A Formal Security Analysis of the Signal Messaging Protocol

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Abstract

Signal is a new security protocol that provides end-to-end encryption for instant messaging. It has recently been adopted by WhatsApp, Facebook Messenger and Google Allo among many others; the first two of these have at least 1 billion active users. Signal includes several uncommon security properties (such as "future secrecy" or "post-compromise security"), enabled by a novel technique called *ratcheting* in which session keys are updated with every message sent. Despite its importance and novelty, there has been little to no academic analysis of the Signal protocol.

We conduct the first security analysis of Signal's Key Agreement and Double Ratchet as a multi-stage key exchange protocol. We extract from the implementation a formal description of the abstract protocol, define a security model which can capture the "ratcheting" key update structure, and prove the security of Signal's core in our model. Our presentation and results can serve as a starting point for other analyses of this widely adopted protocol.

1. Introduction

Recent revelations about mass interception and surveillance of communications have made consumers more privacy-aware. In response, scientists and developers have proposed techniques which can provide security for end users even if they do not fully trust the service providers. For example, the popular messaging service WhatsApp was unable to comply with Brazilian government demands for users' plaintext messages [6] because of its end-to-end encryption.

Early instant messaging systems did not provide much security. While some systems did encrypt traffic between the user and the service provider, the service provider retained the ability to read the plaintext of users' messages. Off-the-Record Messaging [7, 17] was one of the first security protocols for instant messaging: acting as a plugin to a variety of instant messaging applications, users could authenticate each other using public keys or a shared secret passphrase, and obtain end-to-end confidentiality and integrity. One novel feature of OTR was its fine-grained key refreshing: along with each message, users established a fresh ephemeral Diffie–Hellman (DH) shared secret. Since it was not possible to work backward from a later state to an earlier state and decrypt past messages, this technique became known as *ratcheting*; in particular, *asymmetric* ratcheting since it involves asymmetric (public key) cryptography. Unfortunately, OTR saw relatively limited adoption.

Perhaps the first secure instant message protocol to achieve widespread adoption was Apple's iMessage, a proprietary protocol that provides end-to-end encryption. A notable characteristic of iMessage is that it automatically manages the distribution of users' long-term keys, and in particular (as of this writing) users have no interface for verifying friends' keys. iMessage, unfortunately, has a variety of flaws that seriously undermine its security [23].

The Signal Protocol. While there has been a range of activity in end-to-end encryption for instant messaging [19, 45], the most prominent recent development in this space has been the Signal messaging protocol, "a ratcheting forward secrecy protocol that works in synchronous and asynchronous messaging environments" [35, 36]. Signal's goals include end-to-end encryption as well as advanced security properties such as perfect forward secrecy and "future secrecy".

The Signal protocol, and in particular its ratcheting construction, has a relatively complex history. TextSecure [36] was a secure messaging app and the predecessor to Signal. It contained the first definition of Signal's "Double Ratchet", which effectively combines ideas from OTR's asymmetric ratchet and a *symmetric* ratchet (which applies a symmetric key derivation function to create a new key, but does not incorporate fresh DH material) from the first version of the Silent Circle messaging protocol [39]. TextSecure's combined ratchet was referred to as the "Axolotl Ratchet", though the name Axolotl was used by some to refer to the entire protocol.

K.C-G. thanks the Oxford CDT in Cyber Security for their support.
D.S. was supported in part by Australian Research Council (ARC) Discovery Project grant DP130104304, Natural Sciences and Engineering Research Council of Canada (NSERC) Discovery grant RGPIN-2016-05146, and an NSERC Discovery Accelerator Supplement grant.

TextSecure was later merged with RedPhone, a secure telephony app, and the result[1] renamed to Signal, which is now the name of both an instant messaging app and the cryptographic protocol. In the rest of this paper, we will be discussing the cryptographic protocol only.

The Signal cryptographic protocol has seen explosive uptake: it (or a variant) is now used by Google Allo [37], WhatsApp [46], Facebook [20], as well as a host of "secure messaging" apps, including Silent Circle [39], Pond [33], and (via the OMEMO extension [44] to XMPP) Cryptocat v2 [27], Conversations [13], and the next release of ChatSecure [2]. It has driven an unprecedented uptake of encryption in personal communications.

Security of Signal. One might expect this widespread uptake of the Signal protocol by large players to be accompanied by an in-depth security analysis and examination of the design rationale, in order (i) to understand and specify the security assurances which Signal is intended to provide, and (ii) to verify that it provides them. Surprisingly, this is not yet the case.

Frosch et al. [22] performed a security analysis of TextSecure v3, identifying an unknown key share attack against TextSecure because of the way fingerprints are compared. They also analyzed several cryptographic components of TextSecure, showing that the computation of the long-term symmetric key which seeds the ratchet is a secure one-round key exchange protocol, and that the key derivation function and authenticated encryption scheme used in TextSecure are secure. However, they did not cover the security properties of the ratcheting mechanism.

The Signal protocol has changed substantially since this initial analysis of TextSecure. Unfortunately, there currently is little documentation available on the current version of the Signal protocol beyond high-level whitepapers, and no in-depth security analysis. This is in stark contrast to the ongoing development of the next version of the Transport Layer Security protocol, TLS 1.3, which explicitly involves academic analysis in its development [5, 15, 18, 25, 28, 34].

Providing a security analysis for the Signal protocol is challenging for several reasons. First, Signal employs a novel and unstudied design, involving over ten different types of keys and a complex update process which leads to various "chains" of related keys. It therefore does not directly fit into existing analysis models. Second, some of its claimed properties have only recently been formalised [12]. Finally, as a more mundane obstacle, the protocol is not substantially documented beyond its source code.

1.1. Contributions

We provide the first in-depth formal security analysis of the cryptographic core of the Signal messaging protocol, which is used by more than a billion users.

To achieve this, we develop a multi-stage key exchange security model with adversarial queries and freshness conditions that capture the security properties intended by Signal. Compared to previous multi-stage key exchange models which involve a single sequence of stages within each session, our model considers a *tree* of stages to model the various "chains" in Signal. Our security model characterizes many detailed security properties of Signal, thereby providing the first formal definition of Signal's security goals. Among the interesting aspects of our model are the subtle differences between security properties held by keys derived via symmetric and asymmetric ratcheting.

We subsequently prove that the core key exchange protocol of Signal is secure in our model, providing the first formal security guarantees for Signal. We give a proof sketch in Section 5 and the full proof in Section B.2.

In practice, Signal is more than just its key exchange protocol. In Section 6, we describe many other aspects of Signal that are not covered by our analysis, which we believe are a rich opportunity for future research. We hope our presentation of the protocol in Section 2 can serve as a starting point for understanding Signal's core.

1.2. Additional Related Work

Symmetric ratcheting and DH updates (asymmetric ratcheting) are not the only way of updating state to ensure forward secrecy—i.e., that compromise of current state cannot be used to decrypt past communications. Forward-secure public key encryption [10] allows users to publish a short unchanging public key; messages are encrypted with knowledge of a time period, and after receiving a message, a user can update their secret key to prevent decryption of messages from earlier time periods.

Signal's DH updates (asymmetric ratcheting), which it inherits from the design of OTR [7], have been claimed to offer properties such as "future secrecy". Future secrecy of protocols like Signal has been discussed in depth by Cohn-Gordon, Cremers, and Garratt [12]. Their key observation is that Signal's future secrecy is (informally) specified with respect to a passive adversary, and therefore turns out to be implied by forward secrecy. Instead, they observe that mechanisms such as asymmetric ratcheting can be used to achieve a substantially stronger property against an active adversary. They formally define this property as "post-compromise security",

1. TextSecure v1 was based on OTR; in v2 it migrated to the Axolotl Ratchet and in v3 made some changes to the cryptographic primitives and the wire protocol. Signal is based on TextSecure v3 but with additional cryptographic changes.

and show how this substantially raises the bar for resourceful network attackers to attack specific sessions. Furthermore, their analysis indicates that post-compromise security may hold of Signal depending on subtle details related to device state reset and the handling of multiple devices.

Recently, Green and Miers [24] suggest using puncturable encryption to achieve fine-grained forward security with unchanging public keys: instead of deleting or ratcheting the secret key, it is possible to modify it so that it cannot be used to decrypt a certain message. While this is an interesting approach (especially for its relative conceptual simplicity), we focus on Signal due to its widespread adoption.

1.3. Overview

In Section 2 we give a detailed presentation of the Signal protocol. We follow this by a high-level description of its threat model in Section 3, and a formal security model in Section 4. In Section 5 we prove security of Signal's core in our model. As a first analysis of a complex protocol our model has some limitations and simplifying assumptions, discussed in detail in Section 6. We conclude in Section 7.

2. The Core Signal Protocol

Basic setup. The Signal protocol aims to send encrypted messages from one party to another. It assumes each party has a long-term public/private key pair, referred to as the identity key. However, since the parties might be offline at any point in time, standard authenticated key-exchange (AKE) solutions cannot be directly applied. In particular, conventional AKE wisdom says that to achieve perfect-forward secrecy with a DH exchange, the recipient would have to provide an ephemeral key specific for the exchange, but he might be offline at the time of sending.

Instead, Signal implements an asynchronous transmission protocol. This is effectively achieved by requiring potential recipients to pre-send batches of ephemeral public keys during registration. When the sender later wants to send a message, she obtains an ephemeral key for the recipient from an intermediate server (which only acts as a buffer), and performs an AKE-like protocol using the long-term and ephemeral keys to compute a message encryption key.

This basic setup is then extended by making the message keys dependent on all previously performed exchanges between the parties, using a combination of "ratcheting" mechanisms to form "chains". Additionally, further random and secret values are introduced into the computations at various points, which influence the future message keys computed by the communicating partners.

Motivation and Scope. The Signal protocol uses an intricate design whose rationale is currently not documented. Our focus is to study the existing protocol: we aim simply to report what Signal *is*, not why any of its design choices were made.

As of the time of writing, the Signal protocol has no substantial documentation other than its code. Because of the lack of a formal specification and the frequent protocol changes, it is challenging to pin down a precise definition of the intended usage and security properties of Signal. Our descriptions in this section were aided by some documentation but the ultimate authority is the implementation[2] [35].

2.1. Protocol Overview

A party using Signal first registers their long-term key, as well as medium-term keys and some cached ephemeral keys with a key distribution server. Two parties communicate using Signal in long-lived exchanges called *sessions*. A session begins when Alice requests Bob's long-term, medium-term and ephemeral credentials from a key distribution server (perhaps over an authentic channel), optionally verifies them out-of-band, and uses them in a key exchange protocol called the Signal Key Exchange (formerly "TripleDH").

The key exchange outputs a master secret, which in turn is used to derive two symmetric keys: a "root key" and a "sending chain key". As messages are sent and received these keys are frequently updated by passing them through a key derivation function (KDF), at the same time deriving output keys which are used elsewhere in the protocol.

When Alice wishes to encrypt a message for Bob, she advances her sending chain by one step, deriving a replacement sending chain key as well as a message encryption key. She can derive subsequent message encryption keys by repeating this process, advancing the sending chain once per message in order to derive a new key. Similarly, when she receives a message from Bob she advances her receiving chain in order to generate a decryption key.

The root chain is advanced through a separate mechanism: when the session is initialised, Alice also generates an ephemeral key known as her "ratchet key". She attaches this to her messages, authenticated but

2. The tagged releases of libsignal lag behind the current codebase. The commit hash of the state of the repository as of our reading is listed in the bibliography. Note that there are separate implementations in C, JavaScript and Java; the latter is used by Android mobile apps and is the one we have read most carefully.

asymmetric	ipk_A	ik_A	A's long-term identity key pair
	$prepk_B$	$prek_B$	B's medium-term (signed) prekey pair
	$eprepk_B$	$eprek_B$	B's ephemeral prekey pair
	epk_A	ek_A	A's ephemeral key pair
	$rchpk_A^a$	$rchk_A^a$	A's a^{th} ratchet key pair
symmetric		$ck_A^{\mathbf{sym\text{-}ir}:a,y}$	y^{th} key in A's a^{th} send chain
		$ck_A^{\mathbf{sym\text{-}ri}:a,y}$	y^{th} key in A's a^{th} receive chain
		$mk_A^{\mathbf{sym\text{-}ir}:a,y}$	y^{th} message key in A's a^{th} send chain
		$mk_A^{\mathbf{sym\text{-}ri}:a,y}$	y^{th} message key in A's a^{th} receive chain
		rk_A^a	A's a^{th} root key

TABLE 1: Keys used in the Signal protocol. Asymmetric key pairs show public and private components.

not encrypted. When Bob replies to a message, he will send his own "ratchet public key". Upon receiving a new ratchet public key from Bob, Alice advances the root chain *twice*: first with the DH shared secret obtained using her old public key, and second with that using her new. The resulting two outputs of the chain initialise the new receiving and sending chains respectively, and the resulting root chain key replaces the original root chain key.

Figure 3 on page 8 shows an example sequence of stages that one party might go through, with the message encryption keys derived in each stage. For the initiator (resp. responder), $mk^{\mathbf{sym\text{-}ir}:x,y}$ denotes the yth symmetric key on the xth sending (resp. receiving) chain, and $mk^{\mathbf{sym\text{-}ri}:x,y}$ the yth symmetric key on the xth receiving (resp. sending) chain. We use the notation **ir** and **ri** for sending and receiving keys from the initiator of the session's perspective: **ir** is from initiator of the session to responder, so corresponds to a sending key for the initiator and receiving key for the responder, and vice versa for **ri**. The notation inherits its complexity from the underlying protocol, but it does allow us to distinctly name each session key that is generated, and will allow us to make note of the subtly different properties of different keys.

Thus, we can separate the Signal protocol into four phases:

Registration. (Section 2.3)
At installation time, Alice registers her identity with a key distribution server and uploads some cryptographic data.

Session setup. (Section 2.4)
Alice requests and receives some information about Bob (either from the central server or from Bob himself), and uses it to setup a long-lived messaging session and establish initial symmetric encryption keys.

Symmetric-ratchet communication. (Section 2.5)
Alice uses the current symmetric encryption keys of her messaging session for communication with Bob, passing them through a hash function on every iteration. The message keys form a PRF chain, or a "symmetric ratchet".

Asymmetric-ratchet updates. (Section 2.6)
Alice exchanges DH values with Bob, generating new shared secrets and using them to begin new chains of message keys. The exchanged DH values give rise to a sequence of shared secrets, which are hashed into the "root chain" to form the "asymmetric ratchet".

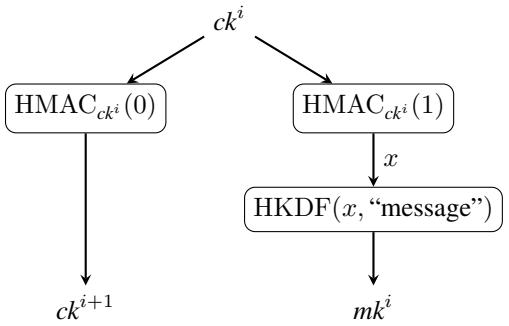
Alice and Bob can run many simultaneous sessions between them, each admitting an arbitrary sequence of stages consisting of symmetric and asymmetric ratcheting. We first explain notation and primitives below, and then discuss each of the four phases in detail in subsequent sections. At the end, we summarize the memory contents as used by Signal.

Table 1 is a glossary to the ten different classes of keys used in the Signal protocol, and Figure 1 shows the key derivations. Figure 2 depicts the operations executed in all stages of the protocol in a pseudocode format.

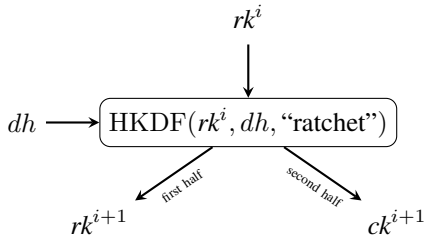
2.2. Notation and Primitives

Groups. Let g denote the generator of a group G of prime order q; we write the group multiplicatively. Signal uses Curve25519 [3].

Sessions. We denote A's ith session by π_A^i.



(a) KDF_m, the KDF for message chain updates. Note that new chain keys are *not* computed using HKDF; instead, they use only a HMAC.



(b) KDF_r, the KDF for root chain updates. dh is the DH value derived for this stage update, and the result is a new root key as well an output chain key.

Figure 1: Key derivation functions for root and chain keys in Signal: keys flow along edges, and boxes apply their functions to their input. The diagrams depict the two functions used to take a chain key, and output a successor chain key and an exported key. In our analysis, we treat these functions as black boxes instead of making specific assumptions. Iterating these functions produces Signal's *chains*.

Stages. Within a session, Signal admits a tree of various different stages, and we adopt a unified notational convention to refer to any of them. All stages are described using a term in [square brackets]; the initial stage is always $[0]$ and contains the key exchange. Subsequent stages occur locally at Alice and Bob, but correspond in the sense of generating matching keys.

Alice and Bob assign different roles to the stages they complete: Alice may consider some stage s as generating a sending key, while Bob considers his version of the same stage as generating a receiving key. To avoid persistent case distinctions, we adopt a *role-agnostic* naming scheme, describing stages as "-ir" if they are used for the initiator to send to the responder, and as "-ri" if they are used for the responder to send to the initiator. This maintains the invariant that stages with the same name generate the same key(s).

There are two types of asymmetric update as part of the DH ratchet; the first uses a received ratchet key to begin a receiving chain, and the second generates a new ratchet key to begin a sending chain. At a given party, we count the number of asymmetric updates in a variable x; thus, we can refer to the x^{th} update of the first type in a session as stage $[\mathbf{asym\text{-}ri}{:}x]$, and of the second type as $[\mathbf{asym\text{-}ir}{:}x]$. Note that the x^{th} "-ri" stage precedes the x^{th} "-ir" stage, because the first asymmetric stage is of type "-ri".

