PFC/JA-86-58

A Fluid Theory of Ion Collection By Probes in Strong Magnetic Fields with Plasma Flow

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October 1986

Submitted to: Physics of Fluids

This work was supported by the U. S. Department of Energy Contract No. DE-AC02-78ET51013. Reproduction, translation, publication, use and disposal, in whole or in part by or for the United States government is permitted.

Abstract

A 1-dimensional fluid theory of Langmuir probe operation in strong magnetic fields is presented. Cross-field diffusion of ions both into and out of the collection region is consistently accounted for. The results differ from previous analyses, which did not account for outward diffusion, by large factors, especially when parallel flow of the external plasma is present. These results provide a more reliable basis for interpretation of recent probe measurements.

I. Introduction

The theory of Langmuir probe operation in strong magnetic fields is of notorious difficulty [1-5]. However, the need for a reliable theory of probe operation in strong fields has become more urgent recently in view of the increased significance attributed to edge conditions in magnetically confined fusion research plasmas and the accompanying proliferation of probe measurements of these edge plasmas [6]. When the ion gyroradius, ρ_i, is substantially smaller than the probe radius, a, ion collection across the field is diffusive even if the parallel flow is dominated by inertial effects. As a result, the quasi-neutral presheath region, in which acceleration of the ions occurs into the sheath, becomes highly elongated along the field, until the cross-field diffusion is able to balance the parallel collection flow.

Since the perpendicular momentum is unimportant in this process, it appears attractive to attempt to simplify the problem by treating the presheath as effectively one-dimensional. One can then seek solutions satisfying Poisson's equation and the equations of motion self-consistently in the parallel direction, treating the perpendicular diffusion equation as a source term in the parallel equations. Stangeby [7,8] has championed this approach recently in applications which have adopted either a fluid or a particle description of the plasma.

The approximations adopted in the 1-dimensional model are that the ion density, n_i, velocity, v_i, and plasma potential ϕ at any parallel position, x, can be regarded as given by single functions of x, representing some kind of mean value of the parameter over the perpendicular extent of the collection region. The radius of the collection region is taken as equal to the probe radius, a. The cross-field diffusion of ions into the collection region may be represented by a source, S, in the ion equations which determine the parallel extent of the collection region. (Without sources a one dimensional presheath would expand to infinity because the quasi neutrality equation would then imply ϕ = constant [9]).

Stangeby and others, in applying this model, have adopted forms of the ion source rate corresponding to 'birth' of ions within the collection region. The resulting equations are then identical to those governing a 1-dimensional plasma discharge, sustained by ionization within the plasma region, between parallel plates. This latter problem has been studied from a kinetic theory plasma viewpoint by Tonks and Langmuir [10] for zero birth velocity and later by Harrison and Thompson [11] who demonstrated that the sheath edge potential and current density are independent of the spatial variation of S. More recently Emmert, et al, [12] have extended these results to the case of finite temperature Maxwellian birth velocity.

These kinetic cases are not easily generalized to the experimentally important situation of a plasma with parallel flow, in which presumably birth with an appropriate flow velocity should be used. Therefore, Stangeby has given a fluid treatment [8] which proves (like the previously mentioned cases) to be analytically integrable, providing compact formulae for ion density and current.

It is, in part, the purpose of the present work to point out that all of these analyses are fatally flawed; that, despite the validity of the theories in treating the 1-dimensional plasma discharge, they cannot be validly applied to treating strong magnetic field probe analysis. The reason lies in the form of the source function adopted. That source is to model the cross field diffusion of ions. As such, it must allow ions not only to diffuse into the collection region, but also to diffuse out. In other words, a diffusion process consists of the exchange of particles between the collection region and the outer plasma. This means that the sources in the equations should model not only outer particles entering the collection region but also (a perhaps smaller number of) particles, which may have spent some time in the collection region, leaving it.

From the point of view of mere ion density all ions are equal, so allowance for ion loss merely causes the source to vary according to the difference between the inside and outside densities. Of itself this would make no difference to the collection current (though it would change the collection region extent) because, as mentioned above, the source rate variation does not affect the current. However, from the point of view of velocity, all ions are <u>not</u> equal. Loss of ions which have already been accelerated in the presheath is not the same as gaining fewer ions at the outer plasma velocity (e.g., at rest). Therefore, to allow the source of ions to be characteristic only of the external velocity, models correctly only the incoming particles not the outgoing. That is, it accounts for 'birth' only, not for 'death' of particles in the collection region.

