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A Fixed-Angle Heat Spreading Model for Dynamic Thermal Characterization of Rear-Cooled Substrates — Source link

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FACULTY OF ENGINEERING
Department of Electronics & Information Systems

A Fixed-Angle Heat Spreading Model for Dynamic Thermal Characterization of Rear-Cooled Substrates

THERMO ELECTRONICS

Bjorn Vermeersch

Seminar 'Physical Electronics'

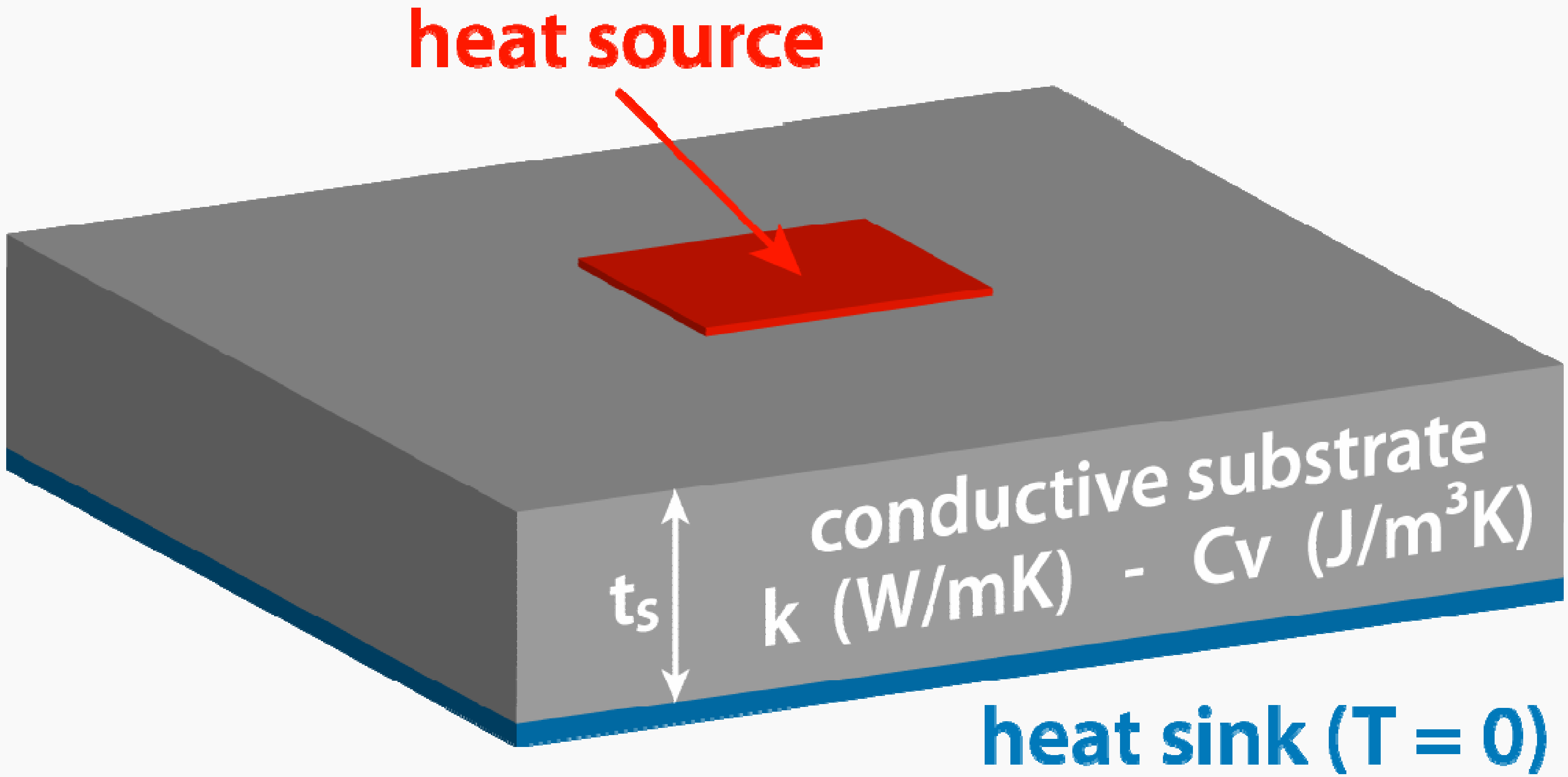
ELIS – 5 April 2007

Outline

- **Introduction**
- Model definition
- Exact calculations
- Results
- Anisotropic substrates
- Conclusions

