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A Feedforward-Feedback Interpretation of a Sliding Mode Control Law[*]

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Abstract

In this paper we provide a feedforward-feedback interpretation of a sliding mode control scheme. Given a desired trajectory, the feedforward signal is generated using a stable inversion method, and the feedback signal includes the switching term of the sliding mode control law. In this manner, we introduce robustness into the stable inversion technique. This approach is motivated by the need to replicate time signals typically in the automobile industry. The application of such an interpretation to a quarter car benchmark model yields encouraging results. Special attention will be given to non-minimum phase systems illustrated by a simulation example of the lunar roving vehicle.

1   Introduction

Sliding Mode Control is a well-known technique capable of making the closed loop system robust with respect to certain kinds of parameter variations and unmodeled dynamics. Sliding mode controllers have proven their effectiveness in several applications diverse in their physical nature and fundamental purpose. Specifically, this method provides an easy way to design trajectory tracking control laws for a plant, linear or nonlinear.

Increasingly stringent performance requirements to be satisfied in a variety of applications justify the fact that precision output tracking remains one of the fundamental problems for control engineers. In the context of linear systems, it is well-known that perfect tracking is relatively easy to achieve in minimum phase systems. However, output tracking for non-minimum phase systems remains a challenging problem due to the fundamental limitations on the transient tracking performance characterized by the number and location of the zeros which are non-minimum phase (Qiu and Davison, 1993). For linear continuous-time systems, Francis and Wonham (1976) show that the asymptotic tracking problem is solvable if, and only if, a set of linear matrix equations is solvable. This was later generalized to nonlinear systems by replacing the linear matrix equations by a set of first order partial differential equations (Isidori and Byrnes, 1990). These approaches asymptotically track any member in a given family of signals
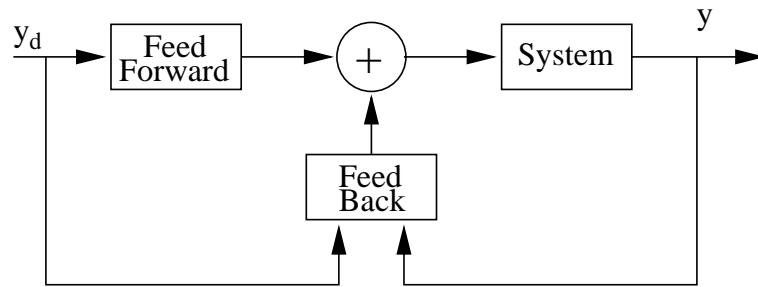
Figure 1: Sliding mode control divided in two parts.

generated by an exosystem. The stable inversion approach was introduced by Devasia *et al.* (1996) to avoid the use of exosystems, and, in the case of non-minimum phase systems, improve the transient performance by using pre-actuation. In Section 2 we summarize this stable inversion approach for nonlinear, possibly non-minimum phase, systems.

It has been suggested (Devasia *et al.*, 1996) that the inverse trajectory becomes a feedforward signal used in conjunction with a more conventional feedback control law in order to make it attractive. The stable inversion approach to output tracking is based on the asumption of perfect knowledge of the system. In the more realistic case of parametric variations and unmodeled dynamics, this approach to the problem seems rather ill-posed. In Section 3, we combine the advantages of the sliding mode and inversion based control techniques to design a control law that not only yields perfect tracking in the case where perfect knowledge of the plant is available, but as well makes the system robust to matched parameter variations and uncertainties. By providing a feedforward-feedback interpretation of a typical sliding mode control law, we fulfill our task in a rather convenient way. This interpretation is illustrated in Fig 1. In this figure the sliding mode control law is expressed as a sum of a feedforward signal and a feedback signal. We compute the feedforward signal using the inversion based technique, and the feedback signal includes the switching part of the sliding mode controller. Thus, the feedback part of the signal attempts to compensate for the inadequacies of the nominal model of the plant. Thereby, we introduce robustness in a natural way to the stable inversion technique which evidently depends on the parameters of the system.

We note that this paper was motivated by the need for time waveform replication which concerns with accurate reproduction of real or synthesized target time histories. Thus complex vibration environments (such as automobile crashes) may be recreated in a test laboratory by simulating field measurements thereby saving precious resources. Other applications include durability tests of, for instance, automobile components, and driving comfort assessment.

We emphasize that other techniques, for instance (J. Sjöberg and Ararwal, 1985; Spall and Cristion, 1998; Hjalmarsson *et al.*, 1998), could be used to compute the feedback signal. In each of these techniques the controller structure is typically fixed *a priori* and the controller parameters are optimally adjusted using the knowledge of the gradient of some cost function computed in different ways. We note that in these techniques the gradient is evaluated experimentally, and therefore no knowledge of the error dynamics is required. However, several experiments are needed, and hence the convergence to the optimal parameters takes some time. On the contrary, no specific controller structure need to be chosen to compensate for specific errors in the case of sliding mode control laws.

