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Infante, I.A.C.; Eliav, E.; Vilkas, M.J.; Ishikawa, Y.; Kaldor, U.; Visscher, L.
published in
Journal of Chemical Physics
2007
DOI (link to publisher)
10.1063/1.2770699

## document version

Publisher's PDF, also known as Version of record

## Link to publication in VU Research Portal

## citation for published version (APA)

Infante, I. A. C., Eliav, E., Vilkas, M. J., Ishikawa, Y., Kaldor, U., \& Visscher, L. (2007). A Fock space coupled cluster study on the electronic structure of the UO2, UO2+, U4+, and U5+ species. Journal of Chemical Physics, 127(12). https://doi.org/10.1063/1.2770699

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# A Fock space coupled cluster study on the electronic structure of the $\mathrm{UO}_{2}, \mathrm{UO}_{2}^{+}, \mathbf{U}^{4+}$, and $\mathbf{U}^{5+}$ species 

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(Received 13 April 2006; accepted 18 July 2007; published online 26 September 2007)


#### Abstract

The ground and excited states of the $\mathrm{UO}_{2}$ molecule have been studied using a Dirac-Coulomb intermediate Hamiltonian Fock-space coupled cluster approach (DC-IHFSCC). This method is unique in describing dynamic and nondynamic correlation energies at relatively low computational cost. Spin-orbit coupling effects have been fully included by utilizing the four-component Dirac-Coulomb Hamiltonian from the outset. Complementary calculations on the ionized systems $\mathrm{UO}_{2}^{+}$and $\mathrm{UO}_{2}^{2+}$ as well as on the ions $\mathrm{U}^{4+}$ and $\mathrm{U}^{5+}$ were performed to assess the accuracy of this method. The latter calculations improve upon previously published theoretical work. Our calculations confirm the assignment of the ground state of the $\mathrm{UO}_{2}$ molecule as a ${ }^{3} \Phi_{2 u}$ state that arises from the $5 f^{1} 7 s^{1}$ configuration. The first state from the $5 f^{2}$ configuration is found above $10000 \mathrm{~cm}^{-1}$, whereas the first state from the $5 f^{1} 6 d^{1}$ configuration is found at $5047 \mathrm{~cm}^{-1}$. © 2007 American Institute of Physics. [DOI: 10.1063/1.2770699]


## I. INTRODUCTION

The study of small actinide molecules presents a challenge for experimental and theoretical chemists. ${ }^{1}$ The nearly degenerate $5 f, 6 d, 7 s$, and $7 p$ orbitals give rise to a multitude of possible configuration interactions and a dense manifold of low-lying states, which complicates computations and renders assignment of experimental spectra difficult. A joint effort of experimentalists and theoreticians is therefore needed to resolve the electronic structure of these systems. An example is the ionization potential (IP) of the $\mathrm{UO}_{2}$ molecule, measured as 5.4 eV by Capone et al. ${ }^{2}$ using the electron impact technique. Theoretical calculations ${ }^{3}$ consistently gave a higher value. Gagliardi et al., ${ }^{4}$ who had done accurate complete active space second order perturbation theory (CASPT2) calculations that gave an IP of 6.27 eV , proposed that the experimental data were in error. A new measurement by Han et al. ${ }^{5}$ using resonantly enhanced multiphoton ionization (REMPI), gave a value of 6.13 eV , in very good agreement with the theoretical values.

Other aspects of these small actinide molecules are, however, less well understood, as different theoretical and experimental techniques give conflicting information. A particularly interesting aspect is the interaction of small actinide
molecules with noble gas matrices. Laser ablation spectroscopy has been used by Andrews and co-workers to trap UO, $\mathrm{UO}_{2}$, and CUO in noble gas matrices ${ }^{3,6-15}$ and measure vibrational frequencies as a function of the matrix composition ( $\mathrm{Ne}, \mathrm{Ar}, \mathrm{Kr}, \mathrm{Xe}$, or mixtures thereof). An intriguing feature of both CUO and $\mathrm{UO}_{2}$ is the large redshift (about $130 \mathrm{~cm}^{-1}$ ) in the antisymmetric stretch found when replacing a neon matrix by an argon matrix. Li et al. suggested that this is due to a change in the electronic ground state, and presented density functional theory (DFT) calculations indicating that a weak bond arises by donation of electron density of the noble gas into the empty uranium $6 d$ orbitals in CUO and $\mathrm{UO}_{2}{ }^{9,15}$ In argon and heavier noble gas matrices, this bonding interaction is strong enough to change the ordering of the ground and first excited states, leading to the observed strong redshifts. A convincing argument was the very good agreement between the calculated and observed asymmetric stretch frequencies. Since in these initial DFT calculations the effect of spin-orbit coupling (SOC) was neglected, theoreticians nevertheless questioned the validity of the simple picture presented. This spurred extensive theoretical work, notably by Gagliardi and co-workers, ${ }^{4,16-18}$ who applied the more sophisticated CASPT2 method and also studied the effect of SOC.

In CUO, the two competing states are the uranyl-like closed shell state, with two formal triple bonds between uranium, carbon, and oxygen, and a triplet in which the $\mathrm{C}-\mathrm{U}$ bonding orbital and the uranium $5 f_{\phi}$ orbital are singly occupied. The latter state has a longer CU bond length, allowing for a more effective interaction with a matrix than the singlet state. In argon, this interaction should then be sufficiently strong to reverse the order of the two states, while in the more weakly interacting neon matrix, the ordering is the same as in the gas phase. This explanation of the large matrix effect is currently generally accepted for CUO, even though the CASPT2 calculations of Roos et al. ${ }^{17}$ gave the triplet ${ }^{3} \Phi$ at slightly lower energy in the gas phase. Our previous work ${ }^{19}$ using the Dirac-Coulomb coupled cluster method, $\operatorname{DC}-\operatorname{CCD}(\mathrm{T})$, gives the closed shell state as ground state in the gas phase, in agreement with the original picture. More important is that all methods predict small energy differences between the two states in the gas phase and do not contradict the explanation given by Andrews and co-workers.

The situation is more complicated for the $\mathrm{UO}_{2}$ molecule. Likely candidates for the ground state are the $5 f^{1} 7 s^{13} \Phi_{u}$ and $5 f^{2}{ }^{3} H_{g}$ states. These states differ in occupation of the $5 f_{\delta}$ orbital (the ${ }^{3} H_{g}$ state) versus the $7 s$ orbital (the ${ }^{3} \Phi_{u}$ state). Both orbitals are nonbonding but the $7 s$ orbital is more diffuse, leading to stronger and shorter bonds in the ${ }^{3} \Phi_{u}$ state. Vibrational spectroscopy gives an asymmetric stretch in the Ne matrix of $915 \mathrm{~cm}^{-1}$ versus $776 \mathrm{~cm}^{-1}$ in the Ar matrix, ${ }^{3,14}$ which suggests that the ordering of states in $\mathrm{UO}_{2}$ also depends on the matrix. Bonding of the noble gas atoms to the ${ }^{3} H_{g}$ state would not only be favored by the longer bond length of that state but also by the lack of repulsive interaction with the electron in the $7 s$ orbital. This picture is corroborated by DFT calculations ${ }^{15}$ of vibrational frequencies for gas phase $\mathrm{UO}_{2}$. The $5 f^{1} 7 s^{13} \Phi_{u}$ and $5 f^{2}{ }^{3} H_{g}$ states do indeed match the experimental frequencies in the neon and argon matrices, respectively. In this case there are, however, also complementary experimental data available. Heaven and co-workers carried out electron spectroscopy in gas phase ${ }^{5,20}$ and in Ar matrices. ${ }^{21}$ These experiments, using the REMPI technique in the gas phase and electronic emission spectroscopy in the matrix, do not indicate a reordering of the states. Both the gas phase and matrix spectra can only be rationalized by assuming that the first excited state lies slightly above the ground state $\left(360 \mathrm{~cm}^{-1}\right.$ in the gas phase, $408 \mathrm{~cm}^{-1}$ in the argon matrix) and is of the same parity. This fits well with the assignment of the ground state as the lower components of the spin-orbit split ${ }^{3} \Phi_{u}$ state. SOC is rather large, leading to significant admixture of ${ }^{3} \Delta_{u}$ character in both the $2 u$ ground state and the $3 u$ first excited state (better described in a $j j$-coupling picture as pure $5 f_{5 / 2}^{1} 7 s_{1 / 2}^{1}$ states). Actual calculations on gas phase $\mathrm{UO}_{2}$ by Chang, ${ }^{22}$ Gagliardi et al. ${ }^{4,18}$ and Fleig et al. ${ }^{23}$ reproduce this splitting well. The manifold of SOC-split grade states does not have two so closely spaced states at low energy. If the ${ }^{3} H_{g}$ state would be the lowest state in the argon matrix, the next gerade state is expected to lie several thousands of $\mathrm{cm}^{-1}$ higher. Han et al. ${ }^{5}$ and later Gagliardi et al. ${ }^{4}$ discussed the difficulties in ex-
plaining both experimental findings but could not present definite theoretical or experimental data to settle the issue of matrix-induced ground state swapping.

A survey of the theoretical and experimental data that is available leads to more questions. For example, the third and fourth excited states in the argon matrix ${ }^{21}$ lie, experimentally, at 1094 and $1401 \mathrm{~cm}^{-1}$, whereas the CASPT2 values ${ }^{18}$ are, respectively, at 2567 and $2908 \mathrm{~cm}^{-1}$, about $1500 \mathrm{~cm}^{-1}$ off. Is this large discrepancy caused by the differences induced by the argon matrix, by deficiencies in the calculation, such as limits on the size of the active space used, or by both? The generalized active space configuration interaction (GASCI) results by Fleig et al. ${ }^{23}$ agree better with experiment and with older spin-orbit configuration interaction calculations of Chang ${ }^{22}$ but both calculations were done in rather modest basis sets and could suffer from basis set incompleteness errors. It is therefore clear that more theoretical work is desirable.

