A dynamic model of endogenous horizontal mergers

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Abstract:

I develop a dynamic model of mergers, where mergers, investment, entry and exit are endogenous variables rationally chosen by firms in order to maximize expected future profits. This model differs from previous analyses in that it incorporates dynamics and endogenizes the merger process. The model generates reasonable predictions: allowing for mergers has the expected effect on entry, exit, investment and surpluses; and changes in tastes and technologies affect industry equilibrium in plausible ways. The results demonstrate that this type of analysis is feasible and can potentially be used as a tool for antitrust policy analysis.

Section 1: Introduction

Because mergers are of significant antitrust policy concern, economists have long been interested in analyzing them and uncovering their many competitive and cost effects. The purpose of this work is to further this analysis with a dynamic, endogenous model of mergers. Most recent advances in merger analysis, including Salant, Switzer and Reynolds (1983), Deneckere and Davidson (1985), Perry and Porter (1985) and Farrell and Shapiro (1990), have used game theory to theoretically model the occurrence and welfare implications of mergers under different policy settings. While these papers have analyzed mergers with a variety of firm interactions, they have generally shared two characteristics: they have examined mergers in a static setting,[1] and they have not modeled an endogenous merger process. Both of these characteristics have limitations, which I discuss below.

Examining mergers in a static instead of dynamic setting often leads to erroneous conclusions because dynamic determinants of firm behavior (principally entry, exit and investment) are likely to have large effects on which mergers occur in an industry and on the welfare consequences of such mergers. For example, even though a static analysis might lead one to conclude that mergers lower consumer welfare by increasing concentration, a dynamic analysis might show that entry into the industry will increase if mergers are allowed, and that this will counterbalance the increase in concentration. Static analyses will also miss the fact that mergers may allow firms to partially internalize the negative externality that their investment exerts on other firms, which will lead these analyses to undervalue the potential profitability of mergers. Additionally, static analyses will not capture the fact that mergers may help otherwise failing firms, and thus prevent the deadweight loss of discarding capital without a significant anti-competitive effect. These are all examples of economically important phenomena that will affect merger outcomes and that can only be analyzed with a dynamic model.

Although perhaps not as obvious, using a non-endogenous merger process is an equally important limitation. While endogenous processes have been studied by the theoretical coalition formation literature (see, for instance, Hart and Kurz, 1981), the merger literature has shied away from an endogenous merger process,[2] because of the complexity of modeling an endogenous process and in reaching sensible conclusions. The reason for the complexity is that, in a merger environment where any firm can merge with any other, it is hard to sort out which mergers will occur from among the many conflicting possibilities. Instead of endogenizing the merger process, most studies have picked particular mergers, and checked whether these mergers are profitable relative to no mergers occurring. Unfortunately, this non-endogenous methodology is also flawed, as I illustrate below.

An analysis of mergers that does not endogenize the merger process will not be able to predict future actions that result from current policies and hence cannot predict the future welfare and industry effects of current policies. Thus, one cannot use a non-endogenous model of mergers to accurately examine the welfare effects of antitrust or other government policy, as this requires a model that predicts how policies affect future entry, exit, investment and mergers, and through that the future structures and performance of the industry. Furthermore, the criterion of "profitable relative to no merger" is not a valid guide of whether a merger will occur: for instance, a merger between two firms A and B that is profitable relative to no merger might not occur because A will wait for B and (a third firm) C to merge, knowing that it will be much better off than with no merger, due to the decreased competition. In general, because the future holds the possibility of mergers, which in turn affects the reservation prices from not merging, a merger that is profitable relative to no merger will not necessarily occur, nor is one assured that a merger that is not profitable relative to no merger will not occur.

More fundamentally, even simpler short-run antitrust policy questions that appear to have nothing to do with endogeneity require an endogenous merger model to obtain the correct answer. As an example, consider a policymaker who is deciding whether or not to block a particular merger. While this policymaker cares only to answer the question of whether this merger is beneficial or harmful and does not care to predict which mergers are likely to occur, the very notions of 'beneficial' and 'harmful' depend on what will transpire in the industry in the future as a function of whether the merger is blocked or not. Since the future holds the possibility of additional mergers, answering this question requires an endogenous merger process. As previous studies have analyzed this type of question using the erroneous assumption that that no future mergers will ever occur, their results will be significantly biased. Thus, while policymakers may not care if they predict the pattern of mergers in an industry, they need a model of endogenous mergers to correctly answer even simple antitrust policy questions.

Because of the limitations of examining mergers in a static non-endogenous setting, I have developed a dynamic game theoretic model of endogenous mergers, which is the focus of this paper. In this model, firms rationally make merger, entry, exit, investment and production decisions every period in order to maximize the expected discounted value (EDV) of net future profits. [3] Although firms can enter and exit, the industry is infinitely lived. Because of random returns to investment and random entry and exit costs, the industry is constantly in flux even in equilibrium. While this merger model can be used with different underlying static production games (such as Cobb-Douglas production functions, differentiated products equilibrium, etc.), for the results presented here, I model firms as capacity-constrained constant marginal costs Cournot competitors in a homogeneous goods market. I solve this dynamic game model using a Markov-Perfect Nash Equilibrium solution concept (see Maskin and Tirole, 1988). Because of the complexity of modeling dynamics and endogeneity, it is not feasible to analytically solve the model that I present. Instead, I computationally solve for the equilibrium of the model for stylized parameter values, and evaluate the implications of the model given equilibrium behavior.

As the endogenous merger process is the crux of the model, I now detail the process that I employ. I use a simple acquisition model for the merger process: every period, the largest firm starts and can buy any one firm that is smaller than it by paying that firm’s offering price; the smaller firms simultaneously set take-it-or-leave-it asking prices.[4] If it successfully acquires another firm, the firms merge and combine their capacities, and the process starts over, with the new largest firm as the potential acquirer. If it does not, then the second largest firm can buy any one firm that is smaller than it with the same simultaneous offer process. This continues until all the firms have had a chance to buy firms smaller than themselves, at which point the entry, exit, investment and production processes occur. In order to avoid non-existence of equilibria (see Section 2), I incorporate a mean zero random cost/synergy payment that accrues to the firm upon realization of the merger. [5]

My primary goal in this research is to develop a model that can be used to analyze the causes and consequences of mergers, in order to serve as a tool for policy analysis. As this is the first dynamic, endogenous model of mergers, there are several issues that it can address that have never even been modeled before. These include: the question of how optimal antitrust policies change when factors such as barriers to entry and increasing returns are present[6] and the effects of antitrust and other government policies on a particular industry.[7] More generally, since this model incorporates many real-world features (such as investment processes, endogenous mergers, etc.) that have not been modeled by previous merger analyses, it can potentially be used to form a theoretical understanding of the effects of mergers under a broad range of tastes and technologies. In this paper, however, I focus on the model itself, and attempt to show that examining a dynamic endogenous model of mergers is a feasible endeavor.

To examine the feasibility of this exercise, I first analyze the equilibrium and short-run behaviors of the model both with and without a merger process, for an arbitrary 'base case' vector of parameters. I find that allowing mergers will change the evolution of an industry, making it generally more concentrated and also increasing the speed at which it can evolve in response to shocks. In addition, mergers will generally increase producer surplus (because of the ability to control for bad investment draws and to prevent the deadweight loss from exit) and lower consumer surplus (because of the increase in concentration) and thus their effect on total surplus is ambiguous. I find that the model generates intuitive results: in the model and in the real world, mergers serve several purposes (from investment risk-sharing to market power) and the model predicts a pattern of mergers and industry evolutions that illustrates these objectives. While these results are certainly not intended as general proofs about this type of model, they are useful because they show that this complex dynamic model yields results that are economically plausible and reasonable.

In addition, I perform some comparative dynamics exercises to examine how the results change as entry, exit, investment, production costs and demand size change. While this is not meant to be an exhaustive exercise of the robustness of any predictions of the model, the results point out that the predictions change in an economically intuitive manner as tastes and technology parameters change. For instance, barriers to entry cause a significant deadweight loss, a loss to consumers and a gain to producers; lower demand sizes or higher marginal costs lead to a concentrated industry and to a significant deadweight loss; and increased investment effectiveness causes more firms with more total capacity to be active.

Having summarized the main points above, I proceed to detail the model and results. The rest of this paper is organized as follows. Section 2 details the industry and merger model and its computation. Section 3 details the results. Finally, Section 4 concludes.

Section 2: Model

The merger literature has generally used the same basic approach:[8] authors have modeled a merger process, followed by a one-period production process, where the producers are the firms that remain in the industry after the merger process. Production has been modeled by some standard (i.e. Cournot, Bertrand or Hotelling) static industrial organization production game. The authors have then solved these models by using backward induction to find the reduced-form profits that accrue to each firm at each information set after the merger process has finished. They have then used these reduced form profits to evaluate the values to firms of different merger choices. Although the model that I present is different in that it is dynamic and uses an endogenous merger process, it has the same basic framework repeated: every period, there is a merger process, followed by exit, a Cournot production game among the remaining firms in the industry, and finally investment and entry.

In the remainder of this section, I discuss the general industry model and state, followed by an analysis of each of the processes (investment and entry, production, exit and merger). Lastly, I briefly outline the computation of the model.

Industry model and state space

I base the industry dynamics in this paper on the Ericson and Pakes (1995) model. The class of models developed by Ericson-Pakes describes an infinite-horizon discrete-time industry with endogenous entry, exit and investment, where firms choose strategies in order to maximize the EDV of their net future profits given their information and some common discount rate β.[9] In this class of models, firms invest with a stochastic R&D type technology in order to increase their production capacities.[10]

As in the Ericson-Pakes model, I use the MPNE solution concept. MPNE, as defined by Maskin and Tirole (1988), restricts actions to be a function of payoff relevant state variables (in this case, the set of capacities for all firms), and thus eliminates many of the vast multiplicity of subgame perfect equilibria that would exist in any dynamic game model. The payoff-relevant state space of this model is the set of capacities for each of up to N different firms active in the industry, and hence is the set of all possible N-tuples, $\left(w_1,\ldots,w_N\right)$. I assume throughout that there are $J \leq N$ firms active. I model the $J < N$ case by setting the capacities of the non-existent firms to 0.

While the state space looks large and complex, it can be simplified in several ways. First, as I detail shortly, the technology ensures that the set of possible capacity levels is discrete and can be mapped onto the set of positive integers using a scale factor τ. Second, each firm's capacity can be restricted to a finite subset of the integers, since Ericson-Pakes show that there is some fixed maximum capacity level reached in equilibrium. Finally, the state space can be restricted to those N-tuples which are weakly descending, i.e. $\left\{\left(w_1,\ldots,w_N\right)\middle| w_j \geq w_{j+1}, 1 \leq j < N\right\}$, since payoffs are exchangeable in the order of one's competitors. While this restriction vastly reduces the size of the state space, it also means that firms in a state are always listed in descending order. With this restriction, if a given state has $J < N$ firms active, then the last $N - J$ components of that state will be zeros.