There are two types of symmetric update, "-ri" and "-ir", depending on whether the chain to which they belong was created by a stage of type $[\mathbf{asym\text{-}ri}{:}x]$ or $[\mathbf{asym\text{-}ir}{:}x]$. At a given party, we count the number of symmetric updates in the x^{th} symmetric chain in a variable y; thus, we can refer to the y^{th} update in the x^{th} symmetric chain as stage $[\mathbf{sym\text{-}ri}{:}x{,}y]$ or $[\mathbf{sym\text{-}ir}{:}x{,}y]$

In Signal, for a fixed x all symmetric stages which a party uses to generate sending keys in chain x occur before the asymmetric stage $x+1$, but symmetric receiving ratchets in chain x can occur at any time after the parent node in the graph has been established. This accommodates out-of-order message receipt. For example, the initiator performs all stages $[\mathbf{sym\text{-}ir}{:}x{,}1], [\mathbf{sym\text{-}ir}{:}x{,}2], \ldots$ before stage $[\mathbf{asym\text{-}ir}{:}x+1]$, but may delay stages $[\mathbf{sym\text{-}ri}{:}x{,}y]$ as much as necessary.

Keys. Signal distinguishes between at least ten different classes of key, so again for ease of reading we adopt a standardised notation. Keys are written in italics and end with the letter k. For asymmetric key pairs, the corresponding public key ends with the letters *pk*, and is always computed by group exponentiation with base g and the private key as the exponent: $pk = g^k$. If the agent A who generates a key is unclear we mark them in subscript (k_A), but omit this where it is clear.

Every stage derives new keys. To identify these keys uniquely, we write the index of the stage deriving a key k in superscript; thus, rk_A^0 would be the root key derived by A in stage $[0]$, and $mk^{\mathbf{sym\text{-}ri}:x,y}$ the message key derived in stage $\mathbf{sym\text{-}ri}{:}x{,}y$. Not all stages derive all keys: for example, there is no $rk^{\mathbf{sym\text{-}ri}:x,y}$, since root keys are not affected by symmetric updates. See Table 1 for a list of keys.

The naming scheme for keys is also role-agnostic: in intended operation, keys will be equal iff they have the same name. As with stages, agents have different intended uses for the same key: for example, the initiator would use the key $mk^{\mathbf{sym\text{-}ir}:x,y}$ for encrypting messages to send, and the responder would use the same key for decrypting received messages.

In our model, there are technically no stages $[\mathbf{sym\text{-}ir}{:}x{,}0]$ or $[\mathbf{sym\text{-}ri}{:}x{,}0]$, but there are keys with these indexes, since the first entry in each sending and receiving chain is created by the asymmetric update starting that chain (see Figure 3). We could equivalently think of Signal only deriving message keys in symmetric stages and allowing $y = 0$, in which case asymmetric stages would not derive message keys. Our formulation simply renumbers keys, so that every stage derives a message key.

Cryptographic functions. The key derivation functions are depicted in Figure 1, and use either HMAC-SHA256 or HKDF using SHA256 [29] as indicated.

AEAD denotes an authenticated encryption scheme with associated data. In Signal, this is an encrypt-then-MAC scheme: encryption is AES256 in CBC mode with PKCS#5 padding, and the MAC is HMAC-SHA256. This is the same combination originally used in TextSecure v3, which was shown by Frosch et al. [22] to have standard authenticated encryption security properties. Since our focus is on the key exchange portion, we omit details of the AEAD and treat it in a black-box fashion.

Sign is related to the Ed25519 signature scheme [4], but modified. Again, we treat it as a black-box signature.

2.3. Registration Phase—Figure 2(a)

Upon installation (and subsequently periodically), all agents generate a number of cryptographic keys and register themselves with a key distribution server.

Specifically, each agent generates the following DH private keys:

(i) a long-term "identity" key *ik*
(ii) a medium-term "signed prekey" *prek*
(iii) multiple short-term "ephemeral prekeys" *eprek*

The public keys corresponding to these values are then uploaded to the server, together with a signature on *prek* using *ik*.

2.4. Session Setup Phase—Figure 2(b)

In the session-setup phase, public keys are exchanged and used to initialise shared secrets in the session memory. The underlying key exchange protocol is a one-round DH protocol called the Signal Key Exchange[3], comprising an exchange of various DH public keys, computation of various DH shared secrets as in Figure 4, and then application of a key derivation function. While many possible variants of such protocols have been explored in-depth in the literature (HMQV [30], Kudla-Paterson [31], NAXOS [32] among many others), the session key derivation used here is new and not based on one of these standard protocols, though it draws some inspiration from [31].

Recall that for asynchronicity Signal uses prekeys: initial protocol messages which are stored at an intermediate server, allowing agents to establish a session with offline peers by retrieving one of their cached messages (in the form of a DH ephemeral public key).

In addition to this ephemeral public key, agents also publish a "medium-term" key, which is shared between multiple peers. This means that even if the one-time ephemeral keys stored at the server are exhausted, the session will go ahead using only a medium-term key. This form of key reuse is studied in [38] and will be modelled in this paper. Thus, session setup in the Signal protocol consists of two steps: first, Alice obtains ephemeral values from Bob (usually via a key distribution server); second, Alice treats the received values as the first message of a Signal key exchange, and completes the exchange in order to derive a master secret.

2.4.1. Receiving ephemerals. The most common way for Alice to receive Bob's session-specific data is for her to query a semi-trusted server for pre-computed values (known as a PreKeyBundle).

When Alice requests Bob's identity information, she receives his identity public key ipk_B, his current signed prekey $prepk_B$, and a one-time prekey $eprepk_B$ if there are any available. Signed pre-keys are stored for the medium term, and therefore shared between everyone sending messages to Bob; one-time keys are deleted by the server upon transmission. Alice's initial message contains identifiers for the prekeys so that Bob can learn which were used.

2.4.2. Building a session. Once Alice has received the above values, she generates her own ephemeral key ek_A, and computes a session key by performing three or four group exponentiations as depicted in Figure 4. She then concatenates the resulting shared secrets and passes them through a key derivation function ($\mathrm{KDF_r}$, Figure 1b) to derive an initial root key rk^0 and sending chain key $ck^{\mathbf{s}:0,0}$. (No DH value is passed to $\mathrm{KDF_r}$ for this initial invocation.) For modelling purposes, we also have Alice generate her initial sending message key $mk^{\mathbf{sym\text{-}ir}:0,0}$ (which is this stage's session key output) and the next sending chain key $ck^{\mathbf{sym\text{-}ri}:0,0}$. Finally, she generates a new ephemeral DH key $rchk^0$ known as her ratchet key.

For Bob to complete[4] the key exchange, he must receive Alice's public ephemeral key epk_A. In the Signal protocol, Alice attaches this value to all messages that she sends (until she receives a message from Bob, since from such a message she can conclude that Bob received epk_A). To disentangle the stages of the model, we

3. The key exchange protocol was originally named TripleDH, from the three DH shared secrets always used in the KDF (although in most configurations four shared secrets are used). The name QuadrupleDH has also been used for the variant which includes the long-term/long-term DH value, not as might be expected the variant which includes the one-time prekey.

4. If the initial message from Alice is invalid, Bob will in fact not complete a session. This does not affect our analysis, which considers only secrecy of session keys, but may become important if e.g. deniability becomes a concern.

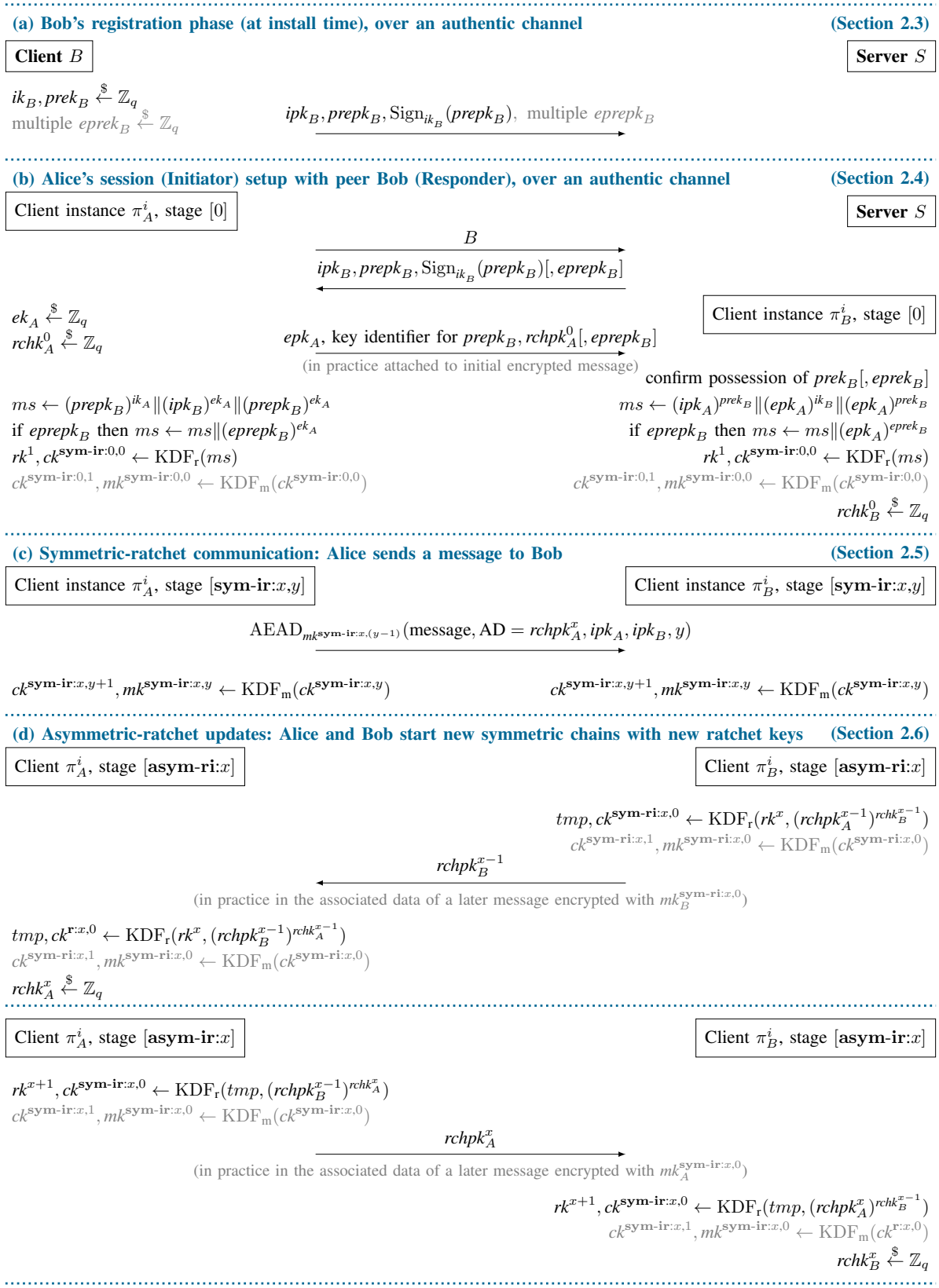


Figure 2: Signal protocol including preregistration of keys. Local actions are depicted in the left and right columns, and messages flow between them. We show only one step of the symmetric and asymmetric ratchets; they can be iterated arbitrarily. Variables storing keys are defined in Table 1, $\mathrm{KDF_r}$ and $\mathrm{KDF_m}$ in Figure 1, and session identifiers in Table 2. Gray text indicates reordered actions in our model, as discussed in Section 5. Each stage derives message keys with the same index as the stage number, and chaining/root keys with the index for the next stage; the latter is passed as state from one stage to the next. State info st in asymmetric stages is defined as the root key used in the key derivation, and for symmetric stages st is defined as the chain key used in key derivation. Symmetric stages always start at $y = 1$ and increment. When an actor sends consecutive messages, the first message is a DH ratchet and then subsequent messages use the symmetric ratchet. When an actor replies, they always DH ratchet first; they never carry on the symmetric ratchet.

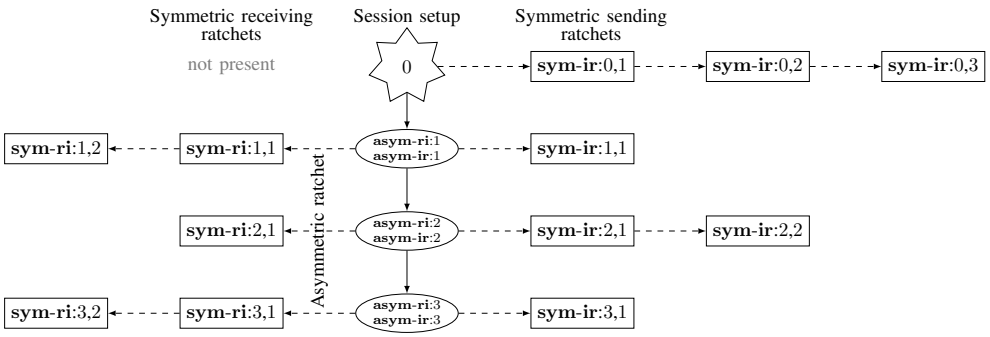


Figure 3: An example tree of *stages* that one party might use in one session of Signal. The content of each node is the stage name; recall that "ir" denotes stages deriving a key used to send from initiator to responder, and vice versa for "ri". In our notation, mk^s denotes the message key derived by stage s. For example, $mk^{\mathbf{sym\text{-}ir}:x,y}$ denotes the yth symmetric key on the xth chain, used by the initiator to encrypt messages and by the responder to decrypt them. Each chain is derived by ratcheting as in Figure 1 with a root or chain key. For the first symmetric ratchets in a session, the initiator of the session only has a sending chain, while the responder only has a receiving chain.

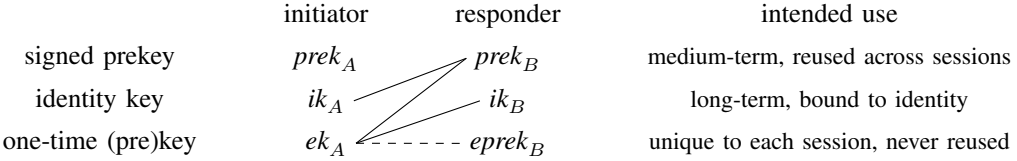


Figure 4: Diffie–Hellman private keys used in the Signal Key Exchange KDF. An edge between two private keys (e.g., ik_A and $prek_B$) indicates that their DH value ($g^{ik_A \cdot prek_B}$) is included in the final KDF computation. The dashed line is optional: it is omitted from the session key derivation if $eprek_B$ is not sent. Note the asymmetry: when Alice initiates a session with Bob, her signed prekey is not used at all. Our freshness conditions in Section 4.3 on page 12 will be partially based on this graph.

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have Alice send epk_A in a separate message; thus, once the session-construction stage is complete, both Alice and Bob have derived their root and chain keys.

When Bob receives epk_A, he first checks that he currently knows the private keys corresponding to the identity, signed pre-, and one-time pre-key which Alice used. If so, he performs the receiver algorithm for the key exchange, deriving the same root key rk^0 and chain key (which he records as $ck^{\mathbf{r}:0,0}$). For modelling purposes, we also have Bob generate his initial receiving message key $mk^{\mathbf{sym\text{-}ir}:0,0}$ (which is this stage's session key output) and the next receiving chain key $ck^{\mathbf{sym\text{-}ir}:0,0}$.

2.5. Symmetric-Ratchet Phase—Figure 2(c)

Two sequences of symmetric keys will be derived using a PRF chain, one for sending and one for receiving. The symmetric chains—to the left and the right in Figure 3—may be advanced for one of two reasons: either Alice wishes to send a new message, or she wishes to decrypt a message she has just received.

In the former case, Alice takes her current sending chain key $ck^{\mathbf{sym\text{-}ir}:x,y}$ and applies the message key derivation function $\mathrm{KDF_m}$ to derive two new keys: an updated sending chain key $ck^{\mathbf{sym\text{-}ir}:x,(y+1)}$ and a sending message key $mk^{\mathbf{sym\text{-}ir}:x,y}$. Alice uses the sending message key to encrypt her outgoing message, then deletes it and the old sending chain key. This process can be repeated arbitrarily.

When Alice receives an encrypted message, she checks the accompanying ratchet public key to confirm that she has not yet processed it, and if not she then performs an asymmetric ratchet update, described below. Regardless, she then reads the metadata in the message header to determine the index of the message in the receiving chain, and advances the receiving chain as many steps as is necessary to derive the required receiving message key; by construction, Alice's receiving message keys equal Bob's sending keys. Unlike for the sending case, Alice cannot delete receiving message keys immediately; she must wait to receive a message encrypted under each one. (Otherwise, out-of-order messages would be undecryptable since their keys would have been deleted.) The open source implementation of Signal has a hard-coded limit of 2000 messages or five asymmetric updates, after which old keys are deleted even if they have not yet been used.

2.6. Asymmetric-Ratchet Phase—Figure 2(d)

The final top-level phase of Signal is the asymmetric-ratchet update. In this phase, Alice and Bob take turns generating and sending new DH public keys and using them to derive new shared secrets. These are

accumulated in the asymmetric ratchet chain, from which new (symmetric) message chains are initialized.

