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October, 2005

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# A Fast Three-step Phase-Shifting Algorithm

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## ABSTRACT

We propose a new three-step phase-shifting algorithm, which is much faster than the traditional three-step algorithm. We achieve the speed advantage by using a simple intensity ratio function to replace the arctangent function in the traditional algorithm. The phase error caused by this new algorithm is compensated for by use of a look-up-table (LUT). Our experimental results show that both the new algorithm and the traditional algorithm generate similar results, but the new algorithm is 3.4 times faster. By implementing this new algorithm in a high-resolution, real-time 3D shape measurement system, we were able to achieve a measurement speed of 40 frames per second (fps) at a resolution of  $532 \times 500$  pixels, all with an ordinary personal computer.

**Keywords:** 3-D shape measurement, phase-shifting algorithm, look-up-table, real time.

## 1. INTRODUCTION

Phase wrapping in the phase-shifting method is the process of determining the phase values of the fringe patterns in the range of 0 to  $2\pi$ .<sup>1</sup> Phase unwrapping, on the other hand, is the process of removing the  $2\pi$  discontinuity to generate a smooth phase map of the object.<sup>2</sup> Many algorithms have been developed to increase the processing speed of phase unwrapping.<sup>3-5</sup> However, few discussions have been focused on increasing the speed of phase-wrapping algorithm. Traditional phase-wrapping algorithms involve the calculation of an arctangent function, which is too slow in the case when real-time processing is required.<sup>6</sup>

In recent years, we have been developing a high-resolution, real-time 3D shape measurement system based on the phase-shifting method.<sup>7,8</sup> As the phase shifting algorithm for the system, the three-step algorithm has been our choice because it requires the minimum number of fringe images among all the existing algorithms. Less number of images means faster image capture as well as processing, which translates into higher measurement speed. However, our experiments show that even with the three-step algorithm, the image processing speed is still not fast enough for real-time 3D shape reconstruction when an ordinary personal computer is used.<sup>6</sup> The bottleneck lies in the calculation of phase, which involves a computationally time-consuming arctangent function. To solve this problem, we propose a new three-step algorithm, which replaces the calculation of the arctangent function with a simple intensity ratio calculation and therefore is much faster than the traditional algorithm. The phase error caused by this replacement is compensated for by use of a look-up-table (LUT). Our experimental results show that both the new algorithm and the traditional algorithm generate similar results, but the new algorithm is 3.4 times faster. The adoption of this new algorithm enabled us to successfully build a high-resolution, real-time 3D shape measurement system that captures, reconstructs, and displays the 3D shape of the measured object at a speed of 40 fps and a resolution of  $532 \times 500$  pixels, all with an ordinary personal computer.<sup>9</sup>

Section 2 describes the principle of the proposed algorithm. Section 3 discusses the error of the algorithm and proposes an error compensation method. Section 4 presents some experimental results. Finally, Section 5 summarizes the work.

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When this work was done, Song Zhang was with the Department of Mechanical Engineering, SUNY at Stony Brook.

## 2. PRINCIPLE

Among the various phase shifting algorithms available,<sup>1</sup> the three-step algorithm requires the minimum number of frames and is the simplest to use. The following equations describe the intensity values of the three measured fringe patterns:

$$I_1(x, y) = I'(x, y) + I''(x, y) \cos [\phi(x, y) - \alpha] , \quad (1)$$

$$I_2(x, y) = I'(x, y) + I''(x, y) \cos [\phi(x, y)] , \quad (2)$$

$$I_3(x, y) = I'(x, y) + I''(x, y) \cos [\phi(x, y) + \alpha] , \quad (3)$$

where  $I'(x, y)$  is the average intensity,  $I''(x, y)$  is the intensity modulation,  $\phi(x, y)$  is the phase, and  $\alpha$  is the phase step size. Even though  $\alpha$  can be any value, the two commonly used ones are  $\alpha = 90^\circ$  and  $\alpha = 120^\circ$ . The fast phase shifting algorithm described in this paper applies only to the case of  $\alpha = 120^\circ$ . For  $\alpha = 120^\circ$ , solving Eqs. (1) to (3) for the phase yields

$$\phi(x, y) = \arctan \left( \sqrt{3} \frac{I_1 - I_3}{2I_2 - I_1 - I_3} \right) . \quad (4)$$

Since computers are not that good at computing the arctangent function, the traditional approach of calculating the phase using Eq. (4) directly is relatively slow.