In my model, there are four stages, with mergers, exit, production, and investment and entry occurring in order. This is again similar to Ericson-Pakes, where there are three stages every period: first exit, then production, and finally investment and entry.

Before examining each of the stages, I first define some notation. Let $V(w, j)$ be the value function for the firm at position j in state w. Given the notation for the state space defined above, if N=4, an example of a value function element would be $V([18,14,9,0],2)$, which would be the value for a firm with capacity 14, which has competitors with capacities 18, 9 and 0.

Additionally, I define some value functions that denote values in the middle of the period, instead of at the beginning of the period. Let $V^{PM}(w, j)$ be the value to the firm immediately after the merger process in any period. The function V^{PM} reflects the value to the firm at the start of the exit and other processes, and has the same domain as the overall value function V. Similarly, let V^{BM} be the value function at the start of various parts of the merger process. I define the domain of V^{BM} in the "Merger process" subsection below.

Lastly, I define a function η which is used to map states back into weakly descending order. This function is necessary because firms can change their capacity more than neighboring firms through investment, entry or merger. To define η, consider any N-tuple w that is *not* necessarily in weakly descending order and any position j. Let $\eta(w, j)$ be the reordering of that state and position into weakly descending order. As an example, consider the above state of $[18,14,9,0]$. Suppose that the second and third firms merge so that they have capacity 23, and I want to examine their combined value. I will refer to this value as $V(\eta[(18,14+9,0,0),2]) \equiv V([23,18,0,0],1)$.

I now detail all of the processes of my model in reverse order. Figure 1 illustrates these processes for a case of three firms. Note that because of randomness, there are several possible outcomes for each process. I have highlighted one outcome down the middle of the figure, and listed others off to the side

.

<u>Entry and investment processes</u>

At the last stage of any period are the entry and investment processes, which occur simultaneously. The investment technology for incumbent firms is such that if firm j has capacity $\tau \cdot w_{jt}$ at time t, then its capacity at time t+1 is $\tau \cdot w_{j,t+1}$, where:

$$(1) \quad w_{j,t+1} = w_{jt} + \nu_{jt} - \bar{\nu}_t .$$

Here, ν_{jt} is the firm's random return to its investment that is distributed:

$$(2) \quad \nu_{jt} = \begin{cases} 1, \text{ with prob. } \alpha x_{jt} / (1 + \alpha x_{jt}) \\ 0, \text{ with prob. } 1/(1 + \alpha x_{jt}) \end{cases},$$

given that x_{jt} is the chosen non-negative level of investment, and α is a parameter. Similarly, $\bar{\nu}_t$ is an industry-wide random depreciation that is distributed:

$$(3) \quad \bar{\nu}_t = \begin{cases} 1, \text{ with prob. } \delta \\ 0, \text{ with prob. } 1 - \delta \end{cases}.$$

Concurrent to the investment process, if there are J<N firms active, one potential entrant receives a draw of an entry cost, which is taken from a distribution $U(x_E^{MIN}, x_E^{MAX})$ and known to the potential entrant before it has to decide whether or not to enter. If the potential entrant pays its entry cost draw, it will enter in the following period, and will have capacity $\tau \cdot (w^E - \bar{\nu}_t)$, for some parameter w^E.

Let me now detail the implications of this simple investment and entry framework. To do this, I define a few more terms. Consider a state $w = (w_1, \ldots, w_N)$ with the first J firms active. Let $\chi_E(w)$ be the probability (over realizations of the entry cost draw) of entry; let e_t be a random variable that equals '1' if entry occurs, and '0' if entry does not occur; let $(x_1(w),\ldots,x_J(w))$ be the vector of investment choices; and let $v_E(w)$ be the gross value of entry (not counting the sunk cost of entry).

Accordingly, consider the set of states w where there are $J < N$ firms active so that there is room for an entrant. For any of these states, an entrant will enter if and only if the net EDV of profits from entering is positive. As entry and investment occur simultaneously, I can analyze the Nash equilibrium conditions for entry by conditioning on the investment choices. I first express $v_E(w)$ as the sum over the potential states next period of the probability of that state occurring given entry, multiplied by the value to the entrant if that state occurs. Thus, the functional equation for $v_E(w)$ is:

$$\text{(4)}\quad \begin{aligned} v_E(w) = \beta \sum_{\nu_{1t},\ldots,\nu_{Jt},\bar{\nu}_t=0,1} &\Pr\left(\nu_{1t},\ldots,\nu_{Jt},\bar{\nu}_t \middle| x_1(w),\ldots,x_J(w)\right)\cdot \\ V\Big(\eta\Big[\big(w_1+\nu_{1t}-\bar{\nu}_t,\ldots,&w_J+\nu_{Jt}-\bar{\nu}_t,0,\ldots,0,w^E-\bar{\nu}_t\big),N\Big]\Big) \end{aligned}$$

Note that I am letting the entrant be in position N initially, which is why the second argument of η is an N. Also, note that the probability in (4) can be easily determined from (2) and (3) since I am conditioning on investment levels.

The potential entrant will enter if the gross value from entry is greater than its draw for the uniform random cost of entry. Thus, for any arbitrary state w (where it is not necessarily the case that $J < N$), one can see that:

$$\text{(5)}\quad \chi_E(w) = \begin{cases} 0, \text{ if } w_N > 0, \text{ i.e. } J = N \\ \max\left\{0, \min\left\{1, \dfrac{v_E(w) - x_E^{MIN}}{x_E^{MAX} - x_E^{MIN}}\right\}\right\}, \text{ if } w_N = 0. \end{cases}$$

Thus, when there are less than N firms active, the probability of entry is 0 if v_E is less than the lowest entry cost x_E^{MIN}, and the probability increases with v_E, until v_E is higher than x_E^{MAX}, at which point entry will always occur. Lastly, note that the indicator of entry, e_t, will equal '1' with probability $\chi_E(w)$.

I now turn to investment. As the incumbent firm chooses to invest in order to maximize its EDV of future profits holding other firms' behaviors fixed, the vector of investment choices $(x_1(w),\ldots,x_J(w))$ must jointly satisfy the functional equations:

$$
\begin{aligned}
&x_j(w) = \max_{\hat{x}_j}\Big\{-\hat{x}_j + \\
(6)\quad &\beta \sum_{\nu_{1t},\ldots,\nu_{Jt},\bar{\nu}_t,e_t=0,1} \Pr\big(\nu_{1t},\ldots,\nu_{Jt},\bar{\nu}_t,e_t \,\big|\, x_1(w),\ldots,x_{j-1}(w),\hat{x}_j,x_{j+1}(w),\ldots,x_J(w),\chi_E(w)\big)\cdot \\
&V\big(\eta\big[\big(w_1+\nu_{1t}-\bar{\nu}_t,\ldots,w_J+\nu_{Jt}-\bar{\nu}_t,0,\ldots,0,e_t\big[w^E-\bar{\nu}_t\big]\big),j\big]\big)\Big\}
\end{aligned}
$$

As production occurs before the investment process, current period profits are unaffected by investment, and so they do not enter into (6). Also, for states where there are N firms active, the value function in (6) does not have components with 0's, nor does it have the last component with a potential entrant.

Equations (5) and (6) together form a system of J+1 equations with J+1 unknowns, $(x_1(w),\ldots,x_J(w),\chi_E(w))$. Although this simultaneous equations problem appears difficult to solve, it is not the case. The basic method is to use the following iterative process: start with an initial guess of $x_1(w),\ldots,x_J(w),\chi_E(w)$, iterate on $\chi_E(w)$ using (5) and each individual $x_j(w)$ using (6), and repeat until this process converges.

To complete my discussion of the solution, I need to detail how to solve for the individual maximization problems in (5) and (6) needed in the iterative solution. Equation (5) has an analytic solution that involves a sum over 2^{J+1} elements, as can be seen from the fact that (4) can be evaluated by summing J+1 random variables over their 0-1 realizations.

I now derive an analytic solution to (6), for the j^{th} firm. First note that the solution to (6) involves a sum over 2^{J+2} elements, of which 2^{J+1} have $\nu_{jt}=1$ (own investment success) and 2^{J+1} have $\nu_{jt}=0$ (own investment failure). To simplify the notation, let $v^0(w,j)$ be the future value of w_j conditional on own investment failure and let $v^1(w,j)$ be its future value conditional on own investment success. Then, as above, $v^0(w,j)$ and $v^1(w,j)$ will each have 2^{J+1} terms. They can be written as

(7a)
$$v^0(w,j) = \sum_{\nu_{1t},\ldots,\nu_{j-1,t},\nu_{j+1,t},\nu_{Jt},\bar{\nu}_t,e_t=0,1} \Pr\left(\nu_{1t},\ldots,v_{j-1,t},v_{j+1,t},\ldots,\nu_{Jt},\bar{\nu}_t,e_t \middle| x_1(w),\ldots,x_{j-1}(w),x_{j+1}(w),\ldots,x_J(w),\chi_E(w)\right)\cdot V\left(\eta\left[\left(w_1+\nu_{1t}-\bar{\nu}_t,\ldots,w_{j-1}+\nu_{j-1,t}-\bar{\nu}_t,w_j-\bar{\nu}_t,w_{j+1}+\nu_{j+1,t}-\bar{\nu}_t,w_J+\nu_{Jt}-\bar{\nu}_t,0,\ldots,0,e_t\left[w^E-\bar{\nu}_t\right]\right),j\right]\right)$$

and

(7b)
$$v^1(w,j) = \sum_{\nu_{1t},\ldots,\nu_{j-1,t},\nu_{j+1,t},\nu_{Jt},\bar{\nu}_t,e_t=0,1} \Pr\left(\nu_{1t},\ldots,v_{j-1,t},v_{j+1,t},\ldots,\nu_{Jt},\bar{\nu}_t,e_t \middle| x_1(w),\ldots,x_{j-1}(w),x_{j+1}(w),\ldots,x_J(w),\chi_E(w)\right)\cdot V\left(\eta\left[\left(w_1+\nu_{1t}-\bar{\nu}_t,\ldots,w_{j-1}+\nu_{j-1,t}-\bar{\nu}_t,w_j+1-\bar{\nu}_t,w_{j+1}+\nu_{j+1,t}-\bar{\nu}_t,w_J+\nu_{Jt}-\bar{\nu}_t,0,\ldots,0,e_t\left[w^E-\bar{\nu}_t\right]\right),j\right]\right)$$

Note that (7a) and (7b) differ only in their value function elements, with an added '1' in the j^{th} component for (7b).