It might be thought that this distinction is mere quibbling about the details of a model which is already admittedly rather approximate. However, in the following sections we will see that the quantitative differences between the model we propose here, which does correctly account for particle exchange, and the models discussed above, are in many cases very large, especially when finite ion velocity in the outer plasma is accounted for. It turns out that the simplest kinetic treatment mentioned above, based on the zero ion birth velocity solution of Harrison and Thompson [11] agrees fairly well with the collection current we shall calculate. However, the results based on finite ion temperature lead to the plainly unphysical result that for $T_i \gg T_e$ the ions are collected at twice the free gas result ($1/4\ n_i \bar{v}_i$), a problem which has been alluded to elsewhere [13]. And when ion drift velocity is allowed, Stangeby's fluid model can be as much as a factor of 4 wrong for Mach numbers up to 1.

It turns out, too, that the equations correctly accounting for particle exchange appear not to be susceptible to exact analytic solution. Therefore, in order to minimize the computational effort and focus on obtaining applicable results, we shall treat a simple fluid model parallel to Stangeby's. That will provide us with a direct comparison which will illustrate the differences.

In Section II we briefly derive the equations; then in Section III a simple approximate analytic solution of the presheath is given, together with an exact numerical integration of the equations for outer drift velocities up to the sound speed. These results provide the data with which probe measurements to determine ion density and drift velocity can be interpreted. Section IV gives a brief discussion and conclusion.

II. The Model

The presheath is modeled as a 1-dimensional, two-fluid plasma, which is quasi-neutral. Thus, Poisson's equation is replaced by the quasi-neutrality equation, $Zn_i=n_e$ (Z is the ion charge). Also, we restrict attention to cases in which the majority of electrons are reflected because the probe is sufficiently negative. Then the electron density can be taken as given by a Boltzmann factor,

$$n_e = Zn_\infty \exp(e\phi/T_e) \ . \qquad (1)$$

Subscript ∞ here denotes quantities in the outer plasma, far from the collection region, where we take the potential $\phi=\phi_\infty$ to be zero. The electron temperature, T_e, is in energy units.

The diffusive exchange of ions between the collection region and the outer plasma we suppose to take place at a rate Ω. That is, the rate of loss of particles per unit length is $\Omega n_i(x)$ and the rate of gain is Ωn_∞. We can regard Ω as being approximated by $D_\perp/a^2$, the diffusive inverse time constant of the collection region for perpendicular diffusion coefficient $D_\perp$. However, the collection current proves to be independent of $D_\perp$ and indeed of the spatial variation of $D_\perp$. Thus Ω determines only the length of the collection region.

The 1-dimensional continuity equation in steady state is therefore

$$\frac{d}{dx} (n_i v_i) = \Omega(n_\infty - n_i) \ . \tag{2}$$

The exchange of momentum between the collection region and the outer plasma is caused by the particles leaving with characteristic momentum $m_i v_i$ and entering with $m_i v_\infty$. Therefore, the momentum equation is

$$n_i m_i v_i \frac{dv_i}{dx} + m_i v_i \Omega(n_\infty - n_i) = n_i ZeE - \frac{dp_i}{dx} + m_i \Omega(n_\infty v_\infty - n_i v_i) \tag{3}$$

where Ze and p_i are the ion charge and pressure respectively, and $E = -d\phi/dx$ is the electric field. By substituting for $d\phi/dx$ from Eq. (1) this becomes

$$n_i v_i \frac{dv_i}{dx} = - c_s^2 \frac{dn_i}{dx} + \Omega\, n_\infty (v_\infty - v_i) \tag{4}$$

In obtaining this equation we have ignored a term dT_i/dx arising from dp_i/dx. Thus, we are adopting a closure of the fluid equations corresponding to isothermal ions. This is hardly justified a priori, and so should be regarded as a simplifying approximation only. The sound speed, c_s, corresponding to this approximation is given by

$$c_s^2 = \frac{ZT_e+T_i}{m_i} \quad . \tag{5}$$

Equations (2) and (4) are now the plasma presheath equations we require to solve. It should be noted that they differ from those of Stangeby [8] only in the final term of equation (2) (i.e. Ωn_i) which is absent in his model. We now make the following non-dimensionalizing transformations

$$\begin{aligned} n &= n_i/n_\infty \\ M &= v_i/c_s \\ y &= \int_0^x \frac{\Omega}{c_s}\, dx' \quad . \end{aligned} \tag{6}$$

