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# A Faster Algorithm for the Steiner Tree Problem<sup>\*</sup>

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**Abstract.** The best algorithm for the Steiner tree problem by Dreyfus and Wagner is 33 years old. We celebrate this occasion and present a variation on this theme. A new algorithm is developed, which improves the running time from  $O(3^k \cdot n^3)$  to  $O((2 + \varepsilon)^k \cdot \text{poly}(n))$ .

Dieses Baums Blatt, der von Osten  
Meinem Garten anvertraut,  
Gibt geheimen Sinn zu kosten,  
Wie's den Wissenden erbaut.

— J.W. v. Goethe

## 1 Prologue

It is commonly believed in the scientific community that NP-hard problems cannot be solved in polynomial time. Nevertheless, we have to deal with many of these in everyday applications. Earlier scholarship has addressed this dilemma in a number of ways, among them approximation, randomized algorithms, parameterized complexity, heuristics and many more. Recently, there has been renewed vigor in the field of exact algorithms, and the exponential run-time bounds for many problems have been improved. Some remarkable examples for such improvement have been achieved with new algorithms for 3-SATISFIABILITY [4], DOMINATING SET [6] and MAX-CUT [3].

The Steiner tree problem on networks is to find a subgraph of minimum total edge weight that connects all nodes in a given node subset. Since we assume positive weights for all edges, this subgraph must be a tree. The respective decision problem, in which we ask for the existence of such a subgraph whose weight does not exceed a given limit, is known to be NP-complete. The optimization problem is also APX-complete, even if the edge weights are restricted to  $\{1, 2\}$  [1]. On the positive side, it is approximable within  $1 + (\ln 3)/2$ , or within 1.28 in the aforementioned restricted variation [7].

The best exact algorithm for the Steiner tree problem known today is due to Dreyfus and Wagner [2]. Given a graph with  $n$  nodes, a set of  $k$  terminals and a function encoding the lengths of the edges, it computes a minimum Steiner tree in time  $O(3^k n + 2^k n^2 + n^2 \log n + nm)$  using dynamic programming. In this paper, we present an algorithm achieving the same in time  $O(c^k \cdot \text{poly}(n))$  for some  $c < 3$ .

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In fact, we can prove bounds of  $O(c^k \cdot \text{poly}(n))$  for arbitrary  $2 < c < 3$ , with large constants hidden in the Landau notation for small values of  $c$ .

For a terminal set  $Y$  of size  $k$ , the Dreyfus-Wagner algorithm works as follows. It first computes optimal Steiner trees for all subsets of  $Y$  of size two, then it uses these results to compute optimal Steiner trees for all subsets of  $Y$  of size three, and so on, until an optimal Steiner tree for  $Y$  itself is found. The crucial lemma behind this algorithm introduces the Dreyfus-Wagner recursion (as it is called in [5]), giving a strategy to find an optimal Steiner tree for a given terminal set quickly by combining two optimal Steiner trees for two subsets with specific properties.

One of the basic ideas used in the Dreyfus-Wagner recursion is the following: if  $T$  is an optimal Steiner tree for some terminal set  $Y \cup \{v\}$  and  $v$  is not a leaf in  $T$ , we can split  $T$  at  $v$  to get optimal Steiner trees  $T_1$  and  $T_2$  for  $Y' \cup \{v\}$  and  $(Y - Y') \cup \{v\}$ , where  $Y'$  is the set of terminals in  $T_1$ . In this paper, we will extend this concept as follows: instead of splitting the tree at a single node  $v$ , we choose an entire set  $X$  of nodes to split the tree at, where  $|X|$  is bounded by  $1/\varepsilon$  for an adjustable parameter  $\varepsilon$ .

An important fact is that there exists such a set  $X$  for which each of the resulting subtrees contains at most  $\varepsilon|Y| + 1$  of the terminals from  $Y$ . Using some additional ideas, this allows for a dynamic programming algorithm that successively combines locally optimal Steiner trees, constructing a globally optimal Steiner tree for the terminal set  $Y \cup X$ . In the case that some optimal Steiner tree for  $Y$  contains all the nodes from  $X$ , the constructed tree is also an optimal Steiner tree for  $Y$ .

