A Formal Characterization of Cellular Networks*

CASOS Technical Report

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September 2005

CMU-ISRI-05-109

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CASOS - Center for Computational Analysis of Social and Organizational Systems

Abstract

We present precise and explicit, definitions and descriptions of pertinent terms and constructs related to a specific type of network topology – the cellular network. First, we clarify the construct of a *cell* and establish the concept of a *cell-core and cell-periphery*; next, we present details of the broader, *cellular network*. Further, we introduce the notion of a *k-cell* subgraph construct. Throughout the report, we introduce several supplementary terms related to these concepts. The terminology and formalization we present can be effectively utilized when conversing about real-world social networks, but may be essential as guidelines when constructing cellular social-networks for virtual experiments. Increasingly, the network form referred to as "cellular" is appearing in empirical social-network studies and is being applied in virtual experiments of social-networks. The presence of a cellular network is particularly prominent in research pertaining to covert and terrorist organizations; although, the form can be found in other less saturnine situations. While the structure of a cellular network is often rather intuitive, to avoid confusion researchers, analysts and especially experimenters have need for formal definitions and precise a description of the topology. To date, no such formalization of the cellular network topology--in the context of social networks--has been published.

* This work was supported by the Department of Defense and National Science Foundation under MKIDS program (IIS0218466) and the Office of Naval Research under Dynamic Network Analysis program (N00014-02-1-0973). The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Department of Defense, the National Science Foundation, the Office of Naval Research, or the U.S. government.

Keywords: social network, cellular network, cell, cell-core, k-cell, covert network

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1. Motivation

Cellular networks are a distinct and important network topology. Although there is a growing body of work referring to cellular networks, there is no complete formal definition. However, there are several papers that seek to describe characteristics of cellular networks (Carley, 2004, forthcoming; Tsvetovat & Carley, 2005). Cellular networks are a critical topology to formally characterize, in part, as they are thought to be a common form for covert networks.

The Research Lab of the Center for Computational Analysis of Social and Organizational Systems (CASOS) frequently conducts experiments involving virtual social-networks. Often these experiments involve generating networks of various forms, such as random, lattice, core-periphery, small-world, scale-free, among many others. In particular, *cellular* networks have recently become quite prominent in experimental-designs as much of the Lab's research involves investigation into covert networks that are often of a cellular structure. Broadly, in the recent past, cellular networks have most often been referred to in the context of biological or electronic-communication systems, but due to the recent ascent of widespread terrorism, researchers and analysts beyond the CASOS Lab have amplified their interest in the cellular form of social networks, albeit of a somewhat different variety than that of the non-social references.

Currently, the sudden addition of *cellular* to the social-science lexicon presently leaves researchers working without a consistent and broadly-accepted formalism. Missing are the precise definitions that are necessary to communicate effectively and to construct cellular networks in a consistent manner. We limit this report solely to a formalization of descriptive definitions pertaining to cellular networks and to its notional parts; while we save for later reports the detailed algorithms for generating data that fit these definitions, and the important in-depth investigation into the network measures and characteristics particular to the cellular topology. The terms and definitions provided in this report are currently used by CASOS researchers and are likely to be useful as well to others in the broader research and analyst communities.

Additionally, we note that the typical definition of cellular networks defines the topology not just in terms of the social network (i.e., connections among actors); but also, in terms of other networks such as the knowledge network (i.e., who knows what). For example, Carley (forthcoming) provides a characterization of a cellular network in which the connections among cells are made by the leaders who know more than other actors and have a broader type of knowledge. This being said, in this report we consider the definition of cellular networks only in terms of a single mode single link network, such as a prototypical social network. Future work needs to extend this definition to a multi-mode context.

2. Introduction

Recently, organizational success appears to be shifting from leaders overseeing the traditional formal and *hierarchical* structured organizations, to those persons who are able to organize people into the less-formal *networked* organizations--either open or covert (Arquilla & Ronfeldt, 2001). While in the 1930's, Max Weber formalized the hierarchical structure, thus establishing it as a broadly understood and widely implemented notion, the recent transition to the more complex networked structures summons similar efforts and necessitates the establishment of research into the complex network structures.