These bring the equations into a form

$$\begin{aligned} M\frac{dn}{dy} + n\frac{dM}{dy} &= 1 - n \\ \frac{dn}{dy} + nM\frac{dM}{dy} &= M_\infty - M \quad . \end{aligned} \tag{7}$$

By elimination we then obtain

$$\begin{aligned} \frac{dn}{dy} &= \frac{(1-n)M-(M_\infty-M)}{M^2-1} \\ \frac{dM}{dy} &= \frac{(M_\infty-M)M-(1-n)}{n(M^2-1)} \end{aligned} \tag{8}$$

and hence

$$\frac{dn}{dM} = n \cdot \frac{(1-n)M-(M_\infty-M)}{(M_\infty-M)M-(1-n)} \tag{9}$$

Notice that Eqs. (8) give singularities at $M=\pm1$. Taking the positive sign to denote flow towards the probe, $M=+1$ is simply the equivalent of the Bohm criterion [1] for sheath formation, and is the required boundary condition at the probe ($y=0$). In view of the other singularity at $M=-1$, there is no regular solution satisfying $M=+1$ at $y=0$ with $M_\infty<-1$. This is a manifestation of the fact that shocks will form in the presheath for supersonic flow. We, therefore, restrict our attention to $|M_\infty|<1$, since this simple model cannot be expected to give an adequate description for supersonic flow. The boundary condition at infinity is $n=1$, $M=M_\infty$.

III. Solutions

The most convenient method of solution is to integrate Eq. (9) to obtain n as a function of M. To do so requires attention to the condition at $M=M_\infty$ (corresponding to $y=\infty$), because both numerator and denominator tend to zero there. A proper treatment of this limit shows that the required derivative at $M=M_\infty$, $n=1$ is

$$\left.\frac{dn}{dM}\right|_\infty = \pm 1 \ . \tag{10}$$

For $|M_\infty|<1$ we expect M to be increasing and n to be decreasing towards the probe; therefore, -1 is the appropriate choice.

An exact analytic solution seems not to be possible in closed form for Eq. (9). However, an approximate solution may be obtained by substituting an appropriate value for n(M) into the fraction on the right hand side and then integrating the equation. If we take $n\approx1-(M-M_\infty)$, as indicated by the boundary condition and perform this integration we get straightforwardly

$$n \approx \exp(M_\infty - M) \tag{11}$$

A somewhat better approximation is obtained by seeking an expansion for n to second order in $(M-M_\infty)$ which makes the appropriate order terms in Eq. (9) zero. This gives $n\approx1-(M-M_\infty)+[(1+M_\infty)/(3+M_\infty)](M-M_\infty)^2$, which when substituted into the fraction in Eq. (9) leads to a solution

$$n = p^\alpha \exp(-q) \tag{12}$$

where

$$\alpha = \frac{1}{8}\,(M_\infty^3+3M_\infty^2+7M_\infty+5)\ \ (M_\infty+1)(M_\infty+5)$$

$$p = 1 + \frac{2(M-M_\infty)}{(M_\infty+1)(M_\infty+3)} \tag{13}$$

$$q = \frac{1}{4}\left[(M_\infty^3 + 3M_\infty^2 + 7M_\infty + 9)(M-M_\infty) + (M_\infty+1)(M-M_\infty)^2\right]$$

The obvious cumbersomeness of such solutions discourages one from pursuing them further. Instead, numerical integration of Eq. (9) has been performed for specified M_∞. The results are shown in Fig. 1.

Since ions flow into the probe sheath at the sound speed the ion current drawn by the probe is proportional to the sheath edge density, i.e., the density at M=1. Figure 2 shows a plot of this density versus the flow Mach number M_∞. Also shown are the results obtained from the approximation Eq. (12) and the corresponding results of Stangeby's model [8]. Notice that the approximate solution is accurate for all but the most negative Mach numbers. Notice also how different the values obtained from Stangeby's model are from our more physically appropriate model.