We encountered this problem in our effort to develop a real-time 3D shape measurement system based on fringe projection and phase shifting.<sup>7,10</sup> As a solution, we proposed a novel phase-shifting method, namely trapezoidal phase-shifting method, that used trapezoidal fringe patterns, instead of the traditional sinusoidal ones.<sup>6</sup> By calculating an intensity ratio using a simple function, instead of phase using the arctangent function, we were able to increase the calculation speed by 4.6 times, which made the real-time reconstruction of 3D shapes possible. However, the use of trapezoidal fringe patterns brought some error due to image defocus, even though the error is small, especially when compared to the traditional intensity-ratio based methods.<sup>11-14</sup> In the course of trying to deal with this error, we discovered that if we apply the algorithm developed for the trapezoidal method to sinusoidal patterns, considering the sinusoidal patterns as the defocused trapezoidal patterns, and then compensate for the small error due to defocus, we could preserve the calculation speed of the trapezoidal method while achieving the same accuracy of the traditional sinusoidal phase shifting algorithm. The principle is described in the following paragraphs.

Figure 1 shows the cross sections of the three phase-shifted sinusoidal patterns with a phase step size of  $120^\circ$ . We divide the period evenly into six regions ( $N = 0, 1, \dots, 5$ ), each covers an angular range of  $60^\circ$ . In each region, the three curves do not cross. Therefore, we can denote the three intensity values to be  $I_l(x, y)$ ,  $I_m(x, y)$ , and  $I_h(x, y)$ , which are the low, median, and high intensity values respectively. From these three intensity values, we can calculate the so-called intensity ratio  $r(x, y)$  as follows:

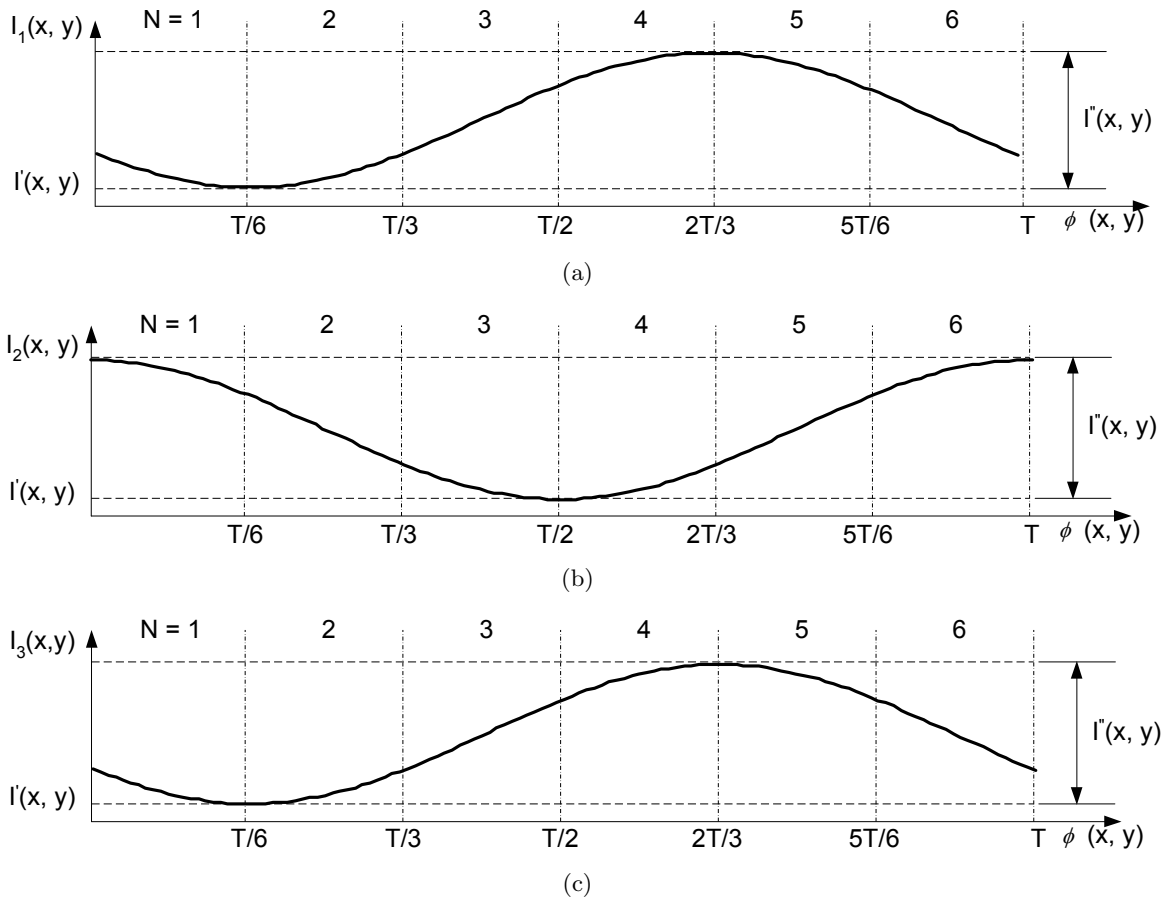
$$r(x, y) = \frac{I_m(x, y) - I_l(x, y)}{I_h(x, y) - I_l(x, y)} , \quad (5)$$