When Alice receives a message from Bob, it may be accompanied by a new (previously unseen) ratchet public key $rchpk_B^{x-1}$. If so, this triggers Alice to enter her next asymmetric ratchet phase $[\mathbf{asym\text{-}ir}{:}x]$. Note that Alice already has stored a previously generated private ratchet key $rchk_A^{x-1}$. Before decrypting the message, Alice updates her asymmetric ratchet as per Figure 2. This consists of two steps. In the first step, denoted $rchk_A^x$, deriving two DH shared secrets $[\mathbf{asym\text{-}ri}{:}x]$, she computes a first DH shared secret (between the received ratchet public key and her old ratchet private key), and combines this with the root chain key to derive a new receiving chain key and receiving message key. In the second step, denoted $[\mathbf{asym\text{-}ir}{:}x]$, she computes a second DH shared secret (between the received ratchet public key and her new ratchet private key), and combines this with the root chain key and the first DH shared secret to derive a new sending chain key and sending message key, as well as the root chain key for the next asymmetric stage.

The message keys in the first and second steps have slightly different security properties, so they are recorded in our model as belonging to distinct stages $[\mathbf{asym\text{-}ri}{:}x]$ and $[\mathbf{asym\text{-}ir}{:}x]$.

Alice then sends her new ratchet public key $rchpk_A^x$ along with future messages to Bob, and the process continues indefinitely.

Bob does the corresponding operations shown in Figure 2 to compute the same DH shared secrets and the corresponding root, chain, and message keys. While symmetric updates can be triggered either by Alice (the session initiator) or Bob (the session responder) and thus could be as in Figure 2(c) or its horizontal flip, asymmetric updates can only be triggered by Alice (the session initiator) receiving a new (previously unseen) ratchet key from Bob (the session responder) and not the other way around, so Figure 2(d) will never be horizontally flipped.

2.7. Memory Contents

Signal is a stateful protocol, and a number of different values are available in Alice's memory at any time. Alice's global state—shared between all of her sessions—contains four different collections of values: identity keys (Alice's own identity private key, and the identity public keys of all her peers), signed prekeys, ephemeral keys, and a list of all sessions.

Furthermore, each session in the collection of sessions above contains the keys used by the protocol. Specifically, a session always has its agent's identity private key and its peer's identity public key, a current root and sending chain key, and a current ratchet key. In addition, it has some number of receiving chain and message keys, corresponding to out-of-order messages not yet received from the peer.

3. Threat Models

We will analyze Signal in the context of a fully adversarially-controlled network. The high-level properties we aim to prove are secrecy and authentication of message keys. Authentication will be implicit (only the intended party could compute the key) rather than explicit (proof that the intended party did compute the key). Forward secrecy and "future" secrecy are not explicit goals; instead, derived session keys should remain secret under a variety of compromise scenarios, including if a long-term secret has been compromised but a medium or ephemeral secret has not (forward secrecy) or if state is compromised and then an uncompromised asymmetric stage later occurs ("future" or post-compromise secrecy [12]). We assume out-of-band verification of identity keys and medium-term keys, and do not consider faulty/malicious random number generators or side channel attacks on implementations.

The finer details of our threat model are ultimately encoded in the so-called freshness predicate, specified in Section 4.3 on page 12, where we provide further information on our threat model design choices.

3.1. On our choice of Threat Model

At the time of writing, Signal does not have a defined threat model nor any formally-specified security properties. As part of our analysis, we therefore had to decide which threat model to assume, based on the informal claims made in some of the documentation. In particular, the README document accompanying the source code [35] states that Signal "is a ratcheting forward secrecy protocol that works in synchronous and asynchronous messaging environments". A separate GitHub wiki page [41] provides some more goals (forward and future secrecy[5], metadata encryption and detection of message replay/reorder/deletion) but to the best of our knowledge no mention of message integrity or authentication is made.

We believe that the threat model we have chosen is realistic, although we discuss later some directions in which it could be strengthened. Parallels can be drawn, for example, with the TLS 1.3 standard [43, Appendix D], which discusses the following properties (where the network is fully adversarially-controlled, and where the adversary may compromise the keys of some participants).

5. Future secrecy is defined as "a leak of keys to a passive eavesdropper will be healed by introducing new DH ratchet keys" [41].

Correctness If Alice and Bob complete an exchange with each other then they should derive the same keys; distinct exchanges should derive distinct keys.

Secrecy If Alice and Bob complete an exchange to generate a key k, nobody other than Alice and Bob should be able to learn anything about k.

(Implicit) Authentication If Alice believes that she shares the key k with Bob, nobody other than Bob should be able to learn anything about k. Note that this property is implied by secrecy.

Forward secrecy An attacker who compromises Alice's long-term secret after a session is complete should not be able to learn anything about the keys derived in that session.

Identity hiding A passive adversary should not learn the identity of partners to a session.

It is common in the authenticated key exchange literature to assume a trusted public key infrastructure (PKI), though some models allow the adversary more control [8]. In Signal the PKI is combined with the network, in the sense that the same server distributes identity and ephemeral keys. Thus, in some sense assuming a trusted PKI also restricts the attacker's control over particular sessions.

Some claims have been made about privacy and deniability in Signal, but these are relatively abstract. In general, signatures are used only for the signed pre-key in the initial handshake, meaning that full deniability is not possible but peer deniability [14] (inability of an observer to prove that Alice intended to communicate *with Bob*) is straightforward.

Additionally, one might consider a threat model that includes imperfect ephemeral random number generators. Signal (like TLS) is not secure in this scenario: since no static-static DH shared secret is included in the KDF, an adversary who knows all ephemeral values can compute all secrets.

The trust assumptions on the registration channel are not defined; Signal has an optional feature whereby participants can verify each other's identity keys through an out-of-band channel, but it is up to implementations whether to require or even support this feature. Without it, an untrusted key distribution server can of course impersonate any agent.

Signal's mechanisms suggest a lot of effort has been invested to protect against the loss of secrects used in specific communications. If the corresponding threat model is an attacker gaining (temporary) access to the device, it becomes crucial if certain previous secrets and decrypted messages can be accessed by the attacker or not: generating new message keys is of no use if the old ones are still recoverable. This, in turn, depends on whether *deletion* of messages and previous secrets has been effective. This is known to be a hard problem, especially on flash-based storage media [42], which are commonly used on mobile phones.

4. Security Model

In this section we present a security model for multi-stage key exchange, which we then apply to model Signal's initial key exchange as well as its ratcheting scheme. Our model allows multiple parties to execute multiple, concurrent *sessions*; each session has multiple *stages*. For Signal, the session represents a single chat between two parties, and each stage represents a new application of the ratchet. Figure 2 depicts, roughly, a single session. There are three types of stage in Signal: the initial key exchange, asymmetric ratcheting, and symmetric ratcheting. In addition, ratcheting stages differ based on whether they are used for generating keys for the initiator to send to the responder (denoted -ir) or vice versa (denoted -ri). For our purposes, every stage generates a session key; depending on the stage, this will be either the sending or the receiving message key.

On the choice of model. We choose to study the security of Signal in the traditional key exchange notion of *key indistinguishability* (albeit a multi-stage variant), as opposed to a monolithic *secure channel* notion such as authenticated and confidential channel establishment (ACCE) [26]. It is often preferable to analyze the key exchange portion independently, and then compose this with authenticated encryption to establish a secure channel [11]; monolithic notions like ACCE are necessary for protocols which use the session key (or values derived from it) in the channel establishment, thus preventing a clean separation for composition. While Signal does use intermediate values between stages, the sending and receiving message keys $mk^{\mathbf{sym\text{-}ir}:x,y}, mk^{\mathbf{sym\text{-}ri}:x,y}$ are not used in subsequent key exchange operations, and thus the key indistinguishability security notion is achievable. This means that unlike in [18, 21], we do not need to modify the protocol algorithm to ensure that the result of testing a session remains consistent with future values.

Another subtlety compared to the multi-stage key exchange model of Fischlin and Günther is that QUIC and TLS 1.3 demand a linear sequence of stages, whereas Signal has a tree of stages, as seen in Figure 3.

Model notation. We present our model as a pseudocode experiment where the primitive in question (the multi-stage key exchange protocol) is modelled as a tuple of algorithms, and then an adversary interacts with the experiment. This approach is commonly used in many other areas of cryptography, but less so in key exchange papers. Compared with key exchange papers in which the security model and experiment is presented in textual format, we argue that this approach makes it easier to see the precise nature of the experiment, and easier to see subtleties in variations.

We adopt the following notational and typographic conventions. Monotype text denotes constants; serif text denotes algorithms and variables associated with the actual protocol (variables are *italicized*); and sans-serif text denotes algorithms, oracles, and variables associated with the experiment. Algorithms and Oracles start with upper-case letters; variables start with lower-case letters. We use object-oriented notation to represent collections of variables. In particular, we will use π_u^i to denote the collection of variables that party u uses in its i^{th} protocol execution ("session"). To denote the variable v in stage s of party u's ith session, we write $\pi_u^i.v[s]$; note s is not (necessarily) a natural number. For Signal, s is [0] for the session setup stage; [**sym-ir**:x,y] or [**sym-ri**:x,y] for symmetric sending or receiving stages; or [**asym-ri**:x] or [**asym-ir**:x] for the 1st and 2nd portions of the xth asymmetric stage. (See also Figure 3 and Figure 2.)

DH protocols conventionally use both ephemeral keys (unique to a session) and long-term keys (in all sessions of an agent). Signal's prekeys do not fit cleanly into this separation, and in order to follow the conventions of the field we refer to them as "medium-term keys".

Generality of our model. Some aspects of our model are quite general, and others are very specific to Signal. Our formulation of a multi-stage key exchange protocol as a tuple of algorithms, as well as the main experiment and oracles in Figure 5, should be applicable to any multi-stage key exchange protocol that includes semi-static (medium term) keys. However, our freshness definition is highly customized to Signal via the clean clauses in Definitions 5, 6 and 8, since we aim to precisely characterize the security properties of Signal session keys.

The level of generality is an important decision when designing a security model for a protocol like Signal: on one hand, a general model allows analysis of and comparison to other protocols; on the other, of necessity it does not allow fine-grained statements about the specific protocol under consideration. Our model lies towards the centre of this spectrum: we aim to keep the overall model structure relatively independent of Signal (though of necessity we added support for the Signal medium-term keys), while the cleanness predicates described later allow us to make fine-grained assertions which capture as much as possible.

Medium-term key authentication. Signal's medium-term keys are signed by the same identity key used for DH, breaking key separation. Although there has been some analysis of this form of key reuse [16, 40], it is nontrivial to prove secure. We instead enforce authentication by fiat, allowing the adversary to select any of the medium-term keys owned by an agent, but not to inject their own. In the game, this is implemented as an extra argument when the adversary creates a new session.

4.1. Multi-Stage Key Exchange Protocol

Definition 1 (Multi-stage key exchange protocol)**.** A *multi-stage key exchange protocol* Π is a tuple of algorithms, along with a keyspace $\mathcal{K}$ and a security parameter λ indicating the number of bits of randomness each session requires. The algorithms are:

- $\mathrm{KeyGen}() \xmapsto{\$} (\mathit{ipk}, \mathit{ik})$: A probabilistic *long-term key generation algorithm* that outputs a long-term public key / secret key pair $(\mathit{ipk}, \mathit{ik})$. In Signal, these are called "identity keys".
- $\mathrm{MedTermKeyGen}(\mathit{ik}) \xmapsto{\$} (\mathit{prepk}, \mathit{prek})$: A probabilistic *medium-term key generation algorithm* that takes as input a long-term secret key *ik* and outputs a medium-term public key / secret key pair $(\mathit{prepk}, \mathit{prek})$. In Signal, these are called "signed prekeys"; in the key exchange literature, they are sometimes called "semi-static keys".
- $\mathrm{Activate}(\mathit{ik}, \mathit{prek}, \mathit{role}) \to (\pi', m')$: A probabilistic *protocol activation algorithm* that takes as input a long-term secret key *ik*, a medium-term secret key *prek*, and a role $\mathit{role} \in \{\texttt{init}, \texttt{resp}\}$, and outputs a state π' and (possibly empty) outgoing message m'.
- $\mathrm{Run}(\mathit{ik}, \mathit{prek}, \pi, m) \to (\pi', m')$: A probabilistic *protocol execution algorithm* that takes as input a long-term secret key *ik*, a medium-term secret key *prek*, a state π, and an incoming message m, and outputs an updated state π' and (possibly empty) outgoing message m'.

Definition 2 (State)**.** A *state* π is a collection of the following variables:

- $\pi.\mathit{role} \in \{\texttt{init}, \texttt{resp}\}$: the instance's role
- $\pi.\mathit{peeripk}$: the peer's long-term public key
- $\pi.\mathit{peerprepk}$: the peer's medium-term public key
- $\pi.\mathit{status}[s] \in \{\varepsilon, \texttt{active}, \texttt{accept}, \texttt{reject}\}$: execution status for each stage s
- $\pi.k[s] \in \mathcal{K}$: the session key output by stage s
- $\pi.\mathit{st}[s]$: any additional protocol state values that a previous stage gives as input to stage s (defined as part of the protocol).
- $\pi.\mathit{sid}[s]$: the identifier of stage s of session π; this is view the actor has of the session π in stage s, as defined in Figure 2.
- $\pi.\mathit{type}[s]$: the type of freshness required for this stage to have security. For Signal, this is `triple`, `triple+DHE`, `asym-ir`, `asym-ri`, `sym-ir` or `sym-ri`.

The state above models "real" variables that an implementation would actually keep track of and use in the execution of the protocol. We will supplement this in the experiment with additional variables that are artificially

added for the experiment. These are administrative identifiers, used to formally reason about what is happening in our security experiment, eg., to identity sessions and partners.

Correctness of a multi-stage key exchange protocol is defined in the natural way—that honest parties compute the same session key in the absence of an adversary—and we omit a formal definition. The multi-stage key exchange framework of Fischlin and Günther [21] defines a security requirement for session matching called "Match security", following the approach of Brzuska et al. [9], which can be used when composing a key exchange protocol with a symmetric key protocol. We omit consideration of Match security, as it is straightforward to prove for Signal.

4.2. Key Indistinguishability Experiment

Having defined what a multi-stage key exchange protocol is, we can now define the experiment for key indistinguishability.

As is typical in key exchange security models, the experiment establishes long-term keys and then allows the adversary to interact with the system. The adversary can direct parties to start sessions with particular medium-term keys, and can control the delivery of messages to parties (including modifying, dropping, delaying, and inserting messages). The adversary can learn various long-term or per-session secret information from parties via reveal queries, and at any point can choose a single stage of a single session to "test". They are then given either the real session key from this stage, or a random key from the same keyspace, and asked to decide which was given. If they decide correctly, we say they win the experiment. This is formalized in the following definition and corresponding experiment.

Definition 3 (Multi-stage key indistinguishability)**.** Let Π be a key exchange protocol. Let $\mathsf{n_P}, \mathsf{n_M}, \mathsf{n_S}, \mathsf{n_s} \in \mathbb{N}$. Let $\mathcal{A}$ be a probabilistic algorithm that runs in time polynomial in the security parameter. Define

$$\mathsf{Adv}^{\mathsf{ms\text{-}ind}}_{\Pi,\mathsf{n_P},\mathsf{n_M},\mathsf{n_S},\mathsf{n_s}}(\mathcal{A}) = \Pr\left[\mathsf{Exp}^{\mathsf{ms\text{-}ind}}_{\Pi,\mathsf{n_P},\mathsf{n_M},\mathsf{n_S},\mathsf{n_s}}(\mathcal{A}) = 1\right] - 1/2$$

where the security experiment $\mathsf{Exp}^{\mathsf{ms\text{-}ind}}_{\Pi,\mathsf{n_P},\mathsf{n_M},\mathsf{n_S},\mathsf{n_s}}(\mathcal{A})$ is as defined in Figure 5. Note $\mathsf{n_S}$ and $\mathsf{n_s}$ are upper bounds on the number of sessions per party and number of stages per session that can be established. We call an adversary efficient if it runs in time polynomial in the security parameter.

Note that $\mathsf{Exp}^{\mathsf{ms\text{-}ind}}$ includes the following global variables:

- b: a challenge bit
- $\mathsf{tested} = (u, i, s)$ or $\perp$: recording the inputs to the query $\mathsf{Test}(u, i, s)$ or $\perp$ if no Test query has happened

Furthermore, $\mathsf{Exp}^{\mathsf{ms\text{-}ind}}$ extends the per-session state π^i_u with the following experiment variables:

- $\pi^i_u.\mathsf{rand}[s] \in \{0,1\}^\lambda$: random coins for π^i_u's sth stage
- $\pi^i_u.\mathsf{peerid} \in \{1, \ldots, \mathsf{n_P}\}$: the identifier of the alleged peer
- $\pi^i_u.\mathsf{peerpreid} \in \{1, \ldots, \mathsf{n_M}\}$: the index of the alleged peer's medium-term key
- $\pi^i_u.\mathsf{rev\_session}[s] \in \{\mathrm{true}, \mathrm{false}\}$: whether $\mathsf{RevSessKey}(u, i, s)$ was called or not; default false
- $\pi^i_u.\mathsf{rev\_random}[s] \in \{\mathrm{true}, \mathrm{false}\}$: whether $\mathsf{RevRand}(u, i, s)$ was called or not; default false
- $\pi^i_u.\mathsf{rev\_state}[s] \in \{\mathrm{true}, \mathrm{false}\}$: whether $\mathsf{RevState}(u, i, s)$ was called or not; default false

We are working in the post-specified peer model, where the peer's identity is learned by the actor during the execution of the protocol, by virtue of learning the peer's public key; and similarly for the peer's semi-static key. Certain aspects of the experiment require the administrative index of the corresponding key, and thus, we assume that $\pi^i_u.\mathsf{peerid}$ is set to the corresponding index upon $\pi^i_u.\mathit{peeripk}$ being set; and similarly for the semi-static key index $\pi^i_u.\mathsf{peerpreid}$ upon $\pi^i_u.\mathit{peerprepk}$ being set. (Recall that experiment-only variables are in sans-serif.)

