Peter D. Hoff

## A First Course in Bayesian Statistical Methods



## Contents

1	Int	roduction and examples	1
	1.1	Introduction	1
	1.2	Why Bayes?	2
		1.2.1 Estimating the probability of a rare event	3
		1.2.2 Building a predictive model	8
	1.3	Where we are going	11
	1.4	Discussion and further references	12
2	$\mathbf{Bel}$	ief, probability and exchangeability	13
	2.1	Belief functions and probabilities	13
	2.2	Events, partitions and Bayes' rule	14
	<b>2.3</b>	Independence	17
	<b>2.4</b>	Random variables	17
		2.4.1 Discrete random variables	18
		2.4.2 Continuous random variables	19
		2.4.3 Descriptions of distributions	21
	2.5	Joint distributions	23
	<b>2.6</b>	Independent random variables	26
	2.7	Exchangeability	27
	2.8	de Finetti's theorem	29
	2.9	Discussion and further references	30
3	One	e-parameter models	
	3.1	The binomial model	31
		3.1.1 Inference for exchangeable binary data	35
		3.1.2 Confidence regions	41
	3.2	The Poisson model	43
		3.2.1 Posterior inference	45
		3.2.2 Example: Birth rates	48
	<b>3.3</b>	Exponential families and conjugate priors	51
	3.4	Discussion and further references	52

4	Mo	nte Carlo approximation	53
	4.1	The Monte Carlo method	53
	4.2	Posterior inference for arbitrary functions	57
	4.3	Sampling from predictive distributions	60
	4.4	Posterior predictive model checking	62
	4.5	Discussion and further references	65
5	The	e normal model	67
	5.1	The normal model	67
	5.2	Inference for the mean, conditional on the variance	69
	5.3	Joint inference for the mean and variance	73
	5.4	Bias, variance and mean squared error	79
	5.5	Prior specification based on expectations	83
	5.6	The normal model for non-normal data	84
	5.7	Discussion and further references	86
6	Pos	sterior approximation with the Gibbs sampler	89
	6.1	A semiconjugate prior distribution	89
	6.2	Discrete approximations	90
	6.3	Sampling from the conditional distributions	92
	6.4	Gibbs sampling	93
	6.5	General properties of the Gibbs sampler	96
	6.6	Introduction to MCMC diagnostics	
	6.7	Discussion and further references	104
7	The	e multivariate normal model1	
	7.1	The multivariate normal density1	
	7.2	A semiconjugate prior distribution for the mean	
	7.3	The inverse-Wishart distribution1	
	7.4	Gibbs sampling of the mean and covariance	
	7.5	Missing data and imputation 1	
	7.6	Discussion and further references	123
8	Gro	oup comparisons and hierarchical modeling	
	8.1	Comparing two groups	
	8.2	Comparing multiple groups1	
		8.2.1 Exchangeability and hierarchical models	
	8.3	The hierarchical normal model	
		8.3.1 Posterior inference	
	8.4	Example: Math scores in U.S. public schools	
		8.4.1 Prior distributions and posterior approximation	
		8.4.2 Posterior summaries and shrinkage	
	8.5	Hierarchical modeling of means and variances	
		8.5.1 Analysis of math score data	
	8.6	Discussion and further references	146

9	Line	ear regression	149			
	9.1	The linear regression model	149			
		9.1.1 Least squares estimation for the oxygen uptake data .	153			
	9.2	Bayesian estimation for a regression model				
		9.2.1 A semiconjugate prior distribution				
		9.2.2 Default and weakly informative prior distributions				
	9.3	Model selection				
	0.0	9.3.1 Bayesian model comparison				
		9.3.2 Gibbs sampling and model averaging				
	9.4	Discussion and further references				
	9.4	Discussion and further references	170			
10	Nor	aconjugate priors and Metropolis-Hastings algorithms	171			
		Generalized linear models				
		The Metropolis algorithm				
		The Metropolis algorithm for Poisson regression				
		Metropolis, Metropolis-Hastings and Gibbs				
	10.4	10.4.1 The Metropolis-Hastings algorithm				
		10.4.2 Why does the Metropolis-Hastings algorithm work?				
	10 5	Combining the Metropolis and Gibbs algorithms				
	10.5					
		10.5.1 A regression model with correlated errors				
	10.0	10.5.2 Analysis of the ice core data				
	10.6	Discussion and further references	192			
11	Line	ear and generalized linear mixed effects models	195			
		A hierarchical regression model				
		Full conditional distributions				
		Posterior analysis of the math score data				
	11.4	Generalized linear mixed effects models	201			
		11.4.1 A Metropolis-Gibbs algorithm for posterior	000			
		approximation				
		11.4.2 Analysis of tumor location data				
	11.5	Discussion and further references	207			
12	Lat	ent variable methods for ordinal data	200			
14		Ordered probit regression and the rank likelihood				
	12.1	12.1.1 Probit regression				
		12.1.1 Proble regression				
	10.0					
	12.2	The Gaussian copula model				
		12.2.1 Rank likelihood for copula estimation				
	12.3	Discussion and further references	223			
Exe	ercis	es	225			
Co	mmo	n distributions	253			
Re	<b>References</b>					
Ind	Index					