Introduction

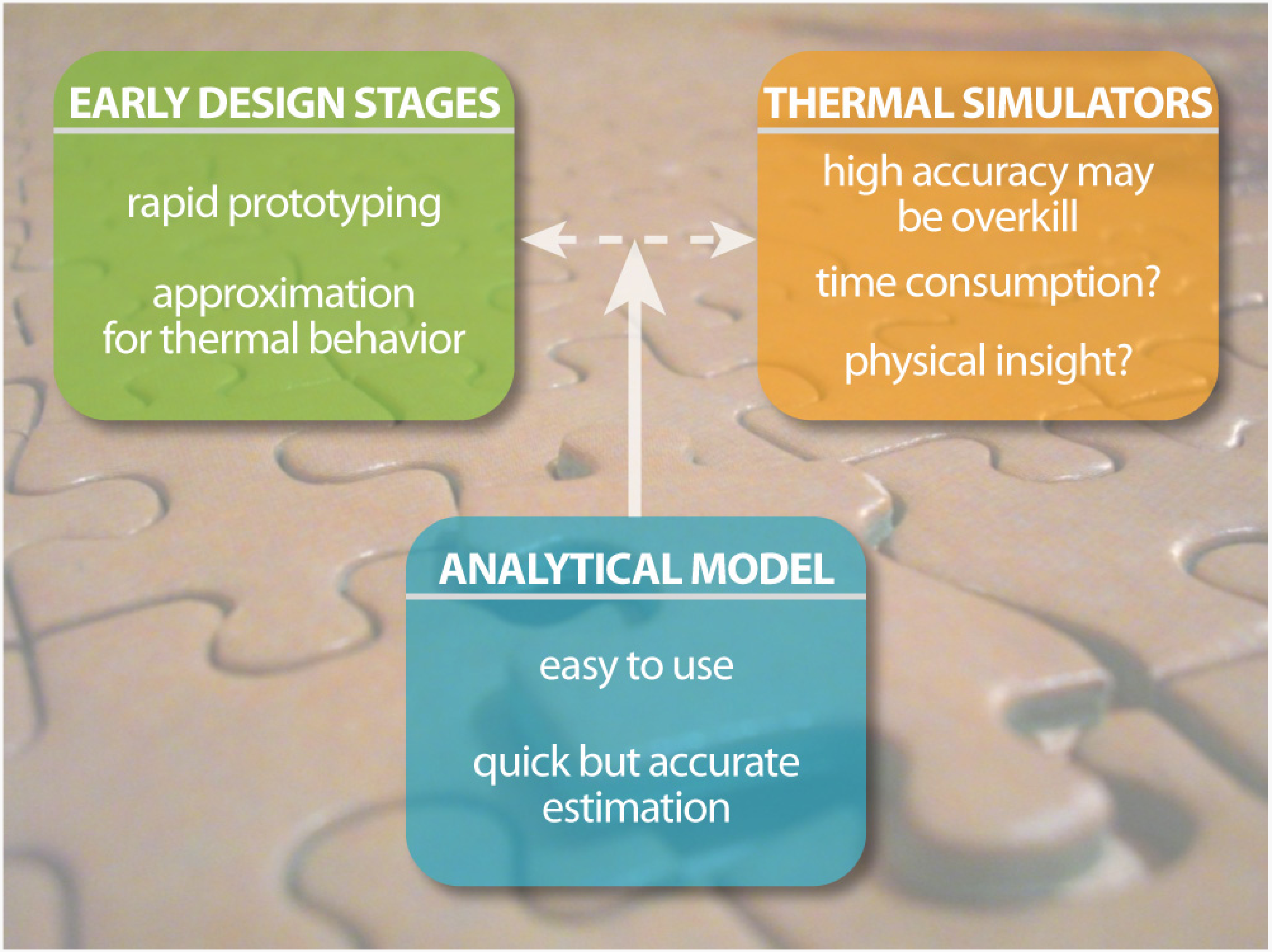
Problem formulation



? **Dynamic behaviour**

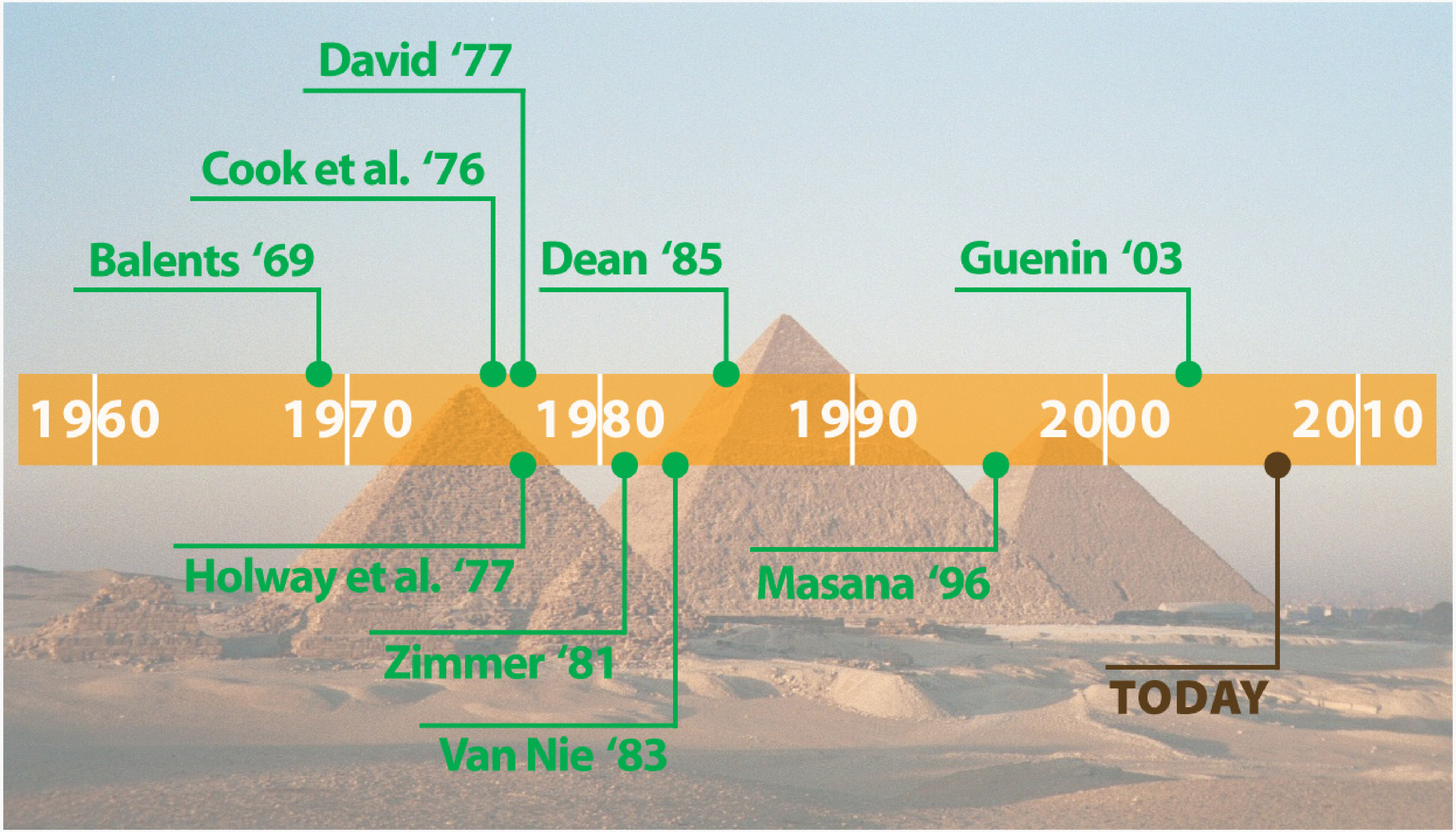
Introduction

Thermal engineering



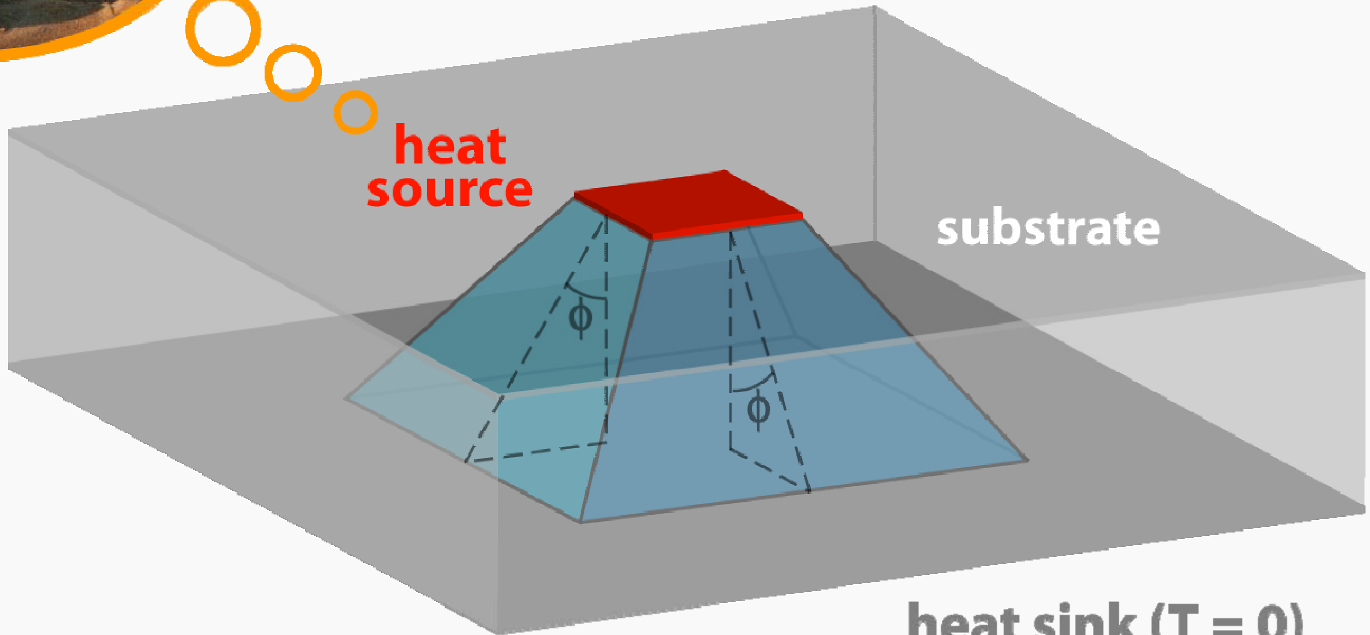
Introduction

Steady state fixed-angle models



Introduction

How does a fixed-angle model look like?



Introduction

Literature overview

REFERENCE	SPREADING ANGLE	HEAT SOURCE	SUBSTRATE LAYERS
Balents '69	45°	□	single
Cook '76	45°	□ multi	multi
David '77	45° , 32.5°	□	single
Holway '77	45°	○	multi
Zimmer '81	32.5°	□	single
Van Nie '83	geo	▭	single
Dean '85	45°	multi ▭	single
Masana '96	geo	○ ▭ assym. pos.	limited in lateral dir.
Guenin '03	45°	□	single

"The 45° heat spreading angle: an urban legend?"

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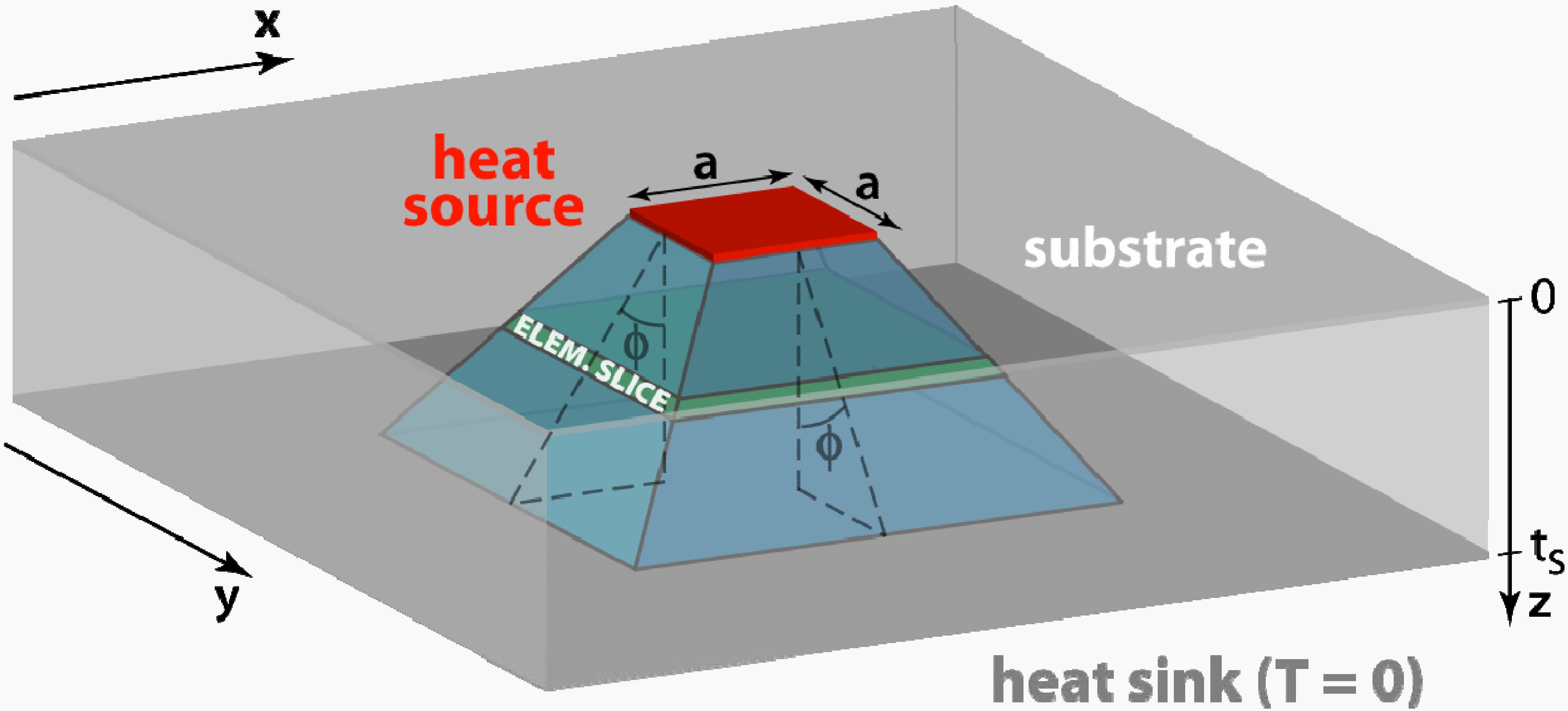
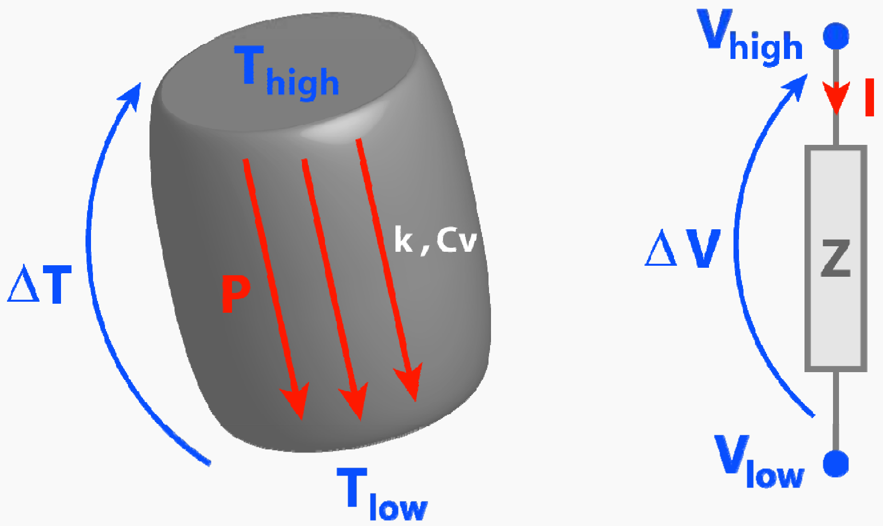
Model definition