Now, noting that $E(A) = E(A|B)\Pr(B) + E(A|B^C)\Pr(B^C)$ one can recover (6) by multiplying $v^0(w,j)$ by the probability of failure (which is $1/(1+\alpha\hat{x}_j)$) and $v^1(w,j)$ by the probability of success (which is $\alpha\hat{x}_j/(1+\alpha\hat{x}_j)$). The maximization problem then simplifies to:

$$(8) \quad x_j(w) = \max_{\hat{x}_j}\left\{-\hat{x}_j + \beta\left(\frac{1}{1+\alpha\hat{x}_j}v^0(w,j) + \frac{\alpha\hat{x}_j}{1+\alpha\hat{x}_j}v^1(w,j)\right)\right\}.$$

As $v^0(w,j)$ and $v^1(w,j)$ are not functions of $\hat{x}_j$, (8) has an easy analytic solution, which is derived in Pakes, Gowrisankaran and McGuire (1993).

Production process

Immediately preceding the investment and entry stage is the production process, which is modeled as a simple capacity-constrained Cournot game with structure:

(9)

Capacity-Constrained Cournot Model for state $(w_1,\ldots,w_N)$
Demand: $Q^D(P) = D - P$
Cost: $mc_j(q_j) = \begin{cases} mc, \text{ if } q_j \le \tau \cdot w_j \\ \infty, \text{ otherwise.} \end{cases}$

From (9), one can see that demand is invariant across periods, and every active firm produces the same homogeneous good in order to satisfy the demand. While firms have no fixed costs, and the same constant marginal costs, mc, they are differentiated by their capacity level: each firm can only produce up to its fixed capacity, w_j, or equivalently, at its capacity level, it has infinite marginal costs. Firms choose their production quantities simultaneously in order to maximize their EDV of net future profits. Because quantity is not a payoff-relevant state variable, the MPNE assumption ensures that the quantities that firms choose will be the Cournot-Nash one-shot quantities, given the demand curve and the cost functions. Hence, for any state $(w_1,\ldots,w_N)$ with $J \le N$ firms active, the equilibrium quantities $(q_1,\ldots,q_J)$ must satisfy the Nash equilibrium conditions:

$$(10) \quad \pi_j(q_j;q_{-j}) = \max_{\hat{q}_j \le \tau \cdot w_j} \left\{ \left[\left(D - q_1 - \ldots - q_{j-1} - \hat{q}_j - q_{j+1} - \ldots - q_J \right) - mc \right] \cdot \hat{q}_j \right\}.$$

Standard results establish that this system has a unique equilibrium, which can easily be solved numerically. Let the reduced-form vector of profits that results from this equilibrium vector of quantities be called $(\Pi_1(w),\ldots,\Pi_J(w),0,\ldots,0)$, where the 0's occur in the case where $J < N$.

Exit process

Directly before the production process, firms simultaneously decide whether or not to exit. Firms receive a scrap value of ϕ from exiting, and hence choose to exit if their continuation EDVs from remaining in the industry are less than ϕ. In order to avoid equilibria where a smaller firm remains active and a larger firm exits, I further stipulate that a firm must exit if any firm at that state with higher capacity has exited; thus for every state there are $J+1$ possible exit decisions (no exit, the smaller firm only exiting, the two smallest exiting, up to all firms exiting) in equilibrium. Finally, if a firm has zero capacity, I force it to exit, in order to avoid the nonsensical possibility of negative capacity in future periods.

To illustrate the exit process, define $\chi_{x_1}(w),\ldots,\chi_{x_J}(w)$ to be a vector of exit indicator functions: $\chi_{x_j}(w)=1$ if the firm remains active and $\chi_{x_j}(w)=0$ if the firm exits. Note that $\chi_x \cdot w$ will be the state that results after the exit decision.[11] Then, given equilibrium strategies $(x_1(\cdot),\ldots,x_J(\cdot),\chi_E(\cdot))$ and equilibrium profits $(\Pi_1(\ ),\ldots,\Pi_N(\ ))$ for all states in the state space, the exit rules must satisfy:

(11)
$$\chi_{x_j}(w)=\begin{cases}1,\text{if } w_j>0,\text{and } -x_j(\overline{w}^j)+\Pi_j(\overline{w}^j)+\beta\left[\frac{1}{1+\alpha x_j(\overline{w}^j)}v^0(\overline{w}^j,j)+\frac{\alpha x_j(\overline{w}^j)}{1+\alpha x_j(\overline{w}^j)}v^1(\overline{w}^j,j)\right]>\phi,\\ 0,\text{otherwise}\end{cases}$$
$$\text{where } \overline{w}^j=\left(\chi_{x_1},\ldots,\chi_{x_{j-1}},1,\chi_{x_{j+1}},\ldots,\chi_{x_J}\right)$$

Note that, unlike in (6), current period profits enter into (11). Using (11), I can express the post-merger V^{PM} value function as:

(12)
$$V^{PM}(w,j)=\chi_{x_j}\cdot\Phi+$$
$$\left(1-\chi_{x_j}\right)\cdot\left(-x_j(\chi_x\cdot w)+\Pi_j(\chi_x\cdot w)+\beta\left[\frac{1}{1+\alpha x_j(\chi_x\cdot w)}v^0(\chi_x\cdot w,j)+\frac{\alpha x_j(\chi_x\cdot w)}{1+\alpha x_j(\chi_x\cdot w)}v^1(\chi_x\cdot w,j)\right]\right).$$

Solving for an equilibrium vector of exit decisions is very straightforward. Although there are J simultaneous decisions being made, I do not have to jointly search for an equilibrium of these decisions. Instead, I can use the following process: start with the Jth firm. Find out if it wants to exit, using (11). If it does not want to exit, there will be no exit. If it does want to exit, continue to the J-1th firm. Find if it wants to exit given that the Jth firm has exited. If it does not want to exit, there will be no further exit. If it does want to exit, continue on to the J-2th firm, etc. The reason that this sequential process works is that firms' values from remaining in the industry are increasing in their position in the state. Thus, if the Jth firm wants to exit thinking that the J-1th firm is staying in (as I computed first) and if the J-1th firm wants to exit given that the Jth firm has exited (as I computed second), then the Jth firm would still want to exit, even knowing that the J-1th firm has exited.

Merger process

The merger process occurs at the beginning of each period, immediately before the exit process. When two firms merge, they combine their capacities and become a larger firm, whose capacity is the sum of the capacities of the two individual firms.[12]

Designing the process by which firms merge is the most problematic part of this paper, and is the reason why no other paper has attempted to develop a fully endogenous, dynamic merger model. There are two broad problems that are likely to occur in any merger process: multiple equilibria and no equilibria. I detail the reasons for both of these problems and the methods I use to resolve them below.

The occurrence of a vast multiplicity of equilibria is likely to be a problem in any endogenous merger process with more than two firms because there will often be several profitable and conflicting opportunities for mergers. Many intuitive and appealing merger processes exhibit this problem. For instance, Kamien and Zang (1990) propose a game where every firm indicates a simultaneous bid for every other firm together with an asking price for itself. In this model, the ‘unmerged’ result is always an equilibrium, as are any outcomes where the mergers that occur are more profitable than no merger. Cooperative bargaining models such as Hart and Kurz (1981) also yield a multiplicity of equilibria.

My solution to the problem of multiple equilibria is to have a sequential merger process.[13] In my model, the largest firm (in terms of capacity) moves first, and can acquire any one smaller firm by paying its asking price; the smaller firms simultaneously set take-it-or-leave-it asking prices. If, on the one hand, it declines to acquire any firm, then the second biggest firm can choose to acquire any one firm that is smaller than it by paying its asking price (with these asking prices also being set simultaneously); then the third biggest firm can; etc. If, on the other hand, the biggest firm chooses to acquire some other firm, then the merger occurs, the firms combine their capacities and a new industry structure results. Again, then, the new biggest firm can choose to merge with any firm that is smaller than it, and then if it chooses not to, the next biggest firm can, etc. Every period, the process continues until there is only one firm left, or the second smallest firm chooses not to buy the smallest firm. Figure 2 presents the merger process graphically for a state where there are three firms active (ignore the 'synergy draws' listed in this figure for now). I choose this simple offer structure at each stage because it is easily computable (as detailed below) and, in equilibrium, it allows for the surplus from the merger to be shared. While it is not true that there will never be multiple equilibria for this model, this structure certainly avoids some of the major sources of multiple equilibria from other models.

For a more subtle reason, non-existence of equilibrium is also likely to be a problem for any endogenous merger process. Existence of equilibrium in this type of model is generally shown by Brouwer's Fixed Point Theorem,[14] where the operator (for which a fixed point is found) maps from the value functions to optimal policies back to the value functions.[15] Brouwer's Fixed Point Theorem requires continuity of the operator in order to prove the existence of a fixed point. In merger models, the reason for non-existence is that the value functions of non-participants to a merger will be discontinuous in the V^{PM} post-merger values to participants because of externalities from the merger.

To illustrate this discontinuity, consider the following example of a stylized industry with three firms, A, B and C, and only one merger possible, that between A and B, so that C is always a non-participant to the merger. Suppose firms A and B have V^{PM} values which make a merger just barely unprofitable. Then, C's (pre-merger) value function incorporates the fact that no merger will occur with probability one. Now suppose that firms A and B have slightly different V^{PM} values which make a merger just barely profitable. Then, as C's value function incorporates the fact that the merger will occur with probability one, it may be very different from the earlier value function. Thus, as a small change in A and B's V^{PM} values results in the probability of merger jumping from zero to one, it can cause a discrete jump in the ex-ante value function of C. This implies that the value function will not be continuous in V^{PM}. This discontinuity is not an artifact of the model. Instead, it is occurring because of the real-world fact that the merging parties impose an externality on the non-participants in the merger.[16]

My solution to the problem of discontinuity is to add a source of randomness to the merger process, which I call a merger cost/synergy. In my model, every time two firms merge, they pay/receive some cost/synergy which is an iid random draw from a mean-zero uniform distribution $U\left(-S^{MAX}, S^{MAX}\right)$. To allow for a randomness in the merger outcome, I assume that the potential acquired firms do not know their synergy draws when they set their asking prices but that the acquirer learns all of its synergy draws (with each of the potential acquired firms) immediately before it makes the acquisition decision. Figure 2 graphically shows how this information structure is manifested.

To bring back the earlier example, if the net value (not including the synergy draw) to A and B of merging is close to zero, then C's pre-merger value function will incorporate the fact that a merger will occur some, but not all, of the time. Small perturbations in A or B's post-merger value functions will change this probability a small amount. Hence, the cost/synergy payment is a convenient though somewhat arbitrary assumption that eliminates the discontinuity to non-participants by smoothing the probability of merger as a function of the prior gain.