Possibly the least understood form of these networked organizations—and perhaps the most feared is the *cellular* network. At present, the cellular form—a network made up of distributed and a seemingly endless number of small human-subgroups—, is associated with the "dark side"

(Ronfeldt & Arquilla, 2001, p. 2) and has the attention of nearly everyone in one way or another as our lives increasingly are affected by widespread terrorism, the spread of drugs, and other forms of illegal activity. The cellular network structure may very well be the underpinning the strength of the new-age covert terrorist organizations (Krebs, 2002). Social network scientists and analysts are rising to the task by studying these terrorist groups and their underlying cellular structure. Following, academic researchers have been restricted to conducting *virtual* experiments on cellular networks as empirical studies are impractical due to the often covert nature of real-life cellular networks.

As more is understood about cellular networks, some traditional structures may ultimately be perceived and to some extent characterized, as having a cellular topology or some cellular characteristics (Mayntz, 2004); albeit, with a depiction more relaxed than described in this report. We conjecture that even traditional groups, such as a hierarchical sales organization, a grade-school playground network, or even a hospital may, from some perspective, maintain a cellular structure--perhaps under the guise of a label such as "silo," for example. Clearly, not enough is understood yet about the characteristics, the dynamics, the implications of, and the formation of cellular networks; nevertheless, fortunately, there is a great deal of research and investigation currently underway.

Along with using traditional empirical research, network researchers may use virtual experimentation to explore social networks in order to generate new ideas and posit new theories for subsequent investigation in the real-world. The exploration into cellular networks is also following this paradigm as virtual experiments are increasingly being carried out (Carley, 2002a, 2003; Carley, et al, 2003, forthcoming; Frantz & Carley, 2005; Tsvetovat, 2005; Tsvetovat & Carley, 2004).

With any new scientific idea, notion, or paradigm, at the outset there is some confusion in the scientific community that needs rectifying. Research into cellular social networks is no exception. The precise definition of a cellular network is one of these missing pieces in the research process. A precise definition of a cellular network is a stringent requisite for virtual experiments to be conducted and shared effectively among the social-network research community.

To date, little has been published that precisely characterizes a cellular network, to the exactness required for virtual experiments. Several academic papers (Carley, 2002b; Carley, Lee, & Krackhardt, 2002; Farley, 2003; Mayntz, 2004; Rothenberg, 2002) have provided guidance--such as cell size and interaction patterns--from anecdotal observation and empirical investigations, while Tsvetovat (2005) has provided an algorithm to generate a virtual cellular network, but these publications lack the specifics needed for a complete formalism that can be encapsulated in computer software and perhaps mathematical formulas--although, we incorporate their observations and prescriptions in the development of this formalism. One effort to document precise algorithms for constructing several different social network topologies, including cellular, is currently underway (see Frantz, Airoldi, Reminga, & Carley forthcoming).

The multifaceted nature of the cellular form requires that a multi-tiered and layered approach be taken to describing the topology. This is contrast to uniform-random networks, hierarchy, scale-free or small-world topologies; while these are complex networks, they are relatively simple models to describe and construct.

The perspective we take on cellar networks is that there are two separate, but connected aspects; a cellular network has simultaneously a *global* and *local* features. Although they are possibly interrelated and certainly connected, only combined do they form a *cellular* network, much of this treatise of cellular networks involves discussion at its separate levels of analysis.

In the following sections of this report, we ascend up the level of analysis of a cellular network by starting with the cell subgroup, which is the principal leadership team of the network's operations unit. We expand on the cell by discussing the cell-periphery subgroup, which expands on the leaders contained in the cell subgroup and encompasses the entire local working subgroup including those involved, but uniformed others connected to the cell, a.k.a., the leadership, by a weak *tether*. Next we discuss the topological level and provide how the cells and cell-periphery subgroups are tied into a single cohesive network structure. Further, we discuss the notion of the k-cell, which provides a perspective on analysis of the network interconnections and structure. We conclude this technical report with a short summary.

The presence of a cell-core subgroup in a network is the feature that most distinguishes a cellular network from other forms of network topologies. It is rather easy to spot in a network matrix both visually and mathematically, and relative to other sub-group configurations, the cell-core may be rather straightforward to observe in the real-world and document accordingly, subject to the covertness of the subjects of course.