A simple Langmuir probe will draw ion current to both upsteam and downstream sides. On the other hand a divided (or 'Janus') probe can measure separately the upstream and downstream collection currents. Therefore, for a particular Mach number of flow the two quantities most useful for diagnosis are the mean collection current and the ratio of the collection currents for $\pm M_\infty$. These quantities, obtained from the numerical solution, are plotted (in terms of sheath-edge densities) in Fig. 3. The flow Mach number may be deduced from the ratio and the density from the mean of the ion saturation currents to either side.

Finally we may return to the spatial equations (8) and perform the integration to obtain y as a function of M (and hence n(y)) giving the (nondimensionalized) spatial variation for the presheath density. This is shown in Fig. 4. The presheath potential is then given by Eq. (1).

Although the presheath parallel length, which is a few times $c_s a^2/D_\perp$, does not enter directly into the ion current deduced, it is important in determining the applicability of the analysis. If the presheath length is greater than the parallel distance to the plasma edge or than the ion-electron collision mean free path, then less ion current will be collected and our treatment will break down. To determine whether or not this occurs requires an estimate of $D_\perp$, but provided the presheath length is small enough our results will be independent of $D_\perp$.

IV. Discussion

The ratio of upstream to downstream ion current deduced from our model has a value of about 12 at $M_\infty \approx 1$ and a slope of 2.1 at $M_\infty=0$. This should be compared with Stangeby's result of 3 at $M_\infty=1$ and a slope of 1 at $M_\infty=0$. These differences are far outside the typical uncertainties inherent in probe measurements and so indicate that use of Stangeby's model will give deduced flow velocities which are in error by a large factor.

It is interesting to note that for example Harbour and Proudfoot [14] found ratios up to about 12 in their measurements of scrape-off flows. Using Stangeby's analysis these results indicate supersonic flow, but using a naive particle model Harbour and Proudfoot offered an alternative subsonic interpretation. Our present results indicate that their highest flow velocities correspond within experimental uncertainties to Mach 1.

The differences in <u>mean</u> ion saturation current between our result and that of Stangeby are less dramatic but still significant. We obtain a particle current density at the sheath edge (for $M_\infty=0$)

$$\Gamma_i = 0.35\, n_\infty c_s \ ,$$

whereas Stangeby gets a coefficient of $^1/_2$. It is often stated that the Bohm formula for ion current is approximately $^1/_2 n_\infty c_s$, from which viewpoint Stangeby's result appears more conventional. Actually this is fallacious. The correct (and original) formula is $^1/_2 n_\infty \surd(ZT_e/m_i)$ which Bohm showed [1] had little dependence on ion energy when $T_i \lesssim ZT_e$, for spherical probes in the absence of magnetic field or collisions. Since our definition of c_s includes an ion temperature term we must have some estimate of T_i before we can relate our current to $n_\infty \surd(ZT_e/m_i)$.

It is clear physically that if the outer plasma ion temperature is much smaller than T_e it must nevertheless be a bad approximation to take T_i=0 in the c_s definition. The reason is that the most important place to obtain c_s correctly is at the sheath edge. However, there the spread of ion particle velocities is from zero to $\surd(-Z_e\phi/m_i)$ corresponding to ions which enter the collection region near or far from the sheath edge respectively. Now the sheath edge potential is $(T_e/e)\ln(0.35) \approx -T_e/e$. Therefore, the ion energy spread at the sheath edge is $\sim ZT_e$ even when T_i=0 outside the collection region. This spread will be increased only a small amount by non-zero external T_i. Therefore, the most appropriate value to use is $c_s \approx \surd(2ZT_e/m_i)$ for $T_i \lesssim ZT_e$.

We conclude, therefore, that the ion current deduced from our model is approximately $0.35\times\surd2 = 0.49$ times $n_\infty \surd(ZT_e/m_i)$, for $T_i \lesssim ZT_e$, recovering a Bohm formula, whereas Stangeby's analysis would give a value about $\surd2$ higher.

The reason why Stangeby's formulation always gives a density (and hence current) which is too high, as illustrated for example by Fig. 2, is that loss of momentum from the accelerated presheath flow has been ignored. In our formulation this loss is properly accounted for, with the result that the sheath edge potential must be more negative in order for the mean ion velocity to reach the sound speed.