Now that I have listed the technology of the merger game, I use dynamic programming to detail the agents' decision criteria. Recall from Figure 2 that, within one period of the dynamic model, the merger process has several stages, with a potential buyer, a set of potential sellers and a set of non-participants at each stage. One can analyze and solve each stage of the merger game separately, using value functions to indicate the payoffs from future stages. By backward induction, one can then compute the equilibrium of the whole merger model, starting with the end stages. I proceed by defining stage game value functions and reservation prices, and then solving for the stage game equilibrium vector of merger asking prices and value functions.

I first define a few terms. Consider any state $w = (w_1, \ldots, w_N)$ with $J \leq N$ firms active. Let $V^{BM}(w, j, i)$ (mentioned earlier) be the value function to firm w_j at the beginning of the stage game where w_i is the potential acquiring firm, i.e. 'the buyer'; let $B(w, i)$ be the reservation price to the buyer w_i if it does not acquire any firm; let $C(w, i, j), i < j$ be the combined value to a merged firm composed of w_j and the buyer w_i; let $S(w, i, j, m), i \neq j \neq m$ be the reservation price to w_m if w_i and w_j were to merge; let $S(w, i, 0, m), 1 < m \le J, i \neq k$ be the reservation price to the firm w_m if w_i were not to merge with any firm; let $s_{ij}, i < j$ be the realized synergy draw at the stage where w_i can potentially acquire w_j; let $(\beta_{i+1}, \ldots, \beta_J)$ denote a vector of merger asking prices or bids; and let $v(\beta_{i+1}, \ldots, \beta_J, w, i, j), i < j$ be the value to potential seller w_j from the vector of bids $(\beta_{i+1}, \ldots, \beta_J)$ when w_i is the potential buyer. Lastly, to ease notation later, let $\hat{w}^{ij}$ be the non-reordered state resulting from a merger between w_i and w_j. By the additive property of capacity, $\hat{w}^{ij} = (w_1, \ldots, w_{i-1}, w_i + w_j, w_{i+1}, \ldots, w_{j-1}, 0, w_{j+1}, \ldots, w_N)$.

The reservation prices B, C, and S can be expressed as functions of later merger stage games. It can be seen that:

$$B(w, i) = \begin{cases} V^{BM}(w, i, i+1), \text{ if } i+1 < J \\ V^{PM}(w, i), \text{ if } i+1 = J \end{cases}$$

$$C(w, i, j) = \begin{cases} V^{BM}(\eta[\hat{w}^{ij}, i], 1) \text{ if } J > 2 \\ V^{PM}(\hat{w}^{ij}, 1) \text{ if } J = 2 \end{cases} \quad (13)$$

$$S(w, i, j, m) = \begin{cases} V^{BM}(\eta[\hat{w}^{ij}, m], 1) \text{ if } j \neq 0 \\ V^{BM}(w, m, i+1), \text{ if } j = 0 \text{ and } i+1 < J \\ V^{PM}(w, m), \text{ if } j = 0 \text{ and } i+1 = J. \end{cases}$$

Note that B,C, and S are not functions of the bid vector for the current state. B is a function of V^{BM} (and not V^{PM}) only when $i+1 < J$ (and not when $i+1 = J$) as it is only when $i+1 < J$ that there is another potential acquired firm and hence another merger stage game left to occur. Similarly, C and S are sometimes a function of V^{BM} and sometimes a function of V^{PM}.

Given these reservation values in (13), I can now define the value to each potential acquired firm of a given vector of bids and use this to solve for the Nash equilibrium vector of bids. To find the value to each potential acquired firm of $(\beta_{i+1}, \ldots, \beta_J)$, one must formalize the decision process of the buyer faced with this vector of bids. Recall that the sellers set their bids before observing the cost/synergy realizations but that the buyer knows the synergy draws when it makes its acquisition decision. One can express the increase in EDV of profit to the buyer from buying any firm w_j relative to buying no firm as:

$$\text{(14)}\quad \text{netgain}_j(\beta_j) = C(w,i,j) - \beta_j - B(w,i) + s_{ij}.$$

By definition, the buyer will choose to buy that firm w_j which yields the highest 'netgain' provided that that netgain is positive. To the potential seller, though, its netgain as well as the netgains from the other potential sellers are uniform random variables. As in equilibrium, the only unknown part of them is the cost/synergy realizations, $\text{netgain}_j(\beta_j) \sim U\left(C(w,i,j) - \beta_j - B(w,i) - S^{MAX}, C(w,i,j) - \beta_j - B(w,i) + S^{MAX}\right)$.

In Figure 3, I represent this concept graphically, for a case where J=5, and firm 1 is the buyer. Here, I show each potential merger as a uniform bar whose range is the minimum and maximum netgains from that merger, depending on the realization of the synergy draw. Even though a merger with firm 5 yields the highest mean netgain in this picture, it is not true that a merger with this firm will always occur. Indeed, even though a merger with firm 5 is the single most probable event, given that the bars are all overlapping and can all be negative, it is possible that firm 1 will choose to acquire any of the other firms, or even no firm, depending on the synergy realizations. It is this randomness in outcomes that smoothes out the values to non-participants and allows for the existence of equilibrium.

I can now use the concept of netgain to examine the Nash equilibrium conditions for the bids of potential acquired firms. The value from a bid vector can be written as:

$$\text{(15)}\quad \begin{aligned} v(\beta_{i+1},\ldots,\beta_J,w,i,j) = &\sum_{k\neq j=i+1}^{J} \Pr\left\{\text{netgain}_k(\beta_k) \geq \max\left\{0, \max_{l=i+1,\ldots,J}\{\text{netgain}_l(\beta_l)\}\right\}\right\} \cdot S(w,i,k,j) \\ &+ \Pr\left\{\text{netgain}_j(\beta_j) \geq \max\left\{0, \max_{l=i+1,\ldots,J}\{\text{netgain}_l(\beta_l)\}\right\}\right\} \cdot \beta_j \\ &+ \Pr\left\{\max_{l=i+1,\ldots,J}\{\text{netgain}_l(\beta_l)\} < 0\right\} \cdot S(w,i,0,j). \end{aligned}$$

In (15), the first term indicates the value to w_j conditional on some other w_k being acquired, the second term indicates the value conditional on w_j itself being acquired, and the third term the value conditional on no firm being acquired. A Nash equilibrium vector of bids $(\beta_{i+1},\ldots,\beta_J)$ then, must satisfy:

$$(16) \quad \beta_j = \max_{\hat{\beta}_j}\left\{v\left(\beta_{i+1},\ldots,\beta_{j-1},\hat{\beta}_j,\beta_{j+1},\ldots,\beta_J,w,i,j\right)\right\}\forall j = i+1,\ldots,J\,.$$

While it is not possible to analytically solve (16), one can use numerical solution methods to solve for an equilibrium vector of bids that satisfies (16), as follows. Because the reservation prices B, C, and S are taken as given, the value to each firm from any vector of bids in (15) can be analytically solved: it is simply the probability that each netgain (corresponding to an acquisition) is both highest and higher than zero multiplied by the value if it is the highest. As the netgains are uniformly distributed, these probabilities have simple analytic solutions, which I detail in Gowrisankaran (1995). Additionally, as an increase in a bid simply shifts the distribution of the netgain left by that amount, the first and second derivatives of the netgains also have analytic solutions. Thus, conditioning on the reservation prices, one can solve for the Nash equilibrium vector of bids by numerically searching for a vector of bids that makes the first derivatives of (15), $dv\left(\beta_{i+1},\ldots,\beta_J,w,i,j\right)/d\beta_j$, $j = i+1,\ldots,J$, equal to zero. This can be done with a standard Newton search method.[17]

Using the equilibrium vector of bids, I can now evaluate $V^{BM}(w,i,i)$. For the potential buyer, this value function is simply its reservation price B plus the expected value of the netgain for each potential acquisition times the probability of that acquisition. This can be formalized as:

$$(17) \quad \begin{aligned} V^{BM}(w,i,i) = B(w,i) + \sum_{k=i+1}^{J} &\Pr\left\{\text{netgain}_k(\beta_k) \geq \max\left\{0, \max_{l=i+1,\ldots,J}\left\{\text{netgain}_l(\beta_l)\right\}\right\}\right\}\cdot \\ &E\left[\text{netgain}_k(\beta_k)\,\middle|\,\text{netgain}_k(\beta_k) \geq \max\left\{0, \max_{l=i+1,\ldots,J}\left\{\text{netgain}_l(\beta_l)\right\}\right\}\right]. \end{aligned}$$

Unlike (15), (16) includes the expectation of each netgain conditional on it being the highest. There is also a simple analytic formula for this expression.

For a potential seller w_j (where $j > i$), the value is simply $V^{PM}(w, j, i) = v(\beta_{i+1}, \ldots, \beta_J, w, i, j)$. And, for a non-participant w_m in the merger, (where m<i), the value is

$$V^{BM}(w, m, i) = \sum_{k=i+1}^{J} \Pr\left\{ \text{netgain}_k(\beta_k) \geq \max\left\{0, \max_{l=i+1,\ldots,J} \{\text{netgain}_l(\beta_l)\}\right\}\right\} \cdot S(w, i, k, m) \tag{18}$$
$$+ \Pr\left\{ \max_{l=i+1,\ldots,J} \{\text{netgain}_l(\beta_l)\} < 0 \right\} \cdot S(w, i, 0, m),$$

where (16) and (18) differ in that the m^{th} firm can never be acquired.

Finally, note that, by definition, $V^{BM}(w,1) = V(w)$, so, using backward induction, I have illustrated how to derive the overall value function given the V^{PM} values.

Computation of the model

Because of its complexity, the model described in this section cannot be analytically solved. However, one can compute an equilibrium of this model by using dynamic programming methods.[18] The method that I use to solve this model is similar to the methods described in Pakes and McGuire (1994). Their methods consist of using dynamic programming to solve for the functional equation at each state. Because it is a dynamic programming algorithm, they use backward induction, computing the profit for each state using (10), entry and investment using (5) and (6) respectively, and exit using (11). The value functions are then computed using (4) and (12). In order to make the problem finite, one must bound the maximum capacity as well as the maximum number of active firms N; these bounds are set high enough so as to not greatly affect the results. By standard dynamic programming methodology, a fixed point of this process yields an equilibrium of the model.