which has a value between 0 and 1, as shown in Figure 2(a). The phase can then be calculated by the following equation:

$$\phi(x, y) = \frac{\pi}{3} \left[ 2 \times \text{round} \left( \frac{N}{2} \right) + (-1)^N r(x, y) \right] , \quad (6)$$

whose value ranges from 0 to  $2\pi$ , as shown in Figure 2(b). As we can see from the figure that the phase calculation is not accurate, but with a small error. In Section 3, we will analyze this error and discuss how this error can be compensated for.

If multiple fringes are used, the phase calculated by Eq. (6) will result in a sawtooth-like shape, just as in the traditional phase shifting algorithm. Therefore, the traditional phase-unwrapping algorithm can be used to obtain the continuous phase map.<sup>2</sup>



**Figure 1.** Cross sections of the three phase-shifted sinusoidal fringe patterns. (a)  $I_1(\alpha = -120^\circ)$ . (b)  $I_2(\alpha = 0^\circ)$ . (c)  $I_3(\alpha = 120^\circ)$

### 3. ERROR ANALYSIS AND COMPENSATION

Fast three-step algorithm has the advantage of fast processing speed over the traditional three-step algorithm. However, this method makes the linear phase value  $\phi(x, y)$  to be nonlinear, as shown in Figure 2(b), which produces error. In this section, we discuss the error caused by applying the fast three-step algorithm for sinusoidal patterns first and then propose an error compensation method.