The multitude of loosely connected, functionally redundant cells, tactically independent and geographically dispersed cells of a covert-type network makes for a robust structure that seems obliviously to random and targeted attacks alike. In the converse of terrorist networks, military organizations have also embraced the cellular form in some of their respective covert operative units, e.g., the US Navy Seals *squad* and *element* groups (autonomous operative teams of 8 or 4 persons, respectively).

3. Cell

A *cell* is a distinct subgroup of actors within a larger cellular network. The presence of at least one cell is fundamental to a network's distinction of being cellular—without at least one cell, a network is not *cellular*. Empirically, a cell often consists of relatively few actors and has a distinct topology that is effortless to identify visually. The actors in a cell can be partitioned into two distinct but intertwined subgroups, namely the *cell-core* and the *cell-periphery*. Figure 1 shows an example of a cell with these two components. These labels are somewhat self-explanatory, but they are formalized in detail below.

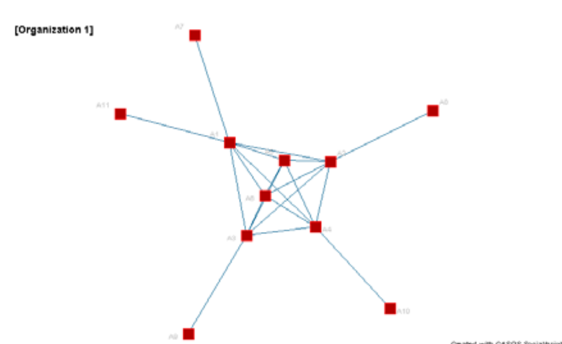

Fig. 1. A Cell (isolated from the larger cellular network) with a Cell-Core and a Cell-Periphery

While it is conceivable that a cell-core may exist without cell-periphery actors, it is rather likely that a cell-periphery will exist empirically, albeit in varied membership-sizes. In our formalism, we require a cell to have a cell-core to meet the definition of a cell, but do not strictly require the presence of cell-periphery actors.

3.1 Cell-Core

We define a *cell-core* as the subgroup of actors, all within a same cell, who form a strict clique. For a particular set of actors to be considered a cell-core these conditions must be met: (a) the actors in the cell-core must be members of the same undirected cellular network, (b) the actors in the cell-core must be members of the same cell, (c) there must be three or more actors in the cell-core, (d) each actor in the subgroup is adjacent to all other actors in the same cell-core subgroup, (e) no other actor in the network can be possibly added to the cell-core and the cell-core retain its strict-clique property, and (f) any cell-core actor is limited to holding membership in only one cell-core.

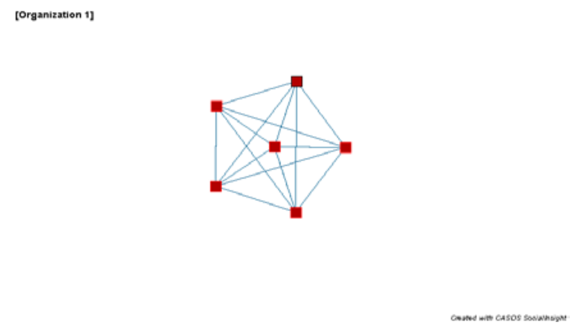

Fig. 2. Cell-Core Subgroup

The strict-clique (Luce & Perry, 1949) prerequisite is fundamental to a cell-core and any relaxation of this requirement of complete adjacency—as in 2-clique (Alba, 1973)—is unacceptable for a subgroup to be considered a cell-core. By relaxing the requirement for each subgroup member to be directly tied to all others in the same subgroup, thus allowing a distance of 2, the notion of a tight and inseparable subgroup is likely violated. Allowing for distance 2 relationships, also may increase the number of actors in the subgroup and risks violating the condition that a cell-core be small and highly interactive within itself. Further, allowing for a distance of 2, while it allows for relatively easy reachability among the members, it also makes dual-membership of a node (into more than one cell) far more likely, which is a clear violation of a cellular network's overall intent to keep groups independent and at a distance from one another.

However, in recognition of the reality of measurement-error in social network studies, an analyst may chose to allow a 2-clique within a cell. Instead, we suggest in such a situation that proxy ties be added to the data to accommodate the strict 1-clique requirement.

Further, a cell is stricter notion than a clique. An actor's membership in a given cell must be *exclusive* without membership in any other cell--within the same network of relations. This implies that a cell is a clique, but is so without allowing overlapping memberships.

