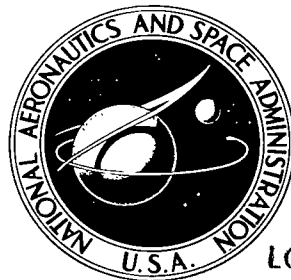


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A FORTRAN IV PROGRAM TO ESTIMATE THE OFF-DESIGN PERFORMANCE OF RADIAL-INFLOW TURBINES

by Carroll A. Todd and Samuel M. Futral, Jr.

Lewis Research Center

Cleveland, Ohio



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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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ABSTRACT

This program computes conventional performance parameters at required combinations of speeds and flow rates. Necessary input information consists of flow areas, diameters, blade angles, and an estimate of design point performance. The flow equations and corresponding computer listings are given. A set of example calculations is included.

A FORTRAN IV PROGRAM TO ESTIMATE THE OFF-DESIGN PERFORMANCE OF RADIAL-INFLOW TURBINES

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SUMMARY

A FORTRAN IV computer program for off-design performance of a radial-inflow turbine is presented. The thermodynamic equations used and the corresponding computer listings are given. Use of the program requires as input information the turbine flow areas, diameters, and blade angles. An estimate of design point performance is also necessary. The output consists of conventional performance parameters at specified flow conditions and speeds. A complete set of calculations for an example turbine is included.

INTRODUCTION

A procedure for predicting the off-design performance of radial-inflow turbines was described by Futral and Wasserbauer in reference 1. It is possible to use this procedure for hand calculations, but it is lengthy and contains several iterations. For this reason a digital computer program analog has been used. The computer program used at Lewis Research Center is given here, with the explanations necessary for its use. The thermodynamic equations are also given. These are the equations of reference 1, except for an improvement in the calculation of rotor incidence loss and the calculation of additional performance parameters.

The program has been used in two ways. (1) For an existing turbine, the performance at design operating point may be known, while information on off-design performance is lacking. In this case, the program provides a convenient way to estimate off-design performance without making actual tests.

(2) The program is also useful as a design guide. For a proposed turbine design, the design point performance can usually be closely estimated as part of the design process. However, if the off-design operation is of sufficient importance, modifications in the design may be considered. In this case, the effect of a series as small design

changes on performance may be studied by repeated use of the program.

In addition to the engineering equations and the FORTRAN IV program, this report includes the input and output listings for an example turbine problem.

ENGINEERING ANALYSIS

The analysis consists of a one-dimensional solution of flow conditions along the mean streamline in the turbine, using perfect gas relations throughout. Equations were written in a step-by-step fashion, beginning at the turbine inlet. The analysis is written for subsonic flow only since stator choking is not expected. Symbols used in the engineering equations are given in appendix A, and the equations are listed in appendix B. Figure 1 shows the location of station subscripts in the turbine.

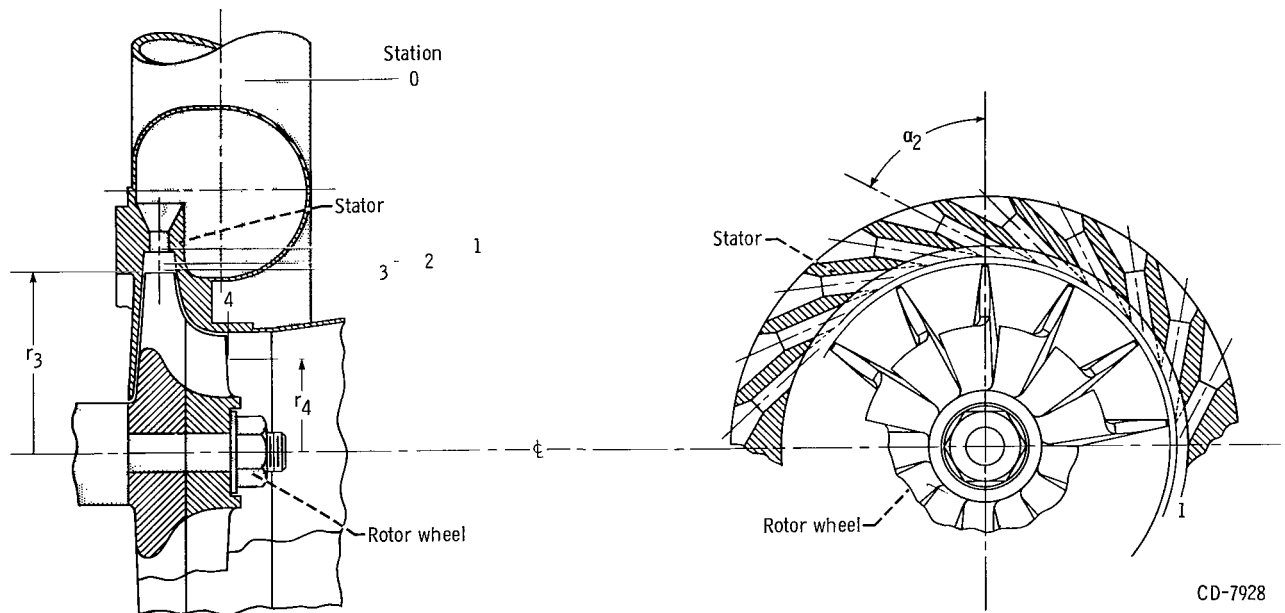


Figure 1. - Turbine stator and rotor.

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Required Information

The geometric features of the turbine which must be specified consist of flow areas, blade angles, and major dimensions. The flow areas needed are those at the turbine inlet, stator exit, rotor inlet, and rotor exit. The turbine inlet area is normal to the flow, upstream of the scroll. The stator exit area and the rotor entrance area are surfaces of circular cylinders, with elements parallel to the turbine axis. The rotor exit area is an annular area, perpendicular to the turbine axis. The stator exit area is located just inside the blade row and the rotor inlet and exit areas are calculated at locations just outside the blade rows.

The blade angles at the stator exit and rotor exit must also be specified. These are angles at the mean streamline.

Diameters needed are the stator exit diameter, rotor inlet tip diameter, and the mean diameter at the rotor exit. The axial blade length at the rotor tip is also necessary.

The required inlet conditions of the working fluid are the total temperature and total pressure at the turbine inlet. The gas properties needed are the specific gas constant, the specific heat at constant pressure, and the specific heat ratio.

Input information which applies to the example turbine is as follows:

Turbine inlet area, sq ft	0.12989
Stator inlet area, sq ft	0.04078
Rotor entrance area, sq ft	0.04175
Rotor exit area, sq ft	0.04719
Stator exit diameter, sq ft	0.42650
Rotor inlet diameter, sq ft	0.41417
Stator exit axial dimension, ft	0.03208
Stator exit angle, deg	72.47
Rotor exit angle, deg	-56.85
Number of blades at rotor tip	22
Turbine inlet total pressure, °R	2060

The thermodynamic properties of the working fluid are as follows:

Specific gas constant	18.437
Specific heat ratio	5/3
Constant pressure specific heat	0.05924

The design point quantities, used in determination of frictional coefficients, are the following:

Mass flow rate, lb/sec	0.7484
Total efficiency	0.897
Wheel speed, rpm	36 000
Total pressure ratio across stator	0.98
Total pressure ratio across turbine	1.7397

Operation of the Program

In solving the equations in appendix B, the computer uses the equations in essentially the order listed. Solutions for the quantities $(V/V_{cr})_0$, $(V/V_{cr})_2$, and $(V/V_{cr})_4$ are accomplished by means of the Newton-Ralphson method since these quantities are implicitly defined.

For operation of the program, the wheel speeds of interest are specified in advance. The initial value for $(V/V_{cr})_1$, the velocity ratio at the stator exit, as well as its increment size are also specified. The final value for this quantity will not be known and is determined by the computer for each wheel speed. For any given wheel speed, repeated increases in the value of $(V/V_{cr})_1$ will eventually result in a flow rate which the turbine cannot pass. Mathematically, this condition is evident in that no solution exists for $(W/W_{cr})_4$, the relative velocity ratio at the rotor exit. When this occurs, the maximum value for $(V/V_{cr})_1$ is computed to five significant figures. The computer prints out a complete set of answers for each value of $(V/V_{cr})_1$ for any given wheel speed. When one wheel speed is finished, the computer goes on to the next speed until all specified speeds have been used.

In this discussion, it is always assumed that the temperature and pressure of the working fluid entering the turbine are constant. This has fulfilled all needs thus far. However, if the program should be required to operate in some other manner, the amount of additional instructions would not be great.

Loss Coefficients

Calculation of rotor incidence loss makes use of the angle ϕ in equation (B17). This is the optimum incidence angle and is that incidence angle associated with minimum rotor incidence loss. The calculation of ϕ follows the method of Stanitz (ref. 2), developed to account for "slip" in centrifugal compressors having radial blading.