Other qualitative differences exist between our solutions and those of Stangeby. We may mention first that our M(y) and n(y) tend smoothly to the external values as $y \to \infty$, whereas Stangeby's have discontinuous derivatives at the point where $M = M_\infty$, $n = 1$, (at finite y) in an obviously unphysical way. Another point is that our results give monotonic variation of n and M with y, whereas Stangeby finds that there is a density (and hence potential) maximum on the downstream side, i.e. for $M_\infty < 0$.

In conclusion, a 1-dimensional fluid model has been presented of ion collection by probes in strong magnetic fields, correctly accounting for diffusion out of, as well as into , the presheath. The results show that previous formulations are in error by large factors, particularly when there is parallel plasma flow velocity. The present results make it possible to diagnose these flows using divided (Janus) probes.

Acknowledgements

I have been greatly assisted by stimulating and challenging discussions with Dr. Bruce Lipchultz. This work was supported under U.S. Department of Energy Contract DE/AC02-78ET51013.

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Figure Captions

Fig. 1 Solutions for the normalized density as a function of ion Mach number, in the presheath. Various cases are shown, for which the external plasma flow velocity is equal to the Mach number when n=1.

Fig. 2 The normalized sheath-edge density as a function of external flow Mach number. The curves are: 'Exact' numerical integrations of Eq. (9); 'Approx.', the approximate analytical formula, Eq. (12); 'Stangeby' the result of using the equations of Ref. [8].

Fig. 3 The ratio and mean value of the sheath-edge density for the side of the probe facing upstream and downstream. The ion saturation current density is equal to c_s times the sheath-edge density.

Fig. 4 Variation of density with non-dimensional distance in the presheath. This can be related to physical distance via Eq. (6).