Preliminary investigations on the application of the combined stable inversion and sliding mode techniques to a quarter car benchmark model are encouraging (Section 4.1). Indeed, the

sliding mode is found to cope reasonably well with nonlinearities and parameter variations despite the fact that the feedforward signal has been computed using only the linearized model. Simulation results for a non-minimum phase system (Section 4.2) show that although the feedforward controller can be computed by a stable inversion technique, the closed loop system (closed with the sliding mode terms) still could show an undesired oscillatory behavior.

2 Inversion-Based Control Law

The stable inversion approach of Devasia *et al.* (1996) aims at providing a bounded inverse of nonlinear, possibly non-minimum phase systems. This bounded inverse is computed by solving a two point boundary value problem obtained via a dichotomic split of the internal dynamics of the system. This results in an acausal input in the case of non-minimum phase systems; the anti-causal part of the input sets up the desired initial condition. In this section we summarize this stable inversion approach. We note that for minimum phase systems this inverse is related to the classical system inverse (Hirschorn, 1979).

Consider the following SISO nonlinear system affine in the input:

$$
\begin{aligned}
\dot{x}(t) &= f(x(t)) + g(x(t))u(t) \\
y(t) &= h(x(t))
\end{aligned}
\tag{1}
$$

where $x(t) \in \mathbb{R}^n$ and $f(x)$, $g(x)$, and $h(x)$ are smooth functions defined on an open set in $\mathbb{R}^n$ (the time variable will be omitted in the rest of the paper). The nonlinear system (1) is said to have a well-defined relative degree r at a point x_0 if $L_g L_f^k h(x) = 0$ for all $k < r - 1$, and for all x in a neighborhood of x_0, and $L_g L_f^{r-1} h(x_0) \neq 0$. Without loss of generality, we assume $x_0 = 0$.

Given a smooth desired trajectory $y_d \in \mathcal{L}_1 \cap \mathcal{L}_\infty$, the stable inversion problem (Devasia *et al.*, 1996) is to find a bounded u_d and a bounded x_d such that $u_d(\pm\infty) = 0$ and $x_d(\pm\infty) = 0$ which satisfies (1) and perfect tracking $y(\cdot) = y_d(\cdot)$ is achieved. Let

$$
\nu_1 \triangleq \left(\begin{array}{cccc} y & \dot{y} & \cdots & y^{(r-1)} \end{array} \right)^T
$$

where $y^{(i)}$ denotes the ith time derivative of y, and ν_2, an $n - r$ dimensional function on $\mathbb{R}^n$, be chosen such that

$$
\left(\begin{array}{c} \nu_1 \\ \nu_2 \end{array} \right) = \psi(x)
\tag{2}
$$

forms a change of coordinates. In these new coordinates, the essence of the system dynamics is represented by:

$$
\begin{aligned}
y^{(r)} &= \alpha(\nu_1, \nu_2) + \beta(\nu_1, \nu_2)u \\
\dot{\nu_2} &= s_1(\nu_1, \nu_2) + s_2(\nu_1, \nu_2)u
\end{aligned}
\tag{3}
$$

The feedback control law

$$
u = \beta^{-1}(\nu_1, \nu_2) \left(y_d^{(r)} - \alpha(\nu_1, \nu_2) \right)
\tag{4}
$$

is well-defined in a neighborhood of the origin and partially linearizes the system. Under this condition,

$$
\nu_1 = \nu_{1d} \triangleq \left(\begin{array}{cccc} y_d & \dot{y}_d & \cdots & y_d^{(r-1)} \end{array} \right)^T
$$

The zero dynamics driven by the reference output trajectory is therefore

$$
\dot{\nu_2} = s_1(\nu_{1d}, \nu_2) + s_2(\nu_{1d}, \nu_2)u \triangleq s(\nu_2, Y_d)
$$

where $Y_d(\cdot)$ represents $\nu_{1d}(\cdot)$ and $y_d^{(r)}(\cdot)$, and

$$u = \beta^{-1}(\nu_{1d}, \nu_2) \left(y_d^{(r)} - \alpha(\nu_{1d}, \nu_2) \right)$$

The inversion problem is thus reduced to finding bounded Carathéodory solutions (Hale, 1980) to the possibly unstable non-linear differential equations $\dot{\nu}_2 = s(\nu_2, Y_d)$ subject to the boundary conditions $\nu_2(\pm\infty) = 0$. It is shown (Devasia *et al.*, 1996; Devasia and Paden, 1998) that bounded solutions exist provided certain regularity conditions on the Jacobian linearization A_s of $s(\nu_2, Y_d)$ are met: s satisfies a local Lipschitz condition, the difference between s and A_s is bounded, and A_s is hyperbolic in the sense that it has no eigenvalues on the imaginary axis. Under these conditions, starting from an initial guess ν_2^0, we iteratively solve for ν_2 as follows:

$$\nu_2^{i+1}(t) = \int_{-\infty}^{\infty} \phi(t-\tau) \left\{ s\left(\nu_2^i(\tau), Y_d(\tau) \right) - A_s \nu_2^i(\tau) \right\} d\tau$$

Here ϕ is the bounded state transition matrix associated with A_s. We note that the convergence of the sequence $\{\nu_2^i\}$ is guaranteed by the regularity conditions stated earlier. Once the desired ν_{2d} is computed, the desired state trajectory is obtained by the inverse coordinate transformation $x_d = \psi^{-1}(\nu_{1d}, \nu_{2d})$ and the desired input trajectory by

$$u_d = b^{-1}(x_d) \left(y_d^{(r)} - a(x_d) \right) \tag{5}$$

where $a(x) = \alpha(\psi(x))$ and $b(x) = \beta(\psi(x))$.