The loss coefficient k_1 applies to the stator and must be evaluated before the main program is used. The determination of k_1 involves simultaneous solution of equations (B1) to (B5) using the design value of mass flow rate. For this purpose, the total pressure ratio across the stator p_1'/p_0' is determined from design data. A separate routine is provided for this solution.

The determination of the loss coefficients m and k were presented as a graphical procedure in reference 1. In the computer determination, this is done by means of a search routine. By repeated calculations, the computer finds a pair of values for m and k which yield design values for mass flow rate, efficiency, and pressure ratio. Design speed and inlet conditions are specified.

Output Information

A sample of the computer output is shown for the example turbine. To assist in reading this, a list of machine output symbols in terms of engineering symbols is included in the section on program output. The information contained in the machine output may be classified under several headings as follows:

- (1) Input information - For each speed, the following quantities are printed out:
 - (a) Design speed
 - (b) Percent design speed
 - (c) Fluid inlet temperature and pressure
 - (d) Turbine dimensions and angles
 - (e) Fluid constants
 - (f) Number of rotor blades (at tip)
- (2) Performance parameters
 - (a) Efficiencies
 - (b) Blade-jet speed ratio
 - (c) Pressure ratios
 - (d) Specific work
 - (e) Ideal work
- (3) Internal variables - Internal variables are listed for each station throughout the turbine as follows:
 - (a) Pressures
 - (b) Temperatures
 - (c) Velocity components
- (4) Additional quantities - Certain ratios have been found useful and are included in the print-out, such as the following:

- (a) $wN\epsilon/\delta$
- (b) $N/\sqrt{T_0}$
- (c) $w\sqrt{T_0}/p'_{in}$
- (d) Γ/p'_{in}
- (e) Losses calculated on a pound mass basis denoted by L with appropriate subscript
- (f) Fractional losses denoted by $\Delta\eta$ where the loss has been divided by the ideal work
- (g) Losses in efficiency, given in equation (B31) as η_E , η_R , and η_I , which are useful in studying loss distributions in the turbine

THE FORTRAN IV PROGRAM

In this section are presented the FORTRAN IV routines, input and output considerations, and the internal documentation. The program operates in three distinct modes, each depending on the type of information available or the desired output. In mode one operation, the loss coefficients are assumed known. Off-design calculations are made by means of changes in speed and changes in the value of $(V/V_{cr})_1$. This mode furnishes performance results, once the loss coefficients are known. In mode two operation, sets of loss coefficients are entered and calculations are made at design values of speed and $(V/V_{cr})_1$. This mode is used for studying the results of variations in frictional effects. In mode three, the program determines the loss coefficients and produces one complete set of calculations representing design conditions of the turbine. This set of calculations verifies the determination of the loss coefficients.

Program Input

Actual input for the example with card format is presented in this section, followed by the definitions of the terms used on the cards.

TITLE						PROJECT NUMBER						ANALYST						SHEET _____ OF _____																																																														
STATEMENT NUMBER	CONT	FORTRAN STATEMENT																				IDENTIFICATION																																																										
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	
MODE ONE FORMAT																																																																																
PREF		WREF																																																																														
.98667		.748																																																																														
A0		A1		A3		A4		D1		D3		XL		AL1																																																																		
.05548		.04078		.04175		.047188		.4265		.414166		.032083		72.13																																																																		
B4		ZBR		PO		TO		U3		U4		G		R																																																																		
-56.55		22.		3588.12		2060.		780.6838		443.245		1.6667		18.437																																																																		
CP		VSTART		DELV		VEND																																																																										
.059242		.2		.05		.9																																																																										
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MODE THREE FORMAT																																																																															
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.98667 .798																																																																															
A0 A1 A3 A4 D1 D3 XL AL1																																																																															
.05548 .04078 .04175 .04718 .4265 .414166 .032083 72.13																																																																															
B4 ZBR PG TG V3 V4 G R																																																																															
-5.6.55 22. 3.588.12 2060. 780.6838 443.245 1.6667 18.437																																																																															
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P.D.S.G.N E.T.A.D P.E																																																																															
1.734 .913 22																																																																															

The definitions of the terms used on the cards are as follows:

Card 1:

PREF estimate of p'_1/p'_0 to determine k_1
WREF estimate of W to determine k_1

Card 2:

AO area of turbine inlet, A_0 , sq ft
A1 area upstream of stator exit, A_1 , sq ft
A3 area upstream of rotor inlet, A_3 , sq ft
A4 area downstream of stator exit, A_4 , sq ft
D1 diameter upstream of stator exit, D_1 , ft
D3 diameter upstream of rotor inlet, D_3 , ft
XL axial distance, l , ft
AL1 absolute gas angle, α_1 , deg

Card 3:

B4 free stream gas flow angle, β_4 , deg
ZBR number of rotor blades at tip of rotor, Z
PO inlet pressure, p'_0 , lb/sq ft
TO inlet temperature, T'_0 , $^{\circ}\text{R}$
U3 design speed upstream of rotor inlet, U_3 , ft/sec
U4 design speed downstream of rotor exit, U_4 , ft/sec
G ratio of specific heat at constant pressure to specific heat at constant volume, γ
R specific gas constant, R , ft-lb/(lb)($^{\circ}\text{R}$)

Card 4:

CP specific heat at constant pressure, c_p , Btu/(lb)($^{\circ}\text{R}$)
VSTART starting value of $(V/V_{cr})_1$
DELV incremental value of $(V/V_{cr})_1$
VEND end value of $(V/V_{cr})_1$

Card 5:
 MODE if MODE = 1, a performance run varying percent speed
 if MODE = 2, manual determination of m and k
 if MODE = 3, automatic determination of m and k

Card 6:		}	MODE ONE
XM	loss coefficient		
XK	loss coefficients		
Card 7+:		}	MODE TWO
PERCT	percent of design speed		
Card 6+:		}	MODE THREE
XM	loss coefficient		
XK	loss coefficient		
Card 6:		}	MODE THREE
PDSGN	specified p'_0/p'_4 or p'_0/p'_4		
ETAD	specified η_t or η_s		
P	if p = 1, p'_0/p'_4 is specified if p = 2, p'_0/p'_4 is specified		
E	if E = 1, η_s is specified if E = 2, η_t is specified		

Description of the Output

A list of the machine output symbols, internal program variables, and their corresponding engineering symbols is given here. Refer to appendix A for the physical interpretation. Following this are sample output listings showing the results of calculations for modes one, two, and three using the input data given previously. It should be noted that each block of output is for a specific value of $(V/V_{cr})_1$.



Machine Output Symbols and Definitions

ALPHA2	α_2
BETA3	β_3
DEE	$\Delta\eta_E$
DEI	$\Delta\eta_I$
DELTA	δ
DER	$\Delta\eta_R$
DES	$\Delta\eta_S$
EQ-SPEED	N_{eq}
ETA-E	η_E
ETA-I	η_I
ETAR	η_R
ETA-S	η_S
ETA-T	η_t
LE	L_E
LI	L_I
NOT	$N/\sqrt{T_0}$
NU	ν
PO, /P4	p'_0/p_4
PO, /P4,	p'_0/p'_4

P1, /PO,	p'_1/p'_0
P2, /PO,	p'_2/p'_0
P2, /P1,	p'_2/p'_1
P4/PO,	p_4/p'_0
P4/P4ID	$p''_4/p''_4, id$
P4/P4,	p_4/p'_4
P4, /PO,	p'_4/p'_0
PHI	φ
PV/PVCR)0	$(\rho V/\rho' V_{cr})_0$
PV/PVCR)1	$(\rho V/\rho' V_{cr})_1$
PV/PVCR)2	$(\rho V/\rho' V_{cr})_2$
PVX/PVCR)3	$(\rho V_x/\rho' V_{cr})_3$
PW/PWCR)4	$(\rho W/\rho'' W_{cr})_4$
R2	r_2
RHO)0	ρ'_0
RHO-VCR)0	$(\rho' V_{cr})_0$
ROTOR-LOSS	L_R
SMALL-W	w
SPEED	N
SPEED-PAR	wN/δ

STATOR-LOSS	L_S
$T_{,,/T,}3$	$(T''/T')_3$
$T3,,$	T''_3
$T4,$	T'_4
$T4,/T4,,$	$(T'/T'')_4$
$T4,,/T3,,$	T''_4/T''_3
THETA-CR	θ_{cr}
TOP	Γ/p'_0
V3	V_3
V4	V_4
$V/V_{CR}0$	$(V/V_{cr})_0$
$V/V_{CR}1$	$(V/V_{cr})_1$
$V/V_{CR}2$	$(V/V_{cr})_2$
$V/V_{CR}3$	$(V/V_{cr})_3$
VCR)0	$V_{cr,0}$
VCR4	$V_{cr,4}$
VU3	$V_{u,3}$
VU4	$V_{u,4}$
$VU/V_{CR}3$	$(V_u/V_{cr})_3$
VX3	$V_{x,3}$