Accurate calculations of the quasidegenerate states of $\mathrm{UO}_{2}$ and similar actinide systems, where $d$ and $f$ orbitals belong to the valence space, are extremely difficult. First principles methods aimed at such systems should not only be based on size-extensive, size-consistent, and balanced treatment of the dynamic and nondynamic correlation effects, but also include the relativistic effects from the outset. The aim of our paper is to reanalyze the $\mathrm{UO}_{2}$ molecule with the relativistic Fock space coupled cluster (FSCC) method that satisfies all these requirements. The FSCC method has been applied to a large number of atoms and molecules, including transition and heavy elements, with experimentally known spectroscopic properties. Examples are atomic gold, ${ }^{24,25} \mathrm{Fr}^{26}$ the lanthanides $\mathrm{La},{ }^{27} \mathrm{Pr},{ }^{28} \mathrm{Yb}$, and $\mathrm{Lu},{ }^{29}$ the actinides $\mathrm{Ac},{ }^{30}$ $\mathrm{Th},{ }^{31}$ and $\mathrm{U},{ }^{28}$ as well as $\mathrm{Hg},{ }^{31} \mathrm{Tl},{ }^{30,32} \mathrm{~Pb},{ }^{33}$ and $\mathrm{Bi} .{ }^{34}$ Good agreement with experimental transition energies (within a few hundreds of wave numbers) was obtained. To quote one example, the average error for the $f^{2}$ levels of $\mathrm{Pr}^{3+}$ was $222 \mathrm{~cm}^{-1}$, four times smaller than that of an extensive multiconfigurational Dirac-Fock calculation. ${ }^{25}$ The quality of results was sufficient to allow reliable predictions for the nature of the ground states and spectra of a number of superheavy elements. Molecules calculated by the method include $\mathrm{AuH},{ }^{35} \mathrm{Au}_{2},{ }^{36} \mathrm{HgH},{ }^{37}$ and TlF. ${ }^{38}$ The FSCC approach has recently been extended by the intermediate Hamiltonian (IH) scheme, ${ }^{39,40}$ which allows the use of much larger P spaces and improves results considerably [see, e.g., the electron affinity of Bi (Ref. 41)]. The new, more accurate IHFSCC method is applied on the electronic spectrum of neptunyl and plutonyl ions, ${ }^{42}$ and is also used in the present work.

An advantage of this approach is its relatively low computational cost, allowing us to use adequate basis sets and active spaces that include all relevant orbitals. The largest calculation performed in the current work had 41 Kramer pairs in the $P$ part of the active space. The method scales, like regular CCSD, as $N^{6}$ in the number of correlated electrons, $N$. This scaling is sufficiently low to make calculations on $\mathrm{UO}_{2}$ surrounded by one or more argon atoms feasible in the near future. Such calculations were, however, not possible with the computers currently available to us.

## II. METHODOLOGY

Benchmark calculations on the U(IV) ion were done using the radial relativistic Fock space CC code of Eliav et al. ${ }^{24}$ and the multi-reference Møller-Plesset (MR-MP) code of Vilkas et al. ${ }^{43-45}$ for the CASPT2 calculations. All-electron single- and multireference correlated calculations on the $\mathrm{UO}_{2}$ molecule were carried out utilizing a locally modified version of the DIRAC04 program. ${ }^{46}$

We considered two types of basis sets, the universal basis set (UBS) of Malli et al., ${ }^{47}$ consisting of $37 s 32 p 24 d 21 f 12 g 10 h 9 i$ uncontracted Gaussians, which provides nearly basis-set-free results in the atomic case, and the more economical $33 s 29 p 21 d 15 f 3 g 1 h$ even-tempered basis set provided by Faegri. ${ }^{48}$ The difference between the atomic results calculated in both sets gives an indication of basis set incompleteness errors, this was furthermore checked by adding additional diffuse functions to the Faegri basis in some of the molecular calculations. The basis set on the oxygen atoms is the uncontracted cc-pVTZ (Refs. 49-51) basis that can be considered sufficient for the accuracy that is targeted. In all cases, the relativistic four-component Dirac-Coulomb (DC) or Dirac-Coulomb-Breit (DCB) Hamiltonians ${ }^{52,53}$ include SOC from the outset, so that mixing of orbitals with different orbital angular momenta occurs already at the Hartree-Fock (HF) level. To facilitate analysis, and for comparison with more conventional approaches, we also used the spin-free modified DC (SFDC) Hamiltonian, ${ }^{54}$ in which SOC is projected out, leaving only the scalar relativistic effects in a four-component framework.

In the atomic calculations, the number of correlated electrons was taken as 51 for $\mathrm{U}^{5+}$ and 52 for $\mathrm{U}^{4+}$. The active virtual space was limited from above by the orbital energy value of +100.0 a.u. The model space $P$ used in the Fock space coupled cluster calculations consisted of all determinants built from the $7-11 s, 7-11 p, 6-10 d, 5-9 f, 7-9 g$, $8-9 h$, and $9 i$ orbitals and was subdivided in a primary $\left(P_{m}\right)$ space that included determinants constructed from $7 s, 7 p$, $6 d$, and $5 f$ orbitals and a secundary $\left(P_{i}\right)$ space that contained the remaining $P$ determinants.

For the molecular correlated calculations we included 12 electrons from the $5 f, 7 s, 6 d$, and $6 p$ orbitals of the uranium atom. The $2 s$ and $2 p$ orbitals of the oxygen atoms, six electrons each, were always taken active given a minimum of 24 electrons that is correlated. The stability of the results with number of electrons correlated was tested by also including the $\mathrm{U} 6 s$ and $5 d$ shells in some calculations. We used two coupled cluster methods that differ in the way the outermost valence electrons are treated. In a conventional singlereference relativistic coupled cluster with explicit inclusion of single and double excitations and perturbative treatment of triples [DC-CCSD(T)] (Refs. 55 and 56) approach, open shell orbitals are generated by a restricted Hartree-Fock method that averages the energy expression of the lowest $5 f 7 s$ or $5 f^{2}$ open shell singlet and triplet (borrowing the nomenclature from nonrelativistic work; a more accurate description is that we place one electron in each of the two highest occupied Kramers spinor pairs). The CC calculation
is then carried out starting from a $\left|5 f_{5 / 2} 7 s_{ \pm 1 / 2}\right|$ reference determinant. A single-reference approach can be used since, in contrast to approaches in which SOC is added a posteriori; the determinants $\left|5 f_{5 / 2} 7 s_{ \pm 1 / 2}\right|$ and $\left|5 f_{5 / 2} 5 f_{3 / 2}\right|$ provide good first approximations to the $2_{u}, 3_{u}$, and $4_{g}$ states. This method is complemented by the genuine multireference FSCC approach ${ }^{57}$ in which we start from a common closed shell reference determinant of the $\mathrm{UO}_{2}^{2+}$ molecule, or the $\mathrm{U}^{6+}$ ion, then add two electrons successively in sectors $(0,1)$ and $(0,2)$. A full CI ( $P$-space diagonalization) is performed in the selected Fock space valence sectors, in order to obtain the nondynamic correlation energy and the multireference wavefunction characteristics of each excited state. The choice of the model space $P$ is nontrivial, the largest $P_{m}$ space for which the FSCC scheme was found to converge, comprising the $7 s$, two of the five $6 d$, and six of the seven $5 f$ spinors, excluding the higher lying $5 f_{\sigma_{1 / 2}}$. Further increase of the model space was not possible, because it leads to intruder states, in particular, in sector (0,2). The CASPT2 approach ${ }^{43-45}$ used in some of the atomic reference calculations is similar, but not identical to the method used by Gagliardi et al. in calculations of the $\mathrm{UO}_{2}$ molecule. ${ }^{18}$ The main difference is in construction of one-electronic orbital space. We have used common set of radial average-state selfconsistent field (SCF) canonical spinors for the ground and all excited states, while in the Roos-Gagliardi CASSCF/ CASPT2 approach a common set of orbitals is used only for states with the same spin and same wave-function symmetry.

To determine the equilibrium geometry of the ground state, we performed FSCC calculations using an evenly spaced ( $0.005 \AA$ ) grid of $\mathrm{U}-\mathrm{O}$ bond distances, spanning the range from 1.680 to $1.840 \AA$. Since DC-IHFSCCSD provides the energy of all states in one calculation, we could obtain the equilibrium geometry of the ground state and that of many excited states. The equilibrium bond distances, found by energy minimization, were $1.739 \AA$ for $\mathrm{UO}_{2}^{+}$and $1.770 \AA$ for $\mathrm{UO}_{2}$. These distances are used to compute adiabatic excitation and ionization energies.

## III. RESULTS AND DISCUSSION

## A. The spectrum of the atomic ions

The FSCC method was first applied by Eliav et al. ${ }^{28}$ for the $5 f^{2}$ states of $\mathrm{U}^{4+}$. Here, we employ a larger basis set and extend the analysis of the excited states to the $\mathrm{U}^{5+}$ ion. We include transitions to $5 f 7 s$ and $5 f 6 d$ states of $\mathrm{U}^{4+}$, because these excitations are important in the electronic spectrum of the neutral $\mathrm{UO}_{2}$ molecule. To compare accuracies of method applied in the molecular case, we report also the atomic CASPT2 energies in Table I.