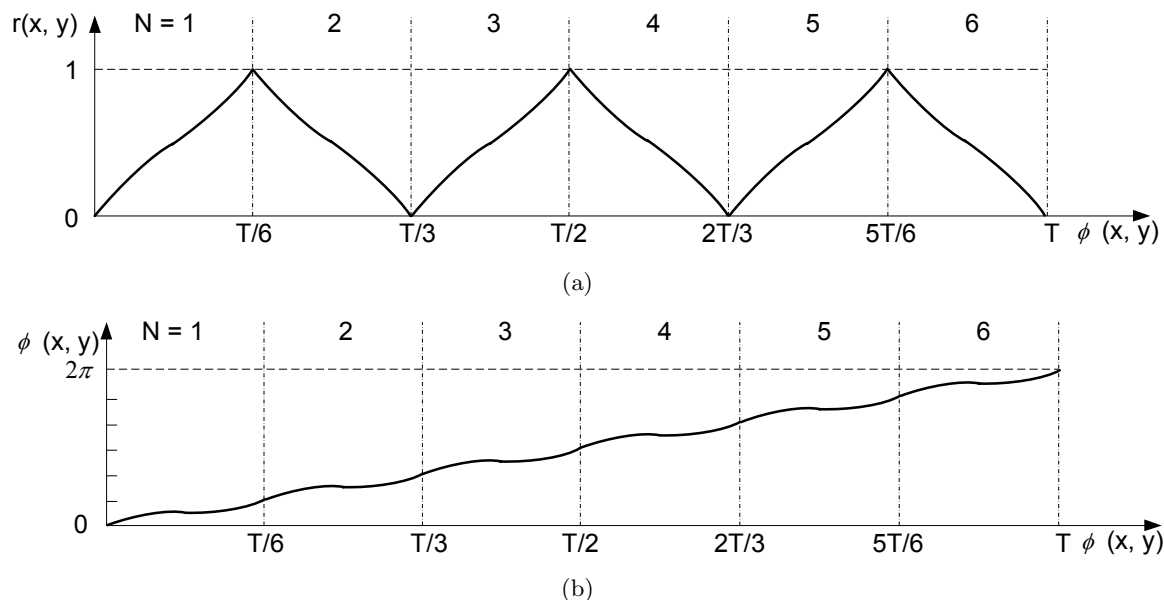
From Figure 2(b), we see that the error is periodical and the pitch is  $\pi/3$ . Therefore, we only need to analyze the error in one period, say  $\phi(x, y) \in [0, \pi/3]$ . In this period,  $I_h(x, y) = I_2(x, y)$ ,  $I_m(x, y) = I_1(x, y)$ , and  $I_l(x, y) = I_3(x, y)$ . By substituting Eqs. (1) to (3) into Eq. (5), we obtain

$$r(\phi) = \frac{I_1 - I_3}{I_2 - I_3} = \frac{1}{2} + \frac{\sqrt{3}}{2} \tan\left(\phi - \frac{\pi}{6}\right). \quad (7)$$

The right-hand side of this equation can be considered as the sum of a linear and a nonlinear terms. That is

$$r(\phi) = \frac{\phi}{\pi/3} + \Delta r(\phi), \quad (8)$$

where the first term represents the linear relationship between  $r(x, y)$  and  $\phi(x, y)$  and the second term  $\Delta r(x, y)$



**Figure 2.** Cross sections of the intensity ratio image and the phase image. (a) Intensity ratio. (b) Phase.

is the nonlinearity error, which can be calculated as follows:

$$\begin{aligned} \Delta r(\phi) &= r(\phi) - \frac{\phi}{\pi/3} \\ &= \frac{1}{2} + \frac{\sqrt{3}}{2} \tan\left(\phi - \frac{\pi}{6}\right) - \frac{\phi}{\pi/3}. \end{aligned} \quad (9)$$

Figure 3(a) shows the plots of both the ideal linear ratio and the real nonlinear ratio. Their difference, which is similar to a sine wave in shape, is shown in Figure 3(b). By taking the derivative of  $\Delta r(x, y)$  with respect to  $\phi(x, y)$  and setting it to zero, we can determine that when

$$\phi(\phi) = \frac{\pi}{6} \pm \cos^{-1}\left(\sqrt{\sqrt{3}\pi/6}\right), \quad (10)$$

the ratio error reaches its maximum and minimum values respectively as

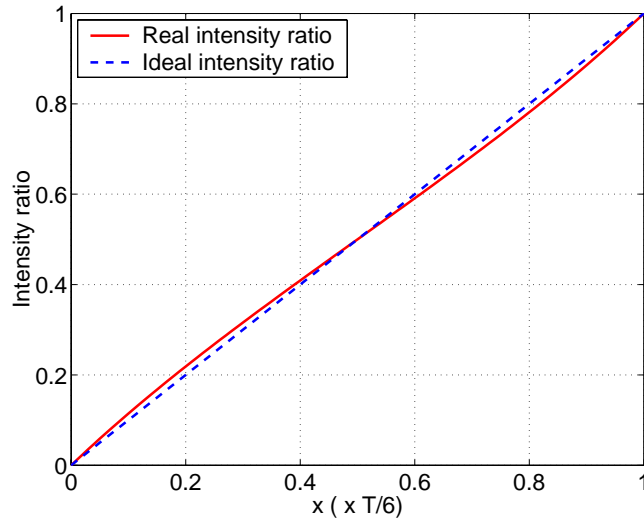
$$\Delta r(\phi)_{max} = \Delta r(\phi)|_{\phi=\phi_1} = 0.0186, \quad (11)$$