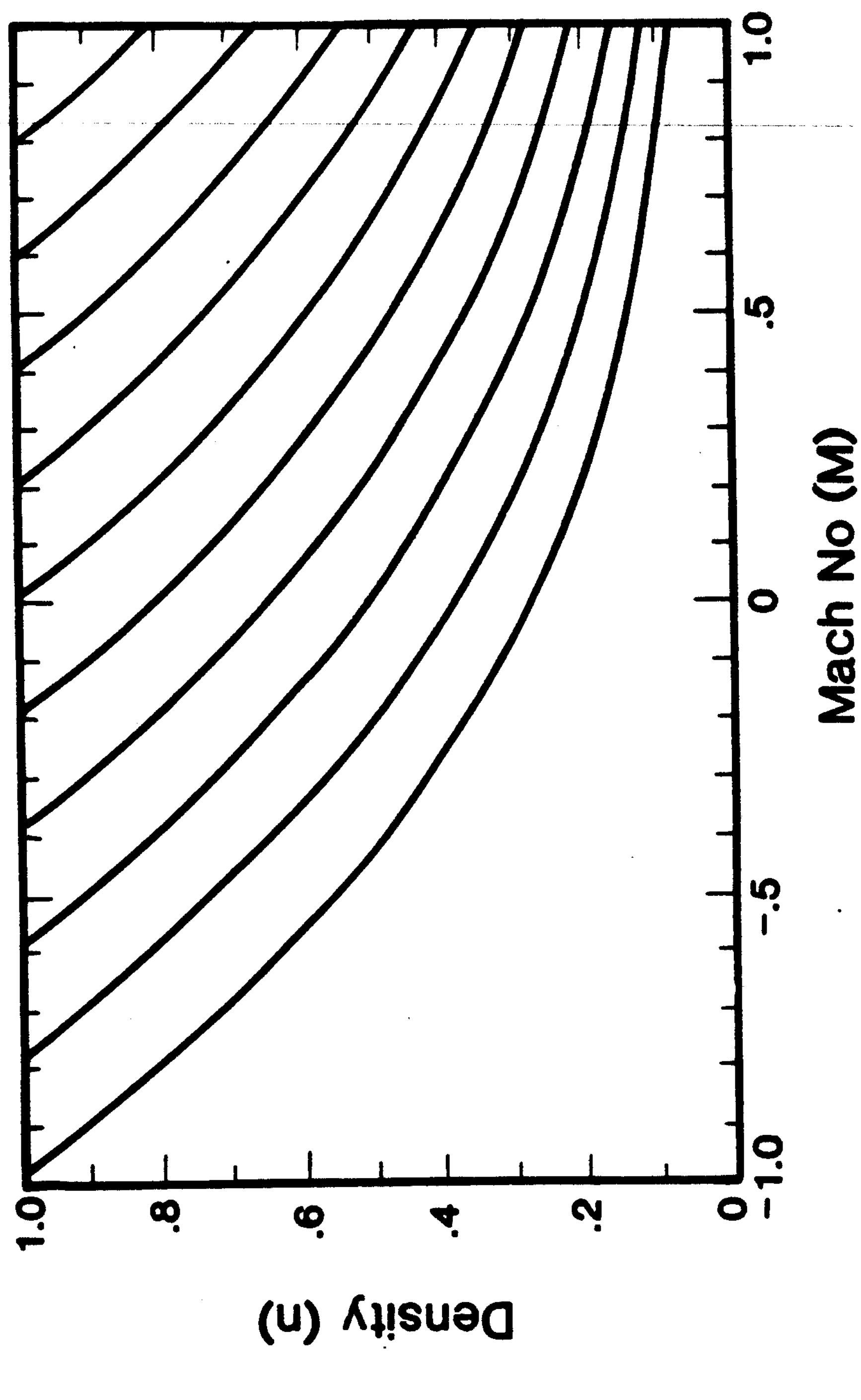
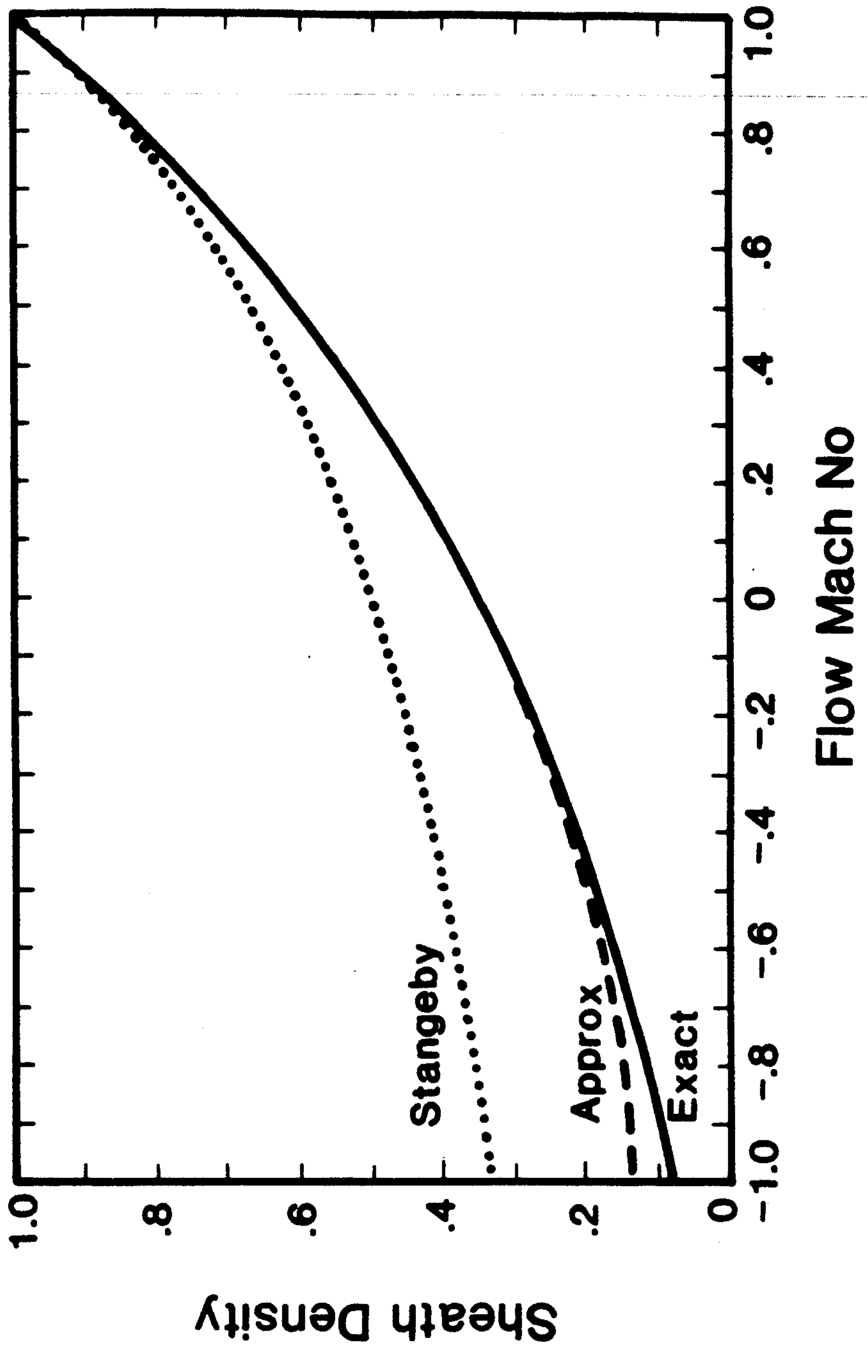