Comparing the different methods with the experimental data of Kaufman and Radziemski, ${ }^{58}$ the excitation energies of the $\mathrm{U}^{5+}$ ion appear to be best described by the XIHFSCCSD scheme (extrapolated intermediate Hamiltonian Fock space coupled cluster with single and double excitations), which within the large UBS basis set gives a mean absolute error (MAE) relative to the experimental data of

TABLE I. The excitation energies $\left(\mathrm{cm}^{-1}\right)$ and mean absolute errors (MAE) relative to the experimental data of $\mathrm{U}^{5+}$ and $\mathrm{U}^{4+}$ ions computed at different levels of theory. For the ground state of $U(I V)$ and $U(V)$ we list the ionization potential $\left(\mathrm{cm}^{-1}\right)$.

| Symmetry | Type | Expt. ${ }^{\text {c }}$ | Universal basis set ${ }^{\text {a }}$ |  |  |  | Faegri basis set ${ }^{\text {b }}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | DCB- <br> CASPT2 <br> (5f6d7s) | $\begin{gathered} \text { DCB- } \\ \text { XIH } \\ \text { FSCC } \end{gathered}$ | DCCASPT2 (5f6d7s) | $\begin{gathered} \text { DC- } \\ \text { XIH } \\ \text { FSCC } \end{gathered}$ | $\begin{gathered} \text { DC- } \\ \text { XIH } \\ \text { FSCC } \end{gathered}$ | $\begin{gathered} \text { DC- } \\ \text { CASPT2 } \\ (5 f 7 s) \end{gathered}$ | DC- <br> CASPT2 <br> (5f6d7s) | $\begin{gathered} \text { DC- } \\ \text { CASPT2 } \\ (5 f 6 d 7 s 7 p) \end{gathered}$ |
| $\mathrm{U}^{5+}$ states |  |  |  |  |  |  |  |  |  |  |
| $6 p^{6} 5 f_{5 / 2 u}$ | $5 f$ | ... | . ${ }^{\text {. }}$ | 508183 | ... | 507326 | 505260 |  |  | ... |
| ${ }^{2} F_{7 / 2}$ | $5 f$ | 7609 | 8226 | 7598 | 8384 | 7833 | 7784 | 8228 | 8228 | 8228 |
| ${ }^{2} D_{3 / 2}$ | $6 d$ | 91000 | 95309 | 90562 | 92989 | 89564 | 88930 | $\cdots$ | 88772 | 88772 |
| ${ }^{2} D_{5 / 2}$ | $6 d$ | 100511 | 105871 | 100107 | 103619 | 99245 | 98586 | . $\cdot$ | 99271 | 99271 |
| ${ }^{2} S_{1 / 2}$ | $7 s$ | 141448 | 144946 | 140211 | 142206 | 139062 | 137660 | 135660 | 135811 | 135811 |
| ${ }^{2} P_{1 / 2}$ | $7 p$ | 193340 | ... | 192351 | ... | 190993 | 194402 | ... | ... | 188322 |
| ${ }^{2} P_{3 / 2}$ | $7 p$ | 215886 | $\cdots$ | 215112 | $\cdots$ | 213698 | 216531 | $\cdots$ | $\cdots$ | 212988 |
|  | MAE |  | $\cdots$ | 651 | $\cdots$ | 1650 | 1603 | $\cdots$ | $\cdots$ | 2948 |
| $\mathrm{U}^{4+}$ states |  |  |  |  |  |  |  |  |  |  |
| ${ }^{3} \mathrm{H}_{4}$ | $5 f^{2}$ | $\ldots$ | 402654 | 381074 | 401337 | 380220 | 378222 | $\cdots$ | $\cdots$ | ... |
| ${ }^{3} \mathrm{~F}_{2}$ | $5 f^{2}$ | 4161 | 3773 | 4202 | 3742 | 4190 | 4202 | 3822 | 3815 | 3815 |
| ${ }^{3} \mathrm{H}_{5}$ | $5 f^{2}$ | 6137 | 6631 | 6070 | 6746 | 6275 | 6223 | 6198 | 6593 | 6596 |
| ${ }^{3} F_{3}$ | $5 f^{2}$ | 8983 | 8897 | 8974 | 8986 | 9147 | 9118 | 8614 | 8907 | 8922 |
| ${ }^{3} F_{4}$ | $5 f^{2}$ | 9434 | 9779 | 9404 | 9892 | 9586 | 9574 | 9598 | 9575 | 9930 |
| ${ }^{3} H_{6}$ | $5 f^{2}$ | 11514 | 12486 | 11420 | 12676 | 11780 | 11713 | 11759 | 12463 | 12466 |
| ${ }^{1} D_{2}$ | $5 f^{2}$ | 16465 | 15106 | 16554 | 15196 | 16785 | 16709 | 15723 | 15479 | 17476 |
| ${ }^{1} G_{4}$ | $5 f^{2}$ | 16656 | 17391 | 16630 | 17599 | 16937 | 16870 | 16755 | 17473 | 17464 |
| ${ }^{3} P_{0}$ | $5 f^{2}$ | 17128 | 15556 | 17837 | 15546 | 17840 | 17941 | 16728 | 16014 | 16014 |
| ${ }^{3} P_{1}$ | $5 f^{2}$ | 19819 | 18426 | 20441 | 18500 | 20570 | 20638 | 19356 | 18844 | 18845 |
| ${ }^{1} I_{6}$ | $5 f^{2}$ | 22276 | 21089 | 22534 | 21306 | 22812 | 23067 | 22950 | 22182 | 22185 |
| ${ }^{3} P_{2}$ | $5 f^{2}$ | 24652 | 23539 | 24991 | 23753 | 25315 | 25300 | 24077 | 23937 | 23923 |
| ${ }^{1} S_{0}$ | $5 f^{2}$ | 43614 | 43361 | 45611 | 43483 | 45765 | 45571 | 45340 | 44454 | 44443 |
| ${ }^{3} \mathrm{H}_{4}$ | $5 f 6 d$ | 59183 | 65821 | 57161 | 63221 | 56289 | 55501 | ... | 58609 | 58612 |
| ${ }^{3} F_{2}$ | $5 f 6 d$ | 59640 | 65172 | 57324 | 62542 | 56475 | 55667 | $\cdots$ | 58003 | 58067 |
| ${ }^{3} G_{3}$ | $5 f 6 d$ | 63053 | 68182 | 61331 | 65353 | 60510 | 59739 | $\ldots$ | 60941 | 60983 |
| ${ }^{1} G_{4}$ | $5 f 6 d$ | 65538 | 72154 | 63336 | 69659 | 62641 | 61791 | $\cdots$ | 65173 | 65176 |
| ${ }^{3} F_{3}$ | $5 f 6 d$ | 67033 | 71826 | 64485 | 69537 | 64141 | 63334 | ... | 65153 | 65146 |
| ${ }^{3} \mathrm{H}_{5}$ | $5 f 6 d$ | 67606 | 75044 | 65755 | 72542 | 65052 | 64282 | ... | 67828 | 67831 |
| ${ }^{3} F_{2}$ | $5 f 7 s$ | 94070 | 97573 | 91410 | 94548 | 90411 | 88841 | 81073 | 89132 | 89673 |
| ${ }^{3} F_{3}$ | $5 f 7 s$ | 94614 | 98083 | 91941 | 95059 | 90965 | 89402 | 81578 | 89727 | 90134 |
| ${ }^{3} F_{4}$ | $5 f 7 s$ | 101612 | 105500 | 98921 | 102614 | 98168 | 96512 | 88572 | 97080 | 97437 |
| ${ }^{1} F_{3}$ | $5 f 7 s$ | 102407 | 105987 | 99713 | 103108 | 98967 | 97492 | 89312 | 97774 | 98302 |
| ${ }^{3} G_{3}$ | $5 f 7 p$ | 139141 | ... | 138614 | ... | 137582 | 138904 | ... | ... | 135528 |
| ${ }^{3} F_{2}$ | $5 f 7 p$ | 140642 | $\cdots$ | 139502 | $\cdots$ | 138380 | 138990 | $\cdots$ | $\cdots$ | 133929 |
| ${ }^{3} G_{4}$ | $5 f 7 p$ | 146926 | $\cdots$ | 145150 | . | 143970 | 147671 | $\cdots$ | $\cdots$ | 143111 |
| ${ }^{3} D_{3}$ | $5 f 7 p$ | 147170 | $\ldots$ | 146413 | $\cdots$ | 145613 | 146180 | $\ldots$ | $\cdots$ | 141555 |
| ${ }^{3} F_{3}$ | $5 f 7 p$ | 156493 | $\ldots$ | 156024 | $\ldots$ | 155028 | 155402 | $\ldots$ | $\cdots$ | 154659 |
|  | MAE $5 f^{2}$ |  | 825 | 357 | 814 | 514 | 507 | 488 | 626 | 654 |
|  | MAE $5 f 6 d$ |  | 6024 | 2110 | 3467 | 2824 | 3623 | ... | 1132 | 1115 |
|  | MAE 5f7s |  | 3610 | 2680 | 657 | 3548 | 5114 | 13042 | 4784 | 4289 |
|  | MAE $5 f 7 p$ |  | ... | 898 | ... | 1924 | 907 | $\cdots$ | $\cdots$ | 4282 |
|  | MAE |  | $\cdots$ | 1191 | $\cdots$ | 1738 | 1956 | $\cdots$ | $\cdots$ | 1.967 |

${ }^{2}$ Reference 47.
${ }^{\mathrm{b}}$ Reference 48.
${ }^{\mathrm{c}}$ References 58 and 59.
$1650 \mathrm{~cm}^{-1}$ without and $651 \mathrm{~cm}^{-1}$ with the Breit interaction. One possible drawback of the Fock space approach is that the starting orbitals to compute the $\mathrm{U}^{5+}$ and $\mathrm{U}^{4+}$ energies are optimized for the highly ionized $\mathrm{U}^{6+}$ ion. Thus, the method must include the full orbital relaxation when computing the excitation energies of the less charged ions. The major part
of the orbital relaxation effects in the present FSCCSD calculations is taken into account by single-electronic excitation amplitudes and two-electronic diagrams with a pair of socalled "spectator" lines (correspond to identical valence orbitals). To account for the rest of the orbital relaxation, which could still be substantial, one must include the contribution


FIG. 1. The electron affinities of $\mathrm{UO}_{2}^{2+}$ molecule for the $5 f, 7 s, 6 d$, and $7 p$ orbitals. On the left, the spin-free and spin-orbit coupling contributions calculated at DC-HF level. On the right, the correlated values from sector $(0,1)$ of the DC-IHFSCCSD calculations. The correlation space was ( $24 e / 6$ a.u.), with a $(17 g, 20 u) P$ model space that includes the $7 p$ orbitals in $P_{m}$. All calculations were performed with the Faegri basis set. The bond distance is $1.770 \AA$.
of higher excitations amplitudes with the appropriate spectator lines [e.g., triples with two pairs of spectators in the case of $(0,2)$ sector] and use a large active space.

