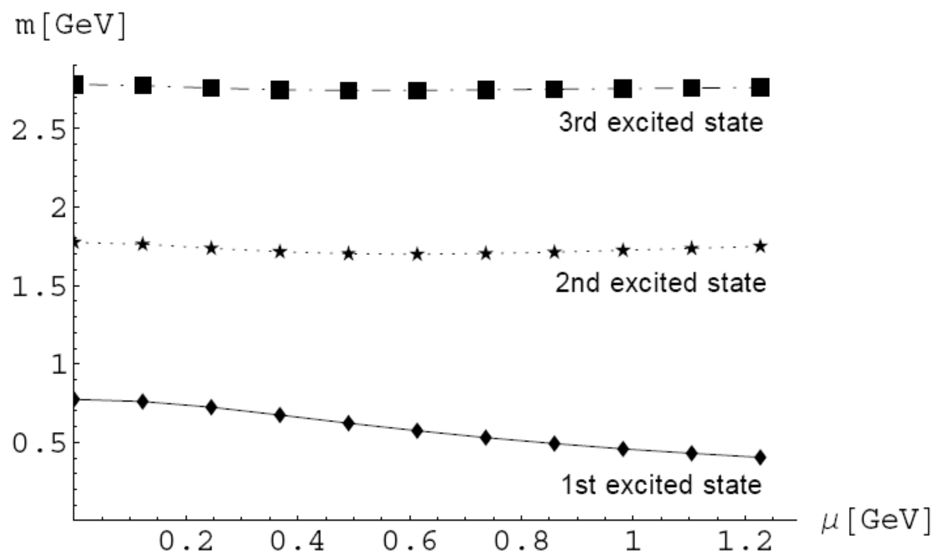
$$S_{tc}^N = \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{tc}} \left( \frac{1}{\epsilon^4} - \frac{1}{z_{IR}^4} + \frac{2\kappa^2 Q^2}{3g^2 R^2} z_{IR}^2 \right)$$



⊙ Mass of the excited vector mesons

$\delta A_\mu = V_\mu(z,p)e^{ip\cdot x}$ in thermal charged AdS.

$$0 = \partial_z^2 V_i - \frac{1}{z}\frac{(1-5q^2z^6)}{(1+q^2z^6)}\partial_z V_i + m_m^2 V_i,$$




| | $\mu=0$ | $\mu=0.245$ | $\mu=0.491$ | $\mu=0.736$ | $\mu=0.982$ | $\mu=1.227$ |
|---|---|---|---|---|---|---|
| mass of the 1st | 0.774 | 0.724 | 0.622 | 0.530 | 0.458 | 0.404 |
| mass of the 2nd | 1.775 | 1.737 | 1.702 | 1.704 | 1.724 | 1.750 |
| mass of the 3rd | 2.782 | 2.758 | 2.743 | 2.747 | 2.755 | 2.762 |

## Summary

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- Thermal charged AdS, which is the zero mass limit of RN AdS BH is proposed as the gravity dual geometry of the hadron phase.
- Phase diagrams close.
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**Thank you for your attention.**