There is one important difference between the Pakes-McGuire method and standard dynamic programming: instead of solving for the equilibrium investment, exit and entry policies conditional on the value functions, Pakes-McGuire solve for the optimal policies conditional on the value functions and the policies of other agents at the same state.[19] The reason for this difference is that they consider a dynamic game model and not a single-agent decision process for which dynamic programming was intended. Thus, while the individually optimal policies can be computed as discussed earlier in this section, a non-linear search would be required to find the equilibrium policy vectors (even conditional on the value function). Thus, the Pakes-McGuire algorithm sets initial value functions and policies, then repeatedly updates the policies and value functions given the old value function *and the old policies*.[20]

My solution algorithm uses the Pakes-McGuire method to compute the investment, entry and exit processes, and adds on an analogous algorithm to compute the merger process. Just as the Pakes-McGuire method starts with an initial guess for the simultaneous investment and exit policies, I start with an initial guess for these as well as for the merger bid prices. I use the Pakes and McGuire method to compute the first iteration of the V^{PM} values. I then use backward induction to recursively solve the V^{BM} games given the bid vector and (17) and (18).[21] I update the bid vector using (15) to allow the algorithm to take one Newton step in the direction of the optimal bid for each individual. At the end of one iteration, I have generated a beginning-of-state V value function, which I then use as starting values for the returns to the investment process in the next iteration. As in Pakes and McGuire (1994), a fixed point of this process will satisfy the functional equations and ensure that each agent is choosing maximizing behavior conditional on the behavior of the other agents. Hence, a fixed point will be a MPNE of my model.

In order to speed convergence of the algorithm, I make use of the stochastic approximation methods developed in Pakes and McGuire (1996). This algorithm computes the solution over the ergodic class instead of over the entire state space, and thus reduces the computation time for cases where the state space is large, such as with 6 firms. Because the properties of this stochastic approximation method are unknown, I always check the results with non-stochastic computations every 1,000,000 iterations, and implement a low stopping tolerance: the algorithm stops only when the state space over the ergodic distribution is at least 99.9% correlated with the subsequent (non-stochastic) iteration, when the percent difference between current and subsequent value functions is less than 0.001%, and when the mean bid derivative is less than 0.01.

Lastly, I note that there is one significant caveat to the Pakes-McGuire method and the method used here: the computation is not a contraction mapping, and hence one cannot guarantee uniqueness of equilibrium. In addition, for my model, I cannot guarantee existence of equilibrium; however, I can show that equilibrium will exist provided that the size of the synergy distribution is sufficiently large.[22] In order to examine the question of uniqueness, I have computed the model with many different starting values and perturbations on the solution method and found the solution to be robust to these conditions. Thus, I do not have reason to believe that there are multiple equilibria for the parameters that I have tried.

Section 3: Results

Since the model that I have developed is fully endogenous and dynamic, it can potentially be used to answer a wide variety of antitrust questions. However, as this paper is the first of its kind, its purpose is not to give definitive and robust answers as to the implications of mergers and antitrust policies in different industries, but rather to show that this type of analysis is feasible and useful. I use two methods to reach this goal. First, I pick a vector of base case parameters and compare the implications of the merger model to those of an industry model without mergers. The purpose of this exercise is to determine whether a complex numerical model can give results that are economically plausible and intuitive. Second, I perform comparative dynamics by examining how the predictions of the model change as I change entry, exit, investment and production costs from the base case. The purpose of this exercise is to examine how the equilibrium changes as tastes and technologies change, again with the aim of examining whether the predictions of the model support and add to the economic intuition on mergers. In the remainder of this section, I evaluate the results of this model using these two methods.

Base case results

Recall that the results that I compute are based on parameter values that specify investment, static demand, production, entry and exit costs, discount and depreciation rates and the synergy distribution. In this subsection, I compute the equilibrium for a base case parameter vector whose values are detailed in Table 1. While the choice of parameters is relatively arbitrary, it was made with three factors in mind. First, as the computation time grows as the number of active firms increases, I chose parameters that result in an industry without many firms. Second, I model a constant returns to scale production technology to avoid having increasing returns as the reason for many mergers. Third, I set entry costs to be roughly 'free entry': I make the cost of capacity to new entrants be roughly the same as for incumbents in the industry. Because entrants gain a discrete chunk of capacity for a fixed cost when they enter while incumbents can only increase their capacity by at most one increment every period, the two technologies are different. However, I compare the two by giving the incumbent the same dollar value as the entrant to spend on capacity over two or three periods, and examining what is the maximum expected increment in capacity that the incumbent can attain with the additional dollar amount.

To evaluate the reasonableness of the predictions of the model, I present the results of the merger model and compare these with the results of an industry without mergers. It is important to note that these results are not meant to provide robust characterizations of the implications of the model, since they are for only one parameter vector.

The results, which are in Tables 2 and 3, are organized as follows. Table 2 takes a starting state where four firms of different sizes (7, 5, 4 and 3) are active and simulates the industry 1 million times given this state in order to show the equilibrium probabilities of finding different numbers of firms active over a 20-year period, both with and without mergers respectively. Table 3 details the long-run ergodic equilibrium industry structure and welfare predictions of the model both with and without mergers, using a 2 million period simulation.

One can observe several properties of the equilibrium by examining Tables 2 and 3. Note that mergers are a common event in equilibrium, occurring in 1.5% of the periods. Note also that the mean realized synergy draw is 0.166. As this is significantly higher than 0, this indicates that firms which are interested in merging wait for high synergy realizations before merging. While there is occasionally more than one merger in the industry, this is a rare event. In addition, firms most often merge to form a duopoly and never merge for monopoly. This is in spite of the fact that firms in this model certainly merge for anticompetitive reasons, and a monopoly can raise prices much more than a duopoly. Thus, given this demand size and the Cournot specification, entry and the bargaining process serve as a sufficient deterrent to discourage merger for monopoly, though not merger for duopoly.

Although merger for monopoly never occurs, one can see from Table 3 that allowing for mergers increase industry concentration significantly, with the median number of firms in the industry dropping from 3 to 2 when mergers are allowed. In addition to increasing the mean concentration in the industry, mergers serve as a relatively quick way for the industry to adjust when needed, compared to entry, exit and investment. This can be seen from Table 2, where the industry with mergers can adjust from four firms to its long-run industry median structure of 2 firms active within only 5 years, while the industry without mergers takes more than 20 years to adjust. The reason for this is that with mergers, superfluous firms do not have to exit the industry (which they will only do if their entire EDV of future profits is lower than the scrap value) for change to occur, but can instead be bought out for a higher price.

As one might expect, having mergers in the industry changes the probabilities of exit and entry in the industry. From Table 3, one can see that these effects are significant: the entry rate increases from 0.3% to 1.6% when mergers are allowed while the exit rate drops from 0.3% to 0.005%. The reason for the exit effect is that mergers are a way of transferring the capacity from weak firms (that would have otherwise exited the industry and only received a scrap value for their capacity) to stronger firms. Thus, exit is lower because allowing for mergers adds another option besides exit for potential exitors. The reason for the entry effect is similar: with mergers, firms that enter do not necessarily have to be profitable enough to survive on their own; it is enough for them to decrease the profits of incumbents sufficiently that the incumbents would want to buy them. Thus, the entry rate is higher, because allowing for mergers adds another option to the potential entrant if it enters, and thus increases the value of entering the industry.

In addition, the industry with mergers has much less investment from R&D by incumbent firms than the industry without mergers. From Table 3, one can see that equilibrium investment by incumbent firms drops by about 40% (from 0.223 to 0.143). The reason for the decline is that mergers allow firms to substitute 'external investment' (i.e. acquisitions), for 'internal investment' (i.e. investment through the R&D process).

Finally, I note that mergers generally have a negative effect on consumer surplus and a positive effect on producer surplus.[23] The reason for the negative effect on consumer surplus is that mergers reduce the number of firms in the industry and thus decrease competition and raise prices. In addition to reducing consumer surplus, the higher prices from less competition will result in a deadweight loss, as they are moving the industry further away from the perfectly competitive solution. However, as noted earlier, the ability of firms to concentrate the industry and raise prices is mitigated by the threat of entry. In contrast, mergers can raise producer surplus by increasing anticompetitive rents, but also by streamlining inefficient investment and exit. Thus, unlike in a simple static model, there is a gain in producer surplus from mergers that is not simply a transfer from consumers. Because mergers can result in an efficiency gain but also in a deadweight loss from anticompetitive behavior, the implications of mergers on total social welfare are, in general, indeterminate. Indeed, the results show that for the parameter values presented, allowing for mergers in this industry actually increases social welfare marginally, by about 0.6% (from 1.926 to 1.939).

In general terms, the above discussion shows that firms choose to merge for several reasons. Mergers decrease the competition in the industry and increase profits that way, as after a merger, the Cournot production game is played with fewer firms. In addition, by decreasing competition, mergers allow firms to partially internalize the negative externality that their investment has on other firms in the industry. Mergers are also a mechanism for increasing capacity, which substitutes for investment. As the investment process as specified allows only for slow growth, mergers are a way in which firms can grow more rapidly. Mergers also serve as a way of transferring the capacity of weak firms that would otherwise have exited the industry and received only the scrap value for their capacity. Finally, mergers are a means of capturing positive synergy draws, as synergy draws can only be obtained by merging. While the decision to merge is a function of all of these reasons confounded together, one can observe separate evidence of each of them, as shown above. The fact that these economically important criteria are guiding the merger decision is evidence that the results are grounded in economic theory and hence that this type of analysis is feasible.

Comparative dynamics

In this subsection, I examine the changes of the predictions of the model to changes in the technologies of investment, exit, entry and production and taste for this good (i.e. size of demand). As can be seen from (10), in the standard Cournot model that I use, an increase in the costs of production is exactly equivalent to a decrease in the demand size D. Hence, the simulations of increases in production costs can equivalently be thought of as decreases in demand size. It is important to note that the comparative dynamics results here are not meant as robust predictions of the effects of different tastes and technologies on industry equilibrium, but rather are intended to give some idea as to whether the predictions of the model can capture and extend economic intuition on mergers.

The results are organized as follows: Table 4 presents the equilibrium predictions of the model with three different investment technologies; Table 5 presents three different exit scrap values; Table 6 three different entry costs; and Table 7 three different production costs/demand sizes. The chosen values of the parameter of interest are indicated in the second row of each table. For Tables 4-6, I have chosen only to increase (and not decrease) the parameter of interest. In Tables 4 and 6 the reason for this is that a decrease in investment effectiveness or entry costs would correspond to a negative real cost of entry; for Table 5 the reason is that a decrease in exit scrap values would have very little effect since there is almost no exit in the base-case results.

In Table 4 one can examine the effects of increasing the effectiveness of the investment technology, α, on the industry welfare and structure. Several effects are apparent: as the investment technology becomes more effective (as α increases), the level of investment decreases, but the amount of excess capacity in the industry increases. Thus, firms are reducing their investment levels to compensate for the better technology, but are not reducing the levels so much as to decrease their excess capacity or even their total capacity. In addition, as the technology becomes more effective, the industry becomes progressively less concentrated and total welfare increases. The reason for this is two-fold: the technology is better, which increases the first-best welfare, and as the industry becomes more profitable, more firms remain active, which increases competition and moves the industry closer to the first-best welfare. Because of the increase in competition, the effect on producer surplus is ambiguous: it first rises and then falls. In addition, as the investment technology becomes more effective, the frequency of mergers and entry diminish vastly. The reason for this is that, relative to investment, mergers and entry are becoming an increasingly poor method of accumulating capacity.