For the $\mathrm{U}^{5+}$ ion, the errors are larger for the more diffuse orbitals, like the $6 d, 7 s$, and $7 p$ shells that show discrepancies from experiment of about $500-1000 \mathrm{~cm}^{-1}$, while the $5 f$ states have errors in the range of $100-200 \mathrm{~cm}^{-1}$. Errors that can be related to incomplete orbital relaxation are also visible in the $\mathrm{U}^{4+}$ ion. The errors relative to the experimental data of Wyart et al. ${ }^{59}$ we obtain with DCB-XIHFSCCSD are small for energy differences among the $5 f^{2}$ states, a MAE of $357 \mathrm{~cm}^{-1}$, but are significant, with a MAE of $2110 \mathrm{~cm}^{-1}$ for transitions to $5 f^{1} 6 d^{1}$ states, and for transitions to the $5 f^{1} 7 s^{1}$ states, a MAE of $2680 \mathrm{~cm}^{-1}$. Despite the fact that these errors are larger than the ones found for the $\mathrm{U}^{5+}$ ion, the overall MAE is still rather low, $1191 \mathrm{~cm}^{-1}$. For comparison, we may look at the CASPT2 method, which does not improve or worsen much, compared to the $\mathrm{U}^{5+}$ ion, and shows errors that are about twice as large as the XIHFSCC values if the largest basis and most accurate Hamiltonian (DCB) are used. It is interesting to note that XIHFSCC and CASPT2 give a qualitatively different error in the calculation of the energy of the $5 f^{1} 6 d^{1}$ manifold relative to the $5 f^{2}$ states: the XIHFSCC values are $2000 \mathrm{~cm}^{-1}$ too low whereas the CASPT2 values are $6000 \mathrm{~cm}^{-1}$ too high.

Table I shows that the effect of the Breit interaction is
much larger in CASPT2 than in the all-order FSCC calculation. Similar large effects are observed in the results of the first FSCC iteration, equivalent to a second order perturbation calculation. The MAEs of atomic excitation energies calculated by CASPT2 increase strongly upon inclusion of the Breit interaction, indicating that the relatively good performance of the CASPT2 method based on the DC Hamiltonian may be due to cancellation of errors. We also investigated the convergence of the CASPT2 energies with the systematic enlargement of the CAS as they may be relevant in discussing the molecular results. It is clear from the table that inclusion of the $6 d$ orbital into the CAS is very important for the quantitatively correct description of the intrashell excitations, while the effect of the $7 p$ orbital is much less pronounced. This points towards the inclusion of the $6 d$ orbital of uranium in the CAS, a procedure that is usually not followed as it leads to prohibitively large CAS spaces in molecular calculations.

The atomic calculations indicate that the FSCC approach is a systematic and precise method to describe the excitation energies of the actinide ions. Inclusion of the Breit term in the Hamiltonian significantly improves the quality of the results, as it should, giving a mean absolute error of $1191 \mathrm{~cm}^{-1}$ for $\mathrm{U}^{+4}$ with all levels coming out in the correct order. It is likely that this accuracy is representative for the errors made when computing the excited states of the $\mathrm{UO}_{2}$ molecule as

TABLE II. The excitation energies at IHFSCC level are given at our best level of approximation for the $\mathrm{UO}_{2}^{+}$ ion using the Faegri basis set with 24 electrons correlated and 6 a.u. as the threshold for the virtual space. The bond distance for our calculation is $1.739 \AA$. The $P$ model space is $(17 g, 20 u)$ with the $7 p$ orbitals included in the $P_{m}$. The full manifold of excited states up to $20000 \mathrm{~cm}^{-1}$ is shown.

| Symmetry | This <br> work | $\left[\mathrm{UO}_{2}(\text { dbm })_{2} \mathrm{DMSO}\right]^{-\mathrm{a}}$ <br> (Ref. 60) | $\left[\mathrm{UO}_{2}(\text { saloph })_{2} \mathrm{DMSO}\right]^{-\mathrm{b}}$ <br> (Ref. 60) | $\left[\mathrm{UO}_{2}\left(\mathrm{CO}_{3}\right)_{3}\right]^{5-}$ <br> (Ref. 60) | $\mathrm{UO}_{2}^{+}$aqueous <br> (Refs. 65 and 66) |
| :--- | ---: | :---: | :---: | :---: | :---: |
| $5 f_{5 / 2 u}^{\phi}$ | 0 | 0 | 0 | 0 | 0 |
| $5 f_{3 / 2 u}^{\delta}$ | 2736 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $5 f_{7 / 2 u}^{\phi}$ | 5751 | 5290 | 5330 | 5560 | $\ldots$ |
| $5 f_{5 / 2 u}^{\delta}$ | 6567 | 6800 | 7140 | 6250 | $\ldots$ |
| $6 d_{3 / 2 g}^{\delta}$ | 15999 | 11600 | 11100 | 8770 | 10638 |
| $7 s_{1 / 2 g}^{\sigma}$ | 17635 | 13500 | 13300 | 10100 | 13550 |
| $6 d_{5 / 2 g}^{\delta}$ | 19774 | 15600 | 15400 | 13200 | $\ldots$ |
| a |  |  |  |  |  |

$\overline{\mathrm{a}} \mathrm{dmb}=$ dibenzoylmethanate, DMSO = dimethyl sulfoxide.
${ }^{\mathrm{b}}$ saloph $=\mathrm{N}, \mathrm{N}^{\prime}$-disalicylidene-o-phenylenediaminate, $\mathrm{DMSO}=$ dimethyl sulfoxide.
these are composed by orbitals that are nonbonding with the oxygen atoms. In the next sections, we will show whether this is the case.

## B. Orbital composition of uranyl and the $\mathrm{UO}_{2}^{+}$excitation energies

In Fig. 1, we sketch the orbital energy and electron affinity diagrams of $\mathrm{UO}_{2}^{2+}$ in the spin-free (SFDC) and full (DC) Dirac-Coulomb approximations. We chose the uranyl, $\mathrm{UO}_{2}^{2+}$, orbital energies, because these present the first step in the Fock space calculation and give a conceptually simple interpretation of the spectrum of singly ionized $\mathrm{UO}_{2}$. In uranyl, the unoccupied metal orbitals are split induced by the oxo ligands to the extent that $5 f$ and $6 d$ manifolds overlap. The $7 s$, that was found at high energy in the U(IV) ion, lies slightly below $5 f$ and becomes the lowest unoccupied molecular orbital. This crude Koopmans picture is improved by the DC-IHFSCCSD method which provides correlated electron affinities. These energies are listed in Fig. 1, where we see a lowering of the $5 f$ manifold that fits with experimental evidence that the unpaired electron resides in the $5 f_{\phi}$ orbital. Inclusion of SOC is important, because it produces a splitting of the same order as the crystal field splitting. This strong SOC effect is also reflected in the composition of the $5 f$ orbitals. The lower $5 f_{5 / 2}$ orbital can be described as $87 \%$ $5 f_{\phi}$ and $13 \% 5 f_{\delta}$, while the upper $5 f_{5 / 2}$ orbital is $13 \% 5 f_{\phi}$ and $87 \% 5 f_{\delta}$. Given these rather large admixtures, it is clear that a $j j$-coupling picture is appropriate.

We computed the $\mathrm{UO}_{2}^{+}$energies as electron affinities of uranyl but may of course also take the $5 f_{\phi, 5 / 2 u}$ ground state energy as reference and list the electron affinities of the remaining states as excitation energies. There is no experimental data of the $\mathrm{UO}_{2}^{+}$molecule in vacuo but one may consider data from this ion bound to different types of ligands. These experimental spectra show five characteristic bands, which are listed in Table II. The first two low-lying peaks are in line with the computed excitation energies and can be assigned to $5 f_{\phi, 5 / 2 u} \rightarrow 5 f_{\phi, 7 / 2 u}$ and $5 f_{\phi, 5 / 2 u} \rightarrow 5 f_{\delta, 5 / 2 u}$ transitions. These excitations are Laporte forbidden in vacuo but are allowed if ligands break the inversion symmetry.

The remaining three bands are found experimentally at different energies, depending of the type of equatorial ligand
bound to the uranium atom. These transitions have been assigned previously by Mizuoka et al. ${ }^{60}$ as $5 f \rightarrow 5 f$ type. Our calculation suggests that these three peaks are related to $5 f \rightarrow 6 d$ and $5 f \rightarrow 7 s$ transitions. This justifies the more intense bands because the transitions are already dipole allowed $(u \rightarrow g)$ in the bare ion. Our values are systematically higher in energy by about $3000 \mathrm{~cm}^{-1}$ which is perhaps not so surprising given the fact that the more diffuse $6 d$ and $7 s$ orbitals will interact stronger with the surrounding ligands in the equatorial region than the $5 f$ orbitals, leading to a larger environmental shift.

The ionization energy of $\mathrm{UO}_{2}$ (calculated in the FSCC approach as the second electron affinity of uranyl) can be readily obtained by subtracting the energy of the lowest $\mathrm{UO}_{2}^{+}$ state from that of the lowest $\mathrm{UO}_{2}$ state. This procedure gives a vertical excitation energy of 5.95 eV and an adiabatic value of 5.92 eV . These values are significantly below the experimental value of $6.13 \mathrm{eV} .{ }^{5}$ A possible reason could be the fact that both states are described using the orbitals from the dication, thus biasing the cation, for which orbital relaxation effects should be smaller than the neutral. To test the influence of this common orbital approximation we compared the multireference approach with the single-reference CCSD approach in which both the cation and the neutral system are treated with optimized orbitals. These DC-CCSD calculations give a vertical ionization energy of 6.00 eV , while DC$\operatorname{CCSD}(\mathrm{T})$ result is 6.04 eV . A calculation with a common set of orbitals (to mimic the FSCC approach) gives indeed a smaller value of 5.95 eV (see also Table III), suggesting that the orbital relaxation effect is present but also indicating that CC-type methods may underestimate the ionization energy of $\mathrm{UO}_{2}$. This is in agreement with earlier work by Majumdar et al. ${ }^{61}$ who reported a ECP-CCD value of 6.01 eV , in good agreement with our DC-CCSD value of 6.00 eV .