Fig. 1

8706.4.3



Sheath Density
1.0
.8
.6
.4
.2
0
Flow Mach No
−1.0
−.8
−.6
−.4
−.2
0
.2
.4
.6
.8
1.0
Stangeby
Approx
Exact

Fig. 2

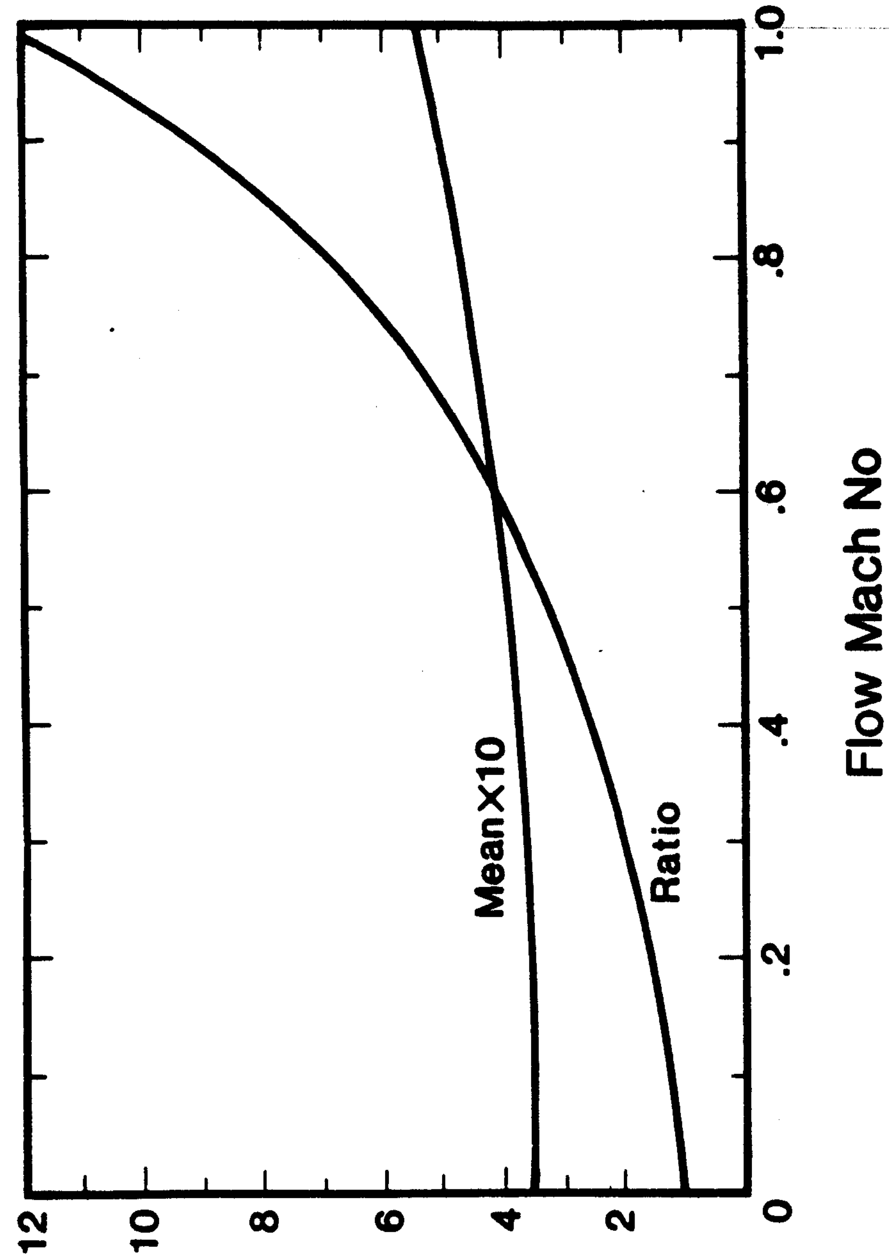