We note that a major drawback of the inversion based approach is that it is applicable for systems that do not have zeros on the imaginary axis, and does not take into account the noise in target time histories. A method that overcomes these problems is suggested in (George *et al.*, 1999) for linear discrete time systems. However, the feedforward controller for the case of non-minimum phase systems generates a signal that is anti-causal, and therefore, such schemes are applicable where trajectory preview is permitted, or in situations wherein the performance with a truncated anti-causal feedforward signal is reasonable.

In this section we have summarized the stable inversion technique for nonlinear systems. The necessary feed-forward input can be computed by obtaining first the desired state trajectory. We emphasize that the stable inversion technique yields perfect tracking for systems where there are no parametric variations or unmodeled dynamics. In the next section we introduce robustness through a sliding mode control law.

3   Sliding Mode Controller

In this section we present a feedforward-feedback interpretation of a sliding mode controller. We introduce a sliding mode controller which turns out to be a combination of the inversion-based control law defined in the previous section and a feedback term (Section 3.1).

Despite the fact that we have a stable inversion technique for inverting non-minimum phase systems, Section 3.2 points out that this does not entirely solve the problems for output-based sliding mode controllers in relation to non-minimum phase systems.

3.1   Sliding Mode Control Law

We now design a sliding mode control law for the system (1). Several definitions for the sliding surface exist. In (Slotine and Li, 1991) the sliding surface is defined as the weighted sum of the

errors between the desired states and the actual states. In this paper we use the definition given in (Lin, 1994):

$$s = e^{(r-1)} + c_{r-1}e^{(r-2)} + \cdots + c_1 e + c_0 \int e \, dt \tag{6}$$

with $e = y - y_d$ (y being the actual system output and y_d the desired system output) and the constants c_i are such that

$$\lambda_r^r + c_{r-1}\lambda_r^{r-1} + \cdots + c_1\lambda + c_0$$

is a Hurwitz polynomial. The (assumed to be) constant r is the relative degree of the system (1) which was introduced in the previous section. The advantage of a sliding surface as defined in (6) compared to one defined in terms of the states is that it is a function of the known output of the system, instead of the generally unknown states. The derivative of s is given by:

$$\dot{s} = e^{(r)} + c_{r-1}e^{(r-1)} + \cdots + c_1\dot{e} + c_0 e$$

By substituting equation (3) (with $a(x) = \alpha(\psi(x))$ and $b(x) = \beta(\psi(x))$) with $e^{(r)} = y^{(r)} - y_d^{(r)}$ in the above equation we get:

$$\dot{s} = a(x) + b(x)u - y_d^{(r)} + \underbrace{c_{r-1}e^{(r-1)} + \cdots + c_0 e}_{=\sum_{i=0}^{r-1} c_i e^{(i)}}$$

If we now choose u to be:

$$u = b^{-1}(x)\left\{ y_d^{(r)} - a(x) - \sum_{i=0}^{r-1} c_i e^{(i)} - k\,\mathrm{sgn}(s) \right\} \tag{7}$$

we fulfill the sliding condition

$$\frac{1}{2}\frac{ds^2}{dt} \leq -\eta|s| \tag{8}$$

if $k > \eta$. If we have a closer look at (7) we recognize that one part of it is the inversion-based control law given in equation (5). Therefore:

$$u = u_{inv} + u_{slid} \tag{9}$$

where

$$u_{inv} \triangleq b^{-1}(x)\left\{ y_d^{(r)} - a(x) \right\} \tag{10}$$

$$u_{slid} \triangleq -b^{-1}(x)\left\{ k\,\mathrm{sgn}(s) + \sum_{i=0}^{r-1} c_i e^{(i)} \right\} \tag{11}$$

The expression u_{inv} is identical to the expression for u_d in the previous section (equation 5) so the same techniques can be applied to determine this control law. The expression for u_{slid} represents the sliding mode strategy. The *switching term* $-b^{-1}(x)k\,\mathrm{sgn}(s)$ drives the system to the sliding surface s and the *sliding term* $-b^{-1}(x)\sum_{i=0}^{r-1} c_i e^{(i)}$ "slides" the system along the sliding surface to the desired (time-varying) position $[y_d^{(r-1)} \quad \cdots \quad \int y_d dt]^T$.