The DC-IHFSCCSD second ionization energy is 14.63 eV , which lies within the interval of $12.8-18.0 \mathrm{eV}$ that was estimated by Cornehl et al. ${ }^{62}$ on basis of thermochemical data. Cornehl et al. also performed pseudopotential coupled cluster calculations that gave an ionization energy of 15.0 eV but argued that this value is probably smaller than the exact value. Our calculations as well as the CASPT2

TABLE III. The first and second ionization potentials (IPs) of the $\mathrm{UO}_{2}$ molecule, calculated at different levels of approximation are given using the Faegri basis set with 24 electrons correlated and 6 a.u. as the threshold for the virtual space. The bond distances used for computing the adiabatic IP values are $1.680 \AA$ for $\mathrm{UO}_{2}^{2+}, 1.739 \AA$ for $\mathrm{UO}_{2}^{+}$, and $1.770 \AA$ for $\mathrm{UO}_{2}$. The vertical IPs are computed at $1.770 \AA$. The $P$ model space is $(17 \mathrm{~g}, 20 \mathrm{u})$ with the $7 p$ orbitals included in the $P_{m}$. The values in parentheses are IPs evaluated using the orbitals of the uranyl dication.

| IP | IH-FSCC <br> adiabatic | IH-FSCC <br> vertical | DC-CCSD <br> vertical | DC-CCSD(T) <br> vertical | CCD <br> vertical |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{UO}_{2} \rightarrow \mathrm{UO}_{2}^{+}$ | 5.92 | 5.97 | 6.00 | CASPT2 <br> adiabatic $^{\mathrm{b}}$ | Expt. $^{\text {c }}$ |

[^1]calculations ( 14.36 eV , see Ref. 18) give a value considerably below 15 eV and do not support that assumption.

## C. The excited states of $\mathrm{UO}_{2}$ : Scalar relativistic results

Results from the SFDC can be directly compared to scalar relativistic results obtained by the Douglas-Kroll-Hess and effective core potential (ECP) approaches. In Table IV, we show the values of the excitation energies up to $35000 \mathrm{~cm}^{-1}$ in the SFDC-FSCC model. Configurations of interest for the interpretation of the experimental spectroscopic data are $5 f^{1} 7 s^{1}, 5 f^{2}, 5 f^{1} 6 d^{1}$, and $5 f^{1} 7 p^{1}$. The $7 s^{2}{ }^{1} \Sigma_{g}^{+}$ state appears also at low energy, but cannot be easily reached from the ground state and should have a strongly repulsive interaction with noble gas atoms in a matrix. When analyzing the eigenvectors of the effective Hamiltonian, we observe significant configurational mixing in many of the states. Comparison with the CASPT2 relative energies given by Gagliardi et al. ${ }^{18}$ shows close agreement between the two methods with respect to singlet-triplet splittings and, in general, all differences between states that arise from the same configuration. The striking difference in the ${ }^{3} \Phi_{u^{-}}{ }^{3} \Delta_{u}$ vertical excitation energy can be explained by considering the large difference in bond distances used in the two calculations. The CASPT2 calculations were done at $1.827 \AA$, while the Fock space calculations were done at $1.770 \AA$. Recalculating the vertical excitation energy at $1.827 \AA$ gives $1219 \mathrm{~cm}^{-1}$, in much better agreement with the CASPT2 value.

From Table IV it is, however, also clear that there are large discrepancies between the methods for energy differences between states that come from different configurations. The FSCC energy of the $5 f^{2}{ }^{3} H_{g}$ state relative to the $5 f^{1} 7 s^{13} \Phi_{u}$ ground state is again smaller (11989 cm ${ }^{-1}$ ) at the longer CASPT2 bond length, but remains much higher than the CASPT2 excitation energy of $5954 \mathrm{~cm}^{-1}$. In general, we find states with primarily $5 f^{2}$ character at rather high energies, sometimes heavily mixed with low-lying $5 f^{1} 7 p^{1}$ and $6 d^{2}$ states. The first state with significant $5 f^{1} 7 p^{1}$ character, the ${ }^{1} \Delta_{g}$, appears at $14865 \mathrm{~cm}^{-1}$ above the ground state.

The ${ }^{3} H_{g}$ excitation was considered important for the reasons we have outlined in Sec. I. Li et al. ${ }^{15}$ have shown that at the ECP-CCSD $(\mathrm{T})$ level of theory on $\mathrm{UO}_{2}(\mathrm{Ar})$, the ${ }^{3} H_{g}$ is sufficiently low in energy that the effect of spin-orbit cou-
pling could be strong enough to change the ground state. Gagliardi et al. ${ }^{18}$ do not disregard this possibility, but point out that this would make it very difficult to interpret the fluorescence and REMPI data of Heaven and co-workers. ${ }^{5,21}$ The relative energy of the $5 f^{2}{ }^{3} H_{g}$ state, as calculated by the SFDC-IHFSCCSD is in qualitative disagreement with these results, as the energy gap with the ground state is so large that matrix-induced ground state switching is improbable.

Our calculations include, however, the $5 f^{1} 6 d^{1}$ states that have not been studied so far. These states could also be of importance to explain the experimental findings. We find the lowest state from that manifold, the ${ }^{3} H_{u}$ state, just $6000 \mathrm{~cm}^{-1}$ above the ground state. This small energy difference makes it likely that the ${ }^{3} H_{u}$ state plays a role in the chemistry of $\mathrm{UO}_{2}$. What important is, furthermore, that transition to the ground state is parity forbidden so that the state might have a sufficiently long lifetime to present an alternative for the suggestion of Gagliardi et al. ${ }^{18}$ that the older experimental value of the ionization potential $(5.4 \mathrm{eV})$ could be due to ionization from a metastable excited state. They proposed that this state would be the ${ }^{3} H_{g}$ state, which would be thermally populated in the production of $\mathrm{UO}_{2}$ in a laser ablation process. Our results indicate a similar possibility, but now based on thermal population of the ${ }^{3} H_{u}$ state, followed by ionization from the $6 d_{\delta}$ orbital.

A more detailed analysis of all aspects of the available spectroscopic data requires, of course, the introduction of SOC, which will perturb the relatively simple scalar relativistic picture sketched above. We will discuss this aspect in the next section.

## D. Analysis of the excited states: Inclusion of spin-orbit coupling

Adding SOC, or rather, not switching it off in our calculations, leads to a more crowded manifold of states below $35000 \mathrm{~cm}^{-1}$ (see Table V). To reduce the number of entries in the table we use dipole selection rules and consider only the allowed $u \rightarrow g$ and $\Delta \Omega=0, \pm 1$ excitations, with the exception of the higher components of the SO-split ${ }^{3} H_{g}$ and ${ }^{3} H_{u}$ states that are relevant in the discussion of spectroscopic data. From the analysis of the eigenvectors of the effective Hamiltonian we may distinguish between single, double, and

TABLE IV. Vertical excitation energies $\left(\mathrm{cm}^{-1}\right)$ for the $\mathrm{UO}_{2}$ molecule at SFDC-IHFSCC level of theory using the Faegri basis set with 24 electrons correlated and 6 a.u. as the threshold for the virtual space. The UO bond distance was taken as $1.770 \AA$. The $P$ model space is $(17 g, 20 u)$ with the $7 p$ orbitals included in the $P_{m}$. CASPT2 values (Ref. 18) refer to calculations done at a bond length of $1.827 \AA$. The full manifold of excited states up to $32000 \mathrm{~cm}^{-1}$ is shown.