## 2 Chopping trees

As already stated in the introduction, our algorithm relies on the fact that there exists a set  $X$  of nodes to split the tree at such that the resulting subtrees contain no more than  $\varepsilon|Y|$  terminals each. This is formalized by the following lemma.

**Lemma 1.** *Let  $\varepsilon > 0$ , and let  $T$  be a tree with root  $r$  and  $k \geq 1/\varepsilon^2$  terminals. Then there exists a set  $X$  of nodes such that*

1. *the subtree rooted at any child of any node  $x \in X \cup \{r\}$  has at most  $\varepsilon k$  terminals that are not part of any subtree rooted at a child of some successor  $x' \in X - \{x\}$  of  $x$ , and*
2. *the size of  $X$  does not exceed  $\lceil 1/\varepsilon \rceil$ .*

*Proof.* The following algorithm computes a set  $X$  that fulfills the conditions described in the lemma:

1. Let  $X := \emptyset$ .
2. If  $T$  contains no more than  $\varepsilon k$  terminals, output  $X$ , and stop.
3. Otherwise, let  $v := r$ .
4. For all the children  $c_1, \dots, c_d$  of  $v$ , count the amount of terminals contained in the subtrees rooted at  $c_1, \dots, c_d$ , and let  $m(v)$  be the maximum of these amounts.
5. If  $m(v) \leq \varepsilon k$ , add  $v$  to  $X$ , remove all the subtrees rooted at  $c_1, \dots, c_d$ , and go to step 2.

6. Otherwise, in the case that  $m(v) > \varepsilon k$ , let  $v$  be one of the  $c_i$  with a maximal amount of terminals in the respective subtree, and go to step 4.

It is easy to see that the algorithm terminates on every input. Clearly, the output  $X$  satisfies the first condition of the lemma, as the algorithm only adds a node to  $X$  if the subtrees rooted at children of that node contain a sufficiently small amount of terminals that have not yet been processed. By cutting off processed subtrees, as done in step 5, the algorithm just avoids to count terminals that lie in subtrees of children of previously chosen nodes.

To see that the second condition is fulfilled, note that the subtree rooted at the currently inspected node  $v$  (in steps 4 through 6) always contains more than  $\varepsilon k$  unprocessed terminals: otherwise, a parent of  $v$  would have been chosen as a node to cut at. Hence, when  $v$  is added to  $X$  in step 5 and all the subtrees rooted at children of  $v$  are cut off, more than  $\varepsilon k - 1$  terminals disappear, where the term  $-1$  is included because  $v$  can be one of the terminals itself. This processing of terminals continues while more than  $\varepsilon k$  terminals remain. In total, and because  $\varepsilon \geq \varepsilon/k$ , we get that

$$|X| \leq \left\lceil \frac{k - \varepsilon k}{\varepsilon k - 1} \right\rceil = \left\lceil \frac{1 - \varepsilon}{\varepsilon - 1/k} \right\rceil = \left\lceil \frac{1}{\varepsilon} \frac{1 - \varepsilon}{1 - 1/\varepsilon k} \right\rceil \leq \left\lceil \frac{1}{\varepsilon} \right\rceil.$$

□

For an example, see Figure 1. Without loss of generality, we assume that henceforth  $1/\varepsilon$  be an integer.