$$\Delta r(\phi)_{min} = \Delta r(\phi)|_{\phi=\phi_2} = -0.0186. \quad (12)$$

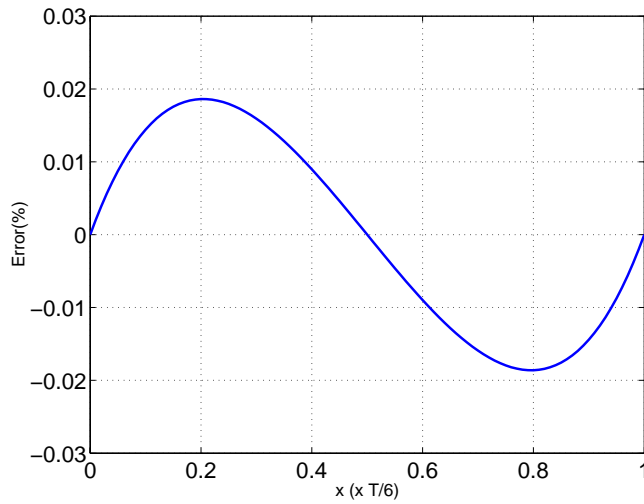
Therefore, the maximum ratio error is  $\Delta r(\phi)_{max} - \Delta r(\phi)_{min} = 0.0372$ . Since the maximum ratio value for the whole period is 6, the maximum ratio error in terms of percentage is  $0.0372/6 = 0.62\%$ . Even though this error is relatively small, it needs to be compensated for when accurate measurement is required. Since the ratio error is a systematic error, it can be compensated for by using an LUT method. In this research, an 8-bit camera is used. Therefore the LUT is constructed with 256 elements, which represent the error values determined by  $\Delta r(\phi)$ . If a higher-bit-depth camera is used, the size of the LUT should be increased accordingly. Because of the periodical nature of the error, this same LUT can be applied to all six regions.

#### 4. EXPERIMENTS

To verify the effectiveness of the proposed algorithm experimentally, we used a projector to project sinusoidal fringe patterns onto the object and an 8-bit black-and-white (B/W) CCD camera with  $532 \times 500$  pixels to capture

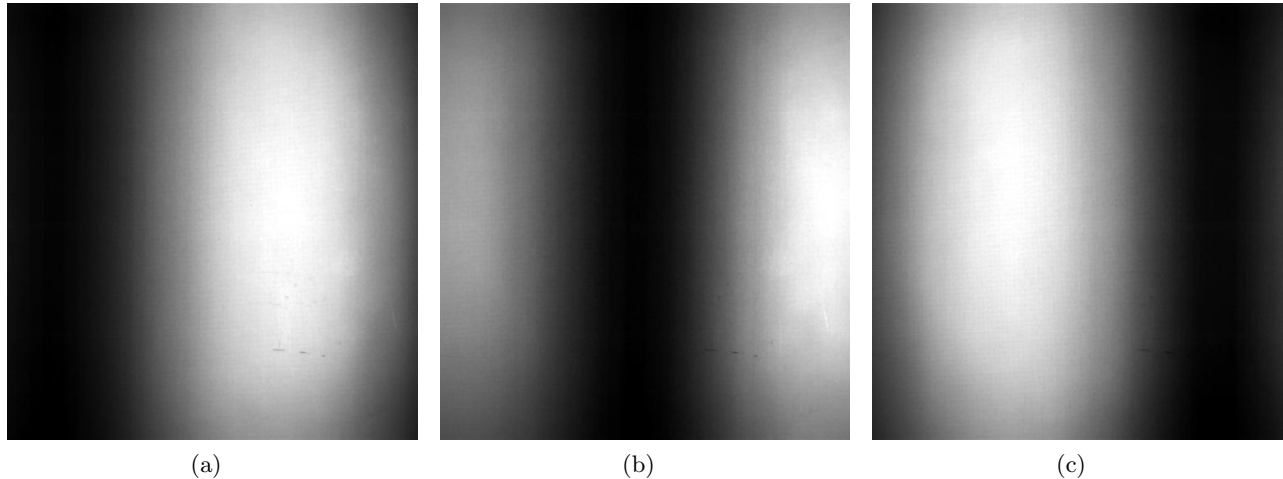


(a)