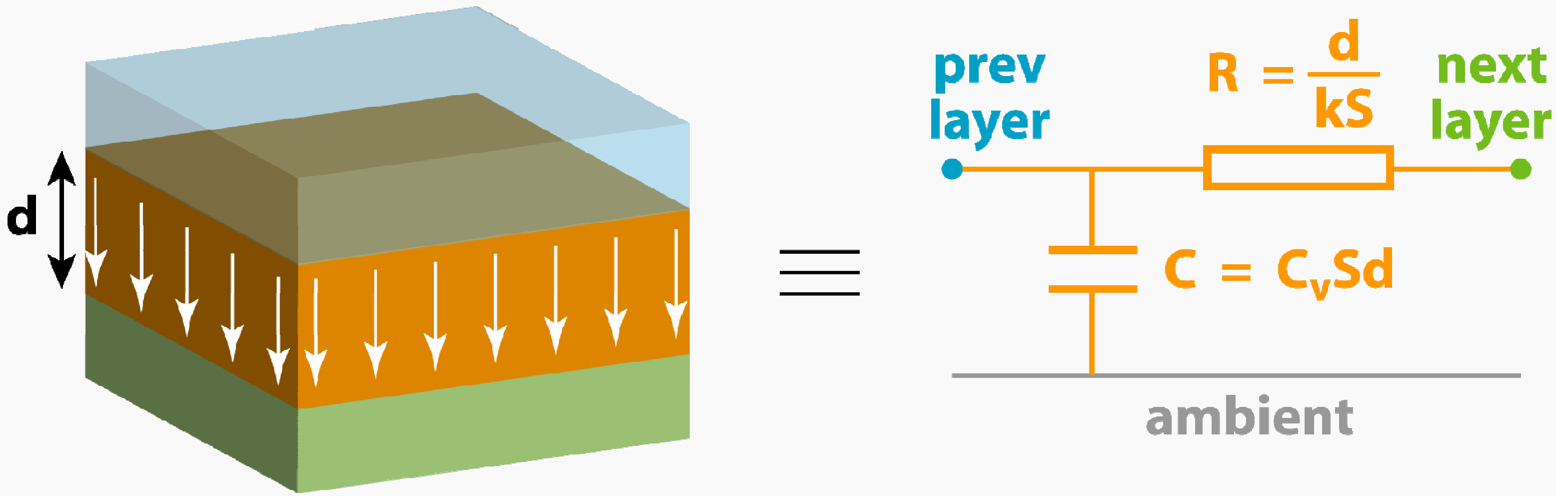
Fixed-angle heat spreading



Model definition

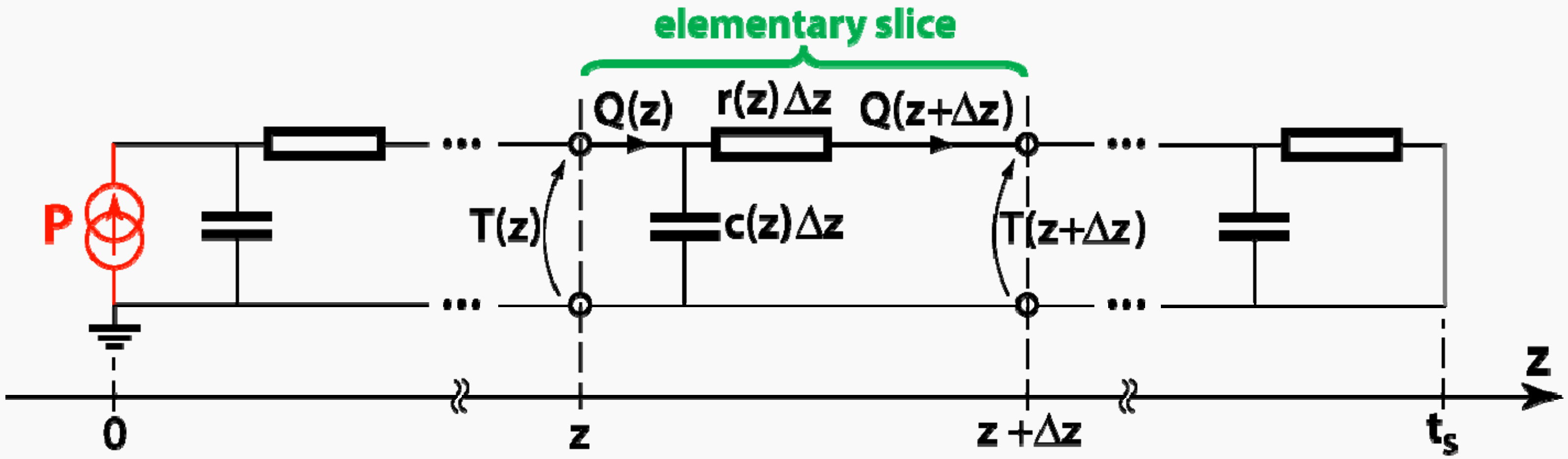
Electrothermal analogy

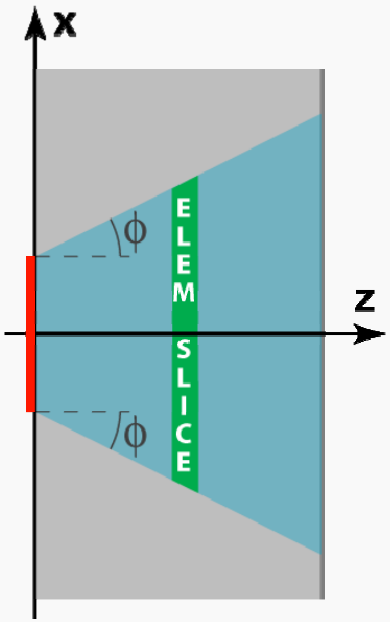




Model definition

Equivalent distributed network





Using phasor representation:

$$\begin{cases} -\dfrac{dQ}{dz} = j\omega c(z) \cdot T(z) \\ -\dfrac{dT}{dz} = r(z) \cdot Q(z) \end{cases}$$

$$r(z) = \frac{1}{k \cdot A(z)} \quad , \quad c(z) = C_v \cdot A(z)$$

$$A(z) = (a + 2z \cdot \tan\phi)^2$$

Model definition

Differential equation

$$\begin{cases} -\dfrac{dQ}{dz} = j\omega c(z)\cdot T(z) \\ -\dfrac{dT}{dz} = r(z)\cdot Q(z) \end{cases} \longrightarrow \boxed{\frac{d^2T}{dz^2} + \frac{2\alpha}{1+\alpha z}\frac{dT}{dz} - \gamma T(z) = 0}$$

$$\alpha = \frac{2\tan\phi}{a}$$

$$\gamma = \frac{j\omega C_v}{k}$$

Substitution: $T(z) = \dfrac{\Psi(z)}{1+\alpha z} \longrightarrow \dfrac{d^2\Psi}{dz^2} - \gamma\cdot\Psi(z) = 0$

$$\boxed{T(z) = C_1\frac{\cosh\left(\sqrt{\gamma}z\right)}{1+\alpha z} + C_2\frac{\sinh\left(\sqrt{\gamma}z\right)}{1+\alpha z}}$$

Boundary conditions: $Q(0) = -\dfrac{1}{r(0)}\dfrac{dT}{dx}\Big|_{x=0} = P \quad , \quad T(t_s) = 0$

Model definition

Thermal impedance

$$T(z) = C_1 \frac{\cosh\left(\sqrt{\gamma z}\right)}{1+\alpha z} + C_2 \frac{\sinh\left(\sqrt{\gamma z}\right)}{1+\alpha z} \qquad \gamma = \frac{j\omega C_v}{k} \;,\; \alpha = \frac{2\tan\phi}{a}$$

- boundary conditions
- **division by dissipated power P**
- normalization ($\rightarrow$ model applicable for any material!)

$$\tilde{Z}_{th}(\tilde{\omega}) = \frac{1}{2\lambda\tan\phi + \sqrt{j\tilde{\omega}}\cdot \mathrm{cotanh}\left(\sqrt{j\tilde{\omega}}\right)} \qquad Z_0 = \frac{t_s}{ka^2} \;,\; \omega_0 = \frac{k}{C_v t_s^2}$$

$$\lambda = \frac{t_s}{a}$$

ϕ → **fitting parameter**

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Exact calculations

Fundamental solution for 3D space

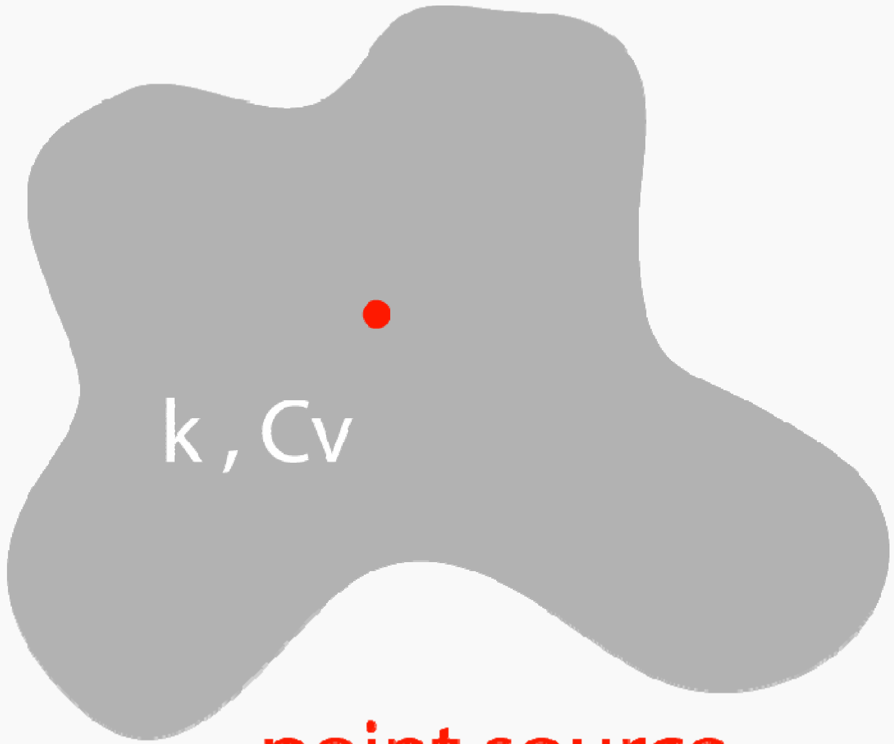
Green's function:

$$G(\vec{r}|\vec{r}\,') = \frac{1}{4\pi k R} \exp\left(-\sqrt{\frac{j\omega C_v}{k}} R \right)$$

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

For distributed heat source:
superposition

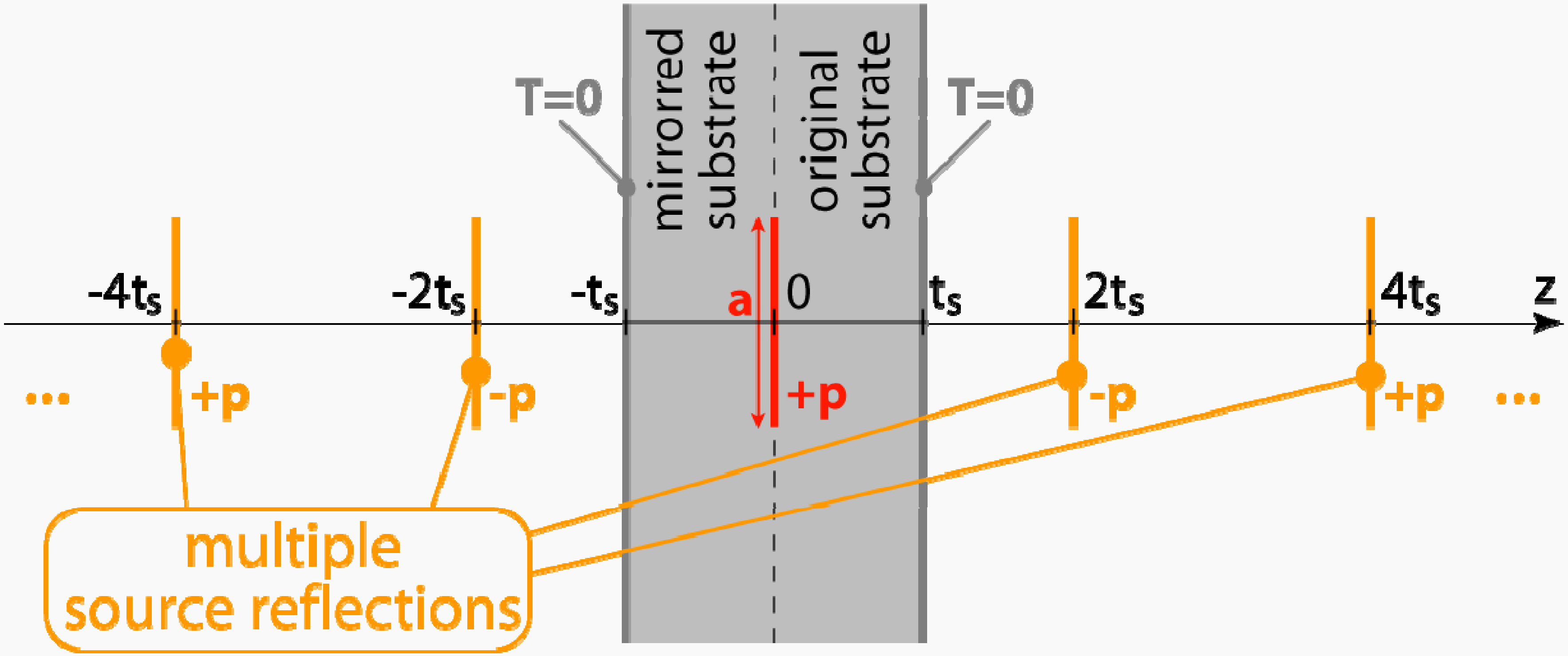
$$T(x,y,z) = \iiint_{\text{source}} p(\vec{r}\,') G(\vec{r}|\vec{r}\,') d\vec{r}\,'$$



point source
$\delta(x',y',z')$

Exact calculations

Multiple reflection technique



$$T_{source}(x,y) = 2\frac{P}{a^2}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2} H(x,x',y,y')dx'dy' \xrightarrow[\text{value}]{\text{average}} Z_{th}$$

$$H = G(z-z'=0) + 2\sum_{n=1}^{\infty}(-1)^n G(z-z'=2nt_s)$$

G = Green's function

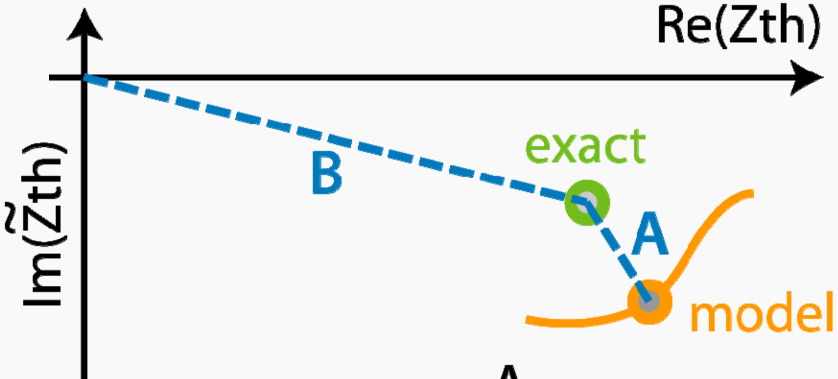
Exact calculations

Model validation

- calculation of sufficient number (N) of impedance points over wide frequency range (logarithmic distribution, 10 points per decade)
- error function:

$$e(\phi) = \frac{1}{N}\sum_{i=1}^{N}\frac{\left|\tilde{Z}^{(i)}_{exact} - \tilde{Z}^{(i)}_{model}(\phi)\right|}{\left|\tilde{Z}^{(i)}_{exact}\right|}$$