While they are very much similar, a cell is not as restrictive as the conception of a *strong alliance* (Wasserman & Faust, 1994); unlike the strong alliance, in the case of a cell, actors' ties to others in the network *are* permitted. Ties to outside the cell are discussed further in the section on the cellular networks, but mentioned briefly here for completeness.

Indeed, a cell can be regarded as an LS set with a strict requirement of being fully complete within itself (Wasserman & Faust, 1994). An LS set is the case in which a subgroup has more ties within itself than outside group ties. We do not consider this characteristic a strict requirement for determination of a subgroup being considered a cell, however, empirically, meeting the LS constraint may prove to be a common occurrence.

In general actors in the cell subgroup have local role equivalence (Winship & Mandel, 1983) however, from some perspective; there is some dissimilarly among the cell group members. As will be discussed in the section on the cellular network, each cell is connected via a tie to at least one other cell (a single-component

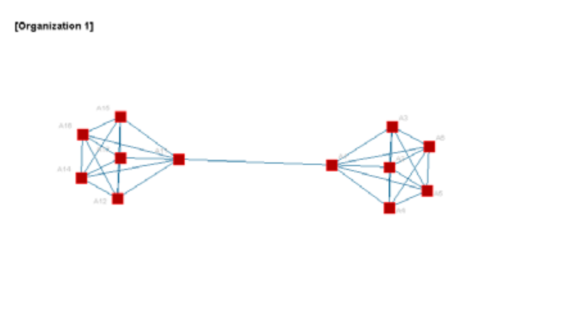

Fig. 3. Two Cell-Core subgroups in the context of a cellular network connected by spanning tie. (node co-membership not permitted)

network requirement). This implies that there is local role equivalence among the cell members, but dissimilarity at the network level. One can only imagine that there is some power prescribed to the actor(s) with ties to other cells, i.e., the greater cellular network

The number of members in the subgroup is considered minimal to accomplish the purpose of the cell. It falls between 4-6 actors at the upper end. This is consistent with the free-forming group size identified by Coleman and James (1961).

3.1.1 Classification of Actor Ties

We classify ties involving cell-core actors into three types according to the adjacent actor's disposition: (a) a *core-tie*; a tie involving an actor in the same cell-core, (b) a *tether*-tie; a tie involving an actor in the core-periphery of the same cell, and (c) a *spanning*-tie; a tie involving an actor in another cell.

3.2 Cell-Periphery

We define the *cell-periphery* as the subgroup of actors who are members of a same cell, but who are excluded from the cell's cell-core. For a set of actors to be considered in the cell-periphery of a cell, these conditions must be met: (a) the actors must be members of the same cellular network, (b) the actors in the subgroup must be members of the same cell, (c) the actors must be excluded from the same cell's cell-core subgroup as specified by the strict-clique requirement, (d) the actors in the subgroup may not be adjacent to one another, and (e) the actors have a unary tie that is exclusive to only one actor in the same cell cell-core and with no other in the broader network. We consider the existence of a cell-periphery subgroup in a cell as optional.

The leadership characteristic of a cell implies that indeed the cell subgroup must be leading someone. These human tools of a cell are instrumental in the carrying out the purposes of the cell and overriding network. Anecdotally, these others are little clued into the strategy of the cell and instead may only see a part of the picture that is favourable to them. These others are expendable and include, possibly, the suicide-bombers. To recognize the significance of and be able to study the characteristics of these followers, we provide the notion of a cell-periphery subgroup.

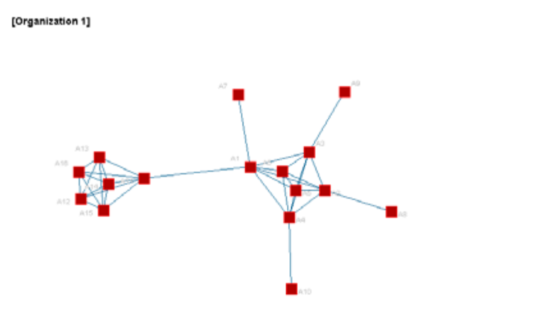

Fig. 4. Cell subgroup (left) and a Cell-Periphery subgroup (on right) connected by 1 spanning tie and 2 spanning nodes

The cell-periphery subgroup is much like an instance of the core-periphery network form (Borgatti & Everett, 1999), but differs because the cell-periphery has stricter requirements. The core-periphery form is characterized by a two-class partition of nodes: those whose are part of a *cohesive* core and those others who are not in the core but are tied to the core. Instead for a cell-periphery subgroup structure, we hold a stricter requirement for the core in that it must be a *cell* structure as previously described. This tightens the core-periphery expectation of a cohesive core (without reference to co-membership) to being a strict 1-clique without co-membership.