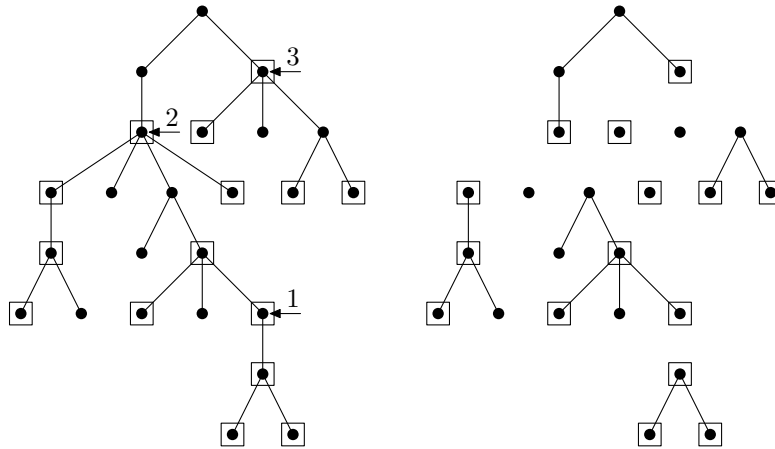


Fig. 1. Example for Lemma 1 ( $k = 15$ ,  $\varepsilon = \frac{1}{5}$ ,  $\varepsilon k = 3$ ;  $|X| = 3 \leq 5$ )

We intend to group nodes from  $X$  in so-called *links*, whenever they serve to delimit a subtree (together with some terminals).

**Definition 1.** Let  $X$  be a set. A multiset  $\mathcal{X} = \{\mathcal{X}_1, \dots, \mathcal{X}_r\}$  with  $\mathcal{X}_1, \dots, \mathcal{X}_r \in 2^X \setminus \{\emptyset\}$  is called a *chain mail* for  $X$  if  $|\mathcal{X}_i \cap \mathcal{X}_j| \leq 1$  for any  $1 \leq i < j \leq r$  and  $G_{\mathcal{X}} := (\{1, \dots, r\}, E_{\mathcal{X}})$ ,  $E_{\mathcal{X}} := \{\{i, j\} \mid \mathcal{X}_i \neq \mathcal{X}_j \wedge \mathcal{X}_i \cap \mathcal{X}_j \neq \emptyset\}$ , is a tree. The elements of a chain mail are called *links*.

Note that a chain mail is a special kind of multiset, in that two links  $\mathcal{X}_i, \mathcal{X}_j$  may be identical only if they are singletons. This makes sense in our context as can be seen from the way we create chain mails in Corollary 1.

To build up the final tree  $T$ , we first want to look at subtrees  $T_i$  that span a link  $\mathcal{X}_i$  and some subset  $Y_i$  of the terminals. Naïvely, we might be inclined to simply select the subtrees cut off in Lemma 1; unfortunately, there might be too many of these to try them all out, since multiple links in a chain mail are allowed to contain the same set of nodes from  $X$ . Thus, in the next lemma, we create bigger subtrees by unifying several links.

**Lemma 2.** *Let  $\varepsilon > 0$ , and let  $T$  be a rooted tree with  $k \geq 1/\varepsilon^2$  terminals. Moreover, let  $X$  be the set given by the algorithm used in the proof of Lemma 1. The subtrees of  $T$ , as cut off by the algorithm, can be mapped to  $3/\varepsilon$  containers such that all the subtrees in each container were cut off at the same node from  $X$  and contain together at most  $\varepsilon k$  terminals.*

*Proof.* Let us extend the algorithm presented in the proof of Lemma 1. Whenever a node is chosen to be added to  $X$ , some amount  $s$  of subtrees with at most  $\varepsilon k$  terminals gets cut off the remaining tree. Create  $s$  new containers, and map each subtree onto one of them. While there are two containers with no more than  $\frac{1}{2}\varepsilon k$  terminals in their subtrees, unite them. When this loop terminates, at most one of the remaining containers has less than  $\frac{1}{2}\varepsilon k$  terminals in its subtrees.