|  | Excited states |  |  |
| :---: | :---: | :---: | :---: |
| Symmetry | CASPT2 ${ }^{\text {a }}$ | $\begin{gathered} \text { IHFSCC } \\ 24 e \\ \text { Faegri } \end{gathered}$ | Composition |
| ${ }^{3} \Phi_{u}$ | 0 | 0 | 97\% ( $5 f_{\phi} 7 s$ ) |
| ${ }^{3} \Delta_{u}$ | 1990 | 575 | 97\% ( $5 f_{\delta} 7 s$ ) |
| ${ }^{1} \Phi_{u}$ | 991 | 826 | $100 \%\left(5 f_{\phi} 7 s\right)$ |
| ${ }^{1} \Delta_{u}$ | 2852 | 1300 | $100 \%\left(5 f_{\delta} 7 s\right)$ |
| ${ }^{1} \Sigma_{u}^{+}$ |  | 5544 | $100 \%\left(5 f_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{3} H_{u}$ |  | 6038 | $100 \%\left(5 f_{\phi} 6 d_{\delta}\right)$ |
| ${ }^{3} \Gamma_{u}$ |  | 7250 | $100 \%\left(5 f_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{3} \Sigma_{u}^{-}$ |  | 7508 | $100 \%\left(5 f_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{1} \Sigma_{g}^{+}$ | 22119 | 7549 | 83 (7s7s), 14\% ( $\left.6 d_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{3} \Pi_{u}$ |  | 9193 | $97 \%\left(5 f_{\phi} 6 d_{\delta}\right)$ |
| ${ }^{3} \Delta_{g}$ |  | 9760 | 91\% $\left(7 s 6 d_{\delta}\right)$ |
| ${ }^{1} H_{u}$ |  | 9910 | 94\% $\left(5 f_{\phi} 6 d_{\delta}\right)$ |
| ${ }^{3} \Sigma_{u}^{+}$ |  | 10244 | $97 \%\left(5 f_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{1} \Gamma_{u}$ |  | 11101 | $100 \%\left(5 f_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{3} H_{g}$ | 5954 | 12863 | 97\% ( $5 f_{\phi} 5 f_{\delta}$ ) |
| ${ }^{1} \Sigma_{u}^{-}$ |  | 13815 | $100 \%\left(5 f_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{1} \Gamma_{g}$ | 11921 | 13865 | $32 \%\left(5 f_{\delta} 5 f_{\delta}\right)+30 \%\left(5 f_{\phi} 5 f_{\pi}\right)+28 \%\left(6 d_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{3} \Sigma_{g}^{-}$ | 7271 | 13926 | $50 \%\left(5 f_{\phi} 5 f_{\phi}\right)+30 \%\left(5 f_{\delta} 5 f_{\delta}\right)+18 \%\left(6 d_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{3} \Pi_{u}$ | 12645 | 14789 | 97\% (7s5f ${ }_{\pi}$ ) |
| ${ }^{1} \Delta_{g}$ |  | 14865 | $52 \%\left(7 s 6 d_{\delta}\right)+20 \%\left(5 f_{\phi} 5 f_{\pi}\right)+20 \%\left(5 f_{\phi} 7 p_{\pi}\right)$ |
| ${ }^{1} \Pi_{u}$ |  | 15459 | $100 \%\left(5 f_{\phi} 6 d_{\delta}\right)$ |
| ${ }^{3} \Sigma_{g}^{-}$ | 15549 | 15526 | $55 \%\left(5 f_{\delta} 5 f_{\delta}\right)+42 \%\left(6 d_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{3} \Pi_{g}$ | 9076 | 15574 | $88 \%\left(5 f_{\phi} 5 f_{\delta}\right)+8 \%\left(5 f_{\pi} 5 f_{\delta}\right)$ |
| ${ }^{1} \Sigma_{g}^{+}$ | 12164 | 16331 | $58 \%\left(5 f_{\phi} 5 f_{\phi}\right)+20 \%\left(5 f_{\delta} 5 f_{\delta}\right)+11 \%\left(6 d_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{1} \Pi_{u}$ | 14320 | 16824 | $87 \%\left(7 s 5 f_{\pi}\right)+8 \%\left(5 f_{\pi} 6 d_{\delta}\right)$ |
| ${ }^{1} \Pi_{g}$ | 13106 | 17410 | $80 \%\left(5 f_{\phi} 5 f_{\delta}\right)+8 \%\left(5 f_{\delta} 5 f_{\pi}\right)+8 \%\left(5 f_{\delta} 7 p_{\pi}\right)$ |
| ${ }^{3} \Gamma_{g}$ | 14262 | 18529 | $54 \%\left(5 f_{\phi} 5 f_{\pi}\right)+44 \%\left(5 f_{\phi} 7 p_{\pi}\right)$ |
| ${ }^{3} \Phi_{g}$ | 17625 | 19373 | $52 \%\left(5 f_{\delta} 5 f_{\pi}\right)+44 \%\left(5 f_{\delta} 7 p_{\pi}\right)$ |
| ${ }^{3} \Delta_{g}$ | 14472 | 19418 | $49 \%\left(5 f_{\phi} 5 f_{\pi}\right)+43 \%\left(5 f_{\phi} 7 p_{\pi}\right)$ |
| ${ }^{1} \Sigma_{g}^{+}$ |  | 20237 | $66 \%\left(5 f_{\delta} 5 f_{\delta}\right)+24 \%\left(6 d_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{3} \Pi_{g}$ | 20676 | 20518 | $52 \%\left(5 f_{\delta} 7 p_{\pi}\right)+37 \%\left(5 f_{\delta} 5 f_{\pi}\right)$ |
| ${ }^{3} \Phi_{u}$ |  | 21486 | $100 \%\left(5 f_{\pi} 6 d_{\delta}\right)$ |
| ${ }^{1} \Phi_{g}$ | 23117 | 21551 | $65 \%\left(5 f_{\delta} 6 p_{\pi}\right)+30 \%\left(5 f_{\delta} 5 f_{\pi}\right)$ |
| ${ }^{1} \Pi_{g}$ | 22081 | 21709 | $52 \%\left(5 f_{\delta} 7 p_{\pi}\right)+24 \%\left(5 f_{\delta} 5 f_{\pi}\right)+16 \%\left(5 f_{\phi} 5 f_{\delta}\right)$ |
| ${ }^{1} \Delta_{g}$ | 17816 | 22015 | $34 \%\left(5 f_{\delta} 7 p_{\pi}\right)+30 \%\left(6 d_{\delta} 7 s\right)+28 \%\left(5 f_{\phi} 5 f_{\pi}\right)$ |
| ${ }^{1} \Gamma_{g}$ |  | 22440 | $44 \%\left(5 f_{\phi} 5 f_{\pi}\right)+44 \%\left(5 f_{\delta} 5 f_{\delta}\right)$ |
| ${ }^{3} \Pi_{u}$ |  | 22598 | $100 \%\left(5 f_{\pi} 6 d_{\delta}\right)$ |
| ${ }^{1} I_{g}$ | 22337 | 23931 | 97\% ( $5 f_{\phi} 5 f_{\phi}$ ) |
| ${ }^{1} H_{g}$ | 20949 | 24396 | $50 \%\left(5 f_{\phi} 5 f_{\delta}\right)$ |
| ${ }^{1} \Phi_{u}$ |  | 24564 | $83 \%\left(5 f_{\pi} 6 d_{\delta}\right)$ |
| ${ }^{1} \Pi_{u}$ |  | 24587 | $92 \%\left(5 f_{\pi} 6 d_{\delta}\right)$ |
| ${ }^{1} \Gamma_{g}$ |  | 25109 | $56 \%\left(6 d_{\delta} 6 d_{\delta}\right)+34 \%\left(5 f_{\delta} 5 f_{\delta}\right)$ |
| ${ }^{3} \Sigma_{g}^{+}$ |  | 26246 | $52 \%\left(5 f_{\phi} 5 f_{\phi}\right)+25 \%\left(6 d_{\delta} 6 d_{\delta}\right)+17 \%\left(5 f_{\delta} 5 f_{\delta}\right)$ |
| ${ }^{1} \Sigma_{g}^{+}$ | 30157 | 31858 | $40 \%\left(5 f_{\phi} 5 f_{\phi}\right)+18 \%\left(5 f_{\pi} 5 f_{\pi}\right)+16 \%\left(6 d_{\delta} 6 d_{\delta}\right)$ |
| ${ }^{3} \Phi_{g}$ | 36917 | 32089 | $90 \%\left(5 f_{\phi} 7 p_{\sigma}\right)$ |
| ${ }^{1} \Phi_{g}$ | 37518 | 32277 | $90 \%\left(5 f_{\phi} 7 p_{\sigma}\right)$ |

${ }^{\mathrm{a}}$ Reference 18 .
mixed single-double excitations relative to the ground state. We assume that double excitations have a negligible intensity and can be discarded in the comparison with experimental data. For convenience, we discuss Table V in different sections according to the main parentage of the states.

## 1. Region $0-3000 \mathrm{~cm}^{-1}: 5 f^{1} 7 s^{1}$ states

As discussed in Sec. I, the first two states are best described in a $j j$-coupling picture as pure $5 f_{\phi, 5 / 2 u}^{1} 7 s_{\sigma, 1 / 2 g}^{1}$ states. The computed energy differences are consistent with the ear-

TABLE V. Vertical excitation energies $\left(\mathrm{cm}^{-1}\right)$ for the $\mathrm{UO}_{2}$ molecule at DC-IHFSCC level of theory using the Faegri basis set with with the Fægri2+ basis, 40 electrons correlated and $45 \mathrm{a} . u$. as the threshold for the virtual space. The $P$ model space is $(17 g, 20 u)$ with the $7 p$ orbitals included in the $P_{m}$. The bond distance is $1.770 \AA$. The composition of each state is given as a percentage. Single reference CCSD and CCSD(T) transition energies are computed with 24 electrons in the active space and a threshold of 6 a.u. Only the excited states that satisfy the selection rule $\Delta \Omega=0$, $\pm 1$ have been selected, with the exception of the first $5_{g}, 6_{g}, 5_{u}$, and $6_{u}$ states. All states composed by double excitations were also removed. Experimental values for which the assignment is not clear are listed in parentheses.