Further, the core-periphery form allows for ties between periphery nodes, while we require that there be absolutely no ties between two periphery nodes. This strict requirement to disallow periphery ties is in conjunction with the often convert nature of cell-periphery subgroups. The cell wants to maintain control over the periphery actors and allowing for relations between them undercuts the cell's power. We hold there is one exception to this strict distancing of the periphery nodes. Empirically, we find that at a point in time, usually for a specific task-related event, the periphery nodes may knowingly or unknowingly come in contact with one another, perhaps only spatially.

The cell-periphery structure is also stricter than the core-periphery form in that only one tie between a periphery actor and a cell node is permitted. This reflects the often desire by the cell to only allow a fragmented view of the cell to the periphery actors. The actor in the cell is sometimes called the periphery's "handler."

The periphery actors may not be ties to any other nodes in the said network. They should be pendulum nodes, i.e., a mutual dyad (in the case of a undirected graph, although if directed, would certainly be an asymmetric dyad with the tie pointing from the cell node to the periphery node.)

The cell-periphery structure allows for an extension of the strict cell specification by including relevant other actors who are involved in the activities of the cell, albeit at a cognitive distance from the leadership making up the cell. The leader-follower paradigm is reflected in the cell-periphery subgroup. The followers are expendable. In context of terrorist networks these periphery are perhaps the suicide bombers – expendable to the cell, yet certainly part of the cell but not the core which must continue to survive.

It is conceivable and acceptable to the definition of a cell-periphery subgroup that there not be any actors in the periphery of the cell-periphery substructure.

3.1.2 Classification of Actor Ties

We classify ties involving cell-periphery actors into a single type according to the adjacent actor's disposition. A *tether-tie* is a tie involving an actor in the core-periphery of the same cell. Cell-periphery actors may only have one tether-tie and no other.

While the *cell*-periphery is somewhat similar to the *core*-periphery network structure described by Borgatti and Everett (1999), the cell-periphery formalism does *not* permit ties across periphery actors. This reflects the frequent controlling nature of the cell-core, whereas cell-core actors likely act more as information brokers rather than as matchmakers, making opportunity for ties among periphery actors unlikely and perhaps even purposely avoided.

4. Cellular Network

We define a *cellular* network as a single-component and undirected network of actors and their relationships, strictly consisting entirely of actors who are members of a specific cell, as previously defined; thus a network in which all actors are a member of a cell. For a network to be considered cellular, these conditions must be met: (a) the ties making up the relations in the network may only be undirected, (b) the network consists of a single component, e.g., there are no isolate actors, and (c) the network consists solely of cell subgroups that are connected via spanning ties, e.g., there are no actor in the network who is not a member of a cell subgroup.

Recall, we are defining cellular networks for virtual experiments requiring the generation of a network in a precise manner. The imprecise real-world networks *may* violate some (or all) aspects of this definition to some extent. More relaxed interpretations of cellular networks, particularly with respect to the inclusion of actors not tied to a specific cell are certainly conceivable and appropriate, but is not included here as we consider such situations as special cases of this purer formalism.

When considering only the ties between cells, the cellular network often has low density, perhaps with small-world-like ties. Each tie, from one cell to another, called a *spanning* tie, represents a connection from a single node within a cell to a single node in the other cell. We do not impose any such restrictions on the density of a cellular network as is done in the formalism of the cell subgroup. There may not be multiple spanning ties from one cell to a same other cell.

While not a requirement for our formalism, real-life networks frequently have a hierarchical form to these cell-to-cell connections. This hierarchy has been found empirically in terrorist networks.

4.1 Spanning Entities

A tie that connects one actor in a cell to an actor in different cell is referred to as a *spanning* tie. An actor in a cell that has a tie to an actor in a different cell is referred to as a *spanning* actor.