The switching term $-k\,\mathrm{sgn}(s)$ will introduce high frequency components to the control signal which may excite high-frequency dynamics which were neglected in the modeling procedure

(Slotine and Li, 1991). One way of reducing this effect is by "softening" the switching law, by for example a saturation function as defined below:

$$u_s = -b^{-1}(x)k\,\mathrm{sat}(\frac{s}{\Phi}) = \begin{cases} -b^{-1}(x)k & s > \Phi \\ -b^{-1}(x)k\frac{s}{\Phi} & |s| < \Phi \\ b^{-1}(x)k & s < -\Phi \end{cases}$$

The high frequency components in the control action are now reduced but stability is only guaranteed outside the region Φ (Slotine and Li, 1991). Also the effect of chattering is reduced by the use of this "softened" switching law.

The switching gain k is a trade-off between accuracy and actuator demands. The gain should be as high as possible to be able to compensate for large modeling errors. On the other hand the gain should be as small as possible to relax the controller demands. For this reason we want the gain k to be large enough to satisfy the sliding condition (we call this value the *optimal gain* k_o). Three ways to determine k are:

- **Known maximum error:** If we assume that there is a modeling error in $a(x)$, we can redefine $a(x)$ by:
$$a(x) = \hat{a}(x) + \Delta a(x)$$
  where $\hat{a}(x)$ is the nominal value of $a(x)$ and $\Delta a(x)$ is bounded to some constant D (thus $|\Delta a(x)| < D$). k should then satisfy

$$k \geq \eta + D$$

  to fulfill the sliding condition (8) (Lin, 1994). This rule is extended to uncertainties in $b(x)$ in (Slotine and Li, 1991).

- **By Experimentation:** In practice, where we generally do not have a known bound on the model-error, the parameter k can be tuned in a few experiments.

- **Adaptive Gain:** In (Lenz *et al.*, 1998) an adaptive gain k is introduced. The switching term $b^{-1}(x)k\,\mathrm{sgn}(s)$ is replaced by the term $b^{-1}(x)k\,\mathrm{sat}(\frac{s}{\Phi})$ and the gain k is given by:

$$k = \int \left(|s| - \Psi\right) dt$$

  where Ψ is constant. In the formula for k one can see that $|s|$ takes care of the increase of k and Ψ takes care of the decrease of k. In (Lenz *et al.*, 1998) the proof is given that k will converge to the optimal gain k_o provided that $0 < \Psi < \Phi$ (Lenz *et al.*, 1998).

  A sliding mode controller can compensate for any matched uncertainty (uncertainties are "matched" if they lie in the subspace spanned by a basis of $g(x)$). However, in the presence of unmatched uncertainties, it is not possible for the sliding control law to steer the system in such a way that the variable s will be arbitrarily small. The adaption mechanism will break down in these circumstances since the gain will increase until s will stay within the defined limits which might not happen.

3.2 Non-minimum phase systems

Sliding mode controllers based on output information cannot be applied to non-minimum phase systems (Edwards and Spurgeon, 1998),(Lin, 1994). One reason for this limitation lies in the fact that output based sliding mode controllers have (as we have exploited in this paper) a

similar structure as inversion based controllers (Lin, 1994). If a non-minimum phase system is straightforwardly inverted then the zeros of the system become the poles of the controller and hence we have designed an unstable controller. This problem can be solved by the use of stable inversion techniques.

The question is now of course whether we have solved the limitation of sliding mode controllers to minimum phase systems or not. The answer seems to be no, which is also demonstrated with the simulation results for the lunar roving vehicle (Section 4.2) which is a non-minimum phase system. By applying the stable inversion technique we have designed a stable controller. But unless the feedforward controller completely cancels the non-minimum phase behavior of the system, the sliding mode feedback term will still act on a non-minimum phase system and therefore the use of the sliding mode controllers should still be limited to minimum phase systems. Interesting to note is that (as can be seen in Section 4.2) the closed loop system is marginally stable, the closed loop system will oscillate but does not become unstable.

4   Simulation Results

The feedforward-feedback interpretation will be demonstrated by the use of two examples. The first example is the quarter car model which describes one quarter of a car placed on a testbench. The second example is the angle control of the lunar roving vehicle which shows a non-minimum phase behavior.

4.1   Application to the Quarter Car Model

In this section we will apply the feedforward-feedback interpretation of a sliding mode controller introduced in the previous section to a special application in the industry of automobiles. To improve reproducibility of test procedures for cars as well as durability tests of new developed cars, one would like to have a test setup where a car is placed on four shakers (actuators) called the base, which simulate the behavior of the car on the road. In other words, the shakers should be controlled in such a way that the car vibrates in the desired way. This "desired way" is given as a reference signal which could be measured on a car during driving conditions.