In total, at most  $\varepsilon k$  nodes are added to  $X$ , and hence, at most  $\varepsilon k$  containers with less than  $\frac{1}{2}\varepsilon k$  terminals in their subtrees remain. All other containers have more than  $\frac{1}{2}\varepsilon k$  terminals, and because there are  $k$  terminals in  $T$ , there can be no more than  $k/(\frac{1}{2}\varepsilon k) = 2/\varepsilon$  of them. Altogether, when our extended algorithm terminates, at most  $3/\varepsilon$  containers remain.  $\square$

**Definition 2.** Let  $\varepsilon > 0$ , and let  $T$  be a tree with terminal set  $Y$ ,  $|Y| \geq 1/\varepsilon^2$ . A chain mail  $\mathcal{X}$  is said to be a *mirror of  $T$*  if there are subsets  $Y_1 \cup \dots \cup Y_r = Y$ , such that there are optimal Steiner trees  $T_i$  for the terminal sets  $\mathcal{X}_i \cup Y_i$  which combine to form the entire tree  $T$ . In this case we will also say that  $\mathcal{X}$  *breaks the terminal set into pieces*  $Y_1 \cup \dots \cup Y_r = Y$ . We call this mirror  *$q$ -granular* if furthermore each subset  $Y_i$  is of size at most  $q$ .

**Corollary 1.** *Let  $\varepsilon > 0$ , and let  $T$  be a tree with  $k \geq 1/\varepsilon^2$  terminals. Then there is an  $(\varepsilon k + 1)$ -granular mirror of  $T$  for a base set  $X$  with  $|X| \leq 1/\varepsilon$ , and  $\mathcal{X}$  contains at most  $3/\varepsilon$  links.*

*Proof.* In the proof of Lemma 2, add to each container its respective cut node from  $X$ , and form new links as a union of old ones in the same way we formed containers from subtrees. It is easy to see that the resulting trees are optimal Steiner trees.  $\square$

### 3 Running in chain mail

The following two lemmata play an important rôle for the runtime analysis of our algorithm. We estimate the number of relevant chain mails for a given set  $X$  as well as the number of steps required to compute optimal Steiner trees for some small terminal sets.

**Lemma 3.** *Let  $X$  be a set of nodes and  $m := |X|$ . The number of chain mails for  $X$  with  $3m$  links is bounded by  $m^{2m-2} \binom{3m-1}{m-1}$ .*

*Proof.* Every chain mail induces a tree  $G_{\mathcal{X}} = (\{\mathcal{X}_1, \dots, \mathcal{X}_r\}, E_{\mathcal{X}})$  where  $E_{\mathcal{X}} = \{\{i, j\} \mid \mathcal{X}_i \neq \mathcal{X}_j \wedge \mathcal{X}_i \cap \mathcal{X}_j \neq \emptyset\}$ . We first prove that there are no more than  $m$  inner nodes in  $G_{\mathcal{X}}$ . This bound is tight, as exemplified by the chain mail  $\mathcal{X} = \{\{1\}, \{1, 2\}, \{2, 3\}, \dots, \{m-1, m\}, \{m\}\}$  for  $X = \{1, \dots, m\}$ .

Being a tree,  $G_{\mathcal{X}}$  can be constructed top-down, starting from an arbitrarily picked root. By definition of  $E_{\mathcal{X}}$ , we also have that every node  $\mathcal{X}_i$  in  $G_{\mathcal{X}}$  other than the root shares exactly one element  $h(i)$  from  $X$  — imagine a hook on which the new link dangles — with its parent. It is easy to see that, at any step in the inductive construction of the tree  $G_{\mathcal{X}}$ , for any leaf  $\mathcal{X}_i$  to be transformed into an inner node, a new leaf  $\mathcal{X}_j$  with  $h(j) \in \mathcal{X}_i \setminus \{h(i)\}$  must be added. Observe that  $h(j)$  cannot be used to turn any other leaf into an inner node later, because  $h(j)$  cannot occur in any previously attached link and will be a hook in any new link which contains this element. Now the claim follows from  $|X| \leq m$ .