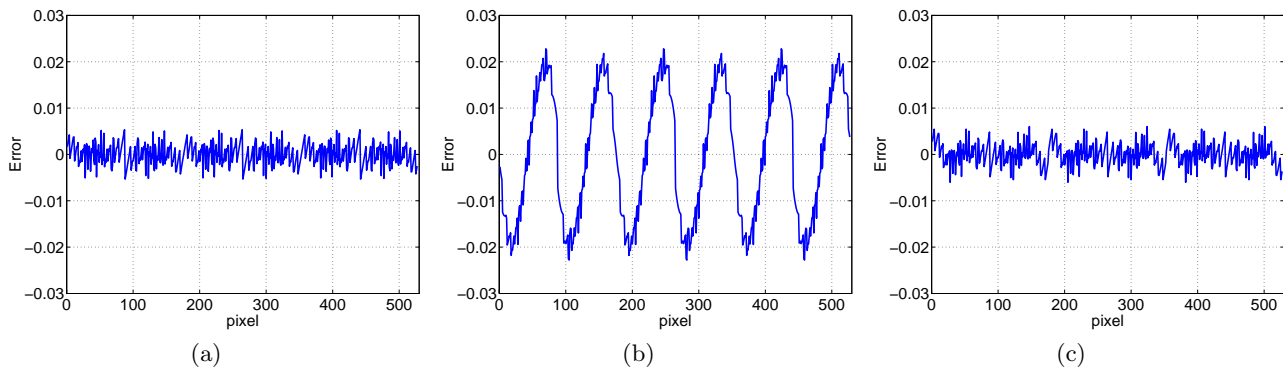


(b)

**Figure 3.** Nonlinearity error caused by the use of the fast three-step algorithm. (a) Intensity ratio in the region  $N = 1$ . (b) Nonlinearity error.