VX/VCR)3	$(v_x/v_{cr})_3$
W1	$\Delta h'$
W3	w_3
W4	w_4
W/WCR)3	$(w/w_{cr})_3$
W/WCR)4	$(w/w_{cr})_4$
WCR3	$w_{cr, 3}$
WCR4	$w_{cr, 4}$
WEQ	w_{eq}
WID	$\Delta h'_{id, s}$
W, ID	$\Delta h'_{id, t}$
WTOP	$w\sqrt{T'_0/p'_0}$

Mode One Sample Output

OFF DESIGN PERFORMANCE OF A RADIAL INFLOW TURBINE

TURBINE GEOMETRY	AQ	0.125887	A1	0.407800E-01	A3	0.417450E-01	A4	0.471880E-01	D1	0.426500
	D3	0.414166	L	0.320830E-01	AL1	72.47000	B4	-56.85000	Z	22.00000
INLET CONDITIONS	PD	3600.000	TO	2060.000						
DESIGN SPEEDS	U3	780.6838	U4	443.2450						
FLUID PROPERTIES	G	1.666700	R	18.43700	CP	0.592420E-01	EPS	0.910949		

MODE 1. PERFORMANCE RUN VARYING PERCENT SPEED

M	3.592625
K	0.754039E-01
K1	0.763553E-01
PERCENT	100.0000
U3	780.6838
U4	443.2450
DESIGN PL./PO.	0.980000
DESIGN SMALL-W	0.748400

V/VCR11	0.200000	P2./PO.	0.998079	V3	253.5031	WCR3	1258.994
VCR10	1235.911	P2./P1.	1.000023	BETA3	-82.29354	W4	132.7295
RHO10	0.947862E-01	ALPHA2	73.31849	T.,/T.,J3	1.037702	WCR4	1217.305
RHO-VCR10	117.1473	R2	0.212270	T3.,	2137.666	VU4	332.1319
PV/PVCR11	0.157008	VU/VCR13	0.196464	T4.,/T3.,	0.934870	T4.,/T4.,	1.016578
SMALL-W	0.283144	VX/VCR13	0.589386E-01	W/WCR13	0.431123	P4./PO.	0.885221
PV/PVCR10	0.186084E-01	V/VCR13	0.205114	PVX/PVCR13	0.580112E-01	VCR4	1227.323
V/VCR10	0.166108E-01	VU3	242.8122	P4/P4ID	0.918390	V4	339.9744
V/VCR12	0.200089	W3	542.7817	PW/PWCR14	0.108550	P4/P4.	0.952730
PV/PVCR12	0.197093	VX3	72.84285	W/WCR14	0.109036	P4/PO.	0.843376
W1	1.651284	ETA-E	0.497547	W, ID	5.808875	T4.,	2031.472
WID	8.338581	ETA-T	0.291155	ETA-S	0.210396	WEQ	0.183813
ETA-I	0.778959	LE	2.308287	DEE	0.287151	LI	2.262151
ROTOR-LOSS	1.689182	STATOR-LOSS	0.928868E-01	DER	0.210134	DES	0.115551E-01
SPFED	36000.00	EQ-SPEED	29695.14	NU	1.730507	THETA-CR	1.469718
DELTA	1.701146	SPEED-PAR	5991.953	W1-EQ	1.150754	PO./P4.	1.129662
NOT	793.1747	WTOP	0.514045	TOP	0.474460E-01	DEI	0.281412
PO./P4	1.185710	PHI	-43.96986	P1./PO.	0.998056	ETA-R	1.066110
NSP	120.8699						

V/VCR11	0.250000	P2./PO.	0.996982	V3	316.8903	WCR3	1249.548
VCR10	1235.911	P2./P1.	1.000035	BETA3	-79.19788	W4	167.5706
RHO10	0.947862E-01	ALPHA2	73.31857	T.,/T.,J3	1.022189	WCR4	1207.532
RHO-VCR10	117.1473	R2	0.212270	T3.,	2105.710	VU4	302.9650
PV/PVCR11	0.244164	VU/VCR13	0.245580	T4.,/T3.,	0.933882	T4.,/T4.,	1.012364
SMALL-W	0.350528	VX/VCR13	0.737055E-01	W/WCR13	0.388769	P4./PO.	0.847779
PV/PVCR10	0.230369E-01	V/VCR13	0.256402	PVX/PVCR13	0.718959E-01	VCR4	1214.974
V/VCR10	0.230415E-01	VU3	303.5152	P4/P4ID	0.926179	V4	316.5768
V/VCR12	0.250111	W3	485.7858	PW/PWCR14	0.137770	P4/P4.	0.958118
PV/PVCR12	0.244267	VX3	91.09343	W/WCR14	0.138771	P4/PO.	0.812273
W1	4.100490	ETA-E	0.626467	W, ID	7.800876	T4.,	1990.797
WID	5.739316	ETA-T	0.525645	ETA-S	0.421024	WFQ	0.227558
ETA-I	0.839335	LE	2.000868	DEE	0.205442	LI	2.073232
ROTOR-LOSS	1.428626	STATOR-LOSS	0.145112	DER	0.146686	DES	0.148996E-01
SPFED	36000.00	EQ-SPEED	29695.14	NU	1.117917	THETA-CR	1.469718
DELTA	1.701146	SPEED-PAR	7417.945	W1-EQ	2.789984	PO./P4.	1.179552
NOT	793.1747	WTOP	0.636380	TOP	0.142408	DEI	0.212872
PO./P4	1.221113	PHI	-37.64624	P1./PO.	0.996947	ETA-R	1.044782

Mode Two Sample Output

OFF DESIGN PERFORMANCE OF A RADIAL INFLOW TURBINE

TURBINE GEOMETRY	AU	0.129887	AI	0.407300E-01	A3	0.417450E-01	A4	0.471880E-01	D1	0.426500
	D3	0.414160	L	0.320330E-01	AL1	72.47000	B4	-56.85000	Z	22.00000
INLET CONDITIONS	PU	3600.000	TU	2060.000						
DESIGN SPEEDS	U3	780.8838	U4	443.2450						
FLUID PROPERTIES	G	1.666700	R	18.43700	CP	0.592420E-01	EPS	0.910949		

MODE 2: MANUAL DETERMINATION OF M AND K
M 3.000000
K 0.500000E-01
K1 0.763553E-01
PERCENT 100.0000
U3 780.8838
U4 443.2450
DESIGN P1,/P0, 0.960000
DESIGN SMALL-W 0.748400

V/VCK1	0.200000	P2,/P0,	0.998695	V3	256.3198	WCR3	1258.537
VCR10	1235.911	P2,/P1,	1.000640	BETA3	-82.25358	W4	130.5060
RHU10	0.947802E-01	ALPHA2	73.51138	T ₃ ,/T ₁ ,3	1.036950	WCR4	1216.832
RHU-VCK10	117.1473	K2	0.212058	T ₃ ,	2136.116	VU4	333.9932
PV/PVCK11	0.197000	VU/VCK13	0.193847	T ₄ ,/T ₃ ,	0.934823	T ₄ ,/T ₄ ,	1.016820
SMALL-W	0.283144	VX/VCK13	0.589232E-01	W/WR13	0.428959	P ₄ ,/P ₃ ,	0.900118
PV/PVCK10	0.186084E-01	V/VCK13	0.207393	PVX/PVCK13	0.579754E-01	VCR4	1227.023
V/VCK10	0.186108E-01	VU3	245.7571	P ₄ /P4ID	0.934413	V4	341.5368
V/VCK12	0.202516	W3	539.8610	PN/PWCR14	0.106788	P4/P4,	0.952278
PV/PVCK12	0.199409	VX3	72.82388	W/WR14	0.107251	P4/PD,	0.857153
W1	1.750150	ETA-E	0.559111	WID	5.030349	T4,	2030.478
W10	7.296770	ETA-T	0.347920	ETA-S	0.239853	WEQ	0.183813
ETA-I	0.805162	LE	2.329552	DER	0.319258	LI	2.233187
RUTUR-LUSS	0.924090	STATUR-LUSS	0.630830E-01	DER	0.126645	DES	0.864533E-02
SPEED	36000.00	EQ-SPEED	29695.14	VJ	1.291541	THETA-CR	1.459718
DELTA	1.701146	SPEED-PAR	5991.953	W1-EQ	1.190812	PD,/P4,	1.110965
NU1	793.1747	WTOP	0.514045	TJP	0.490976E-01	DEI	0.306051
P0,/P4	1.186640	PH1	-43.97732	PL,/PJ,	0.998056	ETA-R	1.184420
NSP	133.5119						