$^{(i)}$ = evaluated at i-th frequency



point error = $\frac{A}{B}$ x 100 [%]

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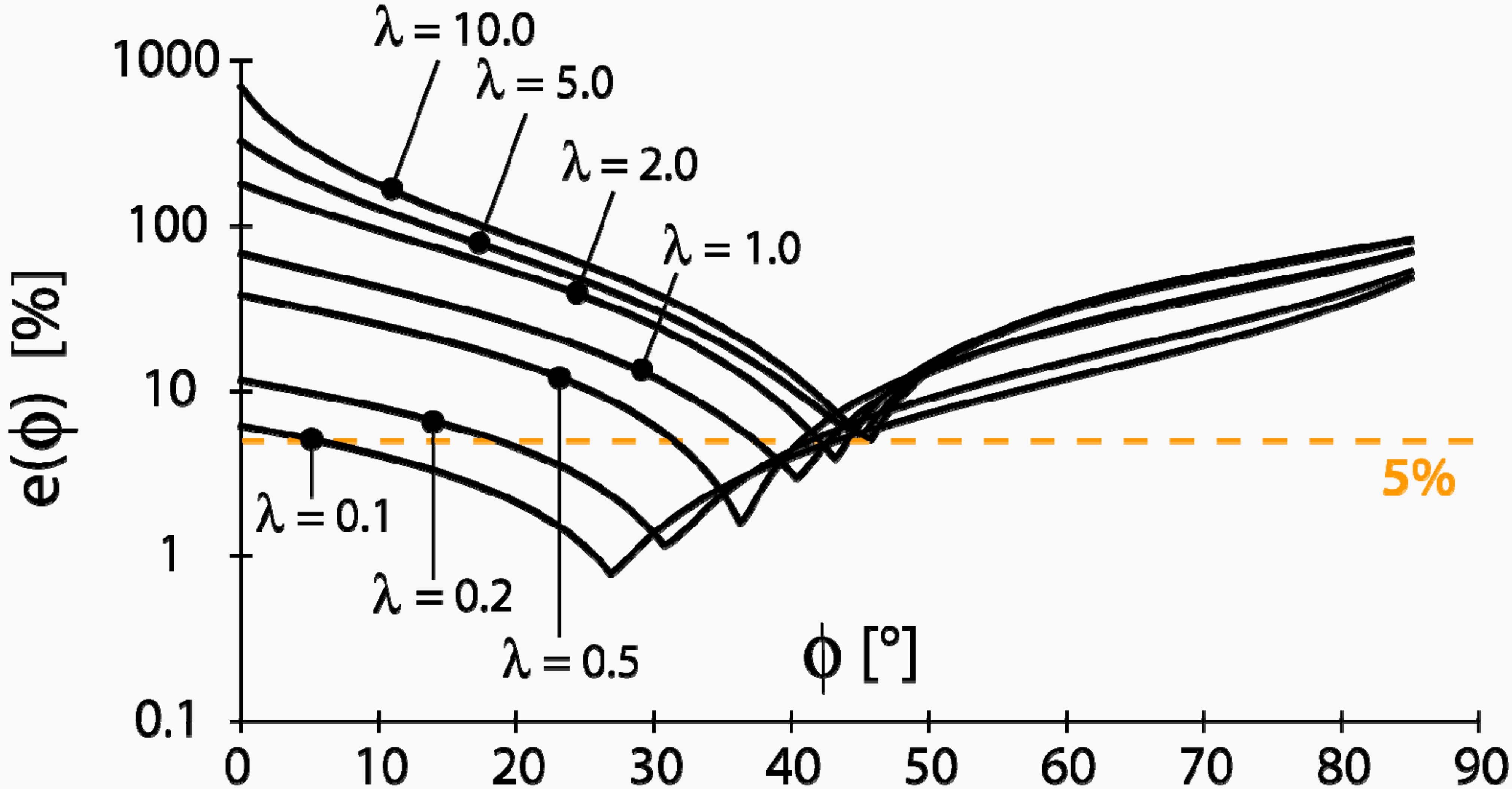
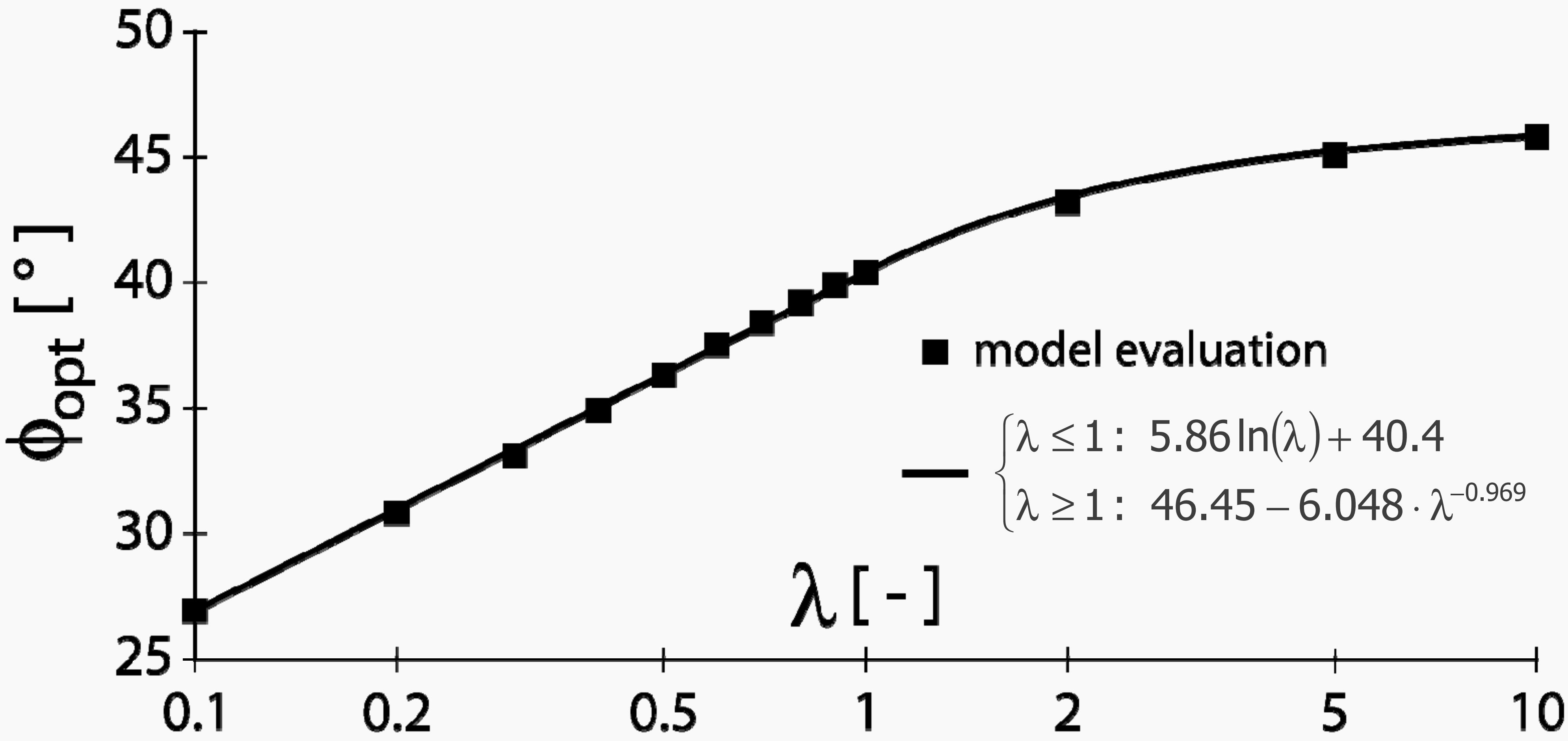
Results