4.2 Cell-Span Topology

We define a cellular network as a graph consisting of connected cell subgroups. While such a network is classified as a *cellular* network, use of this label does not reveal any information about the topology, or structure of how the cells are connected. We consider this important aspect of the cellular network as a separate construct that we call the *cell-span topology*.

The *cell-span topology* can be of any simple or complex network topology conceivable. From a simple line, star, or circle, to a scale-free, small-world, or hierarchical (this is a non-exhaustive list of course), the cell-span topology pertains strictly to the manner in which the cells or cell-periphery subgroups are connected to one another and does not suggest any relevance to the structure within the cell subgroups. The cell-span topology has a large affect on the reachability of the cells and the robustness of the entire network.

5. *k*-cell Subgraph

We define a *k-cell* as a subgraph construct, derived from a cellular network, that consists of only those cells which have a spanning-tie degree of *k* or greater. Each relevant cell in the network is reduced to a single actor representation; all individual cell-core and cell-periphery actors and their ties including any tether ties are removed and replaced with e single node construct. Thus, only ties connecting the cells, a.k.a., the spanning ties, remain in the new construct.

This simplification of the cellular form is especially useful for high-level analysis of the network. Descriptive measures for the *k*-cell can be determined by using classic network techniques and because of its reduced form, the values can be compared to those of other network topologies. The measures for the *k*-cell describe the high-level structure of the network without influence of the internal form of the underlying cells making up the cellular network.

As the value of *k* is increased, the more prominent (in degree connectivity) cells surface, which may lead to identifying the more relevant cells in the overall network. By investigating the different levels of *k*-cell subgraphs, an analyst may uncover important structural properties about the overall cellular network.

The definition of a *k*-cell is fashioned from that of the *k-core* (Seidman, 1983) subgraph. Like the *k*-core, which is intended to be applied to traditional node-tie subgraphs, the *k*-cell is a subgraph of a network of nodes and is based on the ties among individual nodes, i.e., nodal degree; however, in this case, each node represents an entire cell subgroup.

Correspondingly, a 1-cell subgraph is equivalent to its entire original graph, assuming a compliant cellular network; a 2-cell subgraph is one in which only cells with 2 or more spanning ties are included; and so on.

6. Summary

We have provided a formalism of cellular networks—primarily for use in generating networks in virtual experiments—and have introduced terminology that is pertinent to communicating about cellular networks. The cell and cell-periphery subgroups were defined with strict requirements related to constructing them for virtual experiments. The cellular form of a network was defined in terms of the underlying cell and cell-periphery subgroups. We provided a brief discussion on the construction parameters for each construct, which is necessary to generate network data meeting the definition of these subgroups and the cellular topology. Finally, we introduced the k-cell subgraph, which provides for high-level examination of a complex cellular network. Below we summarize the characterization of the four notions we describe in this report:

A *cell* subgroup is characterized as:

- A subgroup of a larger one-mode network of undirected ties
- A clique of three or more actors (each adjacent to all others in the same cell)
- Exclusivity of an actor's membership in a cell
- Ties to others outside the cell-core are permitted
- Consists of the union of two subgroups, the cell-core and the cell-periphery

A *cell-core* subgroup is characterized as:

- A subgroup of actors in the same cell
- A clique of three or more actors (all have ties to all others in the subgroup)
- Exclusivity of an actor's membership in a single cell
- Ties to others outside the cell are permitted

A *cell-periphery* subgroup is characterized as:

- A subgroup of a larger one-mode network of undirected ties
- Consisting of a single cell subgroup, plus a set of periphery nodes
- Each periphery node is uniquely connected (via tether ties) to a single actor in the host cell
- Periphery nodes are not permitted to be connected to one another

A *cellular* network is characterized as:

- A single component graph
- Made up of connected (via spanning ties) cells or cell-periphery subgroups
- Having a topology separate from that of the underlying cells and cell-periphery subgroups

A *k-cell* subgraph is characterized as:

- A subgraph of a cellular network
- All cells that have k or more spanning ties

We also introduce several terms:

Cell-Span Topology – The topology of a 1-cell subgraph.

Core Tie – A tie between two adjacent cell-core actors.

Spanning Tie – A tie between adjacent actors in two different cells.

Spanning Actor – An actor node that has a spanning tie.

Tether Tie – A tie between adjacent cell-core and cell-periphery actors.

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