V/VCK1	0.200000	P2,/P0,	0.997949	V3	320.4066	WCR3	1248.973
VCR10	1235.911	P2,/P1,	1.001005	BETA3	-79.12022	W4	165.0909
RHU10	0.947802E-01	ALPHA2	73.51363	T ₃ ,/T ₁ ,3	1.021249	WCR4	1206.937
RHU-VCK10	117.1473	K2	0.212055	T ₃ ,	2103.772	VU4	305.0409
PV/PVCK11	0.244164	VU/VCK13	0.243558	T ₄ ,/T ₃ ,	0.933821	T ₄ ,/T ₄ ,	1.012692
SMALL-W	0.350528	VX/VCK13	0.736753E-01	W/WR13	0.386049	P ₄ ,/P ₃ ,	0.850185
PV/PVCK10	0.230369E-01	V/VCK13	0.259247	PVX/PVCK13	0.718263E-01	VCR4	1214.572
V/VCK10	0.230415E-01	VU3	307.1957	P ₄ /P4ID	0.940376	V4	318.1267
V/VCK12	0.253144	W3	482.1640	PN/PWCR14	0.135826	P4/P4,	0.957672
PV/PVCK12	0.247086	VX3	91.05610	W/WR14	0.136785	P4/PD,	0.823775
W1	4.180503	ETA-E	0.588837	WID	7.135124	T4,	1989.480
W10	9.105924	ETA-T	0.589524	ETA-S	0.458877	WEQ	0.227558
ETA-I	0.904390	LE	2.021145	DER	0.221959	LI	2.035274
RUTUR-LUSS	0.778077	STATUR-LUSS	0.905512E-01	DER	0.854474E-01	DES	0.108228E-01
SPEED	36000.00	EQ-SPEED	29695.14	VJ	1.156144	THETA-CR	1.459718
DELTA	1.701146	SPEED-PAR	7417.945	W1-EQ	2.843064	PD,/P4,	1.162541
NU1	793.1747	WTOP	0.535380	TJP	0.145117	DEI	0.223511
P0,/P4	1.213924	PH1	-37.65760	PL,/PJ,	0.996947	ETA-R	1.126307

Mode Three Sample Output

OFF DESIGN PERFORMANCE OF A RADIAL INFLOW TURBINE

TURBINE GEOMETRY	AD	0.125887	A1	0.407800E-01	A3	0.417450E-01	A4	0.471880E-01	D1	0.426500
	D3	0.414166	L	0.320830E-01	AL1	72.47000	B4	-56.85000	Z	22.00000
INLET CONDITIONS	PD	3600.000	TD	2060.000						
DESIGN SPEEDS	U3	780.6838	U4	443.2450						
FLUID PROPERTIES	G	1.666700	R	18.43700	CP	0.592420E-01	EPS	0.910949		

MODE 3. AUTOMATIC DETERMINATION OF M AND K

M	3.947835
K	0.683671E-01
K1	0.763553E-01
PFCENT	100.0000
U3	780.6838
U4	443.2450
DESIGN P1./PD,	0.980000
DESIGN SMALL-W	0.748400
DESIGN PD./P4,	1.740000
DESIGN ETA-T	0.917000

V/VCR11	C.615818	P2./PD,	0.981934	V3	783.3564	WCR3	1177.725
VCR10	1235.911	P2./P1,	1.001973	BETA3	-7.711356	W4	444.0692
RHO11	0.947862E-01	ALPHA2	73.38271	T.,./T.,13	0.908058	WCR4	1133.050
RHO-VCR10	117.1473	R2	0.212199	T3.,	1870.599	VU4	71.49717
PV/PVCR11	0.530352	VU/VCR13	0.606932	T4.,/T3.,	0.925572	T4.,/T4.,	0.974083
SMALL-W	0.748445	VX/VCR13	0.182683	W/WCR13	0.193457	P4./PD,	0.575019
PV/PVCR10	0.491883E-01	V/VCR13	0.633829	PVX/PVCR13	0.155864	VCR4	1118.271
V/VCR10	0.492331E-01	V03	750.1139	P4/P41D	0.965595	V4	253.2051
V/VCR12	0.618131	W3	227.8395	PW/PWCR14	0.369567	P4/P4,	0.968264
PV/PVCR12	0.531715	VX3	225.7794	W/WCR14	0.391974	P4/P0,	0.556771
W1	22.12419	ETA-E	0.918337	W,I D	24.23218	T4,	1686.502
W1D	25.48582	ETA-T	0.913009	ETA-S	0.868098	WEQ	0.485880
FTA-I	0.919463	LE	1.280390	DEE	0.502393E-01	LI	0.286848E-01
ROTOR-LOSS	1.342740	STATOR-LOSS	0.901912	DER	0.526858E-01	DES	0.314650E-01
SPFFD	36000.00	EQ-SPEED	29695.14	VU	0.691074	THETA-CR	1.469718
DELTA	1.701146	SPEED-PAR	15838.75	W1-EQ	15.05336	P0./P4,	1.739072
NNT	793.1747	WTOP	1.358794	TOP	1.640600	DEI	0.112552E-02
PO./P4	1.796072	PHI	-17.28718	P1./PD,	0.980000	FTA-R	0.969702
NSP	75.74157						

Main Program Listing

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C A FORTRAN IV COMPUTER ROUTINE TO PREDICT OFF DESIGN PERFORMANCE
C OF A RADIAL INFLOW TURBINE
C
  SUB(X,G)=X*(1.-(G-1.)/(G+1.)*X*X)**(1./(G-1.))
  DATA AC/32.1739/,PI/3.14159/,XJ/778.16/,F1/.01745/
  DATA TOL/.000001/,XNK/9./,XMN/9./
  COMMON /GFK/PREF,WRFF,G,AC,A1,AL1,TO,R,PO
  1, PDSGN,CP,G1,G2,G3,ETAD,U3,U4,VU3,B4,T4T3,WCR3,TT3,W3WC3,B3,
  2 PHI,P2PP1P,P1PP0P
  INTEGER PTEST , ETEST
  RFAD(5,40C)PREF,WRFF
C
C INPUT TURBINE GEOMETRY-INLET CONDITIONS-SPEEDS-FLUID PROPERTIES
C
  1 READ(5,400) AO,A1,A3,A4,D1,D3,XL,AL1,B4,ZBR ,PG,TO,
  U3,U4,      G,R,CP,      VSTART,DELV1,VEND
  U31=U3
  U41=U4
  CALL GETK1(XK1,V1VC1Q)
C
  G1=G+1.
  G2=G-1.
  G2=G2/G1
  EPS=1.4/G*((G1/2.)**(G/G2)/1.8929)
  R1=D1/2.
  R3=C3/2.
C DETERMINE MODE OF OPERATION
C
  READ(5,401)MODE
C MODE=1, IMPLIES A PERFORMANCF RUN VARYING PERCENT SPEED
C MODE=2, IMPLIES INPUTTING VARIOUS VALUES OF M AND K AT DESIGN SPEED
C MODE=3, IMPLIES LETTING THE MACHINE DETERMINE THE VALUE OF M AND K
C
  30 GO TO (2,3,4),MODE
  2 READ(5,400)XM,XK
  5 READ(5,400) PFRCT
  V1VC1=VSTART
  DELV =DELV1
  GO TO 6
  3 READ(5,400) XM,XK
  PERCT=100.
  V1VC1=VSTART
  DELV =DELV1
  GO TO 6
  4 RFAD(5,405) PDSGN,ETAD,PTEST,E TEST
405 FFORMAT(2F10.4,2I1)
  XK=1.E-8
  K=1
  V1VC1=V1VC1Q
  PERCT=100.