Table 5 details the industry equilibrium with different exit scrap values, ϕ. The results show that as the exit scrap value increases, both the probabilities of entry and exit increase. The increase in entry is due to a higher option value from entering the industry because of the potential for exit; the increase in exit is simply due to an increase in the value of this option. In contrast to these entry and exit changes, the probability of mergers first increases and then decreases as the exit scrap value increases. This is because there are two opposing effects: while there are more entrants who may want to be acquired, these entrants have a higher option value from exiting instead of being acquired. The effects of changes in the exit scrap value on industry surpluses are similarly ambiguous, and also small in magnitude.

One can see from Table 6 that unlike the exit scrap value case, changes in entry costs cause a large change in industry concentration: with larger entry barriers, the industry becomes more concentrated, until the structure is most often a monopoly. The reason for this effect is that entry acts as a deterrent for merger to monopoly, since potential entrants will enter under the 'inverted price umbrella' created by the monopolist. Thus, as entry becomes more expensive, it is less of a deterrent, and there is more likely to be a monopoly. This results in an increasing deadweight loss from monopoly. As in a simple static analysis, producer surplus increases at the expense of consumer surplus. Interestingly enough, as entry costs increase, the volume of entry does not always decrease. Instead, for the highest entry cost, there is twice as much entry as for the lowest. The reason is that even though the costs of entry are higher, there is a higher option value from entering which counteracts this. The option value is higher because, as entry costs increase, the median industry structure changes from a duopoly to a monopoly. This monopoly structure increases the rents of the dominant firm which increases its incentives to acquire the entrant in order to preserve its rents, which in turn increases the amount it will pay to acquire the entrant.

Table 7 details the equilibrium with different demand sizes D or equivalently with different marginal costs mc. The results show that increased demand sizes (or decreased marginal costs) cause a huge increase in the number of firms in the industry. Indeed, moving from a demand size of 3 to 5, the mean number of active firms jumps from 1 to 4. The reason for this increase is that the industry is much more profitable since there are much lower costs of production and/or a larger pool of demand. Hence, the sunk entry costs and fixed costs from the investment technology can be much more readily recouped with a larger demand size. Thus, the model indicates that small markets will tend to be monopolized and large markets will tend to be competitive. While it is not meaningful to compare the surpluses across the different demand sizes, one can see that the concentrated industry structure of small markets will cause them to perform much more poorly relative to their social optima than large markets.

In general terms, these comparative dynamics results show that the implications of this model for different tastes and technologies can be explained by economic intuition, which is an indication that the model is potentially useful to analyze policy questions. In addition, because of the interactions between different behaviors, there are often surprising effects that one would not find with a static, non-endogenous analysis. For instance, one might assume that higher entry costs imply lower entry, or that increased exit scrap values will decrease the frequency of mergers, neither of which are found to be true. These examples show that an equilibrium model of mergers is necessary in order to understand how this complex interplay of agents translates into different equilibria for different tastes and technologies.

Section 4: Conclusions and further research

In this paper, I have proposed a model of endogenous horizontal mergers for firms in a dynamic setting. I have detailed the decision processes of the firms, examined the implications of the model for a base case parameter vector and performed some simple comparative dynamics exercises.

As this is the first model of its kind, there are many ways in which it could be made more realistic. One central problem is that the model represents a merged firm as acting as though it were one firm with a capacity that is the sum of the capacities of its component firms. This results in a change in the investment process when a merger occurs: when two firms merge, they go from each having one uncertain R&D investment process, to having only one together. This difference in the stochastic investment realization possibilities may not be reflective of the real-world. The solution to this problem is to allow for multiplant firms.[24] Allowing for multiplant firms will also allow for the examination of mergers in a differentiated products setting. In addition, another major problem is that because of the Markov-Perfect Nash Equilibrium solution concept, I cannot examine collusive behavior, which has often been thought of as a substitute for merger. Finally, there are many aspects of the model that will influence the results in arbitrary ways, including the Cournot firm interactions and the capacity constrained investment process.

In spite of these limitations, the results show that it is possible to develop a dynamic, endogenous model of mergers that generates predictions that are intuitively reasonable. As dynamic, endogenous models are necessary to analyze many important antitrust policy questions, the methods presented here are potentially very useful. While this paper is certainly not the last word on dynamic, endogenous mergers, it does show that this type of analysis is feasible, and hence that future research in this area is potentially beneficial.

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Figure 1

Industry evolution with N=3 and three starting firms

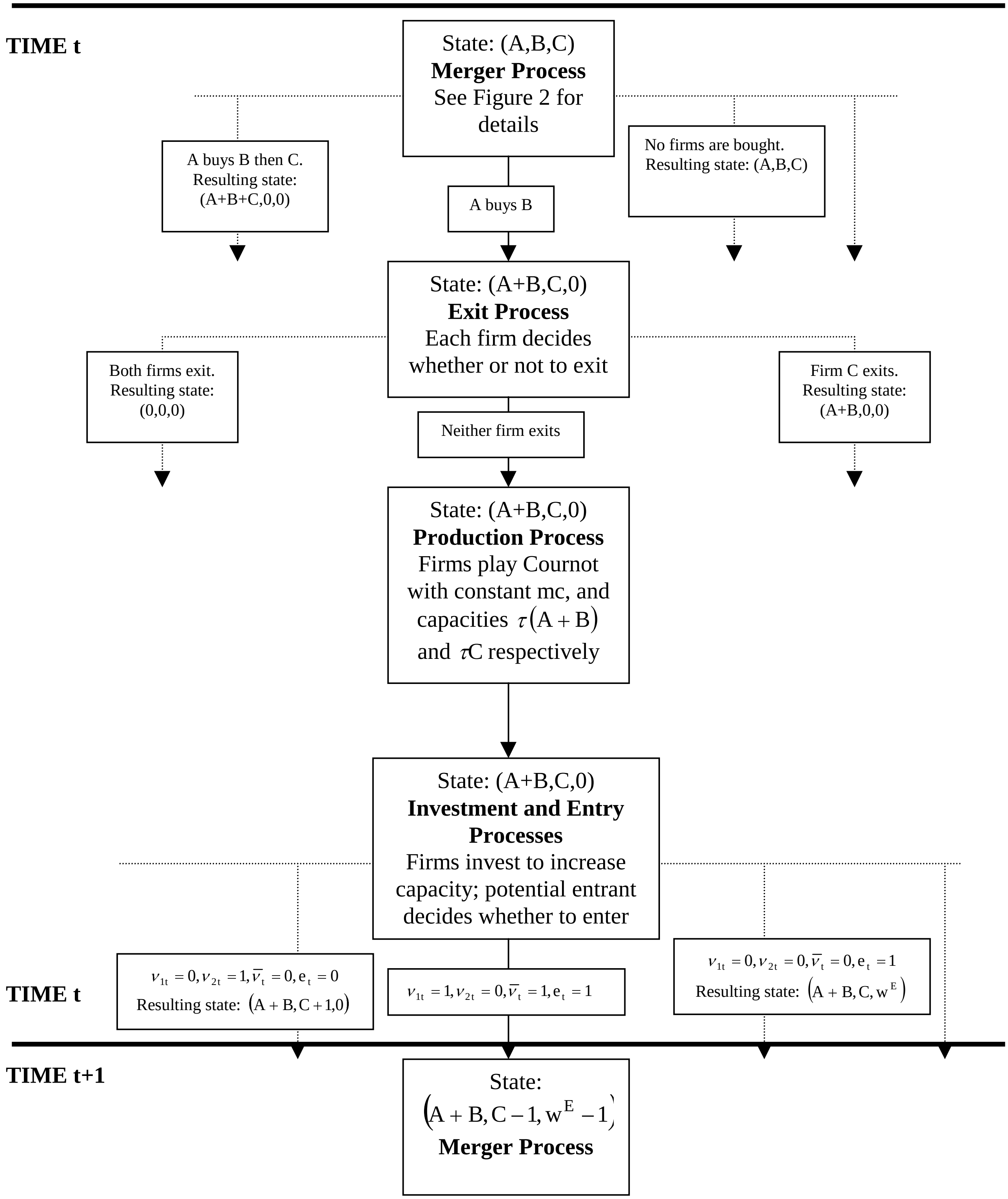

Figure 2

Merger Process for a state with N=3, and three firms A, B, and C, where A > B > C, and B + C > A

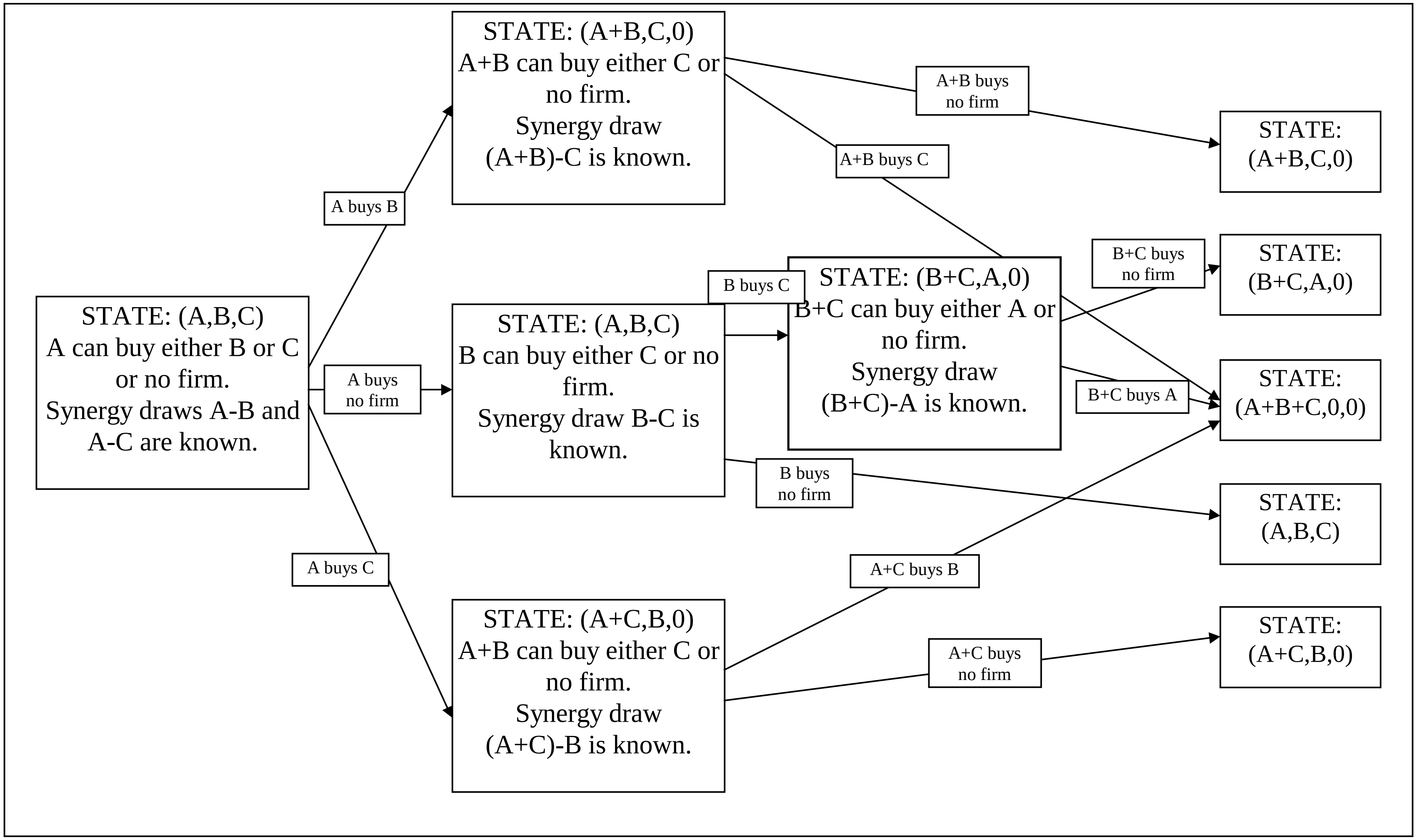

Figure 3

Potential outcomes of a merger process when firm 1 is the potential buyer, and 2 through 5 are the potential sellers. Each seller w_j sets an asking price β_j, which, combined with its synergy draw gives a netgain_j; firm 1 buys the firm with highest netgain_j, if greater than zero.