| Symmetry | Excited states |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | CCSD | $\operatorname{CCSD}(\mathrm{T})$ | IHFSCC | $\begin{gathered} \text { IHFSCC } \\ 40 e \end{gathered}$ | Oscillato | strength |  |
|  | Expt. ${ }^{\text {a }}$ | GASCI ${ }^{\text {b }}$ | CASPT2 ${ }^{\text {c }}$ | Fægri | Fægri | Fægri | +spdl | $f(2 u)$ | $f(3 u)$ | Composition |
| $2_{u}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | $97 \%\left\|5 f_{\phi, 5 / 2 u} 7 s_{s_{\sigma,-1 / 2}}\right\|$ |
| $3_{u}$ | 360 | 427 | 378 | 417 | 384 | 348 | 368 |  |  | $96 \%\left\|5 f_{\phi, 5 / 2 u} 7 s_{\sigma, 1 / 2 g}\right\|$ |
| $1_{u}$ | $1094{ }^{\text {d }}$ | 1089 | 2567 | 1841 | 2052 | 1877 | 2231 | 0 | 0 | $98 \%\left\|5 f_{\delta, 3 / 2 u} 7 s_{\sigma_{\text {c,-1/2g }}}\right\|$ |
| $2_{u}$ | $1401^{\text {d }}$ | 1542 | 2908 | 2224 | 2421 | 2224 | 2588 | 0 | 0 | $96 \%\left\|5 f_{\delta, 3 / 2 u} 7 s_{s_{\sigma,-1 / 2 g}}\right\|$ |
| $4{ }_{u}$ |  |  |  | 3079 | 3521 | 4975 | 5047 | 0 | 0 | $98 \%\left\|5 f_{\phi, 5 / 2 u} 6 d_{\delta, 3 / 2}\right\|$ |
| $4{ }_{u}$ |  |  |  |  |  | 5706 | 6148 | 0 | 0 | $97 \%\left\|5 f_{\phi, 7 / 2 u} 7 s_{\sigma, 1 / 2 g}\right\|$ |
| $3_{u}$ |  |  |  |  |  | 6028 | 6501 | 0 | 0 | $85 \%\left\|5 f_{\phi, 7 / 2 u} 7 s_{\sigma,-1 / 2}\right\|, 10 \%\left\|5 f_{\delta, 512 u} 7 s_{\sigma, 1 / 2 g}\right\|$ |
| $0_{u}$ |  |  |  |  |  | 6470 | 7081 | 0 | 0 | $48 \%\left\|5 f_{\delta, 3 / 2 u} 6 d_{\delta,-3 / 2 g}\right\|, 48 \%\left\|5 f_{\delta, 3 / 2 u} 6 d_{\delta,-3 / 2 g}\right\|$ |
| $3_{u}$ |  |  |  |  |  | 6725 | 7152 | 0 | 0 | $86 \%\left\|5 f_{\delta, 5 / 2 u} 7 s_{\sigma, 1 / 2 g}\right\|, 9 \%\left\|5 f_{\phi, 7 / 2 u} 7 s_{\delta,-1 / 2 g}\right\|$ |
| $2_{u}$ |  |  |  |  |  | 6460 | 7431 | 0 | 0 | $97 \%\left\|5 f_{\delta, 5 / 2 u} 7 s_{\sigma_{\text {, }-1 / 2 g}}\right\|$ |
| $3_{u}$ |  |  |  |  |  | 7291 | 7867 | 0 | 0 | $95 \%\left\|5 f_{\delta, 3 / 2 u} 6 d_{\delta, 3 / 2 g}\right\|$ |
| $5{ }_{u}$ |  |  |  |  |  | 8532 | 8746 | 0 | 0 | $70 \%\left\|5 f_{\phi, 5 / 2 u} 6 d_{\delta, 5 / 2 g}\right\|, 14 \%\left\|5 f_{\phi, 7 / 2 u} 6 d_{\delta, 3 / 2 g}\right\|$ |
| $6_{u}$ |  |  |  |  |  | 12970 | 13458 | 0 | 0 | $98 \%\left\|5 f_{\phi, 7 / 2 u} 6 d_{\delta, 5 / 2 / 2}\right\|$ |
| $4 g$ |  |  |  | 5545 | 6991 | 12103 | 10914 | 0.0004 | 0 | $95 \%\left\|5 f_{\phi, 512 u} 5 f_{\delta, 3 / 2 u}\right\|$ |
| $0_{g}$ |  |  |  | 12012 | 8970 | 10065 | 11436 | 0 | 0 | $73 \%\left\|7 s_{\sigma, 1 / 2 g} 7 s_{\sigma,-1 / 2 g}\right\|, 15 \%\left\|6 d_{\delta, 3 / 2 g} 6 d_{\delta,-3 / 2 g}\right\|$ |
| $1_{g}$ |  |  |  |  |  | 11017 | 12564 | 0 | 0.0273 | $92 \%\left\|7 s_{\sigma,-1 / 2 g} 6 d_{\delta, 3 / 2}{ }^{\prime}\right\|$ |
| $2 g$ |  |  |  |  |  | 11667 | 12958 | 0.0192 | 0 | $62 \%\left\|7 s_{\sigma, 1 / 2 g} 6 d_{\delta, 3 / 2 g}\right\|, 17 \%\left\|7 s_{\sigma,-1 / 2 g} 6 d_{\delta, 5 / 2 g}\right\|$ |
| $1 g_{g}$ |  |  |  |  |  | 15024 | 13919 | 0 | 0.0004 | $79 \%\left\|5 f_{\phi, 5 / 2 u} 5 f_{\delta,-3 / 2 u}\right\|$ |
| 5 g |  |  |  |  |  | 16335 | 15408 | 0 | 0 | $54 \%\left\|5 f_{\phi, 512 u} 5 f_{\delta, 5 / 2 u}\right\|, 46 \%\left\|5 f_{\phi, 7 / 2 u} 5 f_{\delta, 3 / 2 u}\right\|$ |
| 3 g |  |  |  |  |  | 13926 | 15502 | 0 | 0 | $88 \%\left\|7 s_{\sigma, 1 / 28} 6 d_{\delta, 5 / 2 g}\right\|$ |
| 3 g | (18 159) |  |  |  |  | 16753 | 16625 | 0 | 0.1004 | $63 \%\left\|5 f_{\phi, 5 / 2 u} 5 f_{\pi,-1 / 2 u}\right\| 29 \%\left\|5 f_{\phi, 5 / 2 u} 7 p_{\pi, 1 / 2 u}\right\|$ |
| $1 g_{g}$ |  |  |  |  |  | 17345 | 16949 | 0 | 0 | $48 \%\left\|5 f_{\phi,-5 / 2 u} 5 f_{\phi, 7 / 2 u}\right\|, 36 \%\left\|6 d_{\delta, 5 / 22^{\prime}} 6 d_{\delta,-3 / 2 g}\right\|$ |
| $2 g$ | (18 573) |  |  |  |  | 16703 | 17340 | 0 | 0.0680 | $38 \%\left\|5 f_{\phi, 5 / 2 u} 5 f_{\pi,-1 / 2 u}\right\|, 20 \%\left\|5 f_{\phi, 5 / 2 u} 7 p_{\pi, 1 / 2 u}\right\|$ |
| 4 g | 17859 |  |  |  |  | 17120 | 17516 | 0 | 0.0276 | $32 \%\left\|5 f_{\delta, 5 / 2 u} 5 \delta_{\delta, 3 / 2 u}\right\|, 27 \%\left\|6 d_{\delta, 5 / 2 \mathrm{~g}} 6 d_{\delta, 3 / 2 \mathrm{~g}}\right\|$ |
| $6{ }_{6}$ |  |  |  |  |  | 19756 | 18913 | 0 | 0 | $69 \%\left\|5 f_{\phi, 5 / 2 u} 5 f_{\phi, 7 / 2 u}\right\|, 31 \%\left\|5 f_{\delta, 5 / 2 u} 5 f_{\phi, 7 / 2 u}\right\|$ |
| $1_{g}$ |  |  |  |  |  | 20778 | 20801 | 0 | 0 | $40 \%\left\|5 f_{\delta, 3 / 2 u} 5 f_{\pi,-1 / 2 u}\right\|, 34 \%\left\|5 f_{\delta, 3 / 2 u} 7 p_{\pi,-1 / 2 u}\right\|$ |
| $1 g$ | (27 259) |  |  |  |  | 21180 | 21247 | 0 | 0.1686 | $51 \%\left\|5 f_{\phi, 5 / 2 u} 7 p_{\pi,-3 / 2 u}\right\|, 40 \%\left\|5 f_{\phi, 512} 5 f_{\pi,-3 / 2 u}\right\|$ |
| 2 g |  |  |  |  |  | 21443 | 22307 | 0.0116 | 0 | $22 \%\left\|7 s_{\sigma, 1 / 2 g} 6 d_{\delta, 3 / 2 g}\right\|, 22 \%\left\|7 s_{\sigma,-1 / 2 g} 6 d_{\delta, 5 / 2 g}\right\|$ |
| 3 g |  |  |  |  |  | 32644 | 30853 | 0.1281 | 0 | $34 \%\left\|5 f_{\phi, 5 / 2 u} 7 p_{\pi, 12 u}\right\|, 27 \%\left\|5 f_{\phi, 5 / 2 u} 5 f_{\pi, 1 / 2 u}\right\|$ |
| $2 g$ | 31838 |  |  |  |  | 32378 | 31125 | 0 | 0.1021 | $81 \%\left\|5 f_{\phi, 5 / 2 u} 7 p_{\sigma,-1 / 2 u}\right\|, 15 \%\left\|5 f_{\phi, 5 / 2 u} 5 f_{\sigma,-1 / 2 u}\right\|$ |
| 3 g | 31838 |  |  |  |  | 32444 | 31.203 | 0.1013 | 0 | $80 \%\left\|5 f_{\phi, 5 / 2 u} 7 p_{\sigma, 1 / 2 u}\right\|, 15 \%\left\|5 f_{\phi, 5 / 2 u} 5 f_{\sigma, 1 / 2 u}\right\|$ |
| 2 g | (29 700) |  |  |  |  | 33841 | 32071 | 0 | 0.1499 | $31 \%\left\|5 f_{\phi, 5 / 2 u} 7 p_{\pi,-1 / 2 u}\right\|, 27 \%\left\|5 f_{\phi, 512 u} 5 f_{\pi,-1 / 2 u}\right\|$ |
| $1 g_{g}$ |  |  |  |  |  | 35007 | 33189 | 0 | 0.1671 | $47 \%\left\|5 f_{\phi, 5 / 2 u} 5 f_{\pi,-3 / 2 u}\right\|, 28 \%\left\|5 f_{\phi, 5 / 2 u} 7 p_{\pi,-3 / 2 u}\right\|$ |
| 4 g |  |  |  |  |  | 35530 | 33832 | 0.1267 | 0 | $39 \%\left\|5 f_{\phi, 5 / 2 u} 5 f_{\pi, 3 / 2 u}\right\|, 20 \%\left\|5 f_{\phi, 5 / 2 u} 7 p_{\pi, 3 / 2 u}\right\|$ |

${ }^{\text {a }}$ References 5 and 21.
${ }^{\mathrm{b}}$ Reference 23.
${ }^{c}$ Reference 18.
${ }^{\mathrm{d}}$ In Ar matrix.
lier CASPT2 calculations, but the GASCI calculation of Fleig et al. ${ }^{23}$ and also the multireference configuration interaction (MRCI) calculations of Tyagi ${ }^{63}$ give a significantly smaller value for the excitation from $5 f_{\phi, 5 / 2 u}$ to $5 f_{\delta, 3 / 2 u}$. This is probably due to the fact that fewer electrons were correlated in these CI calculations; if we correlate only 14 electrons we also obtain smaller energy differences of 1621 and $1911 \mathrm{~cm}^{-1}$. The experimental data for the transition to the $5 f_{3 / 2}^{1} 7 s_{1 / 2}^{1}$ states is based on the measurements in the argon matrix. ${ }^{21}$ Also, with SOC, the discrepancy between these data and the theoretical values (that were all done in the gas
phase) (Refs. 3, 4, 18, 22, and 23) remains much larger than would be expected from a matrix effect on energy differences between four rather similar states. It is curious that almost the same vibrational frequency $\left(776 \mathrm{~cm}^{-1}\right)$ in the argon matrix that was assigned by Andrews and co-workers ${ }^{3,14}$ to the asymmetric stretch of the ${ }^{3} H_{g}\left(4_{g}\right)$ is also seen as a vibrational band $\left(771 \mathrm{~cm}^{-1}\right)$ in the fluorescence spectra. Lue et al. ${ }^{21}$ assign this band to a symmetric stretch of $776 \mathrm{~cm}^{-1}$ that they deduced from the observation of a $728 \mathrm{~cm}^{-1}$ band of ${ }^{18} \mathrm{OU}{ }^{16} \mathrm{O}$ by Gabelnick et al. ${ }^{64}$ This value is, however, much lower than the ${ }^{3} \Phi_{u}$ symmetric stretch frequency com-
puted by DFT $\left(856 \mathrm{~cm}^{-1}\right){ }^{15}$ large basis set CASPT2 $\left(948 \mathrm{~cm}^{-1}\right),{ }^{18}$ and DC-FSCC $\left(961 \mathrm{~cm}^{-1}\right)$. In the paper of Li et al. ${ }^{15}$ the symmetric stretch of the $4_{g}$ state is reported at $779 \mathrm{~cm}^{-1}$, which does fit the experimental band found in the fluorescence experiment. This interpretation is, however, not corroborated by our calculations, as we compute a symmetric stretch vibration of $911 \mathrm{~cm}^{-1}$ for this state.