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6 U3=U31*PERCT/100.
  U4=U41*PERCT/100.
C CASE CONSTANTS
  GO TO 40
50 VCR0=SQRT(2./G1*AC*TD*G*R)
  RHOOP=PO/R/TO
  PVCRO=VCR0*RHOOP
C STATOR ANALYSIS
  COSAL1=COS(AL1*F1)
  SINAL1=SIN(AL1*F1)
C
23 CONTINUE
  RHOVC1=SUB(V1VC1,G)
  Q1=2.*XK1*RHOVC1*A1*COSAL1/AO/G1
  Y=V1VC1
  X=Y/2.
  Q1=Q1/(1.-Y*Y)
10 P1PPOP=(1.-XK1*G3*(X*X+Y*Y)/(1.-G3*Y*Y))*G/G2
  SW=P1PPOP*RHOVC1*PVCRO*A1*COSAL1
  RHOVCO=SW/PVCRO/AO
  F=SUB(X,G)-RHOVCO
  Q=(F+RHOVCO)/X
  FP=Q-2.*X**2/G1*Q**(2.-G)+X*Q1*P1PPOP**(2.-G)
  X1=X-F/FP
  IF(ABS((X1-X)/X1).LT.TCL) GO TO 9
  X=X1
  GO TO 10
C
C STATOR TO ROTOR ANALYSIS
9 VQVCO=X1
  V2VC2=(XK1+1.)/(XK1+1.)*Y*Y-(XK1-XK1)/(XK1+1.)*X*X
  V2VC2=SQRT(V2VC2)
  R2VC2=SUB(V2VC2,G)
  P2PPOP=(1.-G3*XK1*(VQVCO**2+V2VC2**2)/(1.-G3*V2VC2**2))*G/G2
  P2PP1P=P2PPOP/P1PPOP
  TANAL2=2.*PI*R1*XL*KH2VC2*P2PP1P*SINAL1/(RHCVC1*A1*CCSAL1)
  AL2=ATAN(TANAL2)
  SINAL2=SIN(AL2)
  R2=R1*SINAL1/SINAL2
C
C ROTOR INLET ANALYSIS
C
  VLVC3=R2/R3*V2VC2*SINAL2
  X=V1VC1/2.
12 Q=R2/R3*RH2VC2*COS(AL2)
  F=Q*(1.-G3*(X*X+VUVC3**2))*(-1./G2)-X
  FP=2./G1*X*Q**2/(F+X)-1.
  X1=X-F/FP
  IF(ABS((X1-X)/X1).LT.TCL) GO TO 11
  X=X1
  GO TO 12
11 VXVC3=X1
  V3VC3=SQRT(VXVC3**2+VUVC3**2)
  VU3=VCR0*VUVC3
  VX3=VCR0*VXVC3
  V3=VCR0*V3VC3

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W3=(V3**2-2.*U3*VU3+U3**2)**.5
COSB3=VX3/W3
SINB3=(VU3-U3)/W3
B3=AR SIN(SINB3)
TT3=1.-G3*(2.*U3*VU3-U3**2)/VCR0**2
T3PP=TO*TT3
T4T3=1.-(U3**2-U4**2)/(2.*AC*XJ*CP*T3PP)
W3WC3=VXVC3/(SQRT(TT3)*COSB3)
WCR3=W3/W3WC3
RXVC3=SUB(V3VC3,G)*VXVC3/V3VC3
WX3=W3*COSB3
VU3OPT=U3*(1.-1.98/ZBR)
WU3OPT=VU3OPT-U3
PHI=ATAN(WU3OPT/WX3)
C
IF(MODE.NE.3)GO TO 300
CALL GFIM(XK,XM,1,ETAD,1)
300 CONTINUE
C ROTOR ANALYSIS
C
X=V1VC1/2.
Q=RHXVC3*A3/(A4*COS(B4*F1))/(TT3*T4T3)**(G1/G2/2.)
Q1=W3WC3**2*(XM*XK+SIN(B3-PHI)**2)/T4T3
KCWNT=0
15 Q2=1.-G3*X*X
KCWNT=KCWNT+1
Q3=XM*XK*X*X+Q1
F2=(1.-G3*(Q3)/Q2)**(G/G2)
F3=SUB(X,G)
F=Q/F2-F3
F2P=G/G2*F2**2*(1./G)*(-2.*G3**2*Q3/Q2**2*X-2.*G3*XM*XK*X/Q2)
F3P=-2.*X**2/G1*(F3/X)**(2.-G)+F3/X
FP=-Q/F2**2*F2P-F3P
X1=X-F/FP
C
C CHECK FOR CHOKING CONDITIONS
C
IF((X1.GT.1.).OR.(X1.LT.0.).OR.(KCWNT.GT.300))GO TO 13
IF(ABS((X1-X)/X1).LT.TOL)GO TO 14
X=X1
GO TO 15
14 P4P4ID=F2
RHWCR4=F3
W4WC4=X
WCR4=WCR3*SQRT(T4T3)
W4=WCR4*W4WC4
VU4=U4-W4*ABS(SIN(F1*B4))
WU4=W4*SIN(F1*B4)
T4T4=1.+G3*(U4**2+2.*U4*WU4)/WCR4**2
P4PPOP=P1PPOP*P4P4ID*(TT3*T4T3*T4T4)**(G/G2)*P2PP1P
SINB4=SIN(F1*B4)
COSB4=COS(F1*B4)
VCR4=WCR4*SQRT(T4T4)
V4=SQRT(W4**2*COSB4**2+(U4+W4*SINB4)**2)
P4P4P=(1.-G3*(V4/VCR4)**2)**(G/G2)
P4POP=P4PPOP*P4P4P

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T4P=T4T4*T4T3*T3PP
THETCR=(VCR0/1019.46)**2
WK=VU3*U3-VU4*U4
WK=WK/AC/XJ
WK1=CP*(T0-T4P)
WIDP=CP*T0*(1.-P4POP**G2/G) 84
WID=CP*T0*(1.-P4POP**G2/G) 85
ETAT=WK/WIDP
ETAS=WK/WID
P3P4P=(1.-G3*(V3VC3**2-(VUVC3**2-(U3/VCR0)**2)**2)**G/G2) 86
P3P3P=(1.-G3*V3VC3**2)**G/G2 87
P4PP3P=P4PP0P/P2PP0P
WRIDP=CP*T0*(1.-P4PP3P**G2/G)-(U3-VU3)**2/(2.*AC*XJ) 88
RCTLDS=XM*XK*(W3**2+W4**2)/(2.*AC*XJ)
VC=VCR0*V0VC0
V2=VCR0*V2VC2
STALOS=XK*(V0**2+V2**2)/2./AC/XJ
  CER=RDTLOS/WID
  CES=STALOS/WID
DEL=P0/2116.22
WEQ=SW*SQRT(THETCR)/DEL*EPS 89
WIFQ=WK/THETCR
DEE=V4**2/(2.*AC*XJ*WID)
ETAE=ETAS+DEE
XLF=V4**2/2./XJ/AC
DEI=(13*SIN(B3-PHI))**2/(2.*AC*XJ*WID) 90
XLI=DEI*WID
AL2=57.3*AL2
R3=57.3*B3
PHI=57.3*PHI
FTAI=ETAE+DEI
ETAR=ETAI+DEE
XN=60.*U3/PI/D3
EQSPD=XN/SQRT(THETCR) 91
XNU=U3/SQRT(2.*AC*XJ*WID) 92
SPPAR=SW*XN*FPS/DEL
PCPP4P=1./P4PP0P
XNPT=XN/SQRT(T0) 93
WTOP=SW*SQRT(T0)*144./PO 94
XTOP=SW*WK*3.5616*144./XN/PO*AC*XJ
P4=P4POP*PO
T4=T4P*(1.-G3*(V4/VCR4)**2)
RFO4=P4/R/T4
Q4=SW/RHO4
C0=U3/SQRT(2.*AC*XJ*WIDP) 95
H4=XJ*WIDP
XNSP=XN*SQRT(Q4)/H4**.75 96
SPPAR=SW*XN/DEL
G0D=7537.129*WK*SW/PO/XN*AC*XJ
PCPP4=1./P4POP
IF(MODE.NE.3) GO TO 20
EX=FTAS
IF(ETEF.F0.2) EX=ETAT
IF(K.EQ.6) GO TO 25
CALL (FTMIK,XM,2,EX,K) 106
GO TO 23