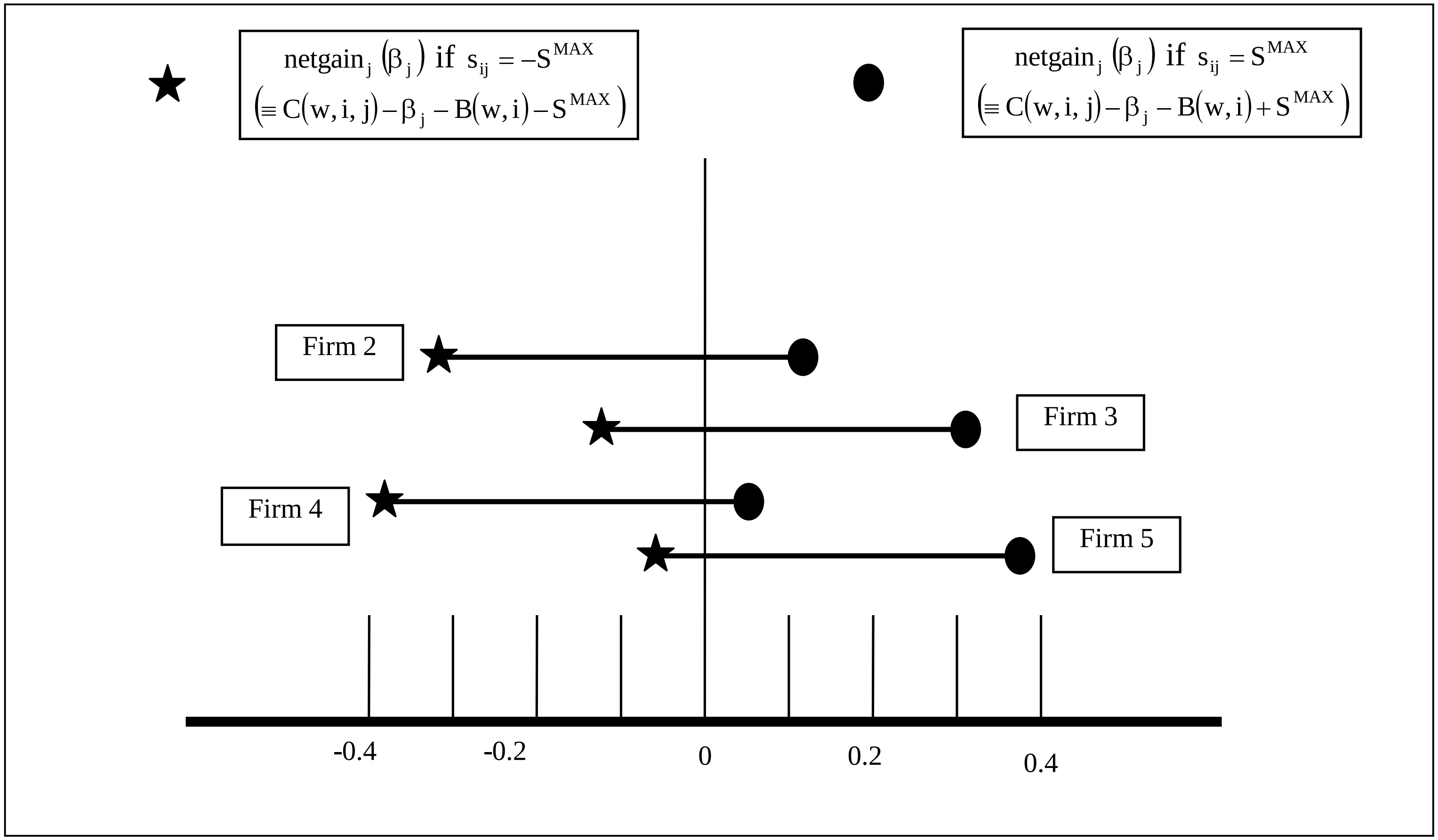

Table 1
Base-case parameters of model

Parameter	Explanation	Value
τ	Ratio of capacity to state	0.1
α	Investment efficiency parameter	10
δ	Industry capacity depreciation rate	0.4
β	Discount rate	0.925
D	Demand for good, i.e. $Q^D(P)=D-P$	4
mc	Marginal cost	1.8
N	Maximum number of firms	6
w^E	Entrant's initial capacity	4
ϕ	Exitor's scrap value	0.1
S^{MAX}	Maximum cost/synergy from merger	0.25
x_E^{MIN}	Minimum entry cost	2.4
x_E^{MAX}	Maximum entry cost	3

Table 2

Distribution of short-run industry evolutions
(Starting state 7-5-4-3-0-0)

Model	Period	Fraction of periods with given number of firms active				Mean entry	Mean exit	Mean mergers
		1	2	3	4			
	1	0.0%	20.2%	49.5%	30.3%	0.000	0.000	0.898
	2	0.0%	39.7%	49.6%	10.6%	0.000	0.000	0.392
	3	0.0%	54.7%	40.8%	4.5%	0.001	0.000	0.211
With	4	0.0%	65.5%	32.2%	2.3%	0.002	0.001	0.131
Mergers	5	0.0%	73.2%	25.4%	1.3%	0.003	0.001	0.088
	8	0.0%	85.8%	13.8%	0.4%	0.006	0.000	0.036
	10	0.0%	89..6%	10.1%	0.2%	0.008	0.000	0.024
	15	0.0%	93..7%	6.2%	0.1%	0.011	0.000	0.015
	20	0.0%	95..2%	4.8%	0.1%	0.013	0.000	0.014
	Long-run	0.0%	95..8%	4.1%	0.1%	0.017	0.000	0.017
	1	0.0%	0.0%	0.0%	100%	0.000	0.000	0.000
	2	0.0%	0.0%	0.0%	100%	0.000	0.000	0.000
	3	0.0%	0.0%	0.0%	100%	0.000	0.000	0.000
Without	4	0.0%	0.0%	0.7%	99.3%	0.000	0.007	0.000
Mergers	5	0.0%	0.0%	1.8%	98.1%	0.000	0.012	0.000
	8	0.0%	0.2%	6.0%	93.8%	0.002	0.016	0.000
	10	0.0%	0.2%	8.7%	91.1%	0.002	0.015	0.000
	15	0.0%	0.4%	14.5%	85.0%	0.003	0.014	0.000
	20	0.0%	0.6%	19.4%	79.9%	0.003	0.012	0.000
	Long-run	0.0%	10.9%	83.4%	5.7%	0.003	0.003	0.000

Table 3
Comparison of equilibrium results with and without mergers

Value in Industry Equilibrium of:	Model with merger process	Model with no merger process
% of periods with 1 firm active	0%	0%
% with 2 firms active	96.3%	10.9%
% with 3 firms active	3.6%	83.4%
% with 4 firms active	0.1%	5.7%
% with 5 or more firms active	0%	0.03%
% of periods with entry	1.6%	0.3%
% of periods with exit	0.005%	0.3%
Mean total investment (standard deviation)	0.182 (0.339)	0.232 (0.199)
Mean inv. by incumbents only (standard deviation)	0.143 (0.083)	0.223 (0.116)
Mean excess capacity (standard deviation)	0.197 (0.184)	0.190 (0.231)
Mean firm production (standard deviation)	0.701 (0.078)	0.510 (0.116)
Mean firm capacity (standard deviation)	0.798 (0.151)	0.574 (0.170)
% of periods with no mergers	98.5%	100%
% of periods with 1 merger	1.5%	0%
% of periods with 2 mergers	0.04%	0%
% of periods with 3 or more mergers	0%	0%
Mean realized synergy draw (standard deviation)	0.166 (0.056)	n/a
Mean consumer surplus (standard deviation)	1.024 (0.118)	1.151 (0.271)
Mean producer surplus (standard deviation)	0.915 (0.324)	0.775 (0.182)
Mean total surplus (standard deviation)	1.939 (0.388)	1.926 (0.340)

Table 4

Comparative Dynamics: Effect of Different Investment Technologies

Value in Industry Equilibrium of:	**Base Case:** α=10	**Experiment:** α=15	**Experiment:** α=20
% of periods with 1 firm active	0%	0%	0%
% with 2 firms active	96.3%	86.8%	0.03%
% with 3 firms active	3.6%	13.1%	98.1%
% with 4 firms active	0.1%	0.1%	1.8%
% with 5 or more firms active	0%	0%	0.03%
% of periods with entry	1.6%	1.4%	0.17%
% of periods with exit	0.005%	0.008%	0.04%
Mean total investment (standard deviation)	0.182 (0.339)	0.145 (0.308)	0.124 (0.133)
Mean inv. by incumbents only (standard deviation)	0.143 (0.083)	0.111 (0.073)	0.120 (0.069)
Mean excess capacity (standard deviation)	0.197 (0.184)	0.299 (0.225)	0.336 (0.299)
Mean firm production (standard deviation)	0.701 (0.078)	0.689 (0.090)	0.528 (0.070)
Mean firm capacity (standard deviation)	0.798 (0.151)	0.829 (0.156)	0.639 (0.160)
% of periods with no mergers	98.5%	98.6%	99.9%
% of periods with 1 merger	1.5%	1.4%	0.12%
% of periods with 2 mergers	0.04%	0.01%	0.003%
% of periods with 3 or more mergers	0%	0%	0%
Mean realized synergy draw (standard deviation)	0.166 (0.056)	0.201 (0.034)	0.203 (0.039)
Mean consumer surplus (standard deviation)	1.024 (0.118)	1.083 (0.111)	1.278 (0.173)
Mean producer surplus (standard deviation)	0.915 (0.324)	0.925 (0.305)	0.827 (0.115)
Mean total surplus (standard deviation)	1.939 (0.388)	2.008 (0.333)	2.105 (0.209)