4.2.1. Session identifiers. We define the session identifiers $sid[s]$ for each stage $[s]$ of Signal in Table 2. It is important to note that these session identifiers only exist in our model, not in the protocol specification itself. We use them to we define restrictions on the adversary's allowed behaviour in our model, so that we can make precise security statements: we will generally restrict the adversary from making queries against any session with the same session identifier as the Test session. If two sessions have equal session identifiers we say that they are "matching".

4.3. Freshness

From a key exchange perspective, the novelty of Signal is the different security goals of different stages' session keys. The subtle differences between those security goals are captured in the details of the threat model. Previously, we provided the adversary with powerful queries with which it can break any protocol. We now define the so-called freshness predicate fresh to constrain that power, effectively specifying the details of the threat model.

$\mathsf{Exp}^{\mathsf{ms\text{-}ind}}_{\Pi,\mathsf{n_P},\mathsf{n_M},\mathsf{n_S},\mathsf{n_s}}(\mathcal{A})$:

$\mathsf{b} \xleftarrow{\$} \{0,1\}$
$\mathsf{tested} \leftarrow \bot$
// generate long-term and semi-static keys
for $u = 1$ **to** $\mathsf{n_P}$ **do**
  $(ipk_u, ik_u) \xleftarrow{\$} \mathrm{KeyGen}()$
  for $\mathsf{preid} = 1$ **to** $\mathsf{n_M}$ **do**
    $(prepk_u^{\mathsf{preid}}, prek_u^{\mathsf{preid}}) \xleftarrow{\$} \mathrm{MedTermKeyGen}(ik_u)$
$pubinfo \leftarrow (ipk_1, \ldots, ipk_{\mathsf{n_P}}, prepk_1^1, \ldots, prepk_{\mathsf{n_P}}^{\mathsf{n_M}})$
$\mathsf{b}' \xleftarrow{\$} \mathcal{A}^{\mathsf{Send},\mathsf{Rev}*,\mathsf{Test}}(pubinfo)$
if $(\mathsf{tested} \neq \bot) \wedge \mathsf{fresh}(\mathsf{tested}) \wedge \mathsf{b} = \mathsf{b}'$ **then**
  return 1 *// the adversary wins*
else
  return 0 *// the adversary loses*

$\mathsf{Send}(u, i, m)$:

if $\pi_u^i = \bot$ **then**
  // start new session and record intended peer
  parse m as $(\pi_u^i.\mathsf{preid}, role)$
  $\pi_u^i.\vec{\mathsf{rand}} \xleftarrow{\$} \{0,1\}^{\mathsf{ns} \times \lambda}$
  $(\pi_u^i, m') \leftarrow \mathrm{Activate}(ik_u, prek_u^{\pi_u^i.\mathsf{preid}}, role; \pi_u^i.\mathsf{rand}[0])$
  return m'
else
  $s \leftarrow \pi_u^i.stage$
  $(\pi_u^i, m') \leftarrow \mathrm{Run}(ik_u, prek_u^{\pi_u^i.\mathsf{preid}}, \pi_u^i, m; \pi_u^i.\mathsf{rand}[s])$
  return m'

$\mathsf{RevSessKey}(u, i, s)$:

$\pi_u^i.\mathsf{rev\_session}[s] \leftarrow \mathrm{true}$
return $\pi_u^i.k[s]$

$\mathsf{RevLongTermKey}(u)$:

$\mathsf{rev\_ltk}_u \leftarrow \mathrm{true}$
return ik_u

$\mathsf{RevMedTermKey}(u, \mathsf{preid})$:

$\mathsf{rev\_mtk}_u^{\mathsf{preid}} \leftarrow \mathrm{true}$
return $prek_u^{\mathsf{preid}}$

$\mathsf{RevRand}(u, i, s)$:

$\pi_u^i.\mathsf{rev\_random}[s] \leftarrow \mathrm{true}$
return $\pi_u^i.\mathsf{rand}[s]$

$\mathsf{RevState}(u, i, s)$:

$\pi_u^i.\mathsf{rev\_state}[s] \leftarrow \mathrm{true}$
return $\pi_u^i.st[s]$

$\mathsf{Test}(u, i, s)$:

// can only call Test *once, and only on accepted stages*
if $(\mathsf{tested} \neq \bot)$ **or** $(\pi_u^i.status[s] \neq \mathtt{accept})$ **then**
  return $\bot$
$\mathsf{tested} \leftarrow (u, i, s)$
// return real or random key depending on b
if $\mathsf{b} = 0$ **then**
  return $\pi_u^i.k[s]$
else
  $k' \xleftarrow{\$} \mathcal{K}$
  return k'

Figure 5: Security experiment for adversary $\mathcal{A}$ against multi-stage key indistinguishability security of protocol Π.

Our goal of the fresh predicate is to describe the best security condition that might be provable for each of Signal's message keys based on the protocol's design; here, "best" is with respect to the maximal combinations of secrets learned by the adversary.

The main motivation for our fresh predicate for the initial stages comes from observing Figure 4 on page 8. In the graph, the edges can be seen as the individual secrets established between initiator and responder, on which the secrecy of the session keys is based. If the adversary cannot learn the secret corresponding to one of these edges, it cannot compute the session key. The adversary can learn the secret corresponding to an edge if it can compromise one of the two endpoints. Thus, if an adversary can learn, e.g., the initiator A's ik_A and ek_A, it can derive the secrets corresponding to all edges. A similar observation can be made for the responder. However, after keys are updated, the situation changes, since additional secrets are introduced, and the adversary may no longer have enough information. Hence we define modified freshness conditions for subsequent stages, thereby encoding Signal's post-compromise security properties.

We define our freshness conditions to exactly exclude those cases for which we can directly observe that the protocol design offers no protection. For example, the design does not include an edge between the long-term keys of the parties (sometimes referred to as the *static Diffie-Hellman key*). This implies that if the long-term keys are secret, but other generated randomness is compromised or bad, Signal offers no protection, since all edges become compromised. Because our freshness conditions are based on fig. 4 and the subsequent key updates, we do not consider this scenario an attack but rather a direct consequence of the design. In this work we aim to prove that, working from the design choices made, Signal indeed achieves the best it can (without introducing further elements in the key derivation function).

The freshness predicate fresh for our experiment works hand-in-hand with a variety of sub-predicates ($\mathsf{clean}_{\mathtt{triple}}$, $\mathsf{clean}_{\mathtt{triple+DHE}}$, $\mathsf{clean}_{\mathtt{asym\text{-}ir}}$, $\mathsf{clean}_{\mathtt{asym\text{-}ri}}$, $\mathsf{clean}_{\mathtt{sym\text{-}ir}}$ and $\mathsf{clean}_{\mathtt{sym\text{-}ri}}$) which are highly specialized to Signal to capture the exact type of security achieved Signal's different types of stages. We define cleanness below.

Definition 4 (Freshness)**.** Define the predicate fresh as follows:

$$\begin{aligned}\mathsf{fresh}(u,i,s) = {} & (\pi_u^i.status[s] = \mathtt{accept}) \\ & \wedge \neg \pi_u^i.\mathsf{rev\_session}[s] \\ & \wedge \left(\forall\ j : \pi_u^i.sid[s] = \pi^j_{\pi_u^i.\mathsf{peerid}}.sid[s] \implies \neg \pi^j_{\pi_u^i.\mathsf{peerid}}.\mathsf{rev\_session}[s]\right) \\ & \wedge \mathsf{clean}_{\pi_u^i.type[s]}(u,i,s)\end{aligned}$$

name	sid	
$sid[0]$	$(\texttt{triple} : ipk_i, ipk_r, prepk_r, epk_i)$	if $type[0] = \texttt{triple}$
	$(\texttt{triple+DHE} : ipk_i, ipk_r, prepk_r, epk_i, eprepk_r)$	if $type[0] = \texttt{triple+DHE}$
$sid[\textbf{asym-ri}{:}x]$	$sid[0] \parallel (\texttt{asym-ri} : rchpk_i^0, rchpk_r^0)$	if $x = 1$
	$sid[\textbf{asym-ir}{:}x-1] \parallel (\texttt{asym-ri} : rchpk_r^{x-1})$	if $x > 1$
$sid[\textbf{asym-ir}{:}x]$	$sid[\textbf{asym-ri}{:}x] \parallel (\texttt{asym-ir} : rchpk_i^x)$	if $x > 0$
$sid[\textbf{sym-ri}{:}x,y]$	does not exist	if $x = 0$
	$sid[\textbf{asym-ri}{:}x] \parallel (\texttt{sym-ri} : y)$	if $x > 0$
$sid[\textbf{sym-ir}{:}x,y]$	$sid[0] \parallel (\texttt{sym} : y)$	if $x = 0$
	$sid[\textbf{asym-ir}{:}x] \parallel (\texttt{sym-ir} : y)$	if $x > 0$

TABLE 2: Definition of session identifiers $sid[s]$ for an arbitrary stage s. Since our stages are named role-agnostically, the definitions for initiator and responder stages coincide; we use i to refer to the identity of the initiator and r for that of the responder. For example, if Alice believes she is responding to Bob, then ipk_i denotes Bob's identity public key and ipk_r denotes Alice's. The initial asymmetric stage sid contains two ratchet keys (instead of one) since they are not used in the initial session key derivation and thus are not contained in $sid[0]$. We note that $sid[\textbf{sym-ir}{:}x,y]$ for $x = 0$ does not exist because the receiver never starts a symmetric chain immediately after the handshake, always first performing a DH ratchet.

Note that fresh and its sub-clauses has access to all variables in the experiment (global, user, session, and stage).

4.3.1. Session setup stage $[0]$**.** Intuitively, a triple-DH or triple-DH+DHE key exchange should be secure as long as at least one of its DH shared secrets is secure; thus the clauses of $\mathsf{clean}_{\texttt{triple}}$ and $\mathsf{clean}_{\texttt{triple+DHE}}$ correspond to those components. Note that $\mathsf{clean}_{\texttt{triple}}$ and $\mathsf{clean}_{\texttt{triple+DHE}}$ only need to be defined for the initial key exchange, i.e., stage $[0]$.

Definition 5 ($\mathsf{clean}_{\texttt{triple}}$)**.** Within the same context as Definition 4, define

$$\begin{aligned}\mathsf{clean}_{\texttt{triple}}(u,i,[0]) = \mathsf{clean}_{\texttt{LM}}(u,i)\\ \vee \mathsf{clean}_{\texttt{EL}}(u,i,0)\\ \vee \mathsf{clean}_{\texttt{EM}}(u,i,0)\end{aligned}$$

Definition 6 ($\mathsf{clean}_{\texttt{triple+DHE}}$)**.** Within the same context as Definition 4, define

$$\begin{aligned}\mathsf{clean}_{\texttt{triple+DHE}}(u,i,[0]) = \mathsf{clean}_{\texttt{triple}}(u,i,[0])\\ \vee \mathsf{clean}_{\texttt{EE}}(u,i,0,0)\end{aligned}$$

For the sub-clauses $\mathsf{clean}_{\texttt{XY}}$ in the above two definitions, our convention is that initiator's key is of type X and the responder's key of type Y, where the possible types are L, M, and E for long-term (*ik*), medium-term (*prek*), and ephemeral (*ek*) keys respectively, as in Figure 4. This necessitates the two definitions below of $\mathsf{clean}_{\texttt{LM}}/\mathsf{clean}_{\texttt{EL}}/\mathsf{clean}_{\texttt{EM}}$ for when the tested session is the initiator or responder.

These three definitions are straightforward for initiator sessions. For responder sessions, the difficult part is that the ephemeral key is now the peer's, not the actor's: to ensure that it is not known by the adversary, we have to ensure the peer session's randomness has not been revealed (identifying the peer session using session identifiers), and that key actually has to come from an honest peer (meaning the peer session must exist). The following clause helps identify that precise situation:

$$\begin{aligned}\mathsf{clean}_{\texttt{peerE}}(u,i,s) = \big(\forall\ j : \pi_u^i.sid[s] = \pi^j_{\pi_u^i.\mathsf{peerid}}.sid[s] \implies \neg\pi^j_{\pi_u^i.\mathsf{peerid}}.\mathsf{rev\_random}[s]\big)\\ \wedge \exists\ j : \pi_u^i.sid[s] = \pi^j_{\pi_u^i.\mathsf{peerid}}.sid[s]\end{aligned}$$

Thus prepared, we can define our various clean predicates. We remark here that this is a non-trivial restriction on the adversary: if the medium-term key is corrupted then we do not permit an attack impersonating Alice to Bob: since the only randomness in a TripleDH handshake is from the initiator (and there is no static-static DH

secret), such an attack will succeed.

$$\mathsf{clean}_{\mathtt{LM}}(u,i) = \begin{cases} \neg\mathsf{rev\_ltk}_u \wedge \neg\mathsf{rev\_mtk}^{\pi^i_u.\mathsf{peerpreid}}_{\pi^i_u.\mathsf{peerid}} & \pi^i_u.\mathit{role} = \mathtt{init} \\ \neg\mathsf{rev\_ltk}_{\pi^i_u.\mathsf{peerid}} \wedge \neg\mathsf{rev\_mtk}^{\pi^i_u.\mathsf{preid}}_u & \pi^i_u.\mathit{role} = \mathtt{resp} \end{cases}$$

$$\mathsf{clean}_{\mathtt{EL}}(u,i,[0]) = \begin{cases} \neg\pi^i_u.\mathsf{rev\_random}[0] \wedge \neg\mathsf{rev\_ltk}_{\pi^i_u.\mathsf{peerid}} & \pi^i_u.\mathit{role} = \mathtt{init} \\ \mathsf{clean}_{\mathtt{peerE}}(u,i,[0]) \wedge \neg\mathsf{rev\_ltk}_u & \pi^i_u.\mathit{role} = \mathtt{resp} \end{cases}$$

$$\mathsf{clean}_{\mathtt{EM}}(u,i,[0]) = \begin{cases} \neg\pi^i_u.\mathsf{rev\_random}[0] \wedge \neg\mathsf{rev\_mtk}^{\pi^i_u.\mathsf{peerpreid}}_{\pi^i_u.\mathsf{peerid}} & \pi^i_u.\mathit{role} = \mathtt{init} \\ \mathsf{clean}_{\mathtt{peerE}}(u,i,[0]) \wedge \neg\mathsf{rev\_mtk}^{\pi^i_u.\mathsf{preid}}_u & \pi^i_u.\mathit{role} = \mathtt{resp} \end{cases}$$

$$\mathsf{clean}_{\mathtt{EE}}(u,i,s,s') = \begin{cases} \neg\pi^i_u.\mathsf{rev\_random}[s] \wedge \mathsf{clean}_{\mathtt{peerE}}(u,i,s') & \pi^i_u.\mathit{role} = \mathtt{init} \\ \mathsf{clean}_{\mathtt{peerE}}(u,i,s) \wedge \neg\pi^i_u.\mathsf{rev\_random}[s'] & \pi^i_u.\mathit{role} = \mathtt{resp} \end{cases}$$

Since we reveal randomness instead of specific keys, this final predicate applies to both the ephemeral keys and the ratchet keys, a fact which we shall use later when defining cleanness of asymmetric stages.

4.3.2. Asymmetric stages $[\mathbf{asym\text{-}ir}{:}x]/[\mathbf{asym\text{-}ri}{:}x]$**.** In Signal, keys are updated via either symmetric or asymmetric ratcheting. Asymmetric ratcheting introduces new DH shared secrets into the state, whereas symmetric ratcheting solely applies a KDF to existing state.

It will be helpful to have the following predicate:

Definition 7 ($\mathsf{clean}_{\mathsf{state}}$)**.** Within the same context as Definition 4, define

$$\begin{aligned} \mathsf{clean}_{\mathsf{state}}(u,i,s,s') = \neg\pi^i_u.\mathsf{rev\_state}[s] \\ \wedge \big(\forall\ j : \pi^i_u.\mathit{sid}[s] = \pi^j_{\pi^i_u.\mathsf{peerid}}.\mathit{sid}[s'] \\ \implies \neg\pi^j_{\pi^i_u.\mathsf{peerid}}.\mathsf{rev\_state}[s']\big) \end{aligned}$$

For brevity, we will write $\mathsf{clean}_{\mathsf{state}}(u,i,s)$ as a shorthand for $\mathsf{clean}_{\mathsf{state}}(u,i,s,s)$.

The state reveal query reveals additional state information that a previous stage gives as input to stage s. For Signal, we define this to mean for asymmetric stages, a state reveal query reveals the root key used in the session key computation that was derived in the previous stage, and for symmetric stages, the state reveal query reveals the chain key derived in the previous stage.

During asymmetric ratcheting, there are actually two substages, in which keys with slightly different properties are derived. In the first substage, the parties apply a KDF to two pieces of keying material: the root key derived at the end of the previous asymmetric stage, and a DH shared secret derived from both party's previous ratcheting public keys. Keys from this substage are marked with $\mathit{sid}[\mathbf{asym\text{-}ri}{:}x]$; they should be secure if either of the two pieces is unrevealed, which is what type `asym-ri` captures. In the second substage, the parties effectively apply a KDF to three pieces of keying material: the root key and DH shared secret from the first substage, as well a DH shared secret derived from one party's previous ratcheting public key and one party's new ratcheting public key. Keys from this substage are marked with $\mathit{sid}[\mathbf{asym\text{-}ir}{:}x]$ and should be secure if at least one of the three pieces is unrevealed, which is what type `asym-ir` captures.