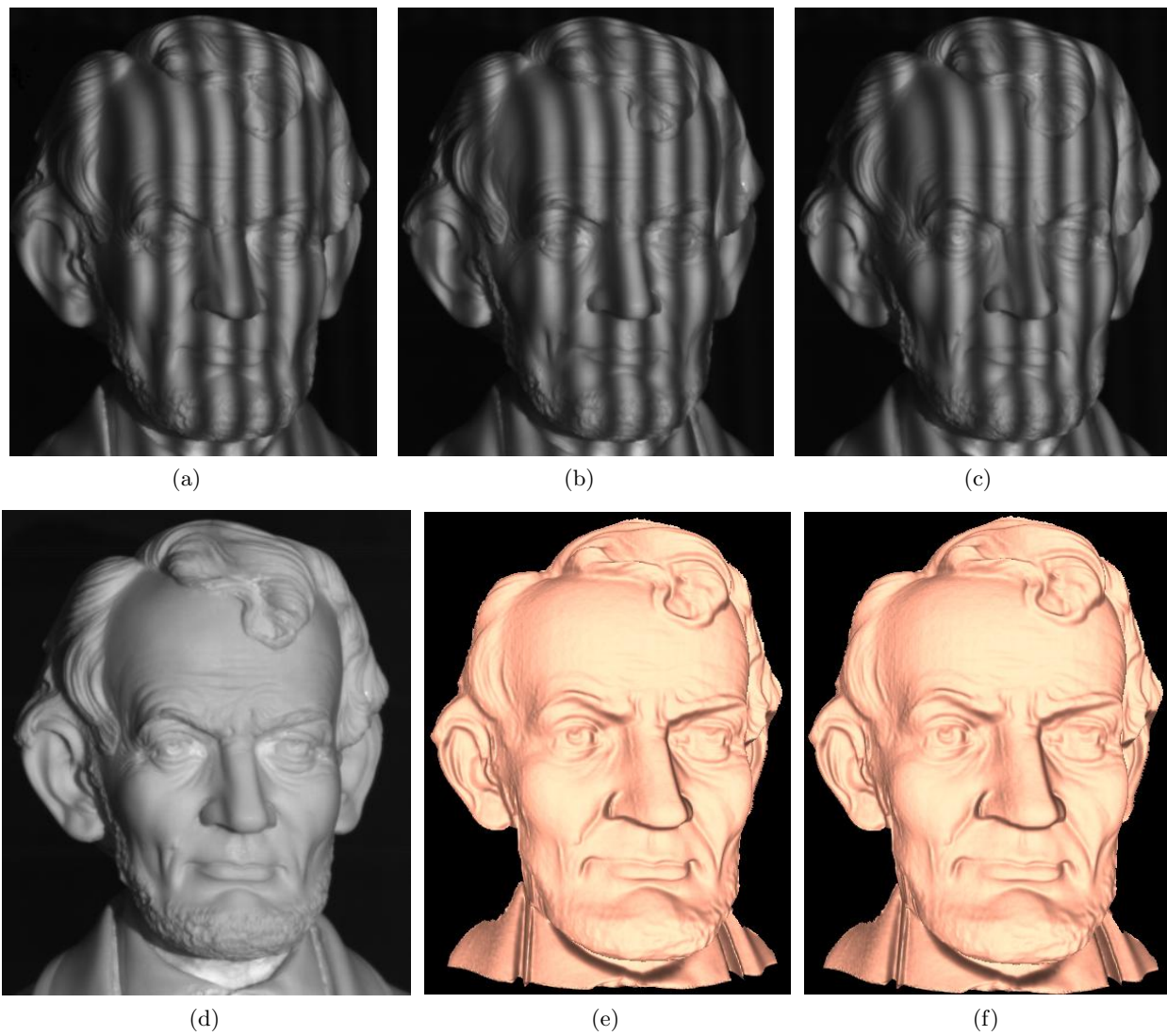


**Figure 4.** Fringe images of a flat board captured by an 8-bit B/W CCD camera. (a) Fringe image  $I_1(\alpha = -120^\circ)$ . (b) Fringe image  $I_2(\alpha = 0^\circ)$ . (c) Fringe image  $I_3(\alpha = 120^\circ)$ .



**Figure 5.** Residual phase error. (a) With the traditional three-step algorithm. (b) With the fast three-step algorithm before error compensation. (c) With the fast three-step algorithm after error compensation.

the three phase-shifted fringe images.<sup>8</sup> First, we used a flat board as the target object. The captured three fringe images are shown in Figures 4(a) – 4(c). For comparison, we applied both the traditional algorithm and the newly proposed algorithm to the same fringe images. Figures 5(a), 5(b), and 5(c) show the error of the traditional three-step algorithm, the error of the fast three-step algorithm before error compensation, and the error of the fast three-step algorithm after error compensation, respectively. From Figure 5(b), we see that the phase error of the fast three-step algorithm before error compensation is approximately 3.7%, which agrees well with the theoretical analysis. After error compensation, this error, which is shown in Figure 5(c), is significantly reduced and is comparable to that of the traditional three-step algorithm as shown in Figure 5(a). Next, we measured an object with more complex surface geometry, a Lincoln head sculpture. The fringe images are shown in Figures 6(a) – 6(c) and the 2D photo of the object is shown in Figure 6(d). The reconstructed 3D shapes based on the traditional and the proposed three-step algorithms are shown in Figures 6(e) and 6(f), respectively. We can see that the two results show almost no difference. In our experiment, we used an ordinary personal computer (Pentium 4, 2.8GHz) for image processing. The traditional three-step algorithm took 20.8 ms, while the proposed new algorithm took only 6.1 ms, which was 3.4 times less. The improvement in processing speed is significant.



**Figure 6.** Measurement results of a Lincoln head sculpture. (a) Fringe image  $I_1(\alpha = -120^\circ)$ . (b) Fringe image  $I_2(\alpha = 0^\circ)$ . (c) Fringe image  $I_3(\alpha = 120^\circ)$ . (d) 2D photo. (e) 3D shape by the traditional three-step algorithm. (f) 3D shape by the fast three-step algorithm.