Table 5
Comparative Dynamics: Effect of Different Exit Scrap Values

Value in Industry Equilibrium of:	**Base Case:** ϕ=0.1	**Experiment:** ϕ=1	**Experiment:** ϕ=2.4
% of periods with 1 firm active	0%	0%	0%
% with 2 firms active	96.3%	93.9%	91.9%
% with 3 firms active	3.6%	6.0%	7.9%
% with 4 firms active	0.1%	0.08%	0.18%
% with 5 or more firms active	0%	0%	0%
% of periods with entry	1.6%	1.9%	2.0%
% of periods with exit	0.005%	0.02%	0.8%
Mean total investment (standard deviation)	0.182 (0.339)	0.199 (0.372)	0.197 (0.378)
Mean inv. by incumbents only (standard deviation)	0.143 (0.083)	0.152 (0.096)	0.148 (0.085)
Mean excess capacity (standard deviation)	0.197 (0.184)	0.139 (0.143)	0.185 (0.184)
Mean firm production (standard deviation)	0.701 (0.078)	0.687 (0.093)	0.685 (0.095)
Mean firm capacity (standard deviation)	0.798 (0.151)	0.755 (0.149)	0.774 (0.160)
% of periods with no mergers	98.5%	98.2%	98.9%
% of periods with 1 merger	1.5%	1.8%	1.1%
% of periods with 2 mergers	0.04%	0.02%	0.01%
% of periods with 3 or more mergers	0%	0%	0%
Mean realized synergy draw (standard deviation)	0.166 (0.056)	0.176 (0.049)	0.184 (0.047)
Mean consumer surplus (standard deviation)	1.024 (0.118)	1.009 (0.133)	1.024 (0.137)
Mean producer surplus (standard deviation)	0.915 (0.324)	0.904 (0.357)	0.917 (0.403)
Mean total surplus (standard deviation)	1.939 (0.388)	1.913 (0.427)	1.941 (0.458)

Table 6

Comparative Dynamics: Effect of Different Entry Costs

Value in Industry Equilibrium of:	**Base Case:** $x_E^{MIN} = 2.4$ $x_E^{MAX} = 3$	**Experiment:** $x_E^{MIN} = 3.4$ $x_E^{MAX} = 4$	**Experiment:** $x_E^{MIN} = 4.2$ $x_E^{MAX} = 5.2$
% of periods with 1 firm active	0%	0.01%	90.8%
% with 2 firms active	96.3%	99.7%	9.2%
% with 3 firms active	3.6%	0.3%	0%
% with 4 firms active	0.1%	0%	0%
% with 5 or more firms active	0%	0%	0%
% of periods with entry	1.6%	0.19%	3.35%
% of periods with exit	0.005%	0.014%	0.008%
Mean total investment (standard deviation)	0.182 (0.339)	0.161 (0.187)	0.203 (0.763)
Mean inv. by incumbents only (standard deviation)	0.143 (0.083)	0.154 (0.085)	0.061 (0.064)
Mean excess capacity (standard deviation)	0.197 (0.184)	0.095 (0.131)	0.228 (0.245)
Mean firm production (standard deviation)	0.701 (0.078)	0.668 (0.111)	1.006 (0.204)
Mean firm capacity (standard deviation)	0.798 (0.151)	0.715 (0.158)	1.215 (0.362)
% of periods with no mergers	98.5%	99.8%	96.7%
% of periods with 1 merger	1.5%	0.17%	3.3%
% of periods with 2 mergers	0.04%	0%	0%
% of periods with 3 or more mergers	0%	0%	0%
Mean realized synergy draw (standard deviation)	0.166 (0.056)	0.668 (0.111)	0.176 (0.046)
Mean consumer surplus (standard deviation)	1.024 (0.118)	0.908 (0.201)	0.607 (0.090)
Mean producer surplus (standard deviation)	0.915 (0.324)	0.965 (0.181)	1.007 (0.766)
Mean total surplus (standard deviation)	1.939 (0.388)	1.874 (0.314)	1.613 (0.779)

Table 7

Comparative Dynamics: Effect of Different Marginal Costs/Demand Sizes

Value in Industry Equilibrium of:	**Experiment:** D=3	**Base Case:** D=4	**Experiment:** D=5
% of periods with 1 or less firm active	100%	0%	0%
% with 2 firms active	0%	96.3%	0%
% with 3 firms active	0%	3.6%	0.05%
% with 4 firms active	0%	0.1%	90.0%
% with 5 or more firms active	0%	0%	9.9%
% of periods with entry	0.2%	1.6%	1.4%
% of periods with exit	0.2%	0.005%	0.03%
Mean total investment (standard deviation)	0.079 (0.118)	0.182 (0.339)	0.334 (0.368)
Mean inv. by incumbents only (standard deviation)	0.074 (0.041)	0.143 (0.083)	0.297 (0.146)
Mean excess capacity (standard deviation)	0.019 (0.048)	0.197 (0.184)	0.376 (0.355)
Mean firm production (standard deviation)	0.475 (0.128)	0.701 (0.078)	0.605 (0.088)
Mean firm capacity (standard deviation)	0.494 (0.154)	0.798 (0.151)	0.697 (0.161)
% of periods with no mergers	100%	98.5%	98.6%
% of periods with 1 merger	0%	1.5%	1.3%
% of periods with 2 mergers	0%	0.04%	0.04%
% of periods with 3 or more mergers	0%	0%	0%
Mean realized synergy draw (standard deviation)	n/a	0.166 (0.056)	0.194 (0.043)
Mean consumer surplus (standard deviation)	0.120 (0.055)	1.024 (0.118)	3.089 (0.379)
Mean producer surplus (standard deviation)	0.248 (0.151)	0.915 (0.324)	1.427 (0.287)
Mean total surplus (standard deviation)	0.368 (0.191)	1.939 (0.388)	4.516 (0.489)

[1] Berry and Pakes (1993) and Cheong and Judd (1993) are exceptions which have been the first attempts at examining mergers in a dynamic setting.

[2] Kamien and Zang (1990) is a notable exception to this. Unfortunately, their endogenous merger process normally results in a vast multiplicity of equilibria.

[3] The dynamic framework of firm behavior in this model is based on the Ericson and Pakes (1995) model.

[4] As this is a dynamic game model, all offering prices and acquisition strategies are rationally made to maximize EDV of future profits, knowing that these actions will affect future exit, entry, investment and mergers.

[5] In Gowrisankaran (1995), I showed that the predictions of the model are generally robust to changes in the order of the acquiring firm, in the placement of the merger process and in the size of the synergy distribution.

[6] Gowrisankaran (1997) analyzes changes in optimal antitrust policies when there are barriers to entry and increasing returns to scale.

[7] For instance, one can combine this model with the Gowrisankaran and Town (1997) dynamic model of the hospital industry, in order to estimate the effects of mergers and antitrust policy on that industry.

[8] This approach is taken by a wide variety of merger papers, such as Salant, Switzer and Reynolds (1983), Deneckere and Davidson (1985), Perry and Porter (1985), Farrell and Shapiro (1990), McAfee, Simons, Williams (1992), Gaudet and Salant (1992) and Kamien and Zang (1990).

[9] The model is solved contingent on parameters that specify the discount rate, technologies and preferences. I detail the parameters in Table 1.

[10] In the Ericson and Pakes (1995) model, the firm’s dynamic state variable is not capacity but any variable where the static profits are increasing in that variable. I chose capacity because it has the property that when two firms merge, they add their capacity and become one firm with a larger capacity, and thus remain within the same state space as unmerged firms. Capacity has been used as a state variable for some industries; see, for instance, Porter and Spence’s (1982) study of the corn wet milling industry.

[11] Because of the restriction on the strategy space, there is no need to reorder this vector – it will already be in weakly descending order, given that w is.

[12] There are other models which have the same state additive property of mergers as capacity and hence could be used with this same framework. For instance, consider a state variable of capital, with capital decreasing the marginal costs of production. In this model, when two firms merge, they would become one firm with the sum of each of their levels of capital. See Perry and Porter (1985) and Gowrisankaran (1997) for an example of such a model.

[13] There is some basis for a sequential type of process in the endogenous coalition formation literature. See, for instance, Bloch (1995 and 1996).

[14] See Ericson and Pakes (1995) for details.

[15] This strategy of proof is similar to that of the classic proof of existence of Nash equilibrium for finite games.

[16] This externality, which has been widely noted in the industrial organization literature, need not be negative. For instance, Stigler (1968) notes that non-participants to a merger benefit because the dominant firm will prop up the price and cause the now famous 'inverted price umbrella'.

[17] As I detail in Gowrisankaran (1995), while the value-bid function of (15) is continuous everywhere and differentiable almost everywhere, it can have a kink if the maximum netgain from a bid is exactly zero and there is no other potential acquired firm with positive netgain. The reason for this is that the right derivative will be zero, since increasing one's bid will have no effect on the (zero) probability of being acquired, while the left derivative will be a bounded away from zero, since, decreasing one's bid will increase one's probability of acquisition proportional to the decrease. I modified the Newton search method to handle this kink by allowing it to find left and right derivatives at the kink point and to stop there if the sign of the left derivative is positive.

[18] C Code for computing this model is available on the author's web site, http://www.econ.umn.edu/~gautam.

[19] This is not the case for the production decision. As the equilibrium production levels do not depend on the value function, the equilibrium reduced-form profit functions Π are computed once and reused at each iteration.

[20] See Pakes, Gowrisankaran and McGuire (1993) for further details regarding the computation of the model.

[21] Within each state, the order of the induction is to start with the case where the J-1th (second to last) firm is the potential buyer, and then to move to the J-2th (third to last) firm, etc. This is because if there is no merger, then the next merger process will be for the same state but with a higher-numbered potential acquiring firm. Across states, the order of induction is to start with states where J=1 (one firm active), and then states with J=2 (two firms active), etc.

The reason for this is that, if a merger does occurs, this will cause a different state of the industry where there will be J-1 (one less) firms active.

[22] See Gowrisankaran (1995) for details.

[23] These results are not true when there are intermediate antitrust policies or increasing returns to scale. Gowrisankaran (1997) shows that surprising results may occur in these cases.

[24] Gowrisankaran (1998) proposes methods that allow the state space for the multi-product differentiated products firm problem to be parsimoniously used in computer algorithms.