Definition 8 ($\mathsf{clean}_{\mathtt{asym\text{-}ir}}, \mathsf{clean}_{\mathtt{asym\text{-}ri}}$)**.** Within the same context as Definition 4, let $s_{ir} = [\mathbf{asym\text{-}ir}{:}x]$, $s_{ri} = [\mathbf{asym\text{-}ri}{:}x]$, $s'_{ir} = [\mathbf{asym\text{-}ir}{:}x-1]$ and $s'_{ri} = [\mathbf{asym\text{-}ri}{:}x-1]$. Define

$$\mathsf{clean}_{\mathtt{asym\text{-}ri}}(u,i,s_{ri}) = \begin{cases} \mathsf{clean}_{\mathtt{EE}}(u,i,[0],[0]) \vee \Big(\mathsf{clean}_{\mathsf{state}}(u,i,s_{ri}) \wedge \mathsf{clean}_{\pi^i_u.\mathit{type}[0]}(u,i,[0])\Big) & x = 1 \\ \mathsf{clean}_{\mathtt{EE}}(u,i,s'_{ri},s'_{ir}) \vee \Big(\mathsf{clean}_{\mathsf{state}}(u,i,s_{ri}) \wedge \mathsf{clean}_{\mathtt{asym\text{-}ir}}(u,i,s'_{ir})\Big) & x > 1 \end{cases}$$

$$\mathsf{clean}_{\mathtt{asym\text{-}ir}}(u,i,s_{ir}) = \begin{cases} \mathsf{clean}_{\mathtt{EE}}(u,i,s_{ri},[0]) \vee \Big(\mathsf{clean}_{\mathsf{state}}(u,i,s_{ir}) \wedge \mathsf{clean}_{\mathtt{asym\text{-}ri}}(u,i,s_{ri})\Big) & x = 1 \\ \mathsf{clean}_{\mathtt{EE}}(u,i,s_{ri},s'_{ir}) \vee \Big(\mathsf{clean}_{\mathsf{state}}(u,i,s_{ir}) \wedge \mathsf{clean}_{\mathtt{asym\text{-}ri}}(u,i,s_{ri})\Big) & x > 1 \end{cases}$$

These clauses capture the "future secrecy" goal of Signal: if a device had been compromised at some prior time (i.e., the party's long-term key, past states, and past ephemeral keys are all compromised, and thus neither the second disjuncts nor $\mathsf{clean}_{\mathtt{EE}}(u,i,s'_{ir},s'_{ri})$ are satisfied), but the current ephemeral keys of both parties are uncompromised and honest ($\mathsf{clean}_{\mathtt{EE}}(u,i,s_{ir},s_{ri})$ *is* satisfied) then the stage is clean. Since our security property requires that the adversary cannot learn keys derived from clean stages, this captures post-compromise security.

Note that $\mathsf{clean}_{\mathtt{EE}}$ is used twice (because cleanness of ephemerals is defined as cleanness of the random numbers): once to show that the randomness is clean when generating ephemerals for the initial key exchange, and once to show that it is clean when generating the first ratchet key pair.

4.3.3. Symmetric stages $[\mathbf{sym\text{-}ir}{:}x{,}y], [\mathbf{sym\text{-}ri}{:}x{,}y]$**.** For stages with only symmetric ratcheting, new session keys should be secure only if the state is unknown to the adversary: this demands that all previous states in this symmetric chain are uncompromised, since later keys in the chain are computable from earlier states in the chain. Thinking recursively, this means that the previous stage's key derivation should have been secure, and that the adversary has not revealed the state linking the previous stage with the current one.

While the symmetric sending and receiving chains derive independent keys and are triggered differently during Signal protocol execution, their security properties are identical and captured by the following predicate; the different forms of the predicate are due to needing to properly name the "preceding" stage.

Definition 9 ($\mathsf{clean}_{\mathtt{sym}}$)**.** Within the same context as Definition 4, there are different freshness conditions on the symmetric part, depending on whether the symmetric stage is used for a message from initiator to responder or vice versa. Moreover, the symmetric stages arising from the initial handshake ($x = 0$) and from subsequent asymmetric stages ($x > 0$) are subtly different.

Specifically, for `sym-ir` we have three cases. Writing $s = [\mathbf{sym\text{-}ir}{:}x{,}y]$,

$$\mathsf{clean}_{\mathtt{sym\text{-}ir}}(u,i,s) = \mathsf{clean}_{\mathsf{state}}(u,i,s,s) \wedge \begin{cases} \mathsf{clean}_{\pi_u^i.type[0]}(u,i,[0]) & x=0, y=1 \\ \mathsf{clean}_{\mathtt{asym\text{-}ir}}(u,i,[\mathbf{asym\text{-}ir}{:}x]) & x>0, y=1 \\ \mathsf{clean}_{\mathtt{sym\text{-}ir}}(u,i,[\mathbf{sym\text{-}ir}{:}x{,}y-1]) & x\geq 0, y>1 \end{cases}$$

For `sym-ri` we have two cases (since there is no stage of type `sym-ri` immediately following the initial handshake). Writing $s = [\mathbf{sym\text{-}ri}{:}x{,}y]$,

$$\mathsf{clean}_{\mathtt{sym\text{-}ri}}(u,i,s) = \mathsf{clean}_{\mathsf{state}}(u,i,s,s) \wedge \begin{cases} \mathsf{clean}_{\mathtt{asym\text{-}ri}}(u,i,[\mathbf{asym\text{-}ri}{:}x]) & x>0, y=1 \\ \mathsf{clean}_{\mathtt{sym\text{-}ri}}(u,i,[\mathbf{sym\text{-}ri}{:}x{,}y-1]) & x>0, y>1 \end{cases}$$

We may write $\mathsf{clean}_{\mathtt{sym}}$ to denote $\mathsf{clean}_{\mathtt{sym\text{-}ir}}$ or $\mathsf{clean}_{\mathtt{sym\text{-}ri}}$ where it is clear which one we mean.

5. Security Analysis

In this section we prove that Signal is a secure multi-stage key exchange protocol in the language of Section 4, under standard assumptions on the cryptographic building blocks.

To formally prove security as in Section 4, we must write out each algorithm compromising the Signal "protocol" in the sense of Definition 1. For the sake of brevity we shall refrain from writing full pseudocode for all of them—Figure 2 contains most of the important algorithms—but we summarise the key points below.

We note as well a few minor reorganizations we make in Figure 2 compared to the actual implementation of Signal. We consider Signal to generate the first message keys for each chain at the same time that it initialises the chain, allowing us to consider these message keys as the session keys of the asymmetric stages. Similarly, we consider Bob to send his own one-time prekey eprepk_B instead of relaying it via the server. We mark these extra steps in gray in Figure 2.

KeyGen and MedTermKeyGen consist of uniform random sampling from the group.

Activate depends on the invoked role. Our prekey reorganization described above means that the roles of initiator and responder are technically reversed: although intuitively Alice initiates a session in our presentation, in fact Bob sends the first message, namely his prekeys (first right-to-left flow of Figure 2(b). Thus, the activation algorithm for the responder (Bob) outputs a single one-time prekey and awaits a response. The activation algorithm for the initiator (Alice) outputs nothing and awaits incoming prekeys.

Run is the core protocol algorithm. It admits various cases, which we briefly describe. If the incoming message is the first, Run builds a session as described previously: for Alice, it operates as in the left side of Figure 2(b) and outputs a message containing epk_A; for Bob, it operates as in the right side of Figure 2(b) and outputs nothing.

After that, there are two cases: Run is either invoked to process an incoming message, or to encrypt an outgoing one. We distinguish between incoming ratchet public keys (causing asymmetric updates) and incoming messages (causing symmetric updates).

(i) *Outgoing message.* Perform a symmetric sending update, modifying the current sending chain key and using the resulting message key as the session key (left side of Figure 2(c)).

(ii) *Incoming ratchet public key.* If this ratchet public key has not been processed before, perform an asymmetric update using it to derive new sending and receiving chain keys as in Figure 2(d). Advance both chains by one step, and output the message keys as the session key for the two asymmetric sub-stages as indicated in the figure.

(iii) *Incoming message.* Use the message metadata to determine which receiving chain should be used for decryption, and which position the message takes in the chain. Advance that chain (according to the right side of Figure 2(c)) as many stages as necessary (possibly zero), storing for future use any message keys that were thus generated. Return as the session key the next receiving message key.

In the Signal protocol, old but unused receiving keys are stored at the peer for an implementation-dependent length of time, trading off forward security for transparent handling of outdated messages. This of course weakens the forward secrecy of the keys, though their other security properties remain the same. We choose not to model this weakened forward secrecy guarantee, passing only the latest chaining key from stage to stage.

With these definitions, we can consider the advantage of an adversary in a multi-stage key exchange security game against our model of the Signal protocol. With definitions in the appendix, our core theorem thus states:

Theorem 1. *The Signal protocol is a secure multi-stage key exchange protocol under the GDH assumption and assuming all* KDF*s are random oracles. That is, if no efficient adversary can break the assumptions with non-negligible probability, then no efficient adversary can win the multi-stage key indistinguishability security experiment for Signal (and thereby distinguish any fresh message encryption key from random) with non-negligible probability.*

Proof (sketch). We give here a proof sketch. The full details can be found in the Appendix. The proof considers of each stage type and sub-clause exhaustively, and is structured using the sequence-of-games technique.

Stage 0. We start by proving the security of the stage 0 key that is output by the `triple` key-exchange during session setup. We show this via taking cases over the disjuncts in the $\mathsf{clean}_{\texttt{triple}}$ clause—over the different ways the session could be clean—noting that one of $\mathsf{clean}_{\texttt{LM}}(u, i)$, $\mathsf{clean}_{\texttt{EL}}(u, i, 0)$, $\mathsf{clean}_{\texttt{EM}}(u, i, 0)$ must be upheld.

We bound each of these probabilities in turn by the advantage of reduction algorithms to the security experiments of our primitives—to DH security using the GDH and ROM assumptions.

Asymmetric stages. Next we consider the security of a stage s key such that stage s has stage type `asym-ir` or `asym-ri`. Again, we take cases over the different ways to satisfy the cleanness predicate, depending on the type of the stage. Most cases are of the form $\mathsf{clean}_{\texttt{EE}}$, and for these we obtain a probability bound by replacing the DH ratchet keys and shared secrets with values from a GDH challenger.

The only case not of this form involves $\mathsf{clean}_{\mathsf{state}}$, which describes a scenario where both recent ratchet keys were compromised but the previous stage was still secure. Secrecy here is intuitive, and the bound follows from an inductive argument: if an adversary could win in this manner, then, assuming GDH and ROM security, there is an adversary which could win against the previous stage.

Symmetric stages. Finally, we consider the security of stage s keys of type `sym`. Here there is no disjunction in the cleanness predicate and hence only one case to consider. We replace the keys used to initialise the current sending chain with uniformly random values, since an adversary who could detect this could win against that previous stage.

Conclusion. The theorem follows by summing probabilities. □

6. Limitations

As a first analysis of a complex protocol, we have chosen (some) simplicity over a full analysis of all of Signal's features. We hope that our presentation and model can serve as a starting point for future analyses.

We discuss here some of the features included in Signal which we have explicitly chosen not to model and observe limitations of our results.

Protocol Components.

Header encryption. In Signal's standard configuration message metadata is transmitted authenticated but unencrypted. For privacy reasons there is a "header encryption variant", in which another key—the header encryption key—is generated from the root chain and used to encrypt messages and their metadata; this variant is used by Pond. Trial decryption is required for Alice to discover which header key was used to encrypt messages. We do not include this capability in our model, since Signal as widely deployed does not use it.

Online key exchange. Signal supports another mechanism for building a session: instead of fetching a bundle of Bob's prekeys, Alice can directly perform an online key exchange with Bob. This mechanism is left over from when Signal used SMS as its transport layer. We do not consider this mechanism for session construction.

Out-of-band key verification. To reduce the trust requirements on the prekey server, Signal supports a fingerprint mechanism for verifying public keys through an out-of-band channel. (It was this mechanism that gave rise to the UKS attack of [22], since the fingerprints do not include identities.) We simply assume that long-term and medium-term public key distribution is honest, and do not analyse the out-of-band verification channel.

Same key for Ed25519 signing and Curve25519 DH. Signal uses the same key *ik* for DH agreement and for signing the medium-term prekeys[6]. [16, 40] prove security of a similar scheme under the Gap-DH assumption, effectively showing that the signatures can be simulated using the hashing random oracle. We conjecture a similar argument could apply here, but do not prove it; instead, we omit the signatures from consideration and

6. This is done in practise by reinterpreting the Curve25519 point as an Ed25519 key, and computing an EdDSA signature.

enforce authentication of the prekeys in the game. This enforced authentication means we do not capture the class of attacks in which the adversary corrupts an identity key and then inserts a malicious signed pre-key.

Out-of-order decryption. If Bob wishes to decrypt out-of-order messages he must store their message keys until they arrive; this obviously reduces their forward security. As discussed in Section 5 we do not consider this storage.

Simultaneous session initiation. Signal has a mechanism to deal silently with the case that Alice and Bob simultaneously initiate a session with each other. Roughly, when an agent detects that this has happened they deterministically choose one party as the initiator (e.g. by sorting identity public keys and choosing the smaller), and then complete the session as if the other party had not acted. This requires a certain amount of trial and error: agents maintain multiple states for each peer, and attempt decryption of incoming messages in all of them. We do not consider this mechanism.

Other Security Goals and Threats. Our model describes key indistinguishability of two-party multi-stage key exchange protocols. There are a number of other security and functionality goals which Signal may address but which we do not study here. These include group messaging properties[7], message sharing across multiple devices, voice and video call security, protocol efficiency (e.g. 0-round-trip modes), privacy, and deniability.

Implementation-specific threats. We make various assumptions on the components used by the protocol. In particular, we do not consider specific implementations of primitives (e.g. the particular choice of curve), instead assuming standard security properties. We also do not consider side-channel attacks.

Tightness of the security reduction. As pointed out in [1], a limitation of conventional game hopping proofs for AKE protocols is that they do not provide tight reductions. The underlying reason is that the reductions depend on guessing the specific party and session under attack. In the case of a widely deployed protocol with huge amounts of sessions, such as Signal, this leads to an extremely non-tight reduction. While [1] develops some new AKE protocols with tight reductions, their protocols are non-standard in their setup and assumptions. In particular, there is currently no known technique for constructing a tight reduction that is applicable to the Signal protocol.

Application Variants. Popular applications using Signal tend to change important details as they implement or integrate the protocol, and thus merit security analyses in their own right. For example, WhatsApp implements a re-transmission mechanism: if Bob appears to change his identity key, clients will resend messages encrypted under the new value. Hence, an adversary with control over identity registration can disconnect Bob and replace his key, and Alice will re-send the message to the adversary.

7. Conclusions and Future Work

In this work we provided the first formal security analysis of the cryptographic core of the Signal protocol. While any first analysis for such a complex object will be necessarily incomplete, our analysis leads to several observations.

First, our analysis shows that the cryptographic core of Signal provides useful security properties. These properties, while complex, are encoded in our security model, and which we prove that Signal satisfies under standard cryptographic assumptions. Practically speaking, they imply secrecy and authentication of the message keys which Signal derives, even under a variety of adversarial compromise scenarios such as forward security (and thus "future secrecy"). If used correctly, Signal could achieve a form of post-compromise security, which has substantial advantages over forward secrecy as described in [12].

Our analysis has also revealed many subtleties of Signal's security properties. For example, we identified six different security properties for message keys (`triple`, `triple+DHE`, `asym-ir`, `asym-ri`, `sym-ir` and `sym-ri`).

One can imagine strengthening the protocol further. For example, if the random number generator becomes predictable, it may be possible to intercept communications with future peers. We have pointed out to the developers that this can be solved at negligible cost by using constructions in the spirit of the NAXOS protocol [32] or including a static-static DH shared secret in the key derivation.

We have described some of the limitations of our approach in Section 6. Furthermore, the complexity and tendency to add "extra features" makes it hard to make statements about the protocol as it is used. Examples include the ability to reset the state [12], encrypt headers, or support out-of-order decryption.

As with many real-world security protocols, there are no detailed security goals specified for the protocol, so it is ultimately impossible to say if Signal achieves its goals. However, our analysis proves that several standard security properties are satisfied by the protocol, and we have found no major flaws in its design, which is very encouraging.

7. The implementation of group messaging is not specified at the protocol layer. If it is implemented using multiple pairwise sessions, its security may follow in a relatively straightforward fashion—however, there are many other possible security properties which might be desired, such as transcript consistency.

Acknowledgements

The authors acknowledge helpful discussions with Marc Fischlin and Felix Günther (TU Darmstadt) and valuable comments from Chris Brzuska (TU Hamburg).

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Appendix A.
On Hardness Assumptions and the Random Oracle Model (ROM)

When performing a game-hopping security proof in an extended Canetti-Krawczyk-style model, after each hop we must show that the resulting game is similar to the original. If certain values have been changed, the queries whose results differ must be simulated in an indistinguishable manner.

In particular, the eCK family of models all contain a query RevSessKey which reveals the session key derived by a targeted session. This models for example cryptanalysis of a large volume of encrypted traffic, or the ability to read certain locations in memory. When replacing certain DH keys with random values, we must ensure that the resulting game is similar to its original. For protocols using only ephemeral DH values g^x and g^y to compute session keys from g^{xy}, replacing g^x and g^y by random does not affect other sessions, and thus other RevSessKey queries are not affected. However, for more complex protocols (such as NAXOS and HMQV) in which the long-term keys are also included in the session key derivation, this game hop becomes more complex. Specifically, when the long-term keys are modified, *all* RevSessKey queries are affected, and their simulation is no longer trivial.