## 5. CONCLUSIONS

In this paper, we proposed a fast three-step phase-shifting algorithm based on intensity ratio calculation and LUT error compensation for real-time 3D shape measurement. This algorithm originated from the previously proposed trapezoidal phase-shifting method, which was aimed at improving the processing speed of the fringe images. In this research, we found that the same algorithm developed for the trapezoidal fringe patterns could be used to process sinusoidal fringe images with a small error, which could be easily eliminated by using an LUT method. This finding resulted in a new three-step phase-shifting algorithm that is 3.4 times faster than and just as accurate as the traditional three-step algorithm. Experimental results demonstrated the effectiveness of the proposed algorithm. We have successfully implemented this algorithm in our real-time 3D shape measurement system, which resulted in a frame rate of 40 fps and a resolution of  $532 \times 500$  points.

## Acknowledgment

This work was supported by the National Science Foundation under grant No. CMS-9900337 and the National Institute of Health under grant No. RR13995.

## REFERENCES

1. D. Malacara, ed., *Optical Shop Testing*, John Wiley and Sons, NY, 1992.
2. D. C. Ghiglia and M. D. Pritt, *Two-Dimensional Phase Unwrapping: Theory, Algorithms, and Software*, John Wiley and Sons, Inc, 1998.
3. M. A. Herráez, D. R. Burton, M. J. Lalor, and M. A. Gdeisat, "Fast two-dimensional phase-unwrapping algorithm based on sorting by reliability following a noncontinuous path," *Appl. Opt.* **41**(35), pp. 7437–7444, 2002.
4. M. A. Herráez, M. A. Gdeisat, D. R. Burton, and M. J. Lalor, "Robust, fast, and effective two-dimensional automatic phase unwrapping algorithm based on image decomposition," *Appl. Opt.* **41**(35), pp. 7445–7455, 2002.
5. A. Asundi and W. Zhou, "Fast phase-unwrapping algorithm based on a gray-scale mask and flood fill," *Appl. Opt.* **37**(23), pp. 5416–5420, 1998.
6. P. Huang, S. Zhang, and F.-P. Chiang, "Trapezoidal phase-shifting method for 3-d shape measurement," in *Proc. SPIE, OpticsEast 5606*, pp. 142–152, (Philadelphia, PA), Oct. 2004.
7. P. S. Huang, C. Zhang, and F. P. Chiang, "High-speed 3-d shape measurement based on digital fringe projection," *Opt. Eng.* **42**(1), pp. 163–168, 2003.
8. S. Zhang and P. Huang, "High-resolution, real-time dynamic 3-d shape acquisition," in *IEEE Computer Vision and Pattern Recognition Workshop (CVPRW'04)*, **3**(3), pp. 28–37, 2004.
9. S. Zhang, *High-resolution, Real-time 3D Shape Measurement*. PhD thesis, State University of New York at Stony Brook, Stony Brook, NY, 11794, 2005.
10. C. Zhang, P. S. Huang, and F.-P. Chiang, "Microscopic phase-shifting profilometry based on digital micromirror device technology," *Appl. Opt.* **41**(8), pp. 5896–5904, 2002.
11. B. Carrhill and R. Hummel, "Experiments with the intensity ratio depth sensor," *Computer Vision, Graphics and Image Processing* **32**, pp. 337–358, 1985.
12. T. Miyasaka, K. Kuroda, M. Hirose, and K. Araki, "Reconstruction of realistic 3d surface model and 3d animation from range images obtained by real time 3d measurement system," in *IEEE 15th International Conference on Pattern Recognition*, pp. 594–598, Sep. 2000.
13. T. Miyasaka, K. Kuroda, M. Hirose, and K. Araki, "High speed 3-d measurement system using incoherent light source for human performance analysis," in *XIXth Congress of the international society for photogrammetry and remote sensing*, pp. 16–23, July 2000.
14. G. Chazan and N. Kiryati, "Pyramidal intensity-ratio depth sensor," Tech. Rep. No. 121, Israel Institute of Technology, Technion, Haifa, Israel, Oct. 1995.