In this paper we only look to one quarter of the car (which is the reason for the name "Quarter Car Model"), only one wheel is considered. A full-car model would be composed of four (or in the case of trucks even more) Quarter Car models which are connected in a fairly complex way to represent the interdependencies between the "Quarters". We model the tire by a simple spring (with stiffness c_w) and the connection of the wheel to the chassis by a combination of a (nonlinear) spring and a (nonlinear) damper (stiffness c_c and damping coefficient d_c), figure 2 demonstrates this. The variables x_c, x_w and x_b represent the car, wheel and base displacement respectively. The input to the system is the base displacement (x_b), the acceleration of the car ($\ddot{x}_c$) is the output.

$$\dot{x} \;=\; f(x,u) \;=\; Ax + \hat{f}(x) + Bu \tag{12}$$
$$y \;=\; h(x) \;=\; Cx + \hat{h}(x) \tag{13}$$

Where:

$$x \;=\; [x_c \quad x_w \quad \dot{x}_c \quad \dot{x}_w]^T$$
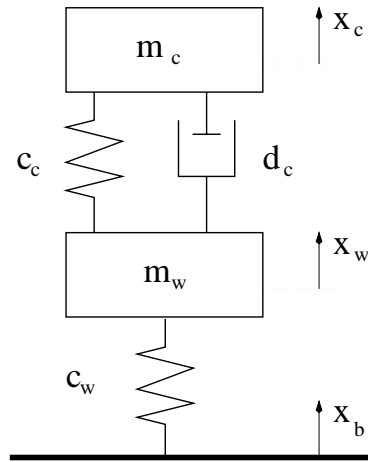
Figure 2: Mechanical diagram of the Quarter Car Model.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{c_c}{m_c} & \frac{c_c}{m_c} & -\frac{d_c}{m_c} & \frac{d_c}{m_c} \\ \frac{c_c}{m_w} & -\frac{c_c+c_w}{m_w} & \frac{d_c}{m_w} & -\frac{d_c}{m_w} \end{bmatrix}$$

$$\hat{f}(x) = \begin{bmatrix} 0 \\ 0 \\ -\gamma\frac{c_c}{m_c}(x_c - x_w)^2 - \delta\frac{d_c}{m_c}(\dot{x}_c - \dot{x}_w)^2 \\ \gamma\frac{c_c}{m_c}(x_c - x_w)^2 + \delta\frac{d_c}{m_c}(\dot{x}_c - \dot{x}_w)^2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{c_w}{m_w} \end{bmatrix}^T$$

$$C = \begin{bmatrix} -\frac{c_c}{m_c} & \frac{c_c}{m_c} & -\frac{d_c}{m_c} & \frac{d_c}{m_c} \end{bmatrix}$$

$$\hat{h}(x) = -\gamma\frac{c_c}{m_c}(x_c - x_w)^2 - \delta\frac{d_c}{m_c}(\dot{x}_c - \dot{x}_w)^2$$

Note that the parameters γ and δ determine the importance of the (quadratic) nonlinearities. If $\gamma = 0$ and $\delta = 0$ the model becomes a linear model.

The derivative of y is obtained by differentiating (13) once with respect to time:

$$\dot{y} = \underbrace{CAx + C\hat{f}(x) + \dot{\hat{h}}(x)}_{= a(x)} + \underbrace{CB}_{= b(x)} u$$

where it is interesting to note that in this case $b(x)$ is a constant, i.e. $b(x) = b$. From the above we can conclude that the Quarter Car has relative degree $r = 1$ so we define the sliding surface as:

$$s = e + c\int e \, dt$$

with $e = y - y_d$ and $c > 0$ to satisfy the requirement that $\lambda + c$ is a Hurwitz polynomial. For the derivative of s we can write:

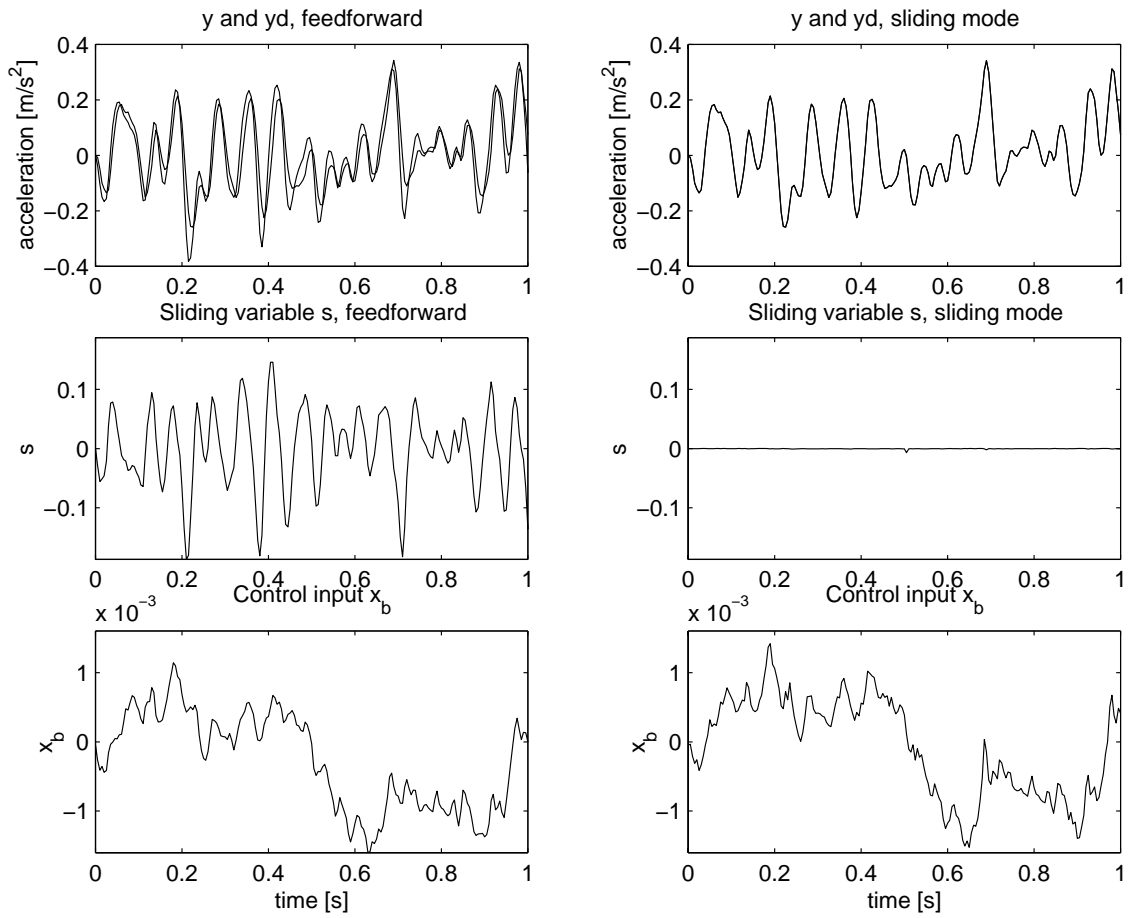$$\dot{s} = \dot{e} + ce = a(x) + bu - \dot{y}_d + ce$$