There is a proof obligation to show that the simulation of these queries does not allow an adversary to distinguish the two games. One way to do this is by using Gap-DH in the random oracle model, assuming that the KDF is a random oracle. In the simulation, whenever the adversary makes a query to the random oracle, the challenger tests the relevant part of the argument using the DDH oracle to determine whether the adversary has successfully derived the DH secret. If so, the simulation can terminate and the challenger uses this value in the Gap-DH game. This is the approach we take.

There are known issues with the ROM. An alternative to Gap-DH is to take a PRF-ODH (pseudorandom function with oracle DH) assumption, which effectively provides an oracle for session key computations, and (roughly) asserts that it is hard to solve computational DH even with access to the oracle. The game hop then takes the computational DH values from the PRF-ODH game, and answers RevSessKey queries by querying the oracle. The probability jump over the game is thus bounded by the PRF-ODH advantage.

There is a further complication in the case of Signal. In most normal DH protocols, there is only one method to compute a session key given a collection of secret inputs; such a method could be called a "combinator". For example, in NAXOS the combinator hashes one long-term key and uses that as a DH exponential. In Signal, on the other hand, there are many different combinators, and the oracle we use must be sufficiently flexible to simulate all of them. Thus, we have the following options:

(i) Define a PRF-ODH game parameterised by the combinator K used to assemble secrets into the arguments to the KDF. For each different type of key in Signal, assume hardness of this game and use this assumption to justify a game hop.
(ii) Assume that the KDF is a random oracle, and justify the game hop directly from the ROM and Gap-DH.

We choose the latter option, since we believe that the former hardness assumption is not necessarily justified. However, we conjecture that a carefully-formulated PRF-ODH game could be proven hard in the ROM, and therefore that one proof could effectively take either option depending on the reader's opinions. We leave such a game for future work.

Definitions of Hardness Assumptions

Our proof of security relies on standard cryptographic hardness assumptions related to DH key exchange. Let $\mathbb{G} = \langle g \rangle$ be a cyclic group of prime order q generated by g, let $\alpha, \beta, \gamma \xleftarrow{\$} \mathbb{Z}_q$, and let $\mathcal{O}_{\text{DDH}}$ be an efficient black box algorithm (oracle) that, on input $(g^\alpha, g^\beta, g^\gamma)$, outputs 1 if $g^\gamma = g^{\alpha\beta}$ and 0 otherwise. For any algorithm $\mathcal{D}$ let

$$\epsilon_{\text{DDH}}(\mathcal{D}) \coloneqq \left| \Pr\left[\mathcal{D}(\mathbb{G}, q, g, g^\alpha, g^\beta, g^{\alpha\beta}) = 1 : \alpha, \beta \xleftarrow{\$} \mathbb{Z}_q\right] - \Pr\left[\mathcal{D}(\mathbb{G}, q, g, g^\alpha, g^\beta, g^\gamma) = 1 : \alpha, \beta, \gamma \xleftarrow{\$} \mathbb{Z}_q\right] \right|$$

$$\epsilon_{\text{CDH}}(\mathcal{D}) \coloneqq \Pr\left[\mathcal{D}(\mathbb{G}, q, g, g^\alpha, g^\beta) = g^{\alpha\beta} : \alpha, \beta \xleftarrow{\$} \mathbb{Z}_q\right]$$

$$\epsilon_{\text{GDH}}(\mathcal{D}) \coloneqq \Pr\left[\mathcal{D}^{\mathcal{O}_{\text{DDH}}}(\mathbb{G}, q, g, g^\alpha, g^\beta) = g^{\alpha\beta} : \alpha, \beta \xleftarrow{\$} \mathbb{Z}_q\right]$$

We make use of the following cryptographic hardness assumptions:

(i) Decisional Diffie-Hellman (DDH): it is hard to distinguish $(g^\alpha, g^\beta, g^{\alpha\beta})$ from $(g^\alpha, g^\beta, g^\gamma)$, i.e., $\epsilon_{\text{DDH}}(\mathcal{D})$ is negligible in $\log(q)$ for any efficient $\mathcal{D}$.
(ii) Computational Diffie-Hellman (CDH): it is hard to compute the value $g^{\alpha\beta}$ from (g^α, g^β), i.e., $\epsilon_{\text{CDH}}(\mathcal{D})$ is negligible in $\log(q)$ for any efficient $\mathcal{D}$.

(iii) Gap Diffie-Hellman (GDH): it is hard to compute the value $g^{\alpha\beta}$ from (g^{α}, g^{β}) even when given black box access to a DDH oracle, i.e., $\epsilon_{\text{GDH}}(\mathcal{D})$ is negligible in $\log(q)$ for any efficient $\mathcal{D}$.

We also make use of the random oracle model (ROM), instantiating all KDFs as black boxes which return a uniformly-random output for any given input.

Appendix B. Security Proof

The proof considers different cases corresponding to the possible behaviour of an adversary. We therefore first describe the high level proof structure in Appendix B.1. We then recall the main theorem and provide the actual proof in Appendix B.2.

B.1. Proof structure overview

Security in this sense means that no efficient adversary can break the multi-stage key-indistinguishability game for the two-party protocol Signal, parametrised by freshness condition fresh, with non-negligible probability. Suppose for contradiction that such an adversary $\mathcal{A}$ exists. Whatever the behaviour of the adversary, trivially (by the definition of the security experiment in Figure 5) it can only succeed when the Tested session $[s]$ is fresh. By Definition 4, this means that the $\mathsf{Test}(u, i, s)$ query satisfies:

(i) $\pi_u^i.\mathit{status}[s] = \texttt{accept}$,
(ii) $\neg\pi_u^i.\mathsf{rev\_session}[s]$,
(iii) for all j such that $\pi_u^i.\mathit{sid}[s] = \pi_v^j.\mathit{sid}[s]$, $\neg\pi_v^j.\mathsf{rev\_session}[s]$, and
(iv) $\mathsf{clean}_{\pi_u^i.\mathit{type}[s]}(u, i, s)$

where v denotes $\pi_u^i.\mathsf{peerid}$, the identity of the intended peer to the Tested session, and $\mathsf{clean}_{\pi_u^i.\mathit{type}[s]}(u, i, s)$ is a cleanness clause as referenced in Definition 4 and subsequent definitions, further restricting the adversary's behaviour. In the following overview, we consider the case that the Tested session is the initiator; the responder is analogous.

Overview of the Case Distinction

A high-level overview of the proof with its main game sequences and case distinctions is given in Figure 6. Signal has many different types of stage, and we analyse each of them separately. Formally, start with a sequence of generic game hops, after which we make a case distinction based on the stage type of the Tested session. Each stage type has its own cleanness predicate, and we deal with them in subcases. For example, if the adversary issues a query $\mathsf{Test}(u, i, [0])$, then we are in the analysis of stage $[0]$, and if the stage type is `triple`, then we consider in turn the subcases where $\mathsf{clean}_{\texttt{LM}}(u, i)$, $\mathsf{clean}_{\texttt{EL}}(u, i, [0])$, or $\mathsf{clean}_{\texttt{EM}}(u, i, [0])$ are true.

For each subcase, we perform an additional game hop, relying on one of the security assumptions in the statement of the theorem. The initial game will be ms-ind, and in the final games the session key will be replaced by a uniformly random value. By summing up the advantages along the way, we can obtain an overall bound on the success probability of the adversary. For ease of reading, we give the high-level structure of the proof here.

We begin by considering the case that the $\mathsf{Test}(u, i, s)$ query was issued on the first stage ($s = [0]$). We break this up into two separate cases:

(i) the initial key exchange had stage type `triple` (so 3 separate pairs of DH shared secrets were used to compute the master secret ms), or
(ii) the initial key exchange had stage type `triple+DHE` (so 4 separate pairs of DH shared secrets were used to compute the master secret ms).

Case 1: `triple`**.** In the first case, where $\mathsf{Test}(u, i, [0])$ and $\pi_u^i.\mathit{type}[0] = \texttt{triple}$, we see by Definition 5 that the following condition must be satisfied:

$$\mathsf{clean}_{\texttt{LM}}(u, i) \quad \vee \quad \mathsf{clean}_{\texttt{EL}}(u, i, [0]) \quad \vee \quad \mathsf{clean}_{\texttt{EM}}(u, i, [0])$$

Case 2: `triple+DHE`**.** In the second case, where $\mathsf{Test}(u, i, [0])$ and $\pi_u^i.\mathit{type}[0] = \texttt{triple+DHE}$, we see by Definition 6 that there is an additional disjunct $\mathsf{clean}_{\texttt{EE}}(u, i, [0])$, and we must have

$$\mathsf{clean}_{\texttt{LM}}(u, i) \quad \vee \quad \mathsf{clean}_{\texttt{EL}}(u, i, [0]) \quad \vee \quad \mathsf{clean}_{\texttt{EM}}(u, i, [0]) \quad \vee \quad \mathsf{clean}_{\texttt{EE}}(u, i, [0])$$

We consider each of these cases in turn, and by a game-hopping argument replace the relevant keys by random values, allowing us subsequently to replace the session keys with random values.

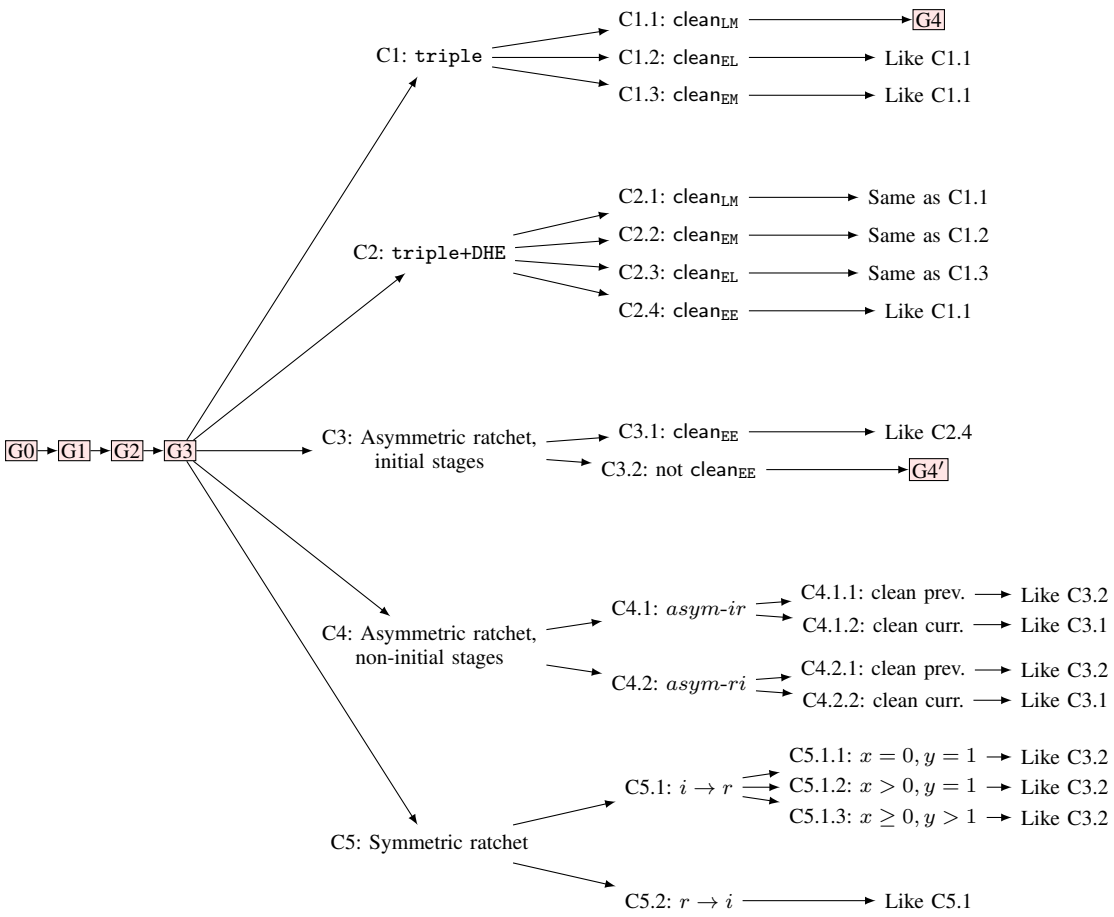

Figure 6: High-level overview of the proof structure. Games are identified by G0, G1, ..., and main case distinctions by C1, C2, ...; G0 denotes the multi-stage security experiment from Section 4.2. In the PDF version of this document, such identifiers can be clicked to jump to the corresponding part of the proof.

Case 3: Asymmetric ratchet, initial stage. Next, we consider the security of the case that the $\mathsf{Test}(u, i, s)$ query was issued on the initial responder-to-initiator asymmetric stage $s = [\mathbf{asym\text{-}ri}{:}1]$. We partition this into two cases corresponding to Definition 8: either the adversary has not issued queries that would break the cleanness of the root key from the first stage $s = [0]$; or the adversary did not inject malicious DH shares in either of the ephemeral shares used in the stage (which in particular, were generated in stage $s = [0]$). That is, we consider the case that $\mathsf{Test}(u, i, s = [\mathbf{asym\text{-}ri}{:}1])$ where $\pi_u^i.\mathit{type}[s] = \texttt{asym-ri}$, and apply Definition 8 to conclude that

$$\Big(\mathsf{clean}_{\pi_u^i.\mathit{type}[0]}(u, i, [0]) \quad \wedge \quad \mathsf{clean}_{\mathsf{state}}(u, i, [\mathbf{asym\text{-}ir}{:}1])\Big) \qquad \vee \qquad \mathsf{clean}_{\mathtt{EE}}(u, i, 0, 0)$$

In a similar fashion to the argument for the initial handshake, we replace certain DH values by values from a GDH challenger, reducing indistinguishability of the session key of this stage to hardness of GDH.

Case 4: Asymmetric ratchet, non-initial stages. We continue, considering the security of the case that the $\mathsf{Test}(u, i, s)$ query was issued in the x^{th} asymmetric responder-to-initiator stage $s = [\mathbf{asym\text{-}ri}{:}x]$. That is, we consider the case that $\mathsf{Test}(u, i, s = [\mathbf{asym\text{-}ri}{:}x])$, where $x \geq 2$ and $\pi_u^i.\mathit{type}[s] = \texttt{asym-ri}$. By Definition 8, we conclude:

$$\Big(\mathsf{clean}_{\texttt{asym-ri}}(u, i, [\mathbf{asym\text{-}ri}{:}x-1]) \wedge \mathsf{clean}_{\mathsf{state}}(u, i, [\mathbf{asym\text{-}ir}{:}x])\Big) \vee \mathsf{clean}_{\mathtt{EE}}(u, i, x-1, x-1)$$

and a similar argument holds.

We must also consider the case that the Test query was issued against an asymmetric stage of type `asym-ir` i.e. a stage used to derive keys for the initiator to encrypt for the responder. The argument in this case is analogous but the cleanness predicates are subtly different and again vary depending on whether the stage is the first of its type. However, the core argument remains the same: we replace certain keys in the Tested session with values from the GDH challenger, in such a way that distinguishing session keys from random would give a GDH advantage.

Case 5: Symmetric ratchet. Finally, we consider symmetric stages. We partition into two cases:

(i) the first symmetric stage $y = 1$, where security follows from the asymmetric stage before it (which could be either the initial handshake or an asymmetric stage)
(ii) a later symmetric stage $y > 1$, where security follows from cleanness of the previous symmetric update

The challenger does not know in advance which adversary behaviour it is up against—only at the end of the game will the challenger know which of the clean predicates were satisfied. That is, the adversary might decide on the fly which session to Test and which clean predicate to satisfy. As such, no global reduction can be given. This is not a problem: in our proof, we are ultimately just ruling out classes of attacks. If an attack exists, it corresponds to some specific adversary, which is covered by one of our cases.

B.2. Proof of the main Theorem

Conventions. We remark on a few conventions which we adopt during the proof.

Many cases technically differ based on whether the actor of the Test session has the initiator or responder role. For example, the first session key derived by the initiator is from a sending chain, while the first one derived by the responder is from a receiving chain. Where the security arguments are identical except for obvious symmetries, we just consider the case of the initiator and leave the responder as analogous.

Signal uses HMAC and HKDF within the KDF invocations. We assume that the KDF invocations themselves (as defined in Figure 1) are random oracles, and thus need not make any assumptions on HMAC and HKDF specifically.

By $break_i$ we mean the probability that the adversary wins game G_i. We aim to show this is close to $1/2$, so that the advantage as defined in Definition 3 is negligible. To avoid overfilling our subscripts, we overload where it is obvious which game is meant.

Theorem 1. *The Signal protocol is a secure multi-stage key exchange protocol under the GDH assumption and assuming all* KDF*s are random oracles. That is, if no efficient adversary can break the assumptions with non-negligible probability, then no efficient adversary can win the multi-stage key indistinguishability security experiment for Signal (and thereby distinguish any fresh message encryption key from random) with non-negligible probability.*

Proof. We begin by performing a series of game hops that affect all potential cases. After these, the game hops diverge depending on which case we are considering.