In order to achieve a non-trivial bound on the number of chain mails, we distinguish two phases in the inductive construction of  $\mathcal{X}$  and  $G_{\mathcal{X}}$ : adding the first  $m$  nodes, some or all of which can become inner nodes in the entire process of building the tree, and attaching  $2m$  leaves. Again, we use both the notion of forming a chain mail and the notion of forming the respective tree.

For the analysis of the first phase, we estimate as follows: Select a hook for each link save the root, and then distribute the remaining nodes from  $X$ . There are  $m-1$  links to select hooks from  $X$  for, yielding  $m^{m-1}$  possibilities. Moreover, at most  $m-1$  nodes remain in so far that they have not been used as hooks, and they have to be assigned to one of  $m$  links, so we can do this in  $m^{m-1}$  ways again. Altogether, the first phase comprises at most  $m^{2m-2}$  alternatives to construct the respective tree.

It remains to estimate the number of ways in which  $2m$  leaves can be added to  $G_{\mathcal{X}}$ . As leaves are singletons (each leaf link in  $\mathcal{X}$  contains exactly one element from  $X$ ), we only need to count the number of ways to choose  $2m$  elements from the set  $\{1, \dots, m\}$  where repetitions are allowed. These are  $\binom{3m-1}{2m}$  many. All in all, we count  $m^{2m-2} \binom{3m-1}{m-1}$  possibilities to generate a chain mail for  $X$ .  $\square$

**Lemma 4.** *Let  $1 > \varepsilon > 0$ ,  $N = (V, E, \ell)$  be a network,  $Y \subseteq V$  a set of terminals with  $|Y| \geq 1/\varepsilon^2$ , and  $\mathcal{X}$  a chain mail for some  $X$  with  $|X| \leq 1/\varepsilon$ . Optimal Steiner trees for all sets of the form  $\mathcal{X}_i \cup Y'$  where  $\mathcal{X}_i \in \mathcal{X}$  and  $Y' \subseteq Y$  with  $|Y'| \leq \varepsilon|Y| + 1$  can be computed in time*

$$O(|\mathcal{X}| \cdot (\varepsilon^\varepsilon(1-\varepsilon)^{1-\varepsilon})^{-|Y|} \cdot 3^{\varepsilon|Y|} \cdot n^3).$$

*Proof.* There are  $|\mathcal{X}|$  links, and for each link all small subsets of  $Y$  have to be considered. By an application of the Stirling formula, these are at most

$$\binom{|Y|}{\varepsilon|Y|+1} = O\left((\varepsilon^\varepsilon(1-\varepsilon)^{1-\varepsilon})^{-|Y|}\right)$$

many. For each combination of a link and such a set the Dreyfus-Wagner algorithm is called, whose running time is bounded by  $O(3^{1/\varepsilon+\varepsilon|Y|+1} \cdot n^3)$  in any such case.  $\square$

## 4 Retrieving the Holy Grail

Having established all the preliminaries, we can now conclude with the central theorem of this paper.

Let  $\mathcal{X} = \{\mathcal{X}_1, \dots, \mathcal{X}_r\}$  be a chain mail for a set of nodes  $X$ . Without loss of generality, let  $\mathcal{X}_1, \dots, \mathcal{X}_r$  be a sequence of links such that its every prefix forms a tree just as  $\mathcal{X}$  itself. Consider the following algorithm, which constructs a table of Steiner trees:

1. For every link  $\mathcal{X}_i \in \mathcal{X}$  and every  $Y' \subseteq Y$  where  $|Y'| \leq \varepsilon|Y| + 1$ , use the Dreyfus-Wagner algorithm to generate an optimal Steiner tree for  $Y' \cup \mathcal{X}_i$  and save it into the table cell  $T(Y' \cup \mathcal{X}_i)$ .
2. For  $s := 2$  to  $|\mathcal{X}|$  do:  
 Let  $X' := \mathcal{X}_1 \cup \dots \cup \mathcal{X}_s$ . Generate a Steiner tree with terminal set  $Y' \cup X'$  for each  $Y' \subseteq Y$  where  $|Y'| \leq s\varepsilon|Y| + s$  as follows: Minimize the expression  $\ell(T(Y'' \cup \mathcal{X}_s) + \ell(T((Y' \setminus Y'') \cup (X' \setminus \mathcal{X}_s)))$  over all  $Y'' \subseteq Y'$  with  $|Y''| \leq \varepsilon|Y| + 1$  and  $|Y' \setminus Y''| \leq (s-1)\varepsilon|Y| + s - 1$ , keeping the best resulting tree. Save this tree into the table cell  $T(Y' \cup X')$ .