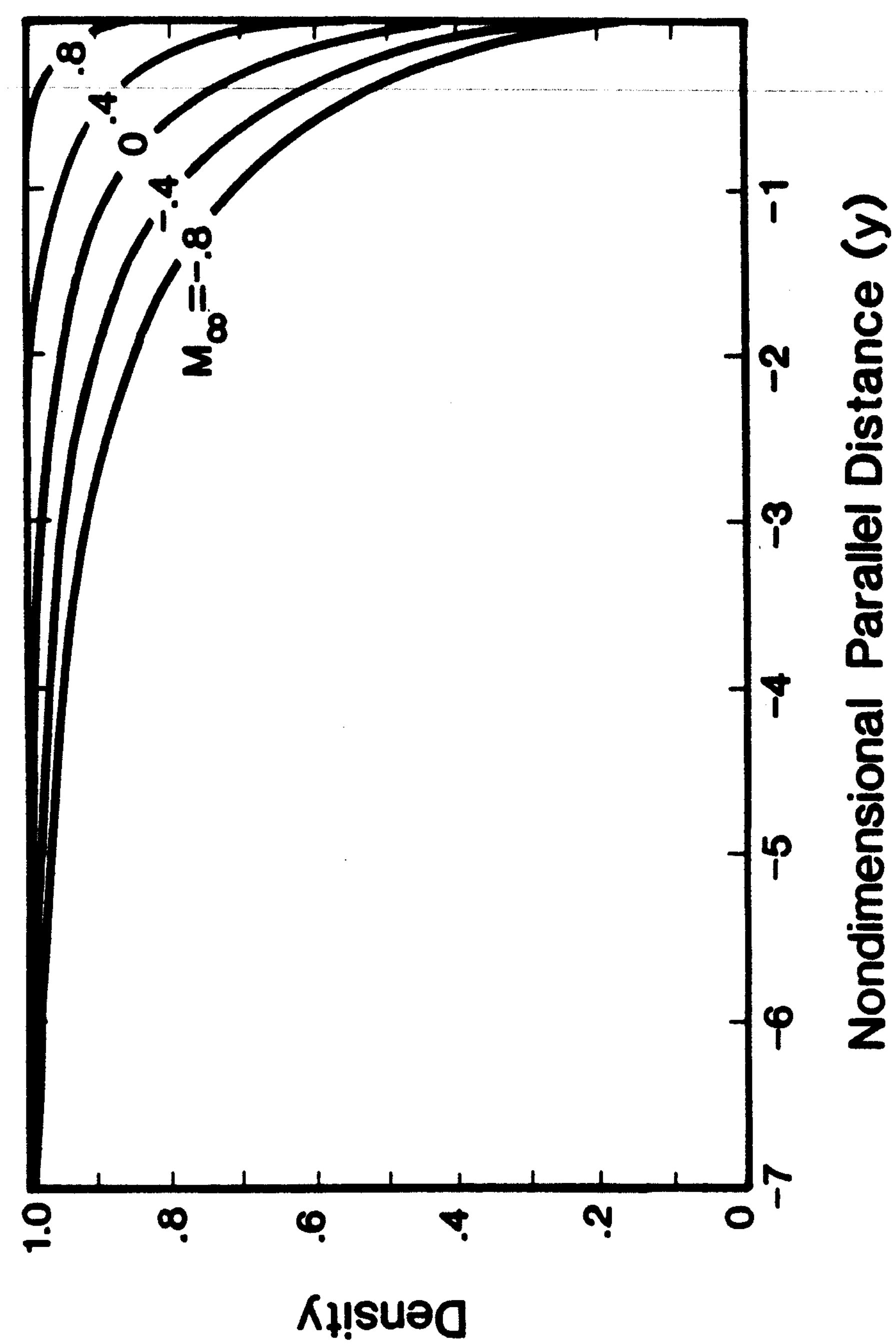


Fig. 3

8706.4.1



Density
1.0
.8
.6
.4
.2
0
−7
−6
−5
−4
−3
−2
−1
Nondimensional Parallel Distance (y)
M_∞=−.8
−.4
0
.4
.8

Fig. 4