Figure 3: Simulation results with switching gain $k = 50$. The left figures represent the situation without feedback, the right figures represent the proposed strategy. The top figures display the desired and the actual system output (y_d and y), the middle figures present the sliding variable (s) and the lower figures display the input ($u = u_{inv} + u_{slid}$).
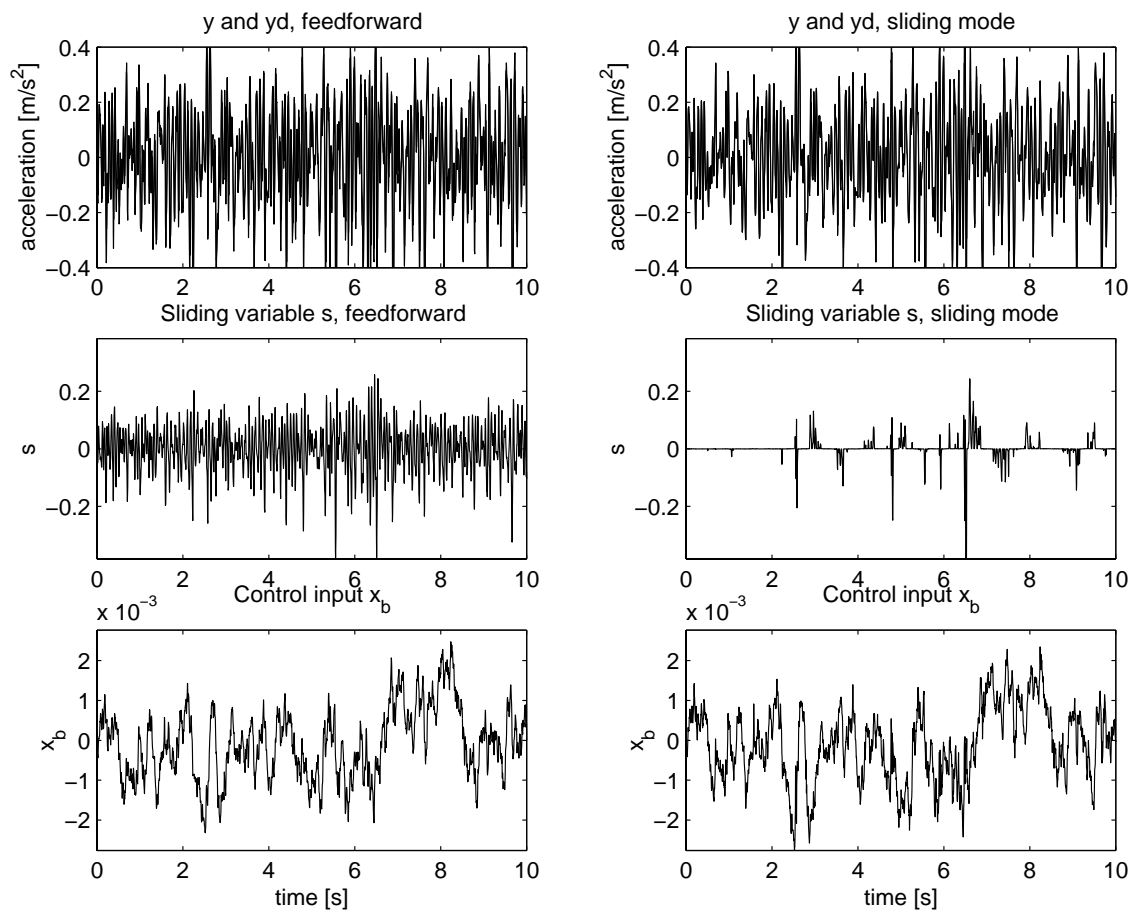
Figure 4: Simulation results with switching gain $k = 50$ for a period of 10 seconds. The left figures represent the situation without feedback, the right figures represent the proposed strategy. The top figures display the desired and the actual system output (y_d and y), the middle figures present the sliding variable (s) and the lower figures display the input ($u = u_{inv} + u_{slid}$).