**Theorem 1.** *Let  $\mathcal{X}$  be an  $(\varepsilon|Y| + 1)$ -granular mirror of an optimal Steiner tree  $\mathcal{T}$  for some terminal set  $Y$  and  $\varepsilon < 0.18$ . Given  $\mathcal{X}$ , the above algorithm computes an optimal Steiner tree for  $Y$  in time*

$$O\left(|\mathcal{X}| \cdot 2^{|\mathcal{X}|} \cdot (\varepsilon^\varepsilon(1-\varepsilon)^{1-\varepsilon})^{-|\mathcal{X}|} \cdot n^3\right).$$

*Proof.* We will reuse the notation from the description of the algorithm. Obviously, the table contains only Steiner trees. The mirror  $\mathcal{X}$  of  $\mathcal{T}$  breaks the terminal set into pieces  $Y_1 \cup \dots \cup Y_{|\mathcal{X}|} = Y$ . The correctness of the algorithm follows from the fact that for each  $1 \leq s \leq |\mathcal{X}|$  the trees  $T(Y' \cup X')$  are optimal for  $Y' = Y_1 \cup \dots \cup Y_s$  and  $X' = \mathcal{X}_1 \cup \dots \cup \mathcal{X}_s$ . In  $\mathcal{T}$  we have an optimal Steiner tree that contains  $X$ , and thus, any optimal Steiner tree for  $Y \cup X$  is a solution to the Steiner tree problem for  $Y$ .

To see the above fact, note first that it is true for  $s = 1$  if we believe in the correctness of the Dreyfus-Wagner algorithm, and that furthermore all the entries computed in the first phase are optimal. For the induction step, consider  $T(Y' \cup X')$  where  $Y' = Y_1 \cup \dots \cup Y_s$  and  $X' = \mathcal{X}_1 \cup \dots \cup \mathcal{X}_s$ . As  $\mathcal{X}$  is a mirror of the optimal Steiner tree  $\mathcal{T}$ , there is a subtree  $T'$  of  $\mathcal{T}$  that is optimal for  $Y' \cup X'$ . In contradiction to our claim, suppose that  $T'$  is cheaper than  $T(Y' \cup X')$ . Obviously,  $\{\mathcal{X}_1, \dots, \mathcal{X}_s\}$  is also a mirror of  $T'$ . Therefore, we can split  $T'$  at the articulation node that is the only element in  $\mathcal{X}_{s-1} \cap \mathcal{X}_s$  to obtain two optimal Steiner trees for the respective subsets of the terminals. Because  $\ell(T') < \ell(T(Y' \cup X'))$ , this implies that at least one of both is better than the respective table entry. This contradicts the induction hypothesis.

On the running time: A numerical analysis of the bound in Lemma 4 shows that the first phase takes less than  $O(2^{|\mathcal{X}|} \cdot n^3)$  for  $\varepsilon \leq 0.18$ . Observe that the precondition  $|Y| \geq 1/\varepsilon^2$  can be assumed to be fulfilled without loss of generality, since the running time is  $O(\text{poly}(n))$  if  $|Y|$  is bounded by a constant.