Game Hops for all Cases

Game 0. This game equals the multi-stage security experiment described in Section 4.2. Thus:

$$\mathsf{Adv}^{\mathsf{ms\text{-}ind}}_{\mathsf{Signal},\mathsf{n_P},\mathsf{n_M},\mathsf{n_S}}(\mathcal{A}) = \Pr(break_0)$$

Game 1. In this game we ensure no collision of honestly generated DH public keys. Specifically, the challenger $\mathcal{C}$ maintains a list L of all DH private values (for $ik, prek, ek, eprek, rchk$) honestly generated during the game. If a DH private value appears twice, $\mathcal{C}$ aborts the simulation and the adversary automatically loses. For an adversary's execution during the game, let $\mathsf{n_P}$ denote the total number of parties, $\mathsf{n_S}$ the maximum number of sessions, $\mathsf{n_M}$ the number of medium-term keys per party, and $\mathsf{n_s}$ the maximum number of stages. We note that there are $\mathsf{n_P}$ long-term keys in the game, $\mathsf{n_M}$ medium-term keys generated for each of the $\mathsf{n_P}$ parties for a total of $\mathsf{n_M n_P}$ medium-term keys, and a maximum of $\mathsf{n_s}$ ephemeral/ratchet keys per session for a total maximum of $\mathsf{n_S n_s}$ ephemeral/ratchet keys. This means a total maximum of $\mathsf{n_P} + \mathsf{n_P n_M} + \mathsf{n_S n_s}$ DH keys in the list, every pair of which must not collide. Therefore we have the following bound:

$$\Pr(break_0) \leq \frac{\binom{\mathsf{n_P}+\mathsf{n_P n_M}+\mathsf{n_S n_s}}{2}}{q} + \Pr(break_1)$$

We know from this game onwards that each honest session uniquely generates DH private keys and thus that each honestly generated DH public key is unique. In future game hops we will replace certain of these values with ones sampled by a challenger; if these replacement values collide, we abort the game. This will appear in e.g. game $G4$.

Game 2. In this game, the challenger guesses in advance the session π^i_u against which the $\mathsf{Test}(u, i, s)$ query is issued: the challenger guesses a pair of indices $(u^*, i^*) \in [1..\mathsf{n_P}] \times [1..\mathsf{n_S}]$, and aborts (and the adversary automatically loses) if the adversary issues a Test query $\mathsf{Test}(u, i, s)$ where $(u, i) \neq (u^*, i^*)$. This will occur with probability $1/\mathsf{n_S n_P}$, and hence:

$$\Pr(break_1) \leq \mathsf{n_S n_P} \cdot \Pr(break_2)$$

Game 3. In this game, the challenger guesses an index $v^* \in [\mathsf{n_P}]$ and aborts if there exists a session π_v^j that matches π_u^i but $v \neq v^*$. Note that it might be the case that no such matching π_v^j exists, but this game ensures that if such a π_v^j does exist, v is unique and known in advance by the challenger.

We must first show that there can exist at most one identity v with the same session identifier as π_u^i (note v may have multiple sessions that match π_u^i as the responder does not contribute freshness in the Triple-DH case). Alice's session identifier for stage $[0]$ contains both ipk_v (the identity public key of the peer) and eprepk_u (their own ephemeral public key). Later sids contain more ephemeral keys. In $G1$ we ensured that all DH values were unique, and hence the claim holds.

It follows that the challenger's guess is correct with probability $1/\mathsf{n_P}$, and so:

$$\Pr(break_2) \leq \mathsf{n_P} \cdot \Pr(break_3)$$

In this game, we do not guess the partner session because the responder does not always contribute an ephemeral key. Only in `triple+DHE` does this occur and indeed in this case we will do another game hop to guess the partner session.

At this point, we need to partition our analysis for individual cases. Since this is $G3$, each different case begins with a hop to some $G4$.

C1: Initial key exchange: $\mathit{type}[0] = \texttt{triple}$

First, we consider the security of Signal in the multi-stage key-indistinguishability game against an adversary $\mathcal{A}$ that issues a $\mathsf{Test}(u, i, [0])$ query with $\pi_u^i.\mathit{type}[0] = \texttt{triple}$. By construction, the only way for the adversary to win (with non-negligible probability) is if $\mathsf{clean}_{\texttt{triple}}(u, i, [0])$ is true. We partition these scenarios into subcases. Note also that a $\mathsf{RevState}(u, i, [0])$ or $\mathsf{RevState}(v, j, [0])$ (where π_v^j is a session matching π_u^i if one exists) query will reveal nothing to the adversary, as there exists no previous state. Moreover, after our game hops, we will have replaced the Tested message key with a uniformly random value that is independent to all other keys, so other issued $\mathsf{RevState}$ queries will only reveal independent root keys and chaining keys. As the state will be independent from the Tested session key, it will not help the adversary distinguish the Test session key from random. How to simulate reveal queries will be dealt with formally in the game hops.

We now begin to separate our analysis based on sub-clauses of the cleanness predicate. Let $E^{\texttt{triple}}$ be the event that an adversary $\mathcal{A}$ wins the ms-ind game by issuing a Test query $\mathsf{Test}(u, i, [0])$, such that $\pi_u^i.\mathit{type} = \texttt{triple}$, and let $E^{\texttt{triple}}_{\mathsf{clean}_{\texttt{LM}}}$ (resp. $E^{\texttt{triple}}_{\mathsf{clean}_{\texttt{EL}}}$, $E^{\texttt{triple}}_{\mathsf{clean}_{\texttt{EM}}}$) be the sub-case in which additionally $\mathsf{clean}_{\texttt{LM}}(u, i)$ (resp. $\mathsf{clean}_{\texttt{EL}}(u, i, [0])$, $\mathsf{clean}_{\texttt{EM}}(u, i, [0])$) is true. By definition of $\mathsf{clean}_{\texttt{triple}}$,

$$\Pr(E^{\texttt{triple}}) \leq \Pr\left(E^{\texttt{triple}}_{\mathsf{clean}_{\texttt{LM}}(u,i)}\right) + \Pr\left(E^{\texttt{triple}}_{\mathsf{clean}_{\texttt{EL}}(u,i,0)}\right) + \Pr\left(E^{\texttt{triple}}_{\mathsf{clean}_{\texttt{EM}}(u,i,0)}\right)$$

C1.1: Case $\mathit{type}[0] = \texttt{triple}$ and $\mathsf{clean}_{\texttt{LM}}(u, i)$:

In this case $\mathcal{A}_{\texttt{triple}}$ issued a $\mathsf{Test}(u, i, [0])$ query such that $\mathsf{clean}_{\texttt{LM}}(u, i)$ is upheld. For Test sessions where $\pi_u^i.\mathit{role} = \texttt{init}$, this requires that $\mathcal{A}_{\texttt{triple}}$ has not issued $\mathsf{RevLongTermKey}(u)$ or $\mathsf{RevMedTermKey}(v, n)$ where $\pi_u^i.\mathsf{peerpreid} = n$. Since we do not consider the signatures over the medium-term prekeys in our model, we may assume that π_u^i has received prepk_v^n without modification.

Recall that an honest session derives a master secret $ms = g^{\mathit{ik}_u \cdot \mathit{prek}_v^n} \| g^{\mathit{ek}_u \cdot \mathit{ik}_v} \| g^{\mathit{ek}_u \cdot \mathit{prek}_v^n}$, and then assigns $\mathit{rk}^1 \parallel \mathit{ck}^{\mathbf{sym\text{-}ir}:0,0} \leftarrow \mathrm{HKDF}(ms)$.

Game 4. In the previous game, all keys are generated as specified in the ms-ind game. In this game, the challenger, acting as the adversary in a Gap-DH game, requests values from a Gap-DH challenger and substitutes them in place of certain actual keys. We have two proof obligations:

(i) to bound the success probability of game $G4$, and
(ii) to bound the difference in probabilities between games $G3$ and $G4$.

We meet these by proving a stronger assertion: we will show that any adversary $\mathcal{A}$ which wins game $G3$ with some probability p has a non-negligible success probability $f(p)$ of winning $G4$. We will then give an upper bound for $f(p)$ in terms of the Gap-DH advantage ϵ_{GDH}, and thereby derive a bound on p.

Defining $G4$. The formal definition of game $G4$ follows. Our challenger $\mathcal{B}_0$, acting as an adversary in a Gap-DH game, begins by requesting a challenge DH pair (g^α, g^β). Via $G2$ and $G3$, $\mathcal{B}_0$ knows the identities u and v involved in the Tested session; additionally, it guesses the index $n \in [1..\mathsf{n_M}]$ of the signed prekey of the peer (prek_v^n) that the Test session will use in the execution of the protocol. $\mathcal{B}_0$ then replaces ipk_u with g^α and prepk_v^n with g^β when they are generated. This is the only difference between the two games.

Because certain keys have been replaced with public keys whose corresponding private values are unknown to $\mathcal{B}_0$, we must define the actions that should be taken when these private values would normally be used in a

computation. Cleanness implies that $\neg\mathsf{rev\_ltk}_u \wedge \neg\mathsf{rev\_mtk}_v^n$, so $\mathcal{B}_0$ will not need to answer any Reveal queries from $\mathcal{A}$ on these values. However, since $\mathcal{B}_0$ has replaced the long-term identity key and a medium-term public key of two parties, if $\mathcal{A}$ decides to direct parties u or v to execute the protocol in a non-Tested session, then $\mathcal{B}_0$ may need to perform concrete computations with the private keys. There are three distinct types of sessions in which $\mathcal{B}_0$ may lack the private keys needed to compute the master secret ms:

(i) a non-Tested session between user u and user v using $prek_v^n$;
(ii) a session between user u and a user other than v; or
(iii) a session between a user other than u and user v using $prek_v^n$.

In each of these types of sessions, $\mathcal{B}_0$ will pick random keys $rk^1, ck^{\mathbf{sym\text{-}ir}:0,0}$ rather them deriving than via $\mathrm{HKDF}(ms)$. $\mathcal{B}_0$ maintains a list of all sessions in which random keys have been substituted: the list contains the random session keys as well as the public keys that should have been used to compute each component of the master secret.

$\mathcal{B}_0$ must also ensure that session key values used are consistent with any queries that $\mathcal{A}$ makes to the random oracle HKDF. We are concerned about queries of the form $g^{x_1}\|g^{x_2}\|g^{x_3}$. Before answering any such query, $\mathcal{B}_0$ goes through each session in the above list: for each entry in the list, it uses its DDH oracle to check if the public keys used to compute each component of the master secret match the corresponding component of this random oracle query. If all components match, then $\mathcal{B}_0$ uses the session keys from the list as the random oracle response; otherwise, $\mathcal{B}_0$ samples a new random value as the random oracle response.

(While the explanation above starts from $\mathcal{B}_0$ picking random session keys when simulating a session and then ensuring random oracle queries are answered consistently, $\mathcal{B}_0$ must also do the reverse: when simulating a session, before picking random keys $\mathcal{B}_0$ analogously use its DDH oracle to check if this matches a previous random oracle query, to ensure correct simulation.)

If it happens that the public keys used to compute a particular component of the master secret are the DH challenge values g^α and g^β, and that DDH oracle indicates that these match the corresponding element of the random oracle query, then $\mathcal{A}$ has found the solution to the GDH problem for us, and $\mathcal{B}_0$ can immediately terminate the simulation and return this answer to its GDH challenger.

Bounding the difference between the games. We must show that an adversary which wins game $G3$ also wins game $G4$ with non-negligible probability. Fortunately, by construction every change made between the two games takes the form of replacing one random value sampled from a distribution with another value sampled from the same distribution. Specifically, two classes of value are changed:

(i) identity and medium-term keys are replaced with values from the GDH challenger; and
(ii) certain session keys are replaced with random values.

For the first class of values, we know that the GDH challenger samples its DH keys uniformly at random from the chosen group, and thus $G4$'s DH keys are uniform. In $G3$, the DH private keys are still selected uniformly, but the change in $G1$ means that $G1$ and $G3$ abort if any DH keys are repeated. Thus, the distribution of DH keys seen by the adversary in $G3$ is slightly different than those seen in $G4$: fortunately this difference is negligible.

For the second class of values, we defined $\mathcal{B}_0$ to sample a value uniformly at random from the domain of the random oracle, and used the DDH oracle to maintain consistency. Thus the distributions of session keys in $G3$ and $G4$ are identical.

Bounding the winning probability of game $G4$. Suppose $\mathcal{A}$ is an adversary which wins game $G3$ with probability p, and consider the execution of $\mathcal{A}$ in game $G4$. We will show that a correct response to the Test query in $G3$ would imply that $\mathcal{B}_0$ submits a correct response to the GDH challenger in $G4$, with nontrivial probability.

First, the game began by guessing the index of the medium-term key, which will be correct with probability $1/\mathsf{n_M}$. Assuming this is correct, the session key of the Test session in $G4$ will depend on $g^{\alpha\beta}$, among other values. Consider an adversary which wins game $G3$. Since the KDF is assumed to be a random oracle, we can conclude that there are three possible ways for $\mathcal{A}$ to win:

(i) *Guessing or collision attack.* There is a collision in the random oracle, or the adversary guesses the session key of the Test session.
(ii) *Key replication attack.* $\mathcal{A}$ issues a RevSessKey query on a session deriving the same session key as the Tested session.
(iii) *Forging attack.* $\mathcal{A}$ queries the random oracle with the master secret of the Test session.

We deal with these in order.

Guessing or collision attack: The first is relatively easy: the probability of a random oracle collision is easily shown negligible (only polynomially-many queries can be made but there are exponentially many possible values), as is the probability of guessing the session key.

Key replication attack: Assuming no random oracle collisions, a key replication attack has negligible probability of success. Recall that the session identifier for an initial session contains all the values used to derive the session key, since the session identifier defined as the concatenation of both agents' identity public keys with the responder's medium-term prekey and the initiator's ephemeral public key. Thus, the only way two sessions with distinct sids can have the same session key is if they use distinct DH values g^x, g^y and g^w, g^z that result in $g^{xy} = g^{wz}$. Since the exponents in the Test session are sampled uniformly at random, the probability of this event is negligible, and thus a RevSessKey query on a non-Test-matching session reveals an unrelated random value with overwhelming probability.

Forging attack: Finally, we show that, if $\mathcal{A}$ conducts a successful forging attack, then with non-negligible probability $\mathcal{B}_0$ submits the correct response to the GDH challenge. If $\mathcal{B}_0$ guessed the index of the medium-term key correctly, then a successful forging attack would require the adversary to query the HKDF random oracle with a value x containing the GDH challenge response $g^{\alpha\beta}$, since this value is input to the key derivation of the Test session. If such a query is made then $\mathcal{B}_0$ will submit $g^{\alpha\beta}$ to the GDH challenger, and this is the correct solution to the GDH challenge.

C1.2: Case *type*$[0] = \texttt{triple}$ and $\mathsf{clean}_{\texttt{EL}}(u, i, 0)$

In this case the adversary $\mathcal{A}_{\texttt{triple}}$ has issued a Test query $\mathsf{Test}(u, i, [0])$ such that $\mathsf{clean}_{\texttt{EL}}(u, i, [0])$ is upheld. For Test sessions such that $\pi_u^i.\textit{role} = \texttt{init}$, this means that $\mathcal{A}_{\texttt{triple}}$ has not issued $\mathsf{RevRand}(u, i, [0])$ and $\mathsf{RevLongTermKey}(v)$ where $v = \pi_u^i.\textit{peeripk}$. For Test sessions such that $\pi_u^i.\textit{role} = \texttt{resp}$, this means that $\mathcal{A}_{\texttt{triple}}$ has not issued $\mathsf{RevLongTermKey}(u)$ and a $\mathsf{RevRand}(v, j, [0])$ such that $\pi_v^j.\textit{sid}[0]$ matches the Test session $\pi_u^i.\textit{sid}[0]$.

The argument for this case is almost identical to that of the previous case. The GDH challenge values g^α, g^β are inserted into the simulation in place of the ephemeral key of the initiator and the long-term key of the responder.

C1.3: Case *type*$[0] = \texttt{triple}$ and $\mathsf{clean}_{EM}(u, i, [0])$

In this case, the adversary $\mathcal{A}_{\texttt{triple}}$ has issued a Test query $\mathsf{Test}(u, i, [0])$ such that $\mathsf{clean}_{\texttt{EM}}(u, i, [0])$ is upheld. For Test sessions such that $\pi_u^i.\textit{role} = \texttt{init}$, this means that $\mathcal{A}_{\texttt{triple}}$ has not issued a $\mathsf{RevRand}(u, i, 0)$ and $\mathsf{RevMedTermKey}(v, \pi_u^i.\mathsf{peerpreid})$. For Test sessions such that $\pi_u^i.\textit{role} = \texttt{resp}$, this means that $\mathcal{A}_{\texttt{triple}}$ has not issued a $\mathsf{RevRand}(v, j, [0])$ such that $\pi_v^j.\textit{sid}[0]$ matches the Test session $\pi_u^i.\textit{sid}[0]$ and $\mathsf{RevMedTermKey}(u, \pi_u^i.\textit{prepk})$.

Again, this is analogous to before. The Gap-DH challenge values g^α, g^β are inserted into the simulation in place of the ephemeral key of the initiator and the particular medium-term key of the responder used in the Test session.