Model validation: error curves



Results

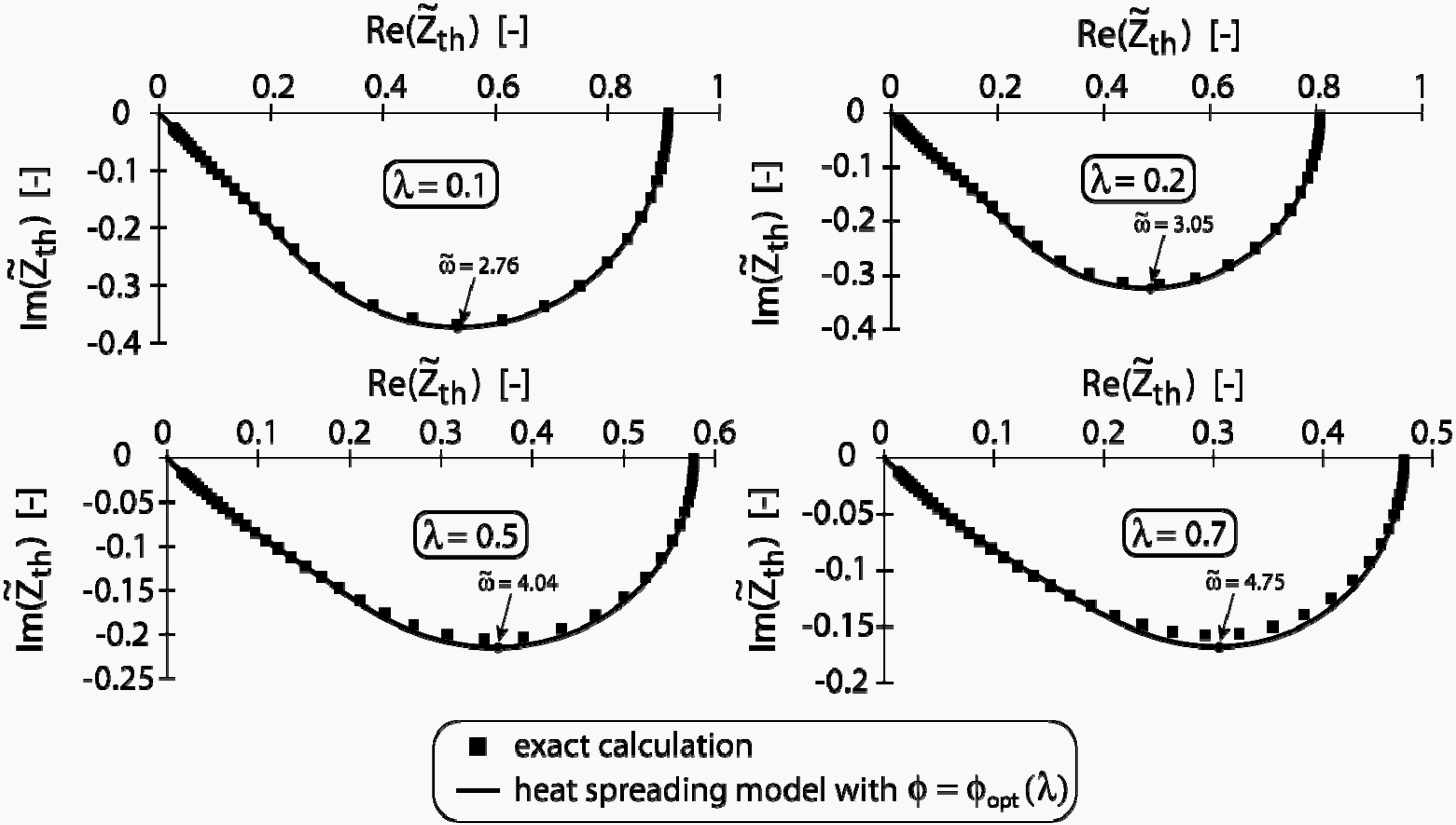
Optimal spreading angle



$e(\phi_{opt})$ ranges between 0.8% and 5.4%

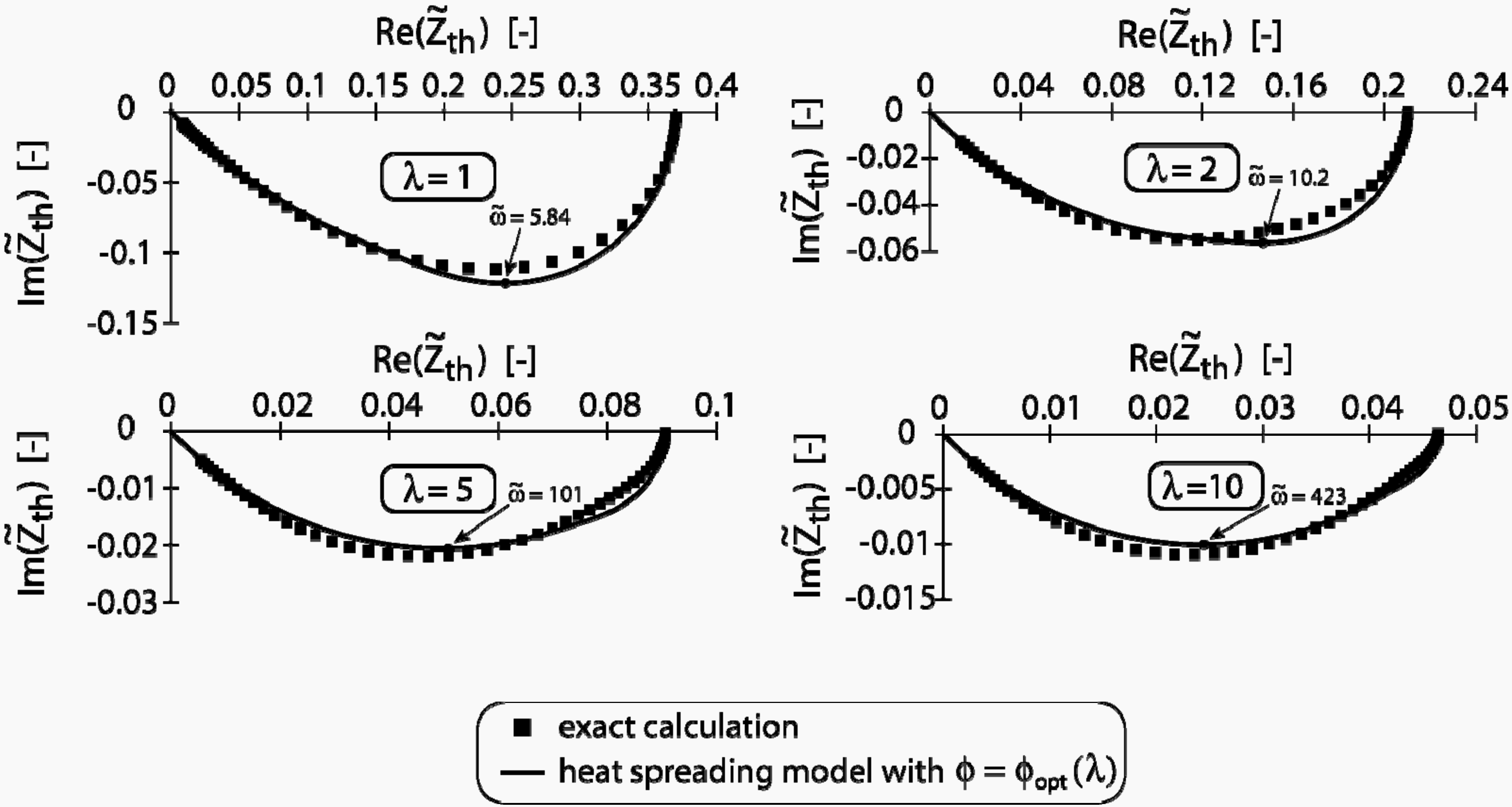
Results

Thermal impedance plots (1)



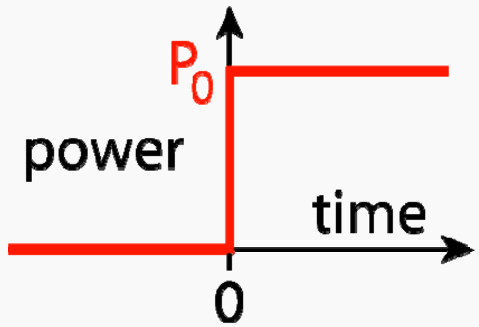
Results

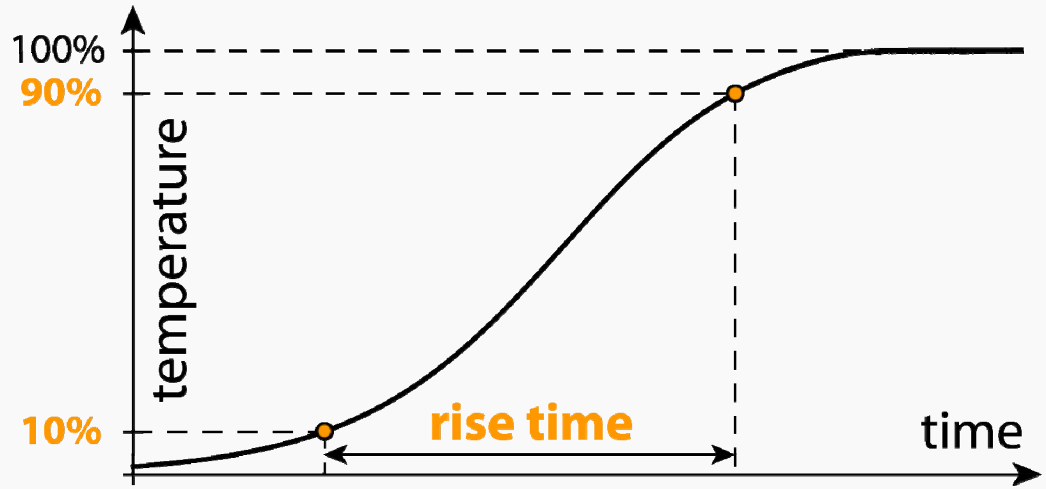
Thermal impedance plots (2)



Results

Step response





$$P(f) = \frac{P_0}{2}\delta(f) + \frac{P_0}{j2\pi f}$$

$$T(t) = \int_{-\infty}^{\infty} Z_{th}(f)\,P(f)\exp(j2\pi ft)\,df$$

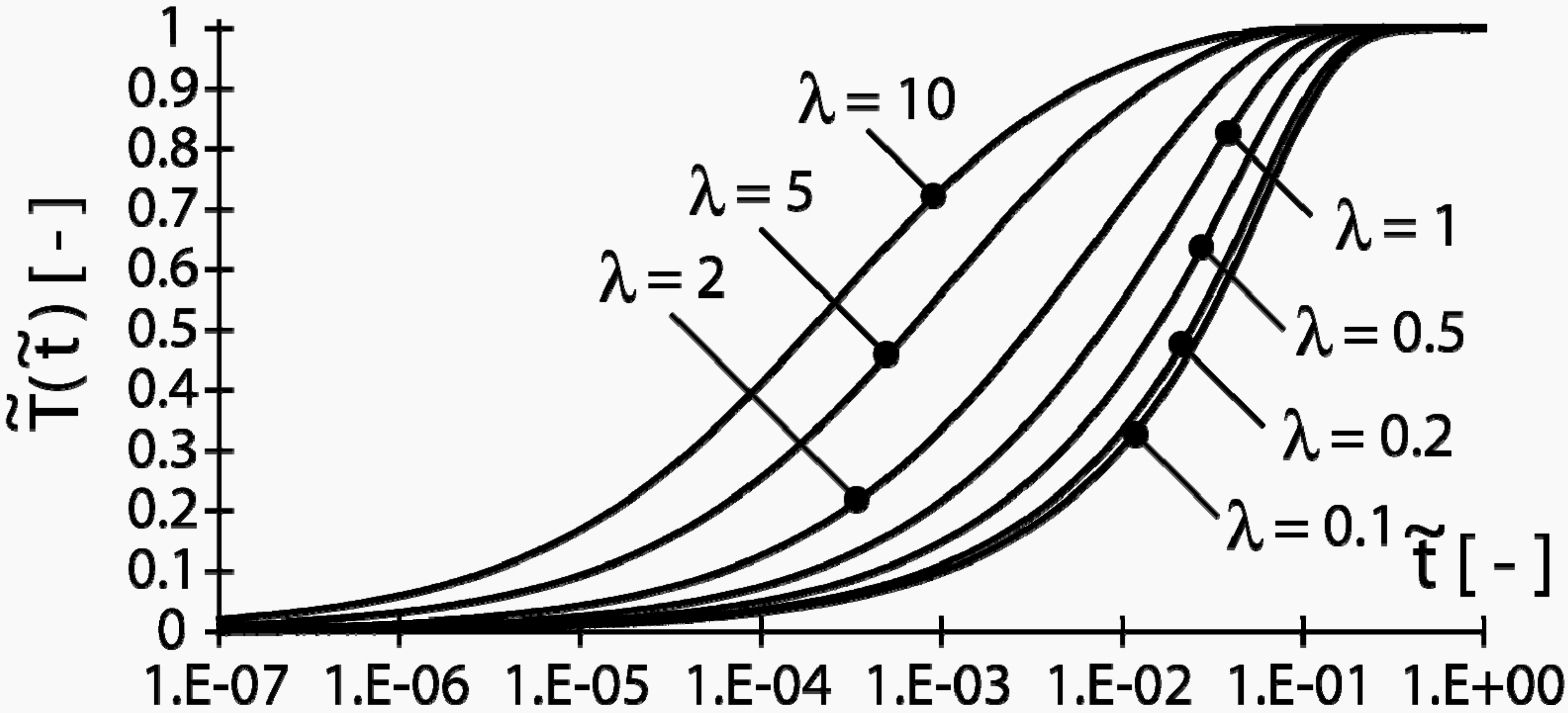
$$\tilde{T}(\tilde{t}) = \frac{1}{2} + \frac{1+2\lambda\tan\phi}{\pi}\int_0^{\infty}\mathrm{Im}\left[\frac{\exp\left(j2\pi\tilde{f}\tilde{t}\right)}{\tilde{f}\left[2\lambda\tan\phi + \sqrt{j\tilde{f}}\,\operatorname{cotanh}\left(\sqrt{j\tilde{f}}\right)\right]}\right]d\tilde{f}$$