The second phase consists of  $|\mathcal{X}| - 1 < |\mathcal{X}|$  steps. In each of these, all the  $Y' \subseteq Y$  of limited size must be looked at. These are less than  $2^{|\mathcal{X}|}$  sets. Again,



for each  $Y'$ , we need to go through all bipartitions where the size of the smaller class  $Y''$  is at most  $\varepsilon|Y| + 1$ . As in the proof of Lemma 4, there are no more than

$$\binom{|Y|}{\varepsilon|Y| + 1} = O\left(\left(\varepsilon^\varepsilon(1 - \varepsilon)^{1-\varepsilon}\right)^{-|Y|}\right)$$

of those.

Altogether, the running time of the algorithm is dominated by the second phase and thus bounded by the product of these three values, multiplied by some polynomial which does not exceed  $n^3$ .  $\square$

Note that this runtime bound still holds if  $\mathcal{X}$  is just a chain mail, but not an  $(\varepsilon|Y| + 1)$ -granular mirror of any optimal Steiner tree. The above algorithm may just not find an optimal Steiner tree in this case.

**Corollary 2.** *Let  $N = (V, E, \ell)$  be a network with  $k$  terminals and  $0 < \delta$ . We can solve the respective Steiner tree problem in time  $O((2 + \delta)^k \cdot \text{poly}(n))$ .*

*Proof.* The crucial factor in the bound from Theorem 1 is  $(\varepsilon^\varepsilon(1 - \varepsilon)^{1-\varepsilon})^{-|Y|}$ . As  $\varepsilon$  approaches 0, the base in this expression converges towards 1. Hence, there is an appropriate  $\varepsilon > 0$  such that  $(\varepsilon^\varepsilon(1 - \varepsilon)^{1-\varepsilon})^{-|Y|} = O((1 + \delta/2)^{|Y|})$ . Using the theorem, we thus get a running time of

$$O(|\mathcal{X}| \cdot (2 + \delta)^{|Y|} \cdot n^3)$$

for each chain mail we consider.

By Corollary 1, there exists a  $(1/\varepsilon + 1)$ -granular mirror  $\mathcal{X}$  of size at most  $3/\varepsilon$  for an optimal Steiner tree in our network, whose base set  $X$  satisfies  $|X| \leq 1/\varepsilon$ . Unfortunately, we do not know how to choose  $\mathcal{X}$  or even  $X$ . However, by Lemma 3, there are no more than

$$O\left(|X|^{2|X|-2} \binom{3|X| - 1}{|X| - 1}\right)$$

different chain mails for a fixed  $X$ . Clearly, there are less than  $n^{1/\varepsilon}$  different sets  $X \subseteq V$  with  $|X| \leq 1/\varepsilon$ . Thus the total number of chain mails to go through is  $O(n^{1/\varepsilon})$ . In total, we get an asymptotic running time of

$$O(n^{1/\varepsilon} \cdot |\mathcal{X}| \cdot (2 + \delta)^{|Y|} \cdot n^3) = O((2 + \delta)^k \cdot \text{poly}(n)).$$

$\square$

## 5 Valor and sincerity

In this section, we take a closer look at the influence of  $\varepsilon$  on the running time.

Corollary 2 states that the Steiner tree problem can be solved in time  $O(c^k \cdot \text{poly}(n))$  for any  $c > 2$  (given that  $k$  is sufficiently large, as  $\varepsilon k \geq 1$  is a precondition for Lemma 1). However, this bound obscures the combinatorial explosion occurring as  $\varepsilon$  approaches 0, and we emphatically point out that we cannot expect the algorithm to be applicable for very small  $\varepsilon$  since it may have to go through too many chain mails.

On the other hand, the estimation on the number of chain mails in Lemma 3 is rather rough. In particular, the analysis of the first phase does not exploit the fact that only those nodes from the base set that were not used as hooks need to be distributed to the links later. Moreover, we consider chain mails  $\mathcal{X}$  of up to  $3/\varepsilon$  links, but we have yet to find an instance where such a huge chain mail is required as an appropriate mirror.