C2: Initial key exchange: *type*$[0] = \texttt{triple+DHE}$

Recall that the initial key exchange can also have type triple+DHE, in which case cleanness requires that

$$\mathsf{clean}_{\texttt{LM}}(u, i) \vee \mathsf{clean}_{\texttt{EL}}(u, i, [0]) \vee \mathsf{clean}_{\texttt{EM}}(u, i, [0]) \vee \mathsf{clean}_{\texttt{EE}}(u, i, [0])$$

We now consider the case that the adversary has issued a Test query $\mathsf{Test}(u, i, [0])$ where the stage $\pi_u^i.\textit{type}[0] = \texttt{triple+DHE}$. We note that the cases are the same as previously, with the additional case $\mathsf{clean}_{\texttt{EE}}(u, i, [0])$. As before, we define

- $E^{\texttt{triple+DHE}}_{\mathsf{clean}_{\texttt{LM}}}$ to be the event that an adversary wins the multi-stage key-indistinguishability game where $\mathcal{A}$ has issued a Test query $\mathsf{Test}(u, i, [0])$ and $\mathsf{clean}_{\texttt{LM}}(u, i)$ is upheld,
- $E^{\texttt{triple+DHE}}_{\mathsf{clean}_{\texttt{EM}}}$ where $\mathcal{A}$ has issued a Test query $\mathsf{Test}(u, i, [0])$ and $\mathsf{clean}_{\texttt{EM}}(u, i, [0])$ is upheld,
- $E^{\texttt{triple+DHE}}_{\mathsf{clean}_{\texttt{EL}}}$ where $\mathcal{A}$ has issued a Test query $\mathsf{Test}(u, i, [0])$ and $\mathsf{clean}_{\texttt{EL}}(u, i, [0])$ is upheld, and
- $E^{\texttt{triple+DHE}}_{\mathsf{clean}_{\texttt{EE}}}$ where $\mathcal{A}$ has issued a Test query $\mathsf{Test}(u, i, [0])$ and $\mathsf{clean}_{\texttt{EE}}(u, i, [0], [0])$ is upheld.

We will bound the adversary's success probability as follows.

$$\Pr(E^{\texttt{triple+DHE}}) \leq \Pr(E^{\texttt{triple+DHE}}_{\mathsf{clean}_{\texttt{LM}}(u,i)}) + \Pr(E^{\texttt{triple+DHE}}_{\mathsf{clean}_{\texttt{EL}}(u,i,[0])}) + \Pr(E^{\texttt{triple+DHE}}_{\mathsf{clean}_{\texttt{EM}}(u,i,[0])}) + \Pr(E^{\texttt{triple+DHE}}_{\mathsf{clean}_{\texttt{EE}}(u,i,[0],[0])})$$

The bounds for the previous summands are proved to be negligible under our cryptographic assumptions exactly as above, yielding the inequalities as desired. As before, the crucial proof step in each case is the Gap-DH assumption. However, for this case it will also make a game hop like Game 3, where we additionally know Bob's unique matching session in advance. We can do this now because Bob has freshness in the handshake.

C2.4: Case $\mathit{type}[0] = \texttt{triple+DHE}$ and $\mathsf{clean}_{\mathtt{EE}}$

The final ephemeral-ephemeral case $E^{\texttt{triple+DHE}}_{\mathsf{clean}_{\mathtt{EE}}}$ is analogous to previous cases except that in Game $G4$ ($E^{\texttt{triple+DHE}}_{\mathsf{clean}_{\mathtt{EE}}}$), we need to replace the ephemeral values of both the initiator and the responder. (Since the simulator in $G4$ will never reuse ephemeral values in different session, the simulation in this case is simpler and will not need to use its DDH oracle to maintain consistency.)

C3: Asymmetric ratcheting, initial stage

We have now proved security of the initial key exchange, optionally including the ephemeral-ephemeral DH computation. We next move on to the asymmetric-ratcheting stages, in which Bob and Alice take turns generating new DH ephemerals and updating their root keys. The first asymmetric-ratcheting stage differs slightly from its successors since it immediately follows the initial handshake, and we deal with it here now. Recall it is of type `asym-ri`, since it is performed when Bob wishes to send a message to Alice.

We consider an adversary $\mathcal{A}$ that issues a $\mathsf{Test}(u, i, s = [\mathbf{asym\text{-}ri}{:}1])$ query, where stage s must have $\mathit{type} = \texttt{asym-ri}$. Note that the initial asymmetric stage is always of type `asym-ri` (messages from Alice to Bob before this stage are sent using the symmetric chain derived from the initial handshake), and thus in this section we do not need to consider *initial* stages of type `asym-ir`. We define

- $E^{\texttt{asym-ri}}$ to be the event that an adversary $\mathcal{A}$ wins the multi-stage key-indistinguishability game by issuing a $\mathsf{Test}(u, i, s = [\mathbf{asym\text{-}ri}{:}1])$ query,
- $E^{\texttt{asym-ri}}_{\mathsf{clean}_{\mathtt{EE}}(u,i,[0])}$ to be the sub-event of $E^{\texttt{asym-ri}}$ satisfying $\mathsf{clean}_{\mathtt{EE}}(u, i, [0], [0])$, and
- $E^{\texttt{asym-ri}}_{\mathsf{clean}_{\pi^i_u.\mathit{type}[0]}(u,i,[0])}$ to be the sub-event of $E^{\texttt{asym-ri}}$ satisfying

$$\mathsf{clean}_{\pi^i_u.\mathit{type}[0]}(u, i, [0]) \wedge \mathsf{clean}_{\mathsf{state}}(u, i, [\mathbf{asym\text{-}ri}{:}1]).$$

It follows from our definition of freshness that

$$\Pr(E^{\texttt{asym-ri}}) \leq \Pr(E^{\texttt{asym-ri}}_{\mathsf{clean}_{\mathtt{EE}}(u,i,[0])}) + \Pr(E^{\texttt{asym-ri}}_{\mathsf{clean}_{\pi^i_u.\mathit{type}[0]}(u,i,[0])}) \tag{1}$$

and we consider these two cases in turn, beginning with the case that $\mathsf{clean}_{\mathtt{EE}}(u, i, [0], [0])$ is upheld.

C3.1: Case $s = [\mathbf{asym\text{-}ri}{:}1]$, $\mathit{type}[s] = \texttt{asym-ri}$ and $E^{\texttt{asym-ri}}_{\mathsf{clean}_{\mathtt{EE}}}$

This case is dealt with in an identical way to case 2.4, with the only substantial difference being that the GDH challenge values are substituted into the ratchet keys $g^{\mathit{rchk}^u_{[0]}}$ and $g^{\mathit{rchk}^v_{[0]}}$. Since we have a randomness-reveal query instead of queries for specific keys, the predicate $\mathsf{clean}_{\mathtt{EE}}$ covers secrecy both of the handshake ephemerals and of the initial ratchet keys, which are generated at the same time.

C3.2: Case $s = [\mathbf{asym\text{-}ri}{:}1]$, $\mathit{type}[s] = \texttt{asym-ri}$ and $E^{\texttt{asym-ri}}_{\mathsf{clean}_{\pi^i_u.\mathit{type}[0]}(u,i,[0])}$

In this case, cleanness comes from the initial key exchange (i.e., from one of its three or four disjuncts), and the fact that the adversary has not revealed the state linking the initial key exchange to this stage. The initial key exchange derives rk_1: we perform one game hop to replace that value with a uniformly random value; the game hop is indistinguishable assuming the security of rk_1, which follows from Cases 1 and 2. Game $4'$ is indicated below.

Game $4'$. In this game we replace the root key rk_1 derived in stage $[0]$ by both the Test session and any matching peers with a uniformly random value.

An adversary which can distinguish $G4'$ from its predecessor $G3$ can distinguish the root key from a random value. The root key was derived in the initial `triple` (or `triple+DHE`) handshake by applying $\mathrm{KDF_r}$ to the master secret ms. In Case 1 (or Case 2), we argued that all values derived from ms using HKDF were indistinguishable from random. Thus, an adversary that wins here contradicts the security of Case 1 (or Case 2).

After replacing the root key rk_1, it is straightforward to see that it is impossible for the adversary to differentiate keys derived in this stage—chaining keys $\mathit{ck}^{\mathbf{sym\text{-}ri}:x,0}$ and $\mathit{ck}^{\mathbf{sym\text{-}ri}:x,1}$, messaging key $\mathit{mk}^{\mathbf{sym\text{-}ri}:x,0}$, and intermediate value tmp from random: these are derived by applying $\mathrm{KDF_r}$ to rk_1 and then $\mathrm{KDF_m}$ to that result. Since both KDFs are modelled as random oracles, and the input to $\mathrm{KDF_r}$ is an independent uniformly random value, the adversary has no advantage is distinguishing this stage's session key from random.

C4: Asymmetric ratcheting, non-initial stages

At this point we move on to arbitrary subsequent asymmetric stages. We assume that the initial handshake was of type `triple`, but the case of `triple+DHE` is analogous. The intuition for this part of the proof is essentially induction and post-compromise security:

- root keys provide security because they come from previous stages which are secure; or
- shared secrets derived from pairs of ephemeral keys provide security even if the root key at the time is compromised.

We first make a case distinction on the direction (`asym-ir` vs. `asym-ri`), and then deal with these cases in turn.

C4.1: Asymmetric ratcheting, $s = [\textbf{asym-ir:}x], x \geq 1$, *type*$[s] =$ `asym-ir`

Definition 8 requires that one of the following conditions must be satisfied if $\mathsf{clean}_{\texttt{asym-ir}}(u, i, [\textbf{asym-ir}{:}x])$ is to hold.

- event $E^{\texttt{asym-ir}}_{\text{clean-prev}}$: $\mathsf{clean}_{\texttt{asym-ri}}(u, i, [\textbf{asym-ri}{:}x-1]) \wedge \mathsf{clean}_{\mathsf{state}}(u, i, [\textbf{asym-ir}{:}x])$
- event $E^{\texttt{asym-ir}}_{\text{clean-cur}}$: $\mathsf{clean}_{\texttt{EE}}(u, i, x-1, x-1)$

C4.1.1: Case $s = [\textbf{asym-ir:}x], x \geq 1$, *type*$[s] =$ `asym-ir` and $E^{\texttt{asym-ir}}_{\textbf{clean-prev}}$

This case follows inductively like Case 3.2. This stage's message key (as well as the next root and chaining key) is derived by applying $\mathrm{KDF_r}$ to tmp, which was derived during stage $[\textbf{asym-ri}{:}x]$, and then $\mathrm{KDF_m}$ to the result. By an argument similar to Case 3.2, we can replace tmp with a random key. Treating the KDF as a random oracle, this stage's message key, as well as the next root key rk_{x+1} and the symmetric chaining keys $ck^{\textbf{sym-ir}:x,0}$ and $ck^{\textbf{sym-ir}:x,1}$, are then indistinguishable from random.

C4.1.2: Case $s = [\textbf{asym-ir:}x], x \geq 1$, *type*$[s] =$ `asym-ir` and $E^{\texttt{asym-ir}}_{\textbf{clean-cur}}$

This case is analogous to Case 3.1, with key indistinguishability following from secrecy of the DH shared secret derived from ratchet keys. We first replace the ratchet public keys with challenge values from the Gap-DH game, noting that $\mathsf{clean}_{\texttt{EE}}$ implies the existence of a unique session at Bob with the same sid as that of Alice's session. As before, an adversary which could distinguish this game from its predecessor allows us to answer the Gap-DH challenge, violating that assumption. Indistinguishability of this stage's message key, as well as the the next root and chaining keys enumerated in Case 4.4.1, then follows from applying the (random oracle) KDF to the (now independent) DH shared secret.

C4.2: Asymmetric ratcheting, $s = [\textbf{asym-ri:}x], x > 1$, *type*$[s] =$ `asym-ri`

Now we come to the case of non-initial asymmetric stages of type `asym-ri`. The proof here is nearly the same as in Case 4.1, except there is an extra KDF application: session keys derived by these stages are computed by first applying a KDF to derive an intermediate value tmp, and second applying another KDF to derive from tmp a session key.

Similarly, we partition our analysis into the following cases.

- event $E^{\texttt{asym-ri}}_{\text{clean-prev}}$: $\mathsf{clean}_{\texttt{asym-ir}}(u, i, [\textbf{asym-ir}{:}x])$
- event $E^{\texttt{asym-ri}}_{\text{clean-cur}}$: $\mathsf{clean}_{\texttt{EE}}(u, i, x, x-1)$

C4.2.1: Case $s = [\textbf{asym-ri:}x], x > 1$, *type*$[s] =$ `asym-ri` and $E^{\texttt{asym-ri}}_{\textbf{clean-prev}}$

Once again the inductive argument here is analogous to Case 3.2: secrecy follows from the root key, and so we begin by replacing the root key with a random value. Detecting this change would violate the security properties of the previous stage, but after it the session key is easily proven indistinguishable from random. This stage's message key $mk^{\textbf{sym-ri}:x,0}$ as well as symmetric chaining keys $ck^{\textbf{sym-ri}:x,0}$ and $ck^{\textbf{sym-ri}:x,1}$ and intermediate value tmp, are all also then indistinguishable from random.

C4.2.2: Case $s = [\textbf{asym-ri:}x], x > 1$, *type*$[s] =$ `asym-ri` and $E^{\texttt{asym-ri}}_{\textbf{clean-cur}}$

For this case, we proceed similarly to Case 3.1. The DH shared secret can be shown indistinguishable under the Gap-DH assumption by replacing the ratchet public keys $rchk_u^x, rchk_v^{x-1}$ of the Test session and its matching peer with values from a GDH challenger. Indistinguishability of the stage's message key, symmetric chaining keys, and intermediate value tmp (as enumerated in case 4.2.1) all follow in turn from applying a (random oracle) KDF to (now independent) secret values.

C5: Symmetric ratcheting: $\textit{type}[s] \in \{\texttt{sym-ir}, \texttt{sym-ri}\}$

We finally arrive at the case of Signal in the multistage key-indistinguishability game against an adversary $\mathcal{A}$ that issues a $\mathsf{Test}(u, i, [\mathbf{sym\text{-}ir}{:}x{,}y])$ or $\mathsf{Test}(u, i, [\mathbf{sym\text{-}ri}{:}x{,}y])$ query against some some symmetric stage.

C5.1: Symmetric ratcheting, $s = [\mathbf{sym\text{-}ir}{:}x{,}y], \textit{type}[s] = \texttt{sym-ir}$

We partition into the three different freshness conditions for the case $[\mathbf{sym\text{-}ir}{:}x{,}y]$. We then cover the case of $[\mathbf{sym\text{-}ri}{:}x{,}y]$ similarly. The intuition is that for the first stage, the symmetric keys are derived from an asymmetric update and their secrecy follows from the previous cases. For subsequent stages, we have security due to the recursive nature of the freshness condition: we can replace the chain key used to derive the message key with randomness; if the simulation did not work, then the adversary could attack the previous stage, which is a contradiction to security of previous cases because the previous stage is fresh. In all symmetric stages, no new ephemeral keying material is introduced, so security depends solely on the chaining state not being leaked (which is guaranteed for these cases by $\mathsf{clean}_{\mathsf{state}}$).

(Recall that the case $y = 0$ is performed as part of the message key derivation in the previous asymmetric update, so that the first symmetric stage derives key number 1.)

C5.1.1: Case $s = [\mathbf{sym\text{-}ir}{:}x{,}y], x = 0, y = 1$, $\textit{type}[s] = \texttt{sym-ir}$

This stage's messaging key is derived by applying a $\mathrm{KDF}_{\mathrm{m}}$ to $ck^{\mathbf{sym\text{-}ir}:0,1}$, which was derived during the initial `triple` or `triple+DHE` handshake. By case 3.2, $ck^{\mathbf{sym\text{-}ir}:0,1}$ is indistinguishable from random. Like the argument in case 3.2, treating $\mathrm{KDF}_{\mathrm{m}}$ as a random oracle, this stage's messaging key, as well as the next chaining key $ck^{\mathbf{sym\text{-}ir}:0,2}$, are thus indistinguishable from random.

C5.1.2: Case $s = [\mathbf{sym\text{-}ir}{:}x{,}y], x > 0, y = 1$, $\textit{type}[s] = \texttt{sym-ir}$

This stage's messaging key is derived by applying a $\mathrm{KDF}_{\mathrm{m}}$ to $ck^{\mathbf{sym\text{-}ir}:x,1}$, which was derived during asymmetric-second-stage $[\mathbf{asym\text{-}ir}{:}x]$. By case 4.1, $ck^{\mathbf{sym\text{-}ir}:x,1}$ is indistinguishable from random. Like the argument in case 3.2, treating $\mathrm{KDF}_{\mathrm{m}}$ as a random oracle, this stage's messaging key, as well as the next chaining key $ck^{\mathbf{sym\text{-}ir}:x,2}$, are thus indistinguishable from random.

C5.1.3: Case $s = [\mathbf{sym\text{-}ir}{:}x{,}y], x \geq 0, y.1$, $\textit{type}[s] = \texttt{sym-ir}$

This stage's messaging key is derived by applying a $\mathrm{KDF}_{\mathrm{m}}$ to $ck^{\mathbf{sym\text{-}ir}:x,y}$, which was derived during symmetric stage $[\mathbf{sym\text{-}ir}{:}x]y - 1$. By case 5.1.1, 5.1.2, or induction on case 5.1.3, $ck^{\mathbf{sym\text{-}ir}:x,y-1}$ is indistinguishable from random. Like the argument in case 3.2, treating $\mathrm{KDF}_{\mathrm{m}}$ as a random oracle, this stage's messaging key, as well as the next chaining key $ck^{\mathbf{sym\text{-}ir}:x,y+1}$, are thus indistinguishable from random.

C5.1: Symmetric ratcheting, $s = [\mathbf{sym\text{-}ri}{:}x{,}y], \textit{type}[s] = \texttt{sym-ri}$

These cases are analogous to case 5.1, by symmetry: cleanness is defined in the same recursive manner for both `sym-ir` and `sym-ri` stages, except that the base cases differ. The initial game hops are thus analogous to those in the `asym-ir` and `asym-ri` respectively, and the subsequent inductive argument is analogous to C5.1.3. □