$$T_0 = \frac{P_0 Z_0}{1+2\lambda\tan\phi}$$

$$t_0 = \frac{2\pi C_v t_s^2}{k}$$

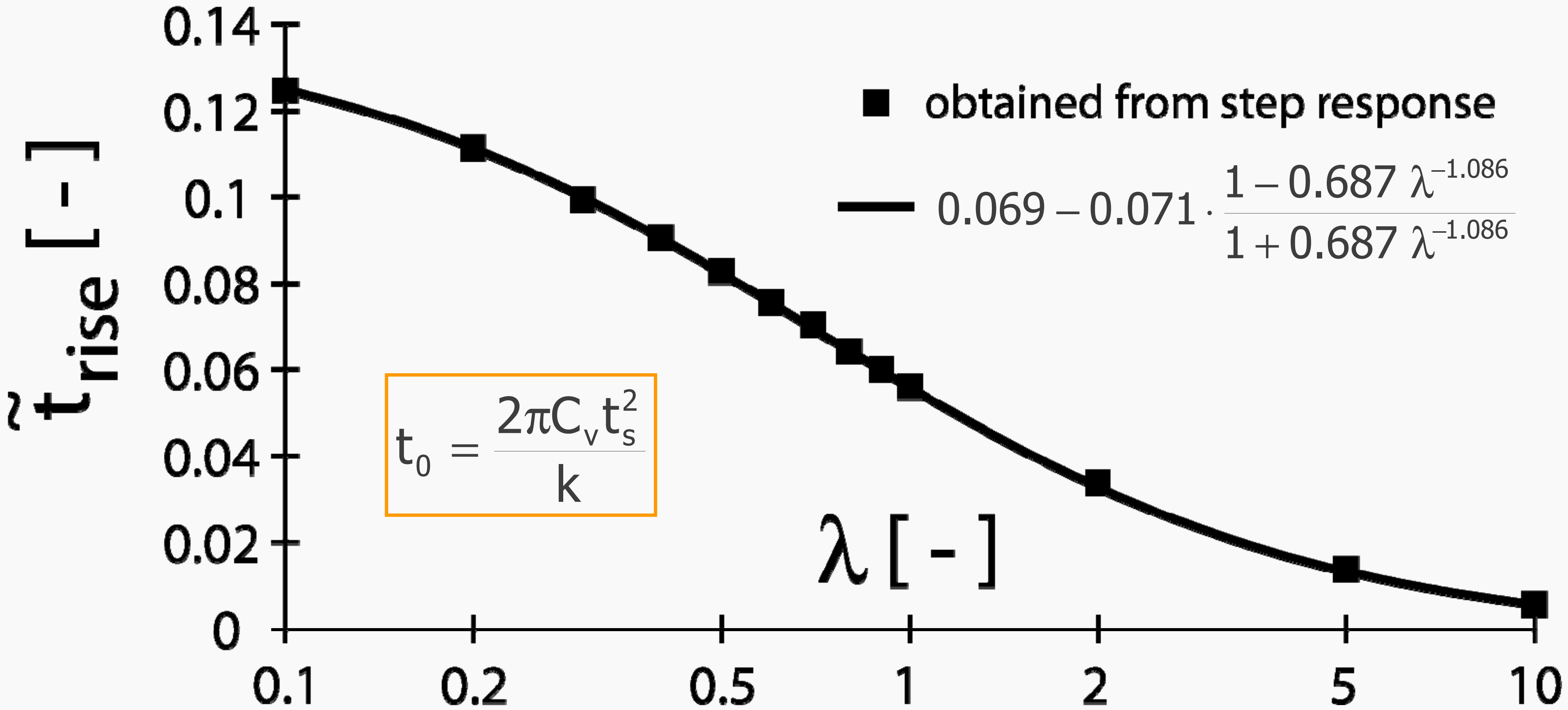
Results

Transient heating curves



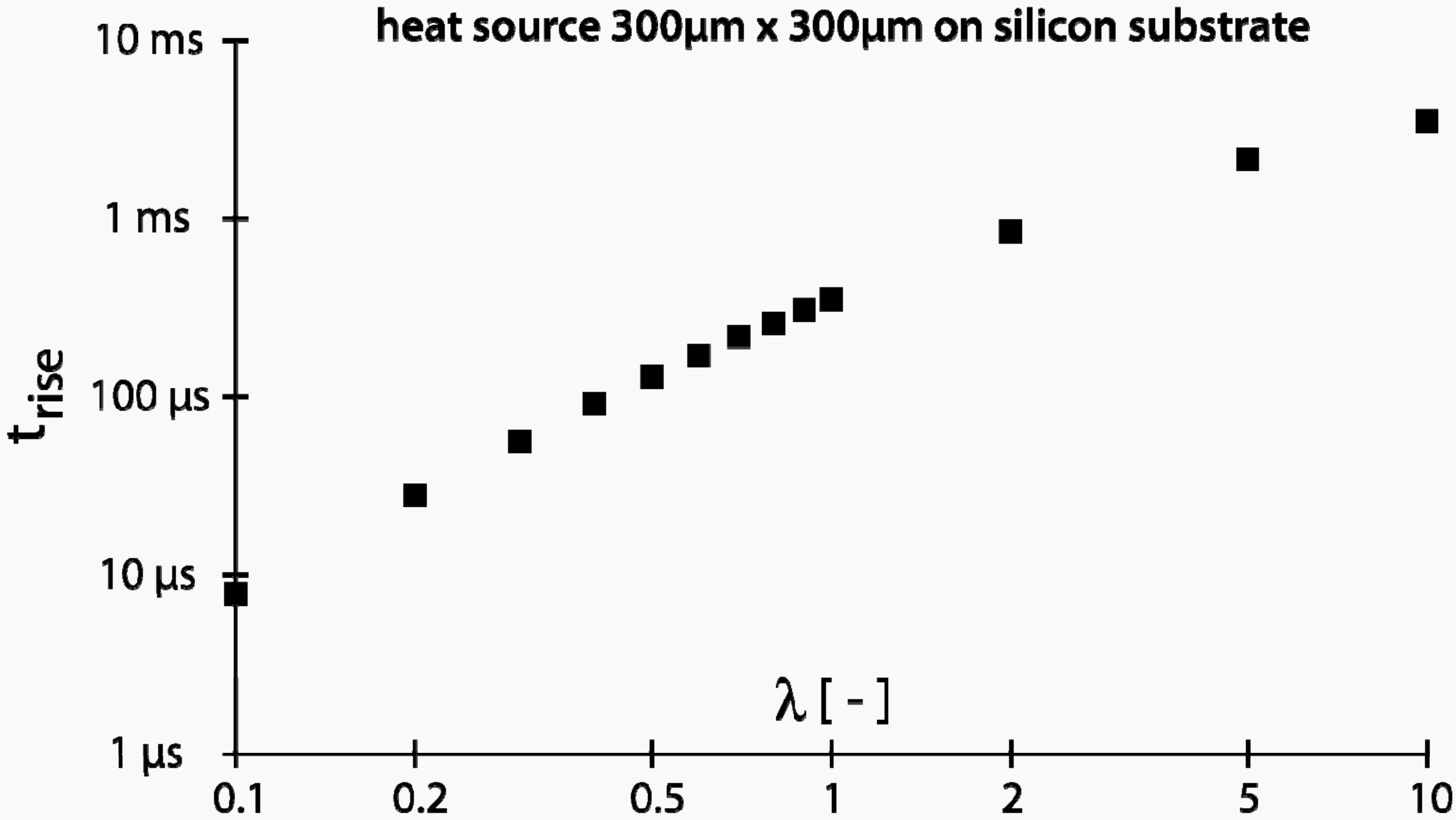
Results

Thermal rise time



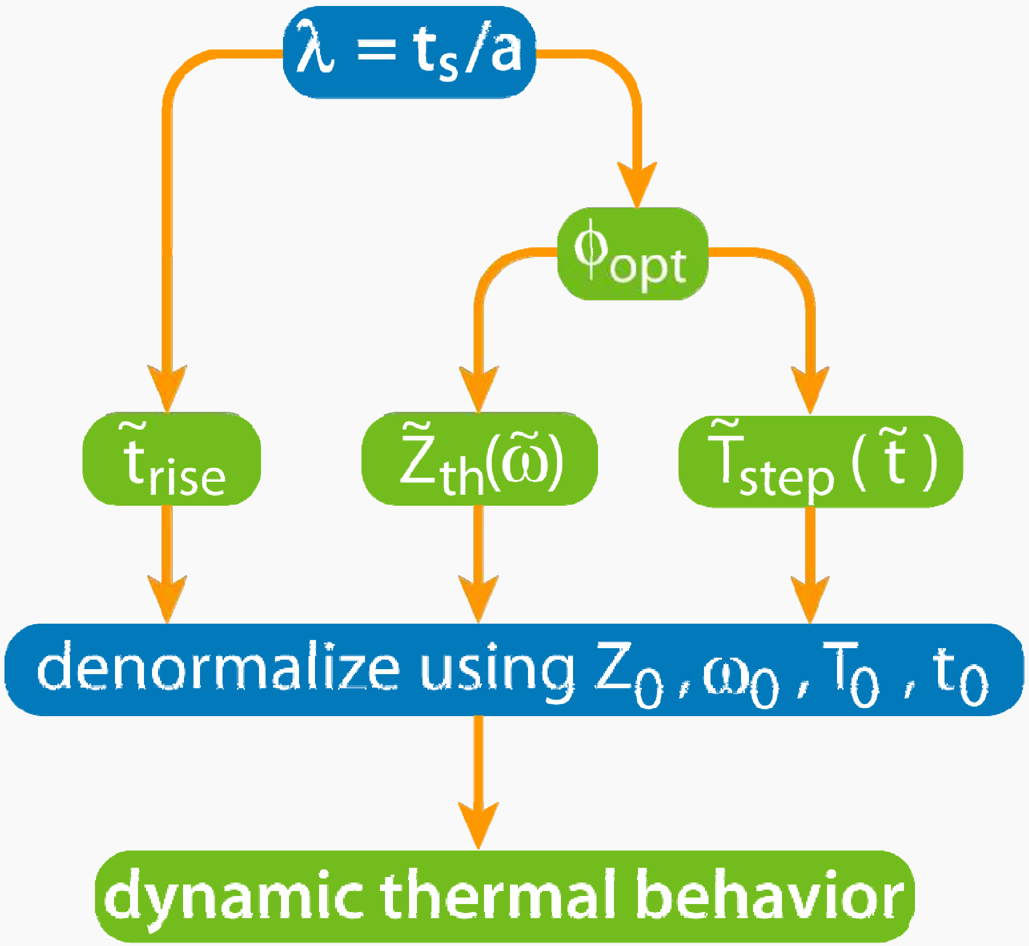
Results

Thermal rise time - denormalized



Results

Global recipe & case study



HEAT SOURCE 300µm x 300µm

	Si	GaAs	Al_2O_3	Cu
k [W/mK]	160	50	22	380
C_V [10^6 J/m³K]	1.78	1.86	2.98	3.47
k / C_V [mm²/s]	90	27	7	110
t_s [µm]	**rise time [ms]**			
50	0.020	0.068	0.246	0.017
100	0.068	0.226	0.821	0.055
150	0.130	0.433	1.575	0.106
300	0.352	1.175	4.277	0.288
500	0.669	2.231	8.125	0.548
1000	1.420	4.77	17.248	1.163
t_s [µm]	**thermal resistance [K/W]**			
50	2.9	9.3	21.2	1.2
100	4.8	15.3	34.9	2.0
150	6.0	19.2	43.6	2.5
300	7.7	24.7	56.0	3.2
500	8.5	27.2	61.9	3.6
1000	9.2	29.4	66.7	3.9