## 2. Region 3 000-9 $000 \mathrm{~cm}^{-1}: 5 f^{1} 7 s^{1}$ and $5 f^{1} 6 d^{1}$ ungerade states

These states have the same parity as the ground state and are therefore difficult to observe directly. The fluorescence data, that were already discussed above, give some bands in the range of $470-600 \mathrm{~nm}$. From these experiments of Lue et al., ${ }^{21}$ it could not be deduced whether these peaks were caused by direct or indirect emission. If the emission occurs from the state excited by the $380.5 \mathrm{~nm}\left(26281 \mathrm{~cm}^{-1}\right)$ laser, there should be accessible states that have energies in the range of $5000-9600 \mathrm{~cm}^{-1}$. Our calculations give both $5 f^{1} 7 s^{1}$ states and $5 f^{1} 6 d^{1}$ states in this range that could be reached by an emission from a $5 f^{1} 7 p^{1}$ state. To check the value of $5047 \mathrm{~cm}^{-1}$ calculated for the $5 f^{1} 6 d^{13} H_{u}$ state, we also performed a single reference $\operatorname{DC}-\operatorname{CCSD}(\mathrm{T})$ calculation on this state. This gives a value of $3521 \mathrm{~cm}^{-1}$ that is comparable to the DC-FSCCSD result and reinforcing the conclusion that the lower $5 f^{1} 6 d^{1}$ states should not be disregarded in the analysis of experimental observations.

## 3. Region 10000-22 $000 \mathrm{~cm}^{-1}$ : $5 f^{2}$ gerade states

The second interpretation of the fluorescence bands that was given is that they are caused by an indirect process, in which an upper level is populated by nonradiative transfer before exhibiting radiative decay to the ground state. This should correspond to a strong emission from a state at $21280 \mathrm{~cm}^{-1}$ to the ground state. In our calculations, we find a $1 g$ state at $21247 \mathrm{~cm}^{-1}$ with a large contribution of the $5 f^{1} 7 p^{1}$ configuration and therefore a large oscillator strength, 0.1686, the most intense found in our calculations. Other states that could contribute to this fluorescence band are the $3 g$ at $16625 \mathrm{~cm}^{-1}$ and the $2 g$ at $17840 \mathrm{~cm}^{-1}$. Since these fluorescence measurements were done in the argon matrix, it is hard to compare directly, but the predicted bands at 470, 577, and 602 nm are not incompatible with the observed fluorescence.

For this region of the spectrum one may, however, also consider the much more precise gas phase REMPI data of Han et al. ${ }^{5}$ On the basis of theoretical data available at that time, ${ }^{22}$ they assigned a state at $17859 \mathrm{~cm}^{-1}$ to a $4 g$ state and two states at 18159 and $18423 \mathrm{~cm}^{-1}$ to $1 g$ states. This assignment is supported by Tyagi ${ }^{63}$ but contested by Gagliardi et al. ${ }^{18}$ who proposed a reassignment of the first transition to $2 g$. Our calculations place a $4 g$ state at $17516 \mathrm{~cm}^{-1}$ which agrees well with the original assignment of this transition as $3 u \rightarrow 4 g$. For the other two transitions, agreement is less good. The most likely candidates are both the $3 g$ at $16625 \mathrm{~cm}^{-1}$ and the $2 g$ at $17340 \mathrm{~cm}^{-1}$ states that have a significant intensity for excitation from the $2 u$ ground state.

Like in the spin-free case, we find significant differences with the CASPT2 results. We find the lowest $5 f^{2}$ state at $10914 \mathrm{~cm}^{-1}$, whereas with CASPT2 this state is only $3330 \mathrm{~cm}^{-1}$ above the ground state. Also the $5 f^{1} 7 p^{1}$ states come out somewhat higher than with CASPT2-SO. As discussed in the previous section, part of the cause for the discrepancy is the difference in bond length used to compute these vertical excitation energies, but this is only a relatively small effect. The same holds for the way in which relatively is treated, such differences usually give rather small deviations for valence properties. This leads us to conclude that differences should be primarily due to the completely different approach to electron correlation that is chosen in both methods. This was already pointed out in the atomic calculations in which case the two methods did also differ from each other by an amount of $6000 \mathrm{~cm}^{-1}$. As the Fock space approach is less well tested in such demanding molecular applications as the one studied here, we decided to also perform some calculations with the more established DC$\operatorname{CCSD}(\mathrm{T})$ method. The two states of interest, the $5 f^{1} 7 s^{1} 2 u$ and the $5 f^{2} 4 g$ states, are both well described by a single determinant reference, which makes it possible to compute the $2 u \rightarrow 4 g$ excitation energy directly. The DC-CCSD calculation gives a rather small excitation energy of $5545 \mathrm{~cm}^{-1}$ that increases to $6991 \mathrm{~cm}^{-1}$ if triple excitations are included with the $\operatorname{DC}-\operatorname{CCSD}(\mathrm{T})$ approach. This is significantly higher than the CASPT2 value, but also significantly lower than the DC-FSCC value. The discrepancy between the two CC approaches can be due to two factors: the $\operatorname{CCSD}(\mathrm{T})$ approach has a large $\mathrm{T}_{1}$ diagnostic value of 0.040 in the $4 g$ calculation which, together with the large effect of the ( T ) correction, could be indicative of a breakdown in the single-reference approach. On the other hand, as already discussed for the calculation of the ionization energy, we base the Fock space approach on orbitals that are obtained in a HF calculation of the dication. This will make the orbitals used in the Fock space calculation tighter, possibly favoring the $5 f^{1} 7 s^{1}$ configuration above the $5 f^{2}$. To assess this effect, we repeated the CCSD calculation using the same orbitals (from the dication, uranyl) as employed in the DC-IHFSCCSD calculation. This resulted in a CCSD excitation energy of $5247 \mathrm{~cm}^{-1}$, indicating that the difference in orbitals is again only marginally important.

## 4. Region $30000-50000 \mathrm{~cm}^{-1}: 5 f^{1} 7 p^{1}$ gerade states

This region includes mainly $5 f^{1} 7 p^{1}$ states. Experimentally, both the absorption REMPI spectrum ${ }^{5}$ and the fluorescence spectrum in Ar matrix ${ }^{21}$ give an intense band at about $27000 \mathrm{~cm}^{-1}$. This is regarded as evidence for the suggestion that the ground state does not change in an argon matrix. In both cases, the transitions should then be due to an intense $7 s \rightarrow 7 p$ transition. Our calculations are in qualitative agreement with this assignment, but give transitions to the $5 f^{1} 7 p^{1}$ states starting around $30000 \mathrm{~cm}^{-1}$ with all excitations in the region between 22000 and 30000 corresponding to double excitations, which should have too little intensity. The difference of $3000 \mathrm{~cm}^{-1}$ could be due to a flaw in the calculations, possibly a remaining basis set deficiency. The $7 p$ orbitals are very diffuse and may require more diffuse functions than the
single shell that we added. We do not expect that the method itself does give large errors in this case, since the ground and excited states are composed of states that have the same main quantum number. In the atomic calculation the errors for such transitions were of the order of few hundreds of wave numbers.

## IV. CONCLUSIONS

We applied the relativistic Fock space coupled cluster method and its intermediate Hamiltonian modifications to analyze the electronic spectrum of the $\mathrm{UO}_{2}$ molecule. The method is size extensive and provides an accurate treatment of dynamic and nondynamic correlation effects, which is crucial for correct calculation and interpretation of the complicated $\mathrm{UO}_{2}$ spectrum. The ease with which a full manifold of excited states is computed in a single run is an important asset of the method. The IHFSCC approach scales as $N^{6}$, but this can be ameliorated by applying linear scaling techniques, because all computationally intensive terms are similar to those that occur in regular CC algorithms.

For the first time, the $6 d$ shell has been included explicitly in the correlated calculation. A $4_{u}$ state arising from the $5 f^{1} 6 d^{1}$ configuration is found at about $5000 \mathrm{~cm}^{-1}$, which makes it an interesting alternative for explaining the low values found in older IP measurements. ${ }^{2}$ The first gerade state, $4_{g}$, is found at more than $10000 \mathrm{~cm}^{-1}$, which is much higher than that predicted by all previous theoretical calculations. This value could be overestimated, but it appears unlikely that this state is the lowest when the $\mathrm{UO}_{2}$ molecule is trapped in an Ar matrix. We propose that it is more likely that the $5 f^{1} 6 d^{1} 4 u$ state is of importance in the heavier noble gas matrices. Like previous calculations, the present work cannot fully explain all experimental observations, but they add more pieces to the complex puzzle that nature has posed.

## ACKNOWLEDGMENTS

Two of the authors (I.I. and L.V.) thank the Netherlands Organization for Scientific Research for financial support via the "Jonge Chemici" programme. Research at TAU is supported by the Israel Science Foundation. This research was performed in part using the Molecular Science Computing Facility (MSCF) in the William R. Wiley Environmental Molecular Sciences Laboratory, a national scientific user facility sponsored by the U.S. Department of Energy Office of Biological and Environmental Research and located at the Pacific Northwest National Laboratory, operated for the Department of Energy by Battelle.
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