We now chose $u = u_{inv} + u_{slid}$ where u_{inv} is given by (10) and u_{slid} by (11).

The parameters used in simulation are: $m_c = 200/190$, $m_w = 33/30$, $c_c = 9000/8000$, $c_w = 200000/220000$, $d_c = 1200/1300$, $d_w = 0/0$, $\gamma = 5/0$ and $\delta = 5/0$ (where the first value is used for the system and the second value is used for the inversion-based control law). One can see in the parameters that the inversion-based controller is designed on a linear model with perturbed parameters. Since there are unmatched uncertainties we cannot use the adaptive gain mechanism as was also discovered in simulations (results are not shown for brevity), so we have selected a constant gain. To reduce the load on the actuators we have selected the saturation function $(\text{sat}(\frac{s}{\Phi}))$ instead of the signum function. The parameters for the controller are: $c = 1$, $\Phi = 0.001$ and $k = 50/50$ (first/second/third experiment).

Figure 3 (result for $k = 50$) shows a remarkable improvement of the tracking performance, things look good. Figure 4 shows the results for $k = 50$ (i.e. same circumstances as in figure 3) for a longer time period, here again some peaks in s appear. This could be the result of the fact that k is still to small but it could also be caused by the unmatched uncertainties.

The figures also show the input signals for both the open loop case (only inversion-based control law) as the proposed control law. These figures look quite similar, there are no substancial high frequency terms on top of the inversion-based control law.

So we may conclude that even in the case where the inversion-based controller is designed on a linear model with perturbed parameters, the systems tracks rather well together with the sliding mode feedback term. The use of the saturation function has resulted in a smooth control signal. A higher gain increases the performance but the error cannot be made arbitrarily small since there are unmatched uncertainties.

4.2   Lunar Roving Vehicle

We will now apply the feedforward-feedback interpretation of a sliding mode controller to the angle control of a moon vehicle. This system can be described by the transfer function (Dorf and Bishop, 1995):

$$G(s) = \frac{2e^{-0.1s}}{0.2s + 1}$$

After approximating the delay by a second order Pade approximation the following state-space model can be found:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where $x \in R^3$, $y \in R$, $u \in R$ and

$$A = \begin{bmatrix} -65 & -1500 & -6000 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 10 & -600 & 12000 \end{bmatrix}$$

The poles of this system are at $s = -5$ and $s = -30 \pm 17.3i$ so the open loop system is stable. The zeros of the system are at $s = 30 \pm 17.3i$ so the system is non-minimum phase. Figure 5 shows the simulation results for the (stable inversion based) feedforward controller to the vehicle under ideal circumstances.

Since the relative degree of the system is $r = 1$ we define the switching surface s like in the quarter car example by:

$$s = e + c \int e \, dt$$

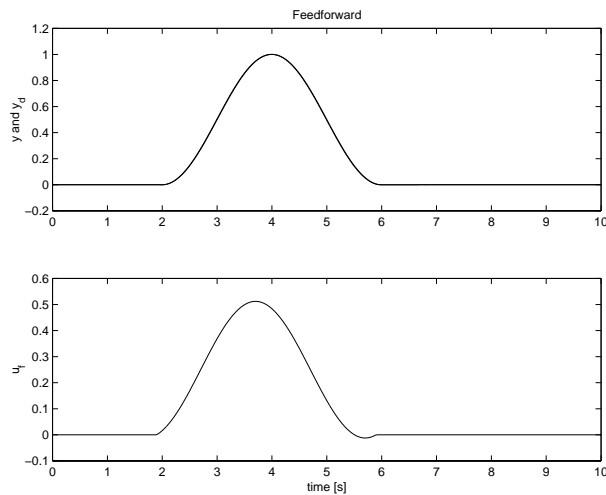with $e = y - y_d$ and $c > 0$ to satisfy the requirement that $\lambda + c$ is a Hurwitz polynomial.

Figure 5: Simulation results for the Lunar Roving Vehicle in feedforward configuration under ideal circumstances (no noise and no model mismatch). The top figure presents the desired and the actual output. The lower figure presents the (bounded) input signal.

Figure 6 shows the simulation results for the feedforward only and the sliding mode controller configuration ($k = 0.9$, $c = 1$ and $\Phi = 1e^{-3}$) where the controllers are designed on the above model but they are applied to the linear system with the matrices $A_s = A + dA$, $B_s = B + dB$ and $C_s = C$ where:

$$ dA = \begin{bmatrix} 6.5 & -100 & 600 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad dB = \begin{bmatrix} -0.1 \\ 0 \\ 0 \end{bmatrix} $$

The figures demonstrate that although the stable inversion technique generates a bounded input signal which would give perfect tracking under ideal circumstances, the feedforward-feedback implementation of a sliding mode controller does not yield the desired robustness. In fact, the closed loop system will start oscillating around the desired position. In this example it can hardly be a surprise that the sliding mode controller runs into problems. A sliding mode controller expects instantaneous switching which is in a system with a delay of course not possible.