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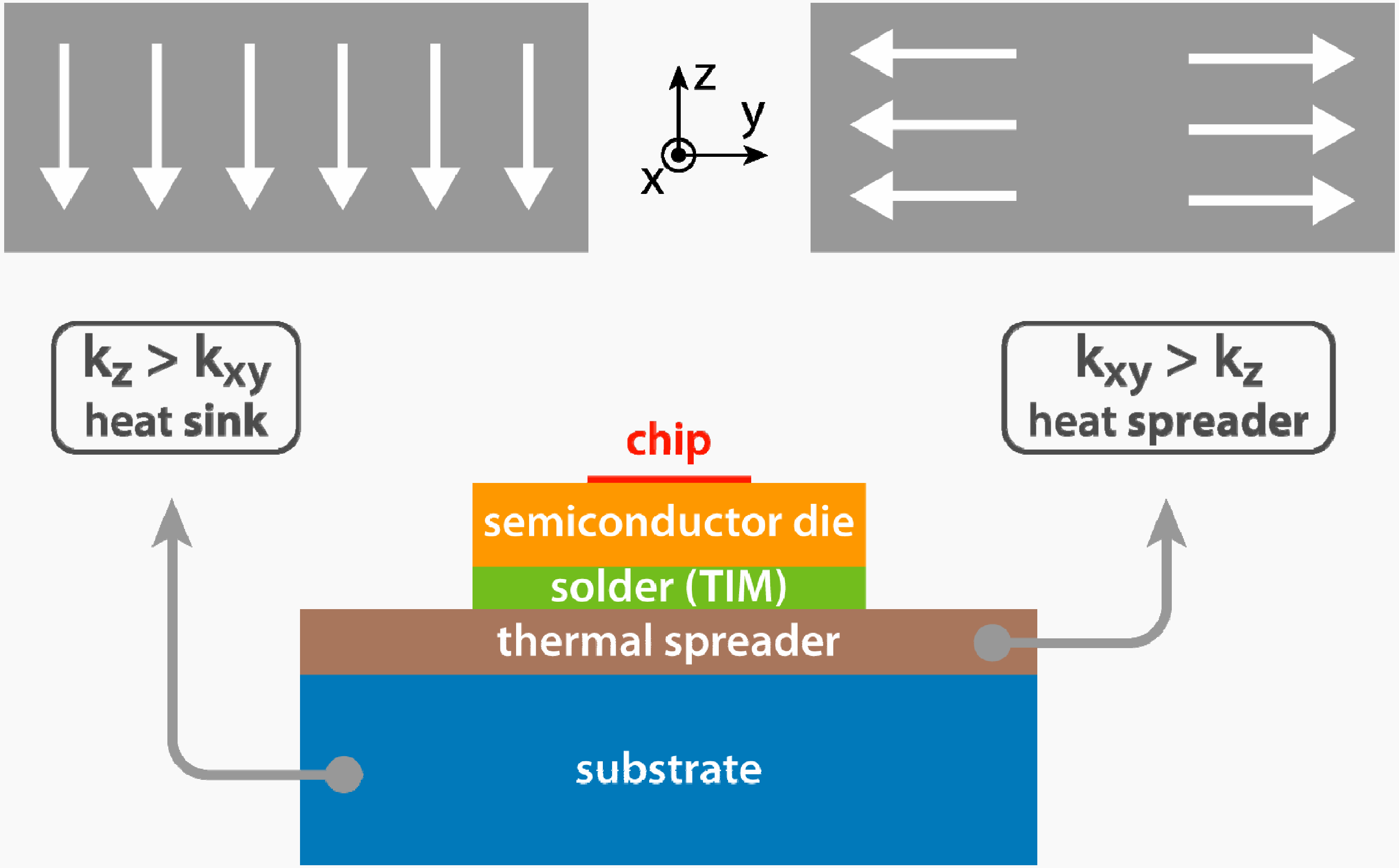
- ▸ **Anisotropic substrates**
- ▸ Conclusions

Anisotropic substrates

Why?

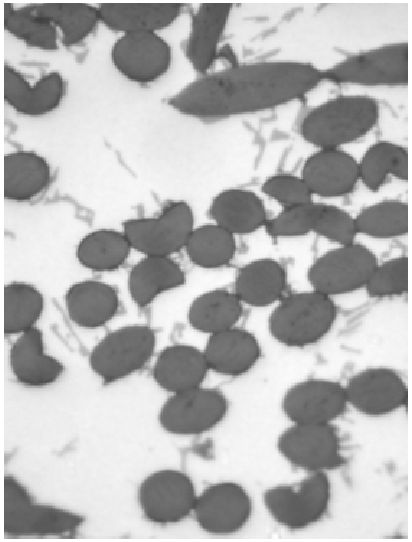
- Inherent property of certain materials
- Thermal engineering: preferential heat flow



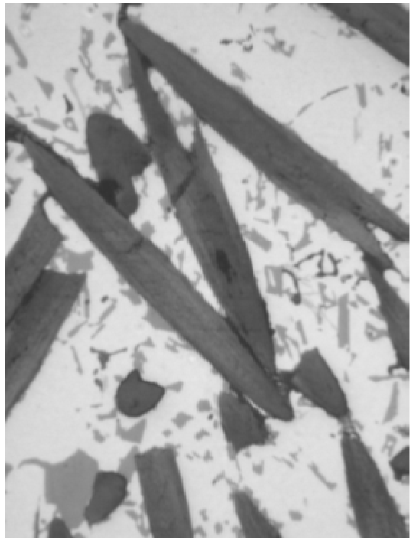
Anisotropic substrates

Practical example

carbon fibres embedded in Al or Cu matrix



In plane vs Z direction



PHYSICAL & MECHANICAL PROPERTIES, (UNITS)	Al filled composite	Cu filled composite	NOTES
Thermal Conductivity, (W/mK)	260 - 300	300 - 340	In-plane values
Thermal Conductivity, (W/mK)	180 - 200	220 - 250	Through-thickness values

Anisotropic substrates

Mathematical treatment (1)

- 3D isotropic heat equation (phasor notation):

$$k\nabla^2 T(x,y,z) - j\omega C_v T(x,y,z) = 0$$

- if $k_{xy} \neq k_z$:

$$k_{xy}\frac{\partial^2 T}{\partial x^2} + k_{xy}\frac{\partial^2 T}{\partial y^2} + k_z\frac{\partial^2 T}{\partial z^2} - j\omega C_v T = 0$$

- coordinate transformation: $\boxed{z' = \beta z}$

$$k_{xy}\frac{\partial^2 T}{\partial x^2} + k_{xy}\frac{\partial^2 T}{\partial y^2} + \beta^2 k_z\frac{\partial^2 T}{\partial z'^2} - j\omega C_v T = 0$$

- Now choose β such that $\beta^2 k_z = k_{xy}$

Anisotropic substrates

Mathematical treatment (2)

- $$\beta = \sqrt{\frac{k_{xy}}{k_z}}$$ "anisotropy factor"

$$k_{xy}\nabla^2 T(x,y,z') - j\omega C_v T(x,y,z') = 0$$

- transformation of boundary conditions:

$$-k_z \left.\frac{\partial T}{\partial z}\right|_0 = \frac{P}{a^2} \quad \Leftrightarrow \quad -k_z \beta \left.\frac{\partial T}{\partial z'}\right|_0 = \frac{P}{a^2}$$

$$\Leftrightarrow \quad -\frac{k_{xy}}{\beta^2}\beta \left.\frac{\partial T}{\partial z'}\right|_0 = \frac{P}{a^2} \quad \Leftrightarrow \quad -k_{xy} \left.\frac{\partial T}{\partial z'}\right|_0 = \beta \frac{P}{a^2}$$

Multiply dissipated power (and hence temperatures) with β

$$T(z = t_s) = 0$$

$$\Leftrightarrow \quad T(z' = \beta t_s) = 0$$

Relate temperatures to appropriate location

Anisotropic substrates

Model modification

- temperature distribution in an anisotropic (k_{xy},k_z) substrate with thickness t_s

  =

  β times the temperature in an isotropic (k_{xy}) substrate with thickness $\beta\, t_s$.
- (!!!) keep factor β in Z_{th} : related to **original** source
- Hence:

$$\tilde{Z}'_{th}(\lambda,\phi) = \beta \cdot \tilde{Z}_{th}(\beta\lambda,\phi') \quad \text{with} \quad \phi' = \phi_{opt}(\beta\lambda)$$

$$\tilde{t}'_{rise}(\lambda) = \tilde{t}_{rise}(\beta\lambda)$$

- (!!!) Don’t forget to reinterpret normalization values

Anisotropic substrates

Impact on steady state

- $$R_{th} = \frac{r_0 t_s}{1 + \alpha t_s} = \frac{t_s}{ka^2} \cdot \frac{1}{1 + 2\lambda \tan\phi}$$ **k = k_{xy} = constant**

$$\longrightarrow \boxed{\frac{R'_{th}}{R_{th}} = \frac{R}{R_{iso}} = \beta^2 \cdot \frac{1 + 2\lambda \tan(\phi_{opt}(\lambda))}{1 + 2\beta\lambda \tan(\phi_{opt}(\beta\lambda))}}$$

- **intuitive expectations** (based on 1-D viewpoint)

"k_z is dominant, pretend isotropic"

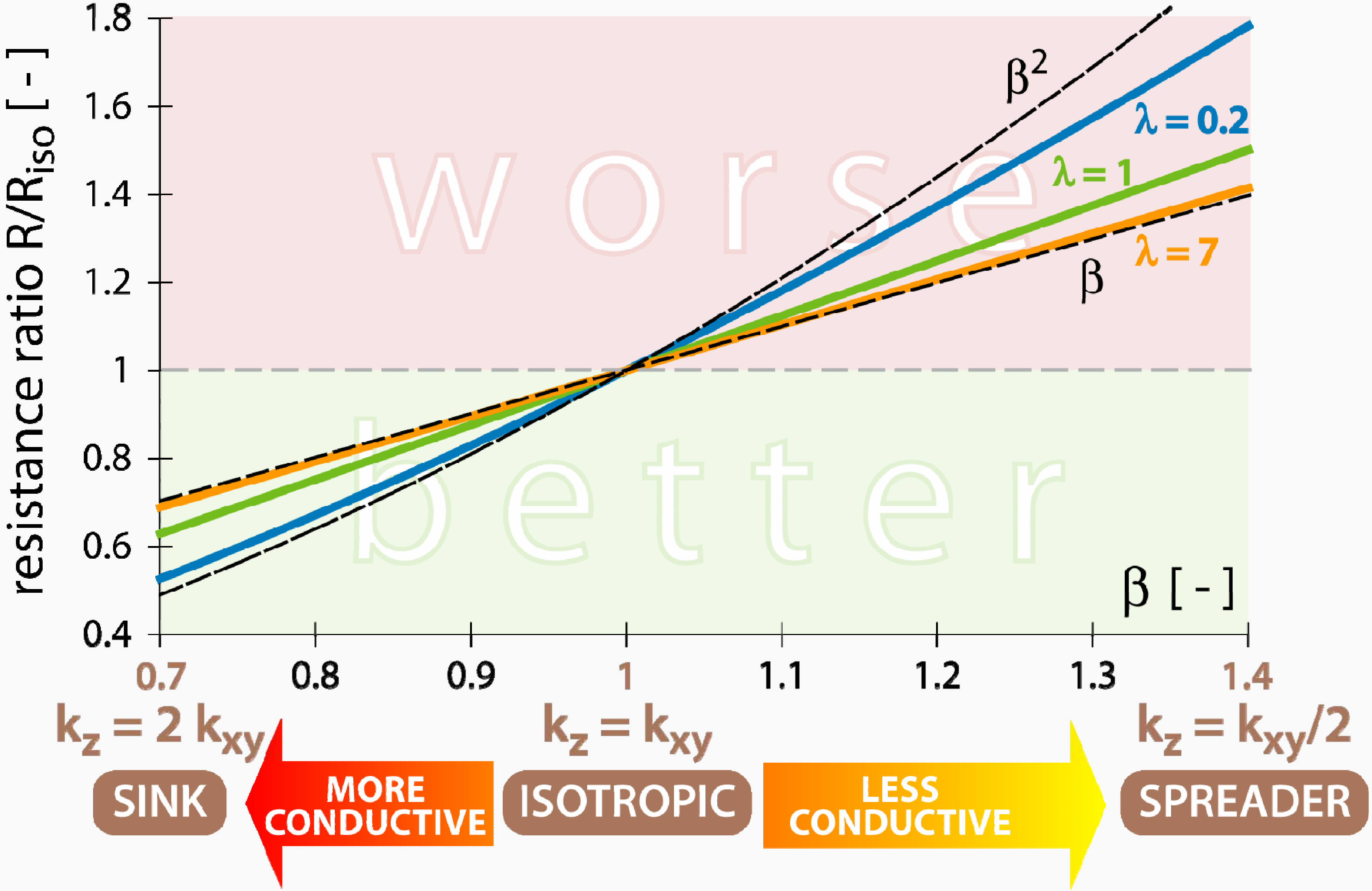
$$R_{th} \div \frac{1}{k} \longrightarrow \boxed{\frac{R}{R_{iso}} = \frac{1/k_z}{1/k_{xy}} = \beta^2}$$