Let us recall some of the estimations from above, and put them together. For the sake of readability, we write  $a/\varepsilon$  instead of  $\lfloor a/\varepsilon \rfloor$ . By Lemma 3, the total number of chain mails to consider is bounded by

$$\binom{n}{1/\varepsilon} \cdot 1/\varepsilon^{2/\varepsilon-2} \binom{3/\varepsilon-1}{1/\varepsilon-1}.$$

We already know that the running time of the core algorithm is dominated by its second phase, as seen in the proof of Theorem 1. It performs at most

$$3/\varepsilon \cdot 2^k \cdot \left( \frac{1}{\varepsilon^\varepsilon (1-\varepsilon)^{1-\varepsilon}} \right)^k$$

many steps per chain mail.

To exemplify the combinatorial explosion, we give the resulting bounds for several values of  $\varepsilon$ . Observe that the runtime bound for the second phase given by Theorem 1 includes the expression  $2^k \cdot (\varepsilon^\varepsilon (1-\varepsilon)^{1-\varepsilon})^{-k}$ . This is less than  $3^k$  for  $\varepsilon$  smaller than about 0.1402. In the following table, the number of chain mails per  $X$  determines the constant factor in the running time, while the number of node sets  $X$  determines the polynomial in the asymptotic bound.

$\varepsilon$	chain mails per $X$	total asymptotic runtime
1/8	$1.0783 \cdot 10^{18}$	$O(2.9152^k \cdot n^{11})$
1/20	$3.8409 \cdot 10^{64}$	$O(2.4392^k \cdot n^{23})$
1/50	$2.1172 \cdot 10^{208}$	$O(2.2061^k \cdot n^{53})$

The picture we come across here is not an unknown one. The tireless wanderer has encountered similar phenomena in the wide field of polynomial time approximation schemes (PTAS). In both cases, small  $\varepsilon$  lead to humongous constants and large exponents for the polynomials. The difference lies in what we try to achieve by choosing smaller and smaller  $\varepsilon$ . Whereas we try to get the base of the exponential part of the runtime bound as close to two as possible, approximationists crave optimality.

## 6 Epilogue

In terms of parameterized complexity, the algorithm presented in this paper is the asymptotically fastest solution to the Steiner tree problem today. While our algorithm is only a bit more complicated than the one by Dreyfus and Wagner, the constants hidden in the Landau notation can become very large if  $\varepsilon$  is chosen too small. This is because we have to go through all the chain mails with  $\lfloor 3/\varepsilon \rfloor$  links for all the  $\lfloor 1/\varepsilon \rfloor$ -element subsets of  $V$ . While there are only  $\binom{n}{\lfloor 1/\varepsilon \rfloor}$  ways to choose an  $X \subseteq V$  with  $|X| = \lfloor 1/\varepsilon \rfloor$ , the number of relevant chain mails grows very rapidly in  $1/\varepsilon$ , as detailed above.

Hence, in practice, our algorithm is very likely to be slower than the Dreyfus-Wagner algorithm for small values of  $n$  or  $k$  (or if  $\varepsilon$  is chosen badly). On the other hand, there is plenty potential for improvement, such as omitting sets  $X$  or chain mails  $\mathcal{X}$  that cannot lead to optimal solutions. To enable more precise statements, work is underway on implementing both algorithms for a comparative analysis. Furthermore, we plan on reducing the number of nodes in  $X$  and of chain mails  $\mathcal{X}$  by dynamically adapting the amount of terminals a link is responsible for.

One might imagine that the basic technique employed lends itself to generalization. That is, it should be worthwhile to look at graph algorithms using a dynamic programming approach similar to the Dreyfus-Wagner one, and try to choose more than one point of combination in order to balance the size of subproblems.

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Knights of the LuFGTI  
 Daniel “Gurnemanz” Mölle  
 Stefan “Gawan” Richter  
 Peter “Parzival” Rossmanith

*Historical Note.* Did you know? When we weigh the authors’ years of birth according to their respective ratio of contribution, their average yields exactly the date the seminal paper by Dreyfus and Wagner was published.

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