5   Conclusions

It was demonstrated that one can easily divide a sliding mode control law in a feedforward term and a feedback term. The feedforward term is identical to an inversion-based control law and the feedback term contains the additional sliding mode terms. A method was demonstrated to determine the inversion-based control law for nonlinear and possibly non-minimum phase systems. The sliding mode feedback law was then added to improve the robustness of the controller.

Results were demonstrated in simulation for an inversion-based control law designed for a linear model and applied to a nonlinear system, for this purpose a Quarter Car model was used. The tracking performance is considerably improved with the added sliding mode feedback terms.

Also simulation results were shown for the lunar roving vehicle which is an example of a non-minimum phase system. The results showed that although the stable inversion technique
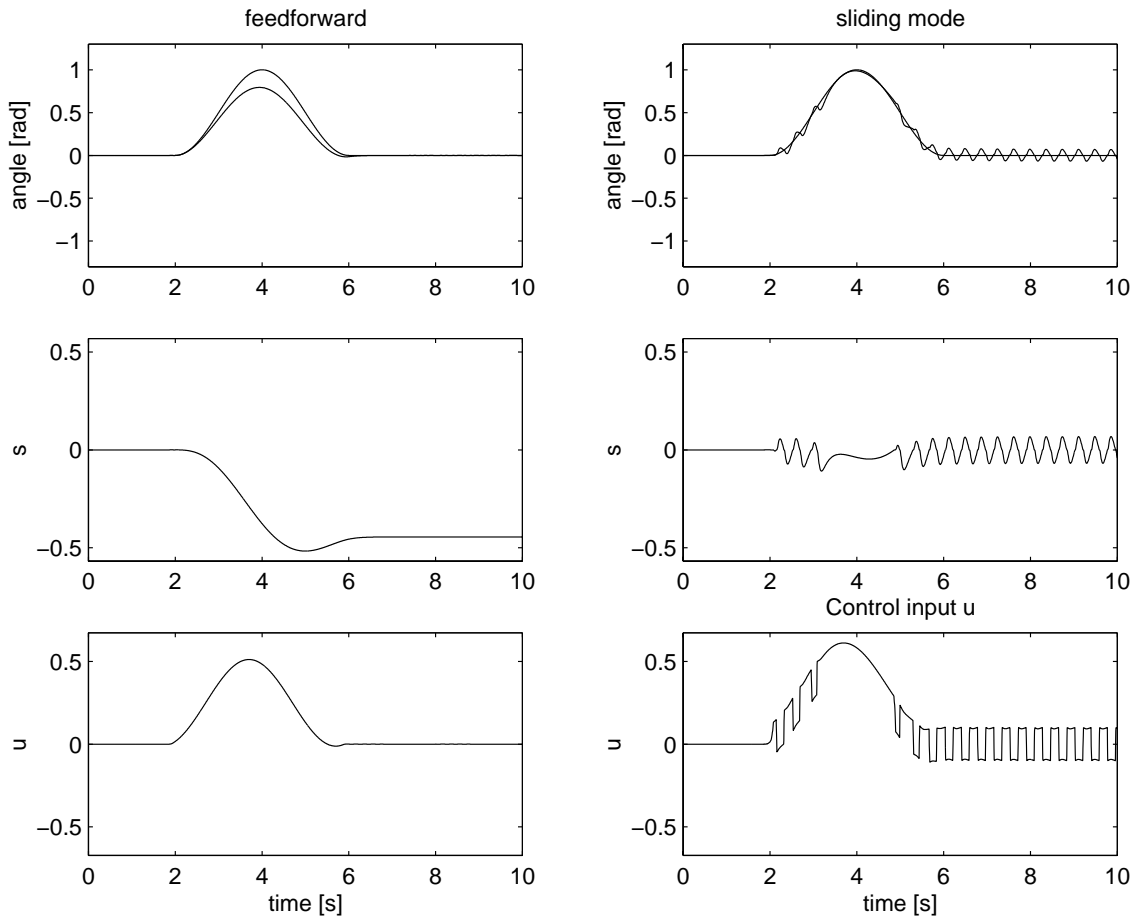
Figure 6: Simulation results for the sliding mode controller applied to the Lunar Roving Vehicle with switching gain $k = 0.5$. The left column of figures represent the feedforward controller only and the right column of figures represent the sliding mode controller results. The top two figures represent the desired and the actual output, the middle figures represent the variables s and the lower figures represent the total input signal in sliding mode $(u_{tot} = u_{inv} + u_{slid})$.

results in a bounded input signal which yields perfect tracking under ideal circumstances (perfect model and no disturbances), the closed loop system suffers from a limit cycle behavior. Further research is required to solve this problem.

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