"anistropic: β times temp. in isotropic substrate k_{xy} but β times thicker"

$$R_{th} \div t_s \longrightarrow \boxed{\frac{R}{R_{iso}} = \beta^2}$$

Anisotropic substrates

R_{th} @ constant k_{xy}: results

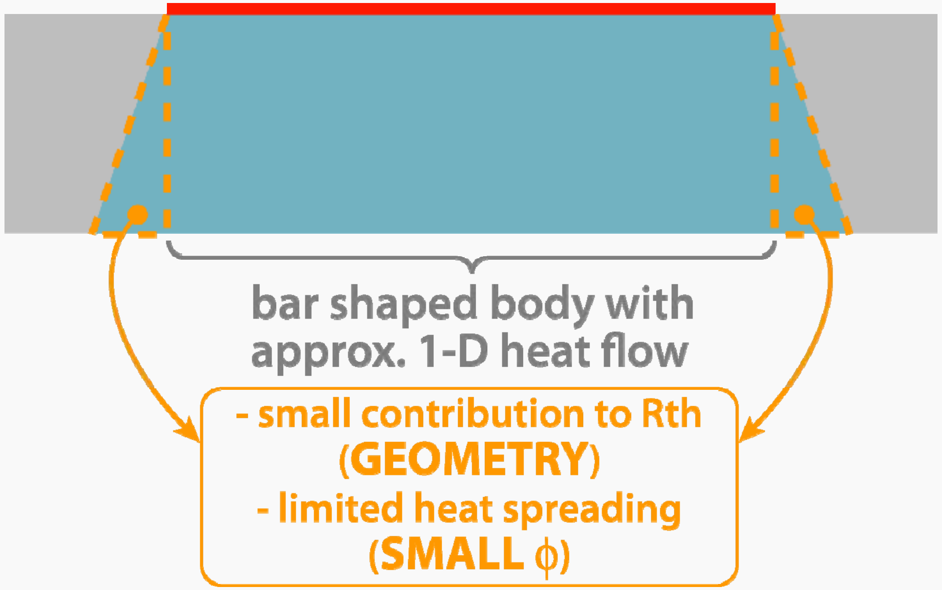


Anisotropic substrates

R_{th} @ constant k_{xy}: analysis

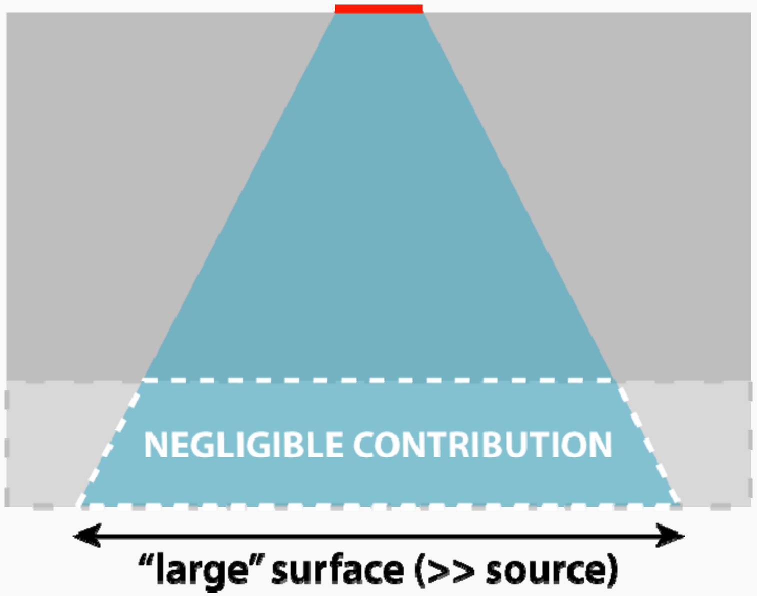
THIN SUBSTRATES ⟷ **THICK SUBSTRATES**

β^2 reasonable for R/R_{iso}



$$\frac{R}{R_{iso}} \approx \frac{\frac{t_s}{k_z a^2}}{\frac{t_s}{k_{xy} a^2}} = \frac{k_{xy}}{k_z} = \beta^2$$

$R/R_{iso} \approx \beta$



$$R = \beta \cdot R_{scaled\ pyr} \approx \beta \cdot R_{iso}$$

Anisotropic substrates

Impact on transient behaviour

- temperature distribution in an anisotropic (k_{xy},k_z) substrate with thickness t_s

  =

  β times the temperature in an isotropic (k_{xy}) substrate with thickness $\beta\, t_s$.

  no influence on trise: time scale unaltered

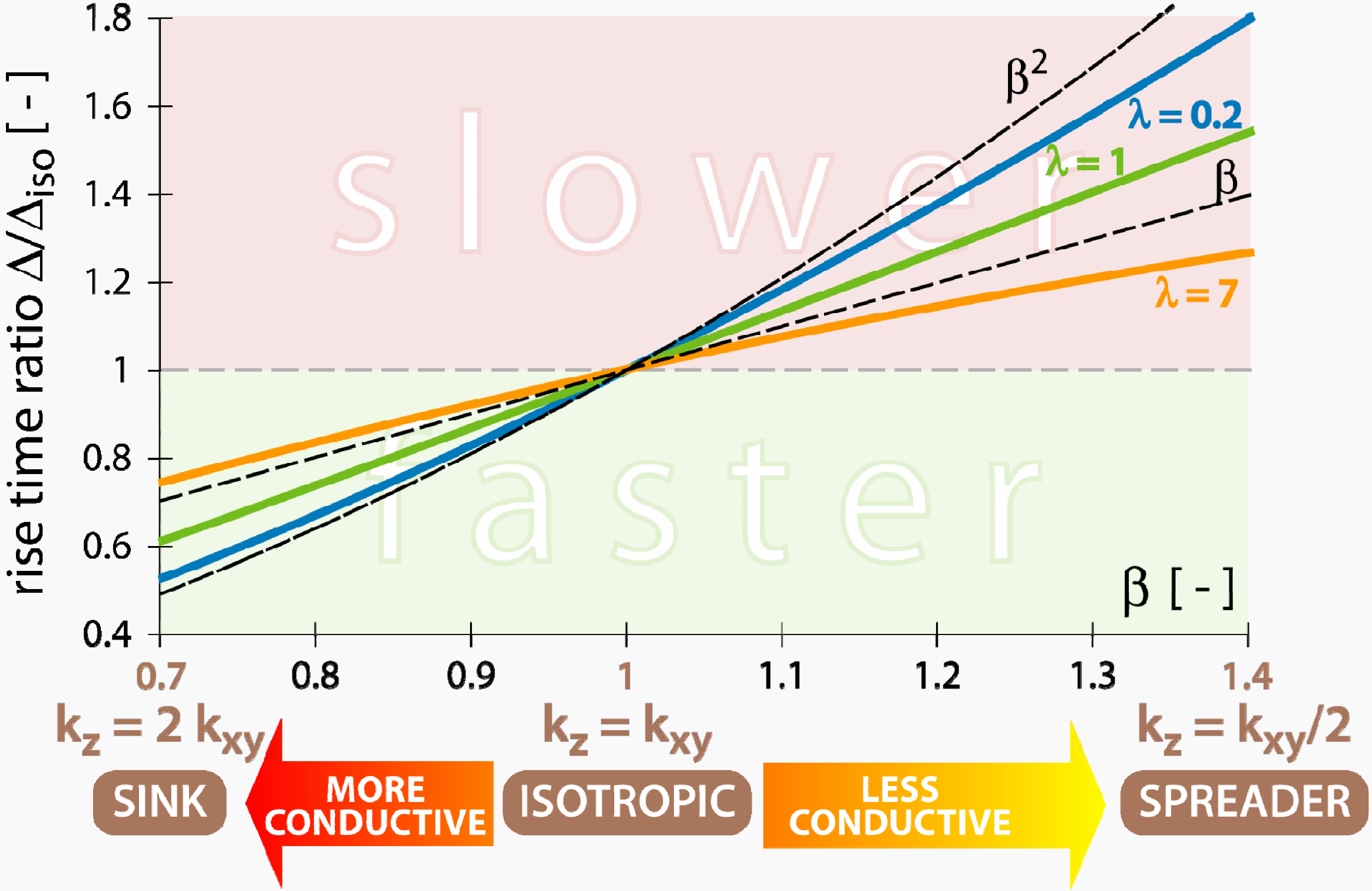
$$\longrightarrow \tilde{t}'_{rise}(\lambda) = \tilde{t}_{rise}(\beta\lambda) \qquad t_0 = \frac{2\pi C_v t_s^2}{k}$$

$$\frac{t'_{rise}}{t_{rise}} = \frac{\Delta}{\Delta_{iso}} = \beta^2 \cdot \frac{0.069 - 0.071 \cdot \dfrac{1 - 0.687(\beta\lambda)^{-1.086}}{1 + 0.687(\beta\lambda)^{-1.086}}}{0.069 - 0.071 \cdot \dfrac{1 - 0.687 \cdot \lambda^{-1.086}}{1 + 0.687 \cdot \lambda^{-1.086}}}$$

$k = k_{xy}$ = constant

Anisotropic substrates

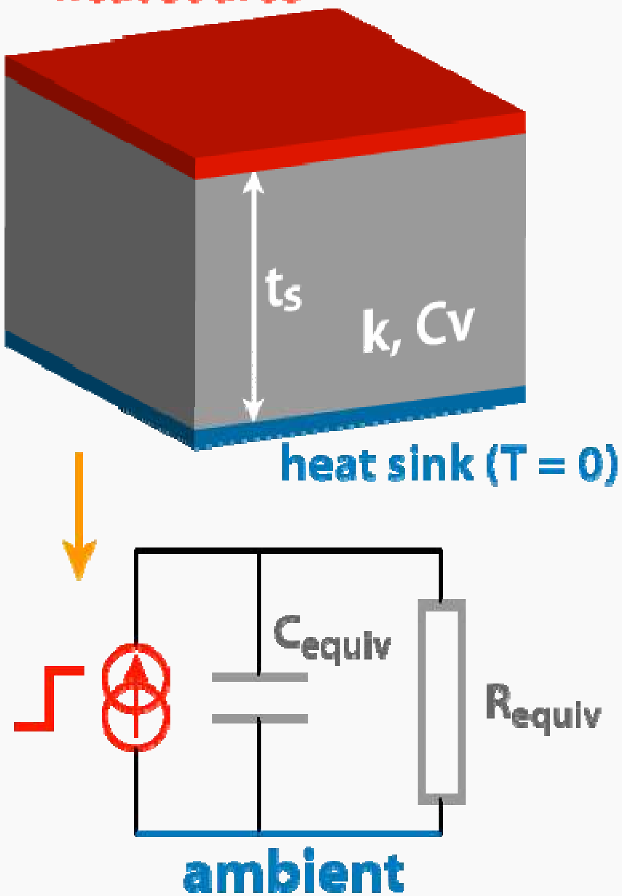
t_{rise} @ constant k_{xy}: results



Anisotropic substrates

t_{rise} @ constant k_{xy}: analysis (1)

THIN SUBSTRATES: β^2 reasonable for Δ/Δ_{iso}



step response for 1-D heat flow

$$\tilde{T} = 1 - \exp\left(-t/\tau_{equiv}\right)$$

$$\tau_{equiv} = R_{equiv} C_{equiv} \approx 0.369 \frac{C_v t_s^2}{k}$$

$$t_{rise} \propto \tau_{equiv}$$

$$\frac{t'_{rise}}{t_{rise}} = \frac{0.369 \frac{C_v t_s^2}{k_z}}{0.369 \frac{C_v t_s^2}{k_{xy}}} = \frac{k_{xy}}{k_z} = \beta^2$$

Anisotropic substrates

t_{rise} @ constant k_{xy}: analysis (2)

- **OTHER CASES**: physical explanation not straightforward at all
- thermal diffusion characterized by whole spectrum of time constants (even for 1-D heat flow)
- trouble:
  - fitting with single exponential impossible if no dominant time constant
  - C_{equiv} not directly proportional to material volume
  - …

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Conclusions

- dynamic fixed-angle heat spreading model
- extends numerous works for steady state
- frequency domain representation (Z_{th}) & step response
- error < 5% if appropriate spreading angle chosen
- simple expressions for ϕ_{opt} & t_{rise}
- valid for wide range of thicknesses (λ = 0.1 … 10)
- can be applied for anisotropic materials
- effect of anisotropy quantified by β + physically explained
- allows quick yet accurate estimation of time-dependent thermal behaviour

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