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21 V1VC1=V1VC1+DELV
   IF(V1VC1.GT.VEND) GO TO 22
   GO TO 23
22 V1VC1=VSTART
   DELV=DELV1
   IF(MODE.EQ.1) GO TO 5
   IF(MODE.EQ.2) GO TO 3
   IF(MODE.EQ.3) GO TO 4
25 CONTINUE
   WRITE(6,509) XM,XK,XK1,PERCT,U3,U4,PREF,WREF
   IF(PTEST.EQ.1) WRITE(6,529) PDSGN
   IF(PTEST.EQ.2) WRITE(6,526) PDSGN
   IF(ETEST.EQ.1) WRITE(6,527) ETAD
   IF(ETEST.EQ.2) WRITE(6,528) ETAD
   GO TO 20
C CHKFC
13 IF(MODE.EQ.2) GO TO 20
   V1VC1=V1VC1-DELV
   DELV=DELV/2.
   IF(DELV.LT.5.F-4) GO TO (5,3),MODE
   GO TO 21
C
C
C          OUTPUT FOR SPEED V1VC1
20 CONTINUE
   WRITE(6,505)
505 FORMAT(1F9)
   WRITE(6,506) V1VC1,P2PPOP,V3,WCR3,VCRO,P2PP1P,B3,W4,RHOOP,
1AL2,TT3,WCR4,PVCR0,R2,T3PP,VU4,RHOVC1,VUVC3,T4T3,T4T4,SW,
2VXVC3,W3WC3,P4PPOP,RHOVCO,V3VC3,RHXVC3,VCRA
C
   WRITE(6,507) VOVCO,VU3,P4P4ID,V4,V2VC2,W3,RHWCR4,P4P4P,
1KH2VC2,VX3,W4WC4,P4POP,WK,ETA,EAE,WIDP,T4P,WID,ETAT,ETAS,WEQ,
2ETA1,XLF,DEF,XLI,ROTLOS,STALOS,DER,DES,XN,EQSPD,XNU,THETCR
C
   WRITE(6,508) DEL,SPPAR,W1EC,POPP4P,XNOT,WTOP,XTOP,DEI,
1POPP4,PHI,P1PPOP,ETAR,XNSP
C
   GO TO 21
C
C          OUTPUT INPUT
40 WRITE(6,501) A0,A1,A3,A4,D1,D3,XL,AL1,B4,ZBR,PO,TO,
1U31,U41,G,R,CP,EPS
   IF(MODE.EQ.1) WRITE(6,502)
   IF(MODE.EQ.2) WRITE(6,503)
   IF(MODE.EQ.3) GO TO 525
509 FORMAT(14X,1HM,G15.6/14X,1HK,G15.6/13X,2HK1,G15.6/7X,8H PERCENT,
1G15.6 /13X,2HU3,G15.6/13X,2HU4,G15.6/1X,14HDESIGN P1,/PO, G15.6 /
2 1X,14HDESIGN SMALL-W,G15.6 )
   WRITE(6,509) XM,XK,XK1,PERCT,U3,U4,PREF,WREF
   WRITE(6,505)
   GO TO 50
525 WRITE(6,504)
   GO TO 50
C
C          FORMATS
529 FORMAT(2X,13HDESIGN PO,/P4 G15.6)
526 FORMAT(1X,14HDESIGN PO,/P4, G15.6)
527 FORMAT(2X 13HDESIGN ETA-S G15.6)

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528 FORMAT(2X,13HDESIGN ETA-T G15.6)
400 FORMAT(8E10.5)
501 FORMAT(1H1,
1 36X,49HOFF DESIGN PERFORMANCE OF A RADIAL INFLOW TURBINE //
121H TURBINE GEOMETRY AN,G16.6,2X,2HA1,G16.6,2X,2HA3,G16.6,2X,
22FA4,G16.6,2X,2HD1,G16.6 / 19X,2HD3,G16.6,2X,1HL,G17.6,2X,3HAL1,
3G16.6,1X,2HB4,G16.6,2X,1HZ,G16.6 /21H INLET CONDITIONS PO,
4G16.6,2X,2HT0,G16.6 /14H DESIGN SPEEDS,5X,2HU3,G16.6,2X, 2HU4,
5G16.6/20H FLUID PROPERTIES G,G17.6,2X,1HR,G17.6,2X,2HCP,G16.6 ,
62X,3HFPS,G16.6)
401 FORMAT(11)
502 FORMAT( 8HKMODE 1,2X,37HPERFORMANCE RUN VARYING PERCENT SPEED )
503 FORMAT( 8HKMODE 2,2X,31HMANUAL DETERMINATION OF M AND K )
504 FORMAT( 8HKMODE 3,2X,34HAUTOMATIC DETERMINATION OF M AND K )
C
506 FORMAT(12H V/VCR)1,G15.6,9X,7HP2,/PO,,G15.6,14X,2HV3,G15.6,
11X,4HWCR3,G15.6/7X,5HVCR)0,G15.6,9X,7HP2,/P1,,G15.6,11X,5HBETA3,
2G15.6,13X,2HW4,G15.6/8X,5HRHO)0,G15.6,10X,6HALPHA2,G15.6,8X,
38FT,./T.)3,G15.6,11X,4HWCR4,G15.6/12H RHO-VCR)0,G15.6,14X,2HR2,
4G15.6,12X,4HT3,.,G15.6,12X,3HVU4,G15.6/12H PV/PVCR)1,G15.6,
58X,8HVU/VCR)3,G15.6,7X,9HT4,./T3,.,G15.6,7X,8HT4,/T4,.,G15.6 /
612H SMALL-W,G15.6,8X,8HVX/VCR)3,G15.6,9X,7HW/WCR)3,G15.6,6X,
77FP4,/PO,,G15.6 /12H PV/PVCR)0,G15.6,9X,7HV/VCR)3,G15.6,6X,
81CHPVX/PVCR)3,G15.6,11X,4HVCR4,G15.6 )
507 FORMAT(12H V/VCR)0,G15.6,13X,3HVU3,G15.6,9X,7HP4/P4ID,G15.6,
11X,2HV4,G15.6 /12H V/VCR)2,G15.6,14X,2HW3,G15.6,7X,
29HPW/PWCR)4,G15.6,9X,6HP4/P4,,G15.6 /12H PV/PVCR)2,G15.6,13X,
33FVX3,G15.6,9X,7HW/WCR)4,G15.6,9X,6HP4/PO,,G15.6/10X,2HW1,G15.6,
411X,5HETA-E,G15.6,12X,4HW,1D,G15.6,12X,3HT4,,G15.6/9X,3HW1D,G15.6,
511X,5HETA-T,G15.6,11X,5HETA-S,G15.6,12X,3HWEQ,G15.6 /
61F 6X,5HFTA-I,G15.6,14X,2HLE,G15.6,13X,3HDEF,G15.6,13X,
72HLI,G15.6/12H ROTOR-LOSS,G15.6,5X,11HSTATOR-LOSS,G15.6,
812X,3FDER,G15.6,12X,3HDES,G15.6/1H ,6X,5HSPEED,G15.6,8X,
X8FEQ-SPEED,
9G15.6,14X,2HNU,G15.6,7X,8HTHETA-CR,G15.6 )
C
508 FORMAT(7X,5HDELTA,G15.6,7X,9HSPEED-PAR,G15.6,
111X,5HW1-EQ,G15.6,8X,7HPO,/P4,,G15.6/9X,3HNOT,G15.6,12X,4HWTOP,
2G15.6,13X,3FTOP,G15.6,12X,3HDEI,G15.6/
31F 5X,6HPO,/P4,G15.6,13X,3HPHI,G15.6,9X,7HP1,/PO,,G15.6,
410X,5HETA-R,G15.6/1H 8X,3HNSP,G15.6)
END

```

Internal Program Variables and Definitions

AO	A_0
A1	A_1
A3	A_3
A4	A_4
AC	acceleration constant
AL1	α_1
AL2	α_2
B3	β_3
B4	β_4
COSAL1	$\cos \alpha_1$
COSB3	$\cos \beta_3$
COSB4	$\cos \beta_4$
CP	c_p (input variable)
D1	D_1
D3	D_3
DEE	$\Delta \eta_E$
DEI	$\Delta \eta_I$
DEL	δ
DELV	incremental value of $(V/V_{cr})_1$ (input variable)



DER	$\Delta\eta_R$
DES	$\Delta\eta_S$
DM	Δm
ETAD	design value of efficiency
ETA _E	η_E
ETA _I	η_I
ETA _R	η_R
ETA _S	η_S
ETA _t	η_t
EPS	ϵ
F	temporary storage
F1	temporary storage
F2	temporary storage
F2P	temporary storage
F3	temporary storage
F3P	temporary storage
FP	temporary storage
G	γ
G1	$\gamma + 1$
G2	$\gamma - 1$
G3	$(\gamma - 1)/(\gamma + 1)$
H4	temporary storage

KMAX	maximum number of k's
MODE	if MODE = 1, performance run varying percent speed if MODE = 2, manual determination of m and k if MODE = 3, automatic determination of m and k (input variables)
PO	p'_0
POPP4	p'_0/p_4
POPP4P	p'_0/p'_4
P1PPOP	p'_1/p'_0
P2PPOP	p'_2/p'_0
P2PP1P	p'_2/p'_1
P3P3P	p_3/p'_3
P3P4P	p_3/p'_4
P4	p_4
P4POP	p_4/p'_0
P4P4ID	$p''_4/p''_{4, id}$
P4P4P	p_4/p'_4
P4PPOP	p'_4/p'_0
P4PP3P	p'_4/p'_3
PDSGN	specified p'_0/p'_4 or p'_0/p_4 (input variable)
PERCT	percent of design speed (input variable)
PHI	φ

PIN	$p'_0/144$
PREF	estimate of p'_1/p'_0 to determine k_1 (input variable)
PVCRO	$(\rho'V_{cr})_0$
Q	temporary storage
Q1	temporary storage
Q2	temporary storage
Q3	temporary storage
Q4	temporary storage
R	R (input variable)
R1	r_1
R2	r_2
R3	r_3
RH2VC2	$(\rho V/\rho'V_{cr})_2$
RHOOP	ρ'_0
RHOVCO	$(\rho V/\rho'V_{cr})_0$
RHWCR4	$(\rho W/\rho''W_{cr})_4$
RHXVC3	$(\rho V_x/\rho'V_{cr})_3$
ROTLOS	L_R
SINAL1	$\sin \alpha_1$
SINAL2	$\sin \alpha_2$
SINB3	$\sin \beta_3$

SPPAR	wN/δ
STALOS	L_S
TO	T'_0
T4	T_4
T4P	T'_4
T3PP	T''_3
T4T3	T''_4/T''_3
T4T4	$(T'/T'')_4$
THETCR	θ_{cr}
TT3	$(T''/T')_3$
U3	U_3 (input variable)
U31	initial U_3
U4	U_4 (input variable)
U41	initial U_4
VO	V_0
V2	V_2
V3	V_3
V4	V_4
VOVCO	$(V/V_{cr})_0$
V1VC1	see input



V2VC2	$(V/V_{cr})_2$
V3VC3	$(V/V_{cr})_3$
VCRO	$V_{cr,0}$
VCR4	$V_{cr,4}$
VEND	end value of $(V/V_{cr})_1$ (input variable)
VSTART	starting value of $(V/V_{cr})_1$ (input variable)
VU3	$V_{u,3}$
VU4	$V_{u,4}$
VU3OPT	eq. (B16b)
VUVC3	$(V_u/V_{cr})_3$
VXVC3	$(V_x/V_{cr})_3$
W3	W_3
W4	W_4
W3WC3	$(W/W_{cr})_3$
WCR3	$W_{cr,3}$
WEQ	w_{eq}
WEQD	see input
WID	$\Delta h'_{id,s}$
WIDP	$\Delta h'_{id,t}$
WREF	estimate of W to determine k_1 (input variable)

WTOP	$w\sqrt{T'_0/p'_0}$
WU3OPT	eq. (B16c)
WU4	$W_{u,4}$
WX3	$W_{x,3}$
X	temporary storage
X1	temporary storage
XJ	constant, 778.16
XK1	k_1
XLE	L_E
XN	N
XNOT	N/T'_0
XNSP	NSP
XNV	ν
XTOP	Γ/p'_{in}
ZBR	Z (input variable)

Subroutine GETK1(XK1:X1)

This routine evaluates the loss coefficient k_1 and a design $(v/v_{cr})_1$ with a specified p'_0/p'_1 and weight flow w where

XK1 loss coefficient k_1

X1 design $(v/v_{cr})_1$

The subroutine is as follows:

```

SUBROUTINE GETKI(XK1,X1)
COMMON /GETK/ P,W,G,A0,A1,AL,T,R,PD
SUB(X,G)=X*(1.-(G-1.)/(G+1.))*X**((1./(G-1.))
VCRO=(64.34*G*R*T/(G+1.))**.5
RFOO=PD/R/T
Q=W/(RHOO*A0*VCRO)
ROCO=Q
X=.1
1 F=SUB(X,G)-Q
F1=(F+Q)/X
FP=-2.*X*X/(G+1.)*F1**(2.-G)+F1
X1=X-F/FP
IF(ABS((X1-X)/X1).LT.1.E-5) GO TO 2
X=X1
GO TO 1
2 VC=X1
Q=W/(P*RHO0*VCRO*A1*COS(AL*.01745))
X=.1
3 F=SUB(X,G)-Q
ROC1=F+Q
F1=(F+Q)/X
FP=-2.*X*X/(G+1.)*F1**(2.-G)+F1
X1=X-F/FP
IF(ABS((X1-X)/X1).LT.1.E-5) GO TO 4
X=X1
GO TO 3
4 XK1=(P**((G-1.)/G)-1.)*(1.-(G-1.)/(G+1.))*X1**2)/
1((G-1.)/(G+1.))*(VO**2+X1**2))*(-1.)
RETURN
END

```

Subroutine GETM(XK, XM, IGO, ETEST, K)

For IGO = 1, this routine computes a value of XM as a function of XK by using equations (B17b) to (B24b). For IGO = 2 this routine varies the value of XK by false positioning until all design specifications are met. The following are defined for this subroutine:

XK	loss coefficient
XM	loss coefficient
IGO	index for mode of operation
ETEST	design value of η_t or η_s
K	indicator used in method of false positioning

The subroutine is as follows:

```

SUBROUTINE GETM(XK,XM,IGD,ETEST,K)
COMMON/GETK/PREF,WREF,G,A0,A1,AL1,TO,K,PO,P4PPOZ,CP,G1,G2,G3,
LEIAT,U3,U4,VU3,B4,T4T3,WCR3,TT3,W3WC3,B3,PHI,P2PF1P,P1PPOP
IF(IGD.GT.1) GO TO 100
P4PPOP=1./P4PPOZ
DIFD=CP*TO*(1.-P4PPOP** (G2/G))
DIF=CHID*ETAT
VL4=(DIF*25033.-VU3*U3)/U4
VL4=-VU4
W4=(VU4-U4)/ABS(SIN(.01745*B4))
W4=-W4
WCR4=WCR3*SQR(T4T3)
W4WC4=W4/WCR4
WU4=W4*SIN(.01745*B4)
T4T4=1.+G3*(U4**2+2.*U4*WU4)/WCR4**2
P4P4ID=P4PPOP/(P2PF1P*P1PPOP*(TT3*T4T3*T4T4)** (G/G2))
T=W3WC3**2/T4T3
T1=-(P4P4IC** (G2/G)-1.)*(1.-G3*W4WC4**2)/G3
XMK=(T1-T*(SIN(B3-PHI))**2)/(W4WC4**2+T)
XM=XMK/XK
RETURN
100 GO TO (101,102),K
101 XK1=XK
E1=ETEST
DIF1=ETEST-ETAT
K=2
XK=XK+.005
RETURN
102 XK2=XK
E2=ETEST
DIF2=ETEST-ETAT
IF(DIF2*DIF1) 106,105,104
104 E1=E2
XK1=XK2
XK=XK+.005
DIF1=DIF2
RETURN
105 XK=XK?
K=6
RETURN
106 XK=-(XK2-XK1)/(DIF2-DIF1)*DIF1+XK1
K=6
RETURN
END

```

Lewis Research Center,
 National Aeronautics and Space Administration,
 Cleveland, Ohio, December 6, 1968,
 125-23-02-10-22.

APPENDIX A

SYMBOLS

A	area, sq ft
c_p	specific heat at constant pressure, Btu/(lb)(°R)
D	diameter, ft
g	dimensional constant, 32.1741 ft/sec ²
$\Delta h'$	specific turbine work, Btu/lb
$\Delta h'_{id,s}$	ideal turbine work based on inlet total to exit static pressure ratio, Btu/lb
$\Delta h'_{id,t}$	ideal turbine work based on inlet total to exit total pressure ratio, Btu/lb
J	mechanical equivalent of heat, 778.029 ft-lb/Btu
k_1, k, m	loss coefficients, dimensionless
L	kinetic energy loss, Btu/lb
l	axial distance, ft
N	turbine speed, rpm
NSP	specific speed
p	pressure, lb/sq ft
p_{in}	turbine inlet pressure, psia
R	gas constant, (ft-lb)/(lb)(°R)
r	radius, ft
T	absolute temperature, °R
U	blade speed, ft/sec
V	absolute velocity of gas, ft/sec
W	gas velocity relative to rotor, ft/sec
w	mass flow rate, lb/sec
Z	number of rotor blades at tip of rotor
α	absolute gas angle, angle between absolute velocity vector and meridional plane at mean channel, positive when direction of tangential velocity component and direction of wheel velocity agree and negative when they disagree

β	relative gas angle, angle between the velocity vector relative to wheel and meridional plane at mean channel; same sign convention applies as for α
Γ	torque, ft-lb
γ	ratio of specific heat at constant pressure to specific heat at constant volume
δ	ratio of turbine inlet total pressure to U.S. standard atmospheric pressure, $p'_0/2116.22$
ϵ	function of γ used in relating parameters to those using air inlet conditions at U.S. standard sea-level conditions
	$\frac{\gamma^*}{\gamma} \left[\frac{\left(\frac{\gamma + 1}{2} \right)^{\gamma/(\gamma-1)}}{\left(\frac{\gamma^* + 1}{2} \right)^{\gamma^*/(\gamma^*-1)}} \right]$
η_E, η_I, η_R	modified efficiency figures used in studying loss distribution in turbine, dimensionless
η_S	efficiency based on ratio of inlet total to exit static pressure
η_t	efficiency based on ratio of inlet total to exit total pressure
θ_{cr}	squared ratio of critical velocity at turbine inlet to critical velocity at U.S. standard atmospheric temperature, $\left(V_{cr, 0} / 1019.46 \right)^2$
ν	blade-jet speed ratio, $U_3 / \sqrt{2gJ \Delta h'_{id, s}}$
φ	optimum rotor incidence angle, deg
ρ	gas density, lb/cu ft
Subscripts:	
cr	condition corresponding to Mach 1
E	exit
eq	air equivalent (U.S. standard sea level)
I	incidence
id	ideal
opt	optimum
R	rotor
S	stator
u	tangential component, positive when its direction agrees with that of wheel velocity and negative when it disagrees

- x meridional component, component in plane containing axis of rotation
- 0 station at turbine inlet
- 1 station immediately upstream of stator exit
- 2 station where flow from stator exit is assumed to occupy entire cylindrical area
- 3 station immediately upstream of rotor inlet
- 4 station immediately downstream of rotor exit

Superscripts:

- (') absolute total state
- ('') total state relative to rotor
- (*) U.S. standard sea-level conditions (temperature, 518.67⁰ R; pressure, 14.696 psia)

APPENDIX B

THERMODYNAMIC EQUATIONS

The equations in this appendix are listed in the order used by the computer. A complete set of calculations is made for each specified value of $(V/V_{cr})_1$ at each speed.

$$\left(\frac{\rho V}{\rho' V_{cr}}\right)_1 = \left(\frac{V}{V_{cr}}\right)_1 \left[1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{V}{V_{cr}}\right)_1^2 \right]^{1/(\gamma-1)} \quad (B1)$$

Equations (B2) to (B5) are to be solved simultaneously for p'_1/p'_0 :

$$w = \frac{p'_1}{p'_0} \left(\frac{\rho V}{\rho' V_{cr}}\right)_1 (p' V_{cr})_0 A_1 \cos \alpha_1 \quad (B2)$$

$$\left(\frac{\rho V}{\rho' V_{cr}}\right)_0 = \frac{w}{(\rho' V_{cr} A)_0} \quad (B3)$$

Calculate $(V/V_{cr})_0$ from

$$\left(\frac{\rho V}{\rho' V_{cr}}\right)_0 = \left(\frac{V}{V_{cr}}\right)_0 \left[1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{V}{V_{cr}}\right)_0^2 \right]^{1/(\gamma-1)} \quad (B4)$$

$$\frac{p'_1}{p'_0} = \left\{ 1 - \frac{\frac{\gamma - 1}{\gamma + 1} k_1 \left[\left(\frac{V}{V_{cr}}\right)_0^2 + \left(\frac{V}{V_{cr}}\right)_1^2 \right]}{1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{V}{V_{cr}}\right)_1^2} \right\}^{\gamma/(\gamma-1)} \quad (B5)$$

$$\left(\frac{V}{V_{cr}}\right)_2 = \left[\frac{k_1 + 1}{k + 1} \left(\frac{V}{V_{cr}}\right)_1^2 - \frac{k - k_1}{k + 1} \left(\frac{V}{V_{cr}}\right)_0^2 \right]^{1/2} \quad (\text{B6a})$$

$$\left(\frac{\rho V}{\rho' V_{cr}}\right)_2 = \left(\frac{V}{V_{cr}}\right)_2 \left[1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{V}{V_{cr}}\right)_2^2 \right]^{1/(\gamma-1)} \quad (\text{B6b})$$

$$\frac{p'_2}{p'_0} = \left\{ 1 - \frac{\frac{\gamma - 1}{\gamma + 1} k \left[\left(\frac{V}{V_{cr}}\right)_0^2 + \left(\frac{V}{V_{cr}}\right)_2^2 \right]}{1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{V}{V_{cr}}\right)_2^2} \right\}^{\gamma/(\gamma-1)} \quad (\text{B7})$$

$$\frac{p'_2}{p'_1} = \frac{\frac{p'_2}{p'_0}}{\frac{p'_1}{p'_0}} \quad (\text{B8})$$

$$\alpha_2 = \tan^{-1} \left[\frac{\pi D_1 l \left(\frac{\rho V}{\rho' V_{cr}}\right)_2 \left(\frac{p'_2}{p'_1}\right) \sin \alpha_1}{\left(\frac{\rho V}{\rho' V_{cr}}\right)_1 A_1 \cos \alpha_1} \right] \quad (\text{B9})$$

$$\gamma_2 = \gamma_1 \frac{\sin \alpha_1}{\sin \alpha_2} \quad (\text{B10})$$

$$\left(\frac{V_u}{V_{cr}}\right)_3 = \frac{\gamma_2}{\gamma_3} \left(\frac{V}{V_{cr}}\right)_2 \sin \alpha_2 \quad (\text{B11})$$

Determine $\left(\frac{V_x}{V_{cr}}\right)_3$ from

$$\left(\frac{V_x}{V_{cr}}\right)_3 = \frac{\frac{\gamma_2}{\gamma_3} \left(\frac{\rho V}{\rho' V_{cr}}\right)_2 \cos \alpha_2}{\left\{1 - \frac{\gamma - 1}{\gamma + 1} \left[\left(\frac{V_x}{V_{cr}}\right)_3^2 + \left(\frac{V_u}{V_{cr}}\right)_3^2 \right]\right\}^{1/(\gamma-1)}} \quad (\text{B12})$$

$$\left(\frac{V}{V_{cr}}\right)_3 = \left[\left(\frac{V_x}{V_{cr}}\right)_3^2 + \left(\frac{V_u}{V_{cr}}\right)_3^2 \right]^{1/2} \quad (\text{B13a})$$

$$\left(\frac{\rho V}{\rho' V_{cr}}\right)_3 = \left(\frac{V}{V_{cr}}\right)_3 \left[1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{V}{V_{cr}}\right)_3^2 \right]^{1/(\gamma-1)} \quad (\text{B13b})$$

$$V_{u,3} = V_{cr,0} \left(\frac{V_u}{V_{cr}}\right)_3 \quad (\text{B14a})$$

$$V_{x,3} = V_{cr,0} \left(\frac{V_x}{V_{cr}}\right)_3 \quad (\text{B14b})$$

$$V_3 = V_{cr,0} \left(\frac{V}{V_{cr}}\right)_3 \quad (\text{B14c})$$

$$W_3 = \left(V_3^2 - 2U_3 V_{u,3} + U_3^2 \right)^{1/2} \quad (\text{B14d})$$

$$\cos \beta_3 = \frac{V_{x,3}}{W_3} \quad (\text{B14e})$$

$$\beta_3 = \sin^{-1} \frac{V_{u,3} - U_3}{W_3} \quad (\text{B14f})$$

$$\left(\frac{T''}{T'}\right)_3 = 1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{2U_3 V_{u,3} - U_3^2}{V_{cr,0}^2} \right) \quad (\text{B15a})$$

$$T''_3 = T'_0 \left(\frac{T''}{T'}\right)_3 \quad (\text{B15b})$$

$$\frac{T''_4}{T''_3} = 1 - \frac{U_3^2 - U_4^2}{2gJc_p T''_3} \quad (\text{B15c})$$

$$\left(\frac{W}{W_{cr}}\right)_3 = \frac{\left(\frac{V_x}{V_{cr}}\right)_3}{\left(\frac{T''}{T'}\right)_3^{1/2} \cos \beta_3} \quad (\text{B15d})$$

$$\left(\frac{\rho V_x}{\rho' V_{cr}}\right)_3 = \left(\frac{\rho V}{\rho' V_{cr}}\right)_3 \frac{\left(\frac{V_x}{V_{cr}}\right)_3}{\left(\frac{V}{V_{cr}}\right)_3} \quad (\text{B15e})$$

Calculate the angle φ as follows:

$$\frac{V_{u,3,opt}}{U_3} = 1 - \frac{1.98}{Z} \quad (\text{B16a})$$

$$V_{u, 3, \text{opt}} = U_3 \left(\frac{V_{u, 3, \text{opt}}}{U_3} \right) \quad (\text{B16b})$$

$$W_{u, 3, \text{opt}} = V_{u, 3, \text{opt}} - U_3 \quad (\text{B16c})$$

$$\varphi = \tan^{-1} \frac{W_{u, 3, \text{opt}}}{V_{x, 3}} \quad (\text{B16d})$$

The following three equations are to be solved simultaneously for $(W/W_{\text{cr}})_4$ and $p''_4/p''_{4, \text{id}}$:

$$\left(\frac{\rho W}{\rho'' W_{\text{cr}}} \right)_4 = \frac{\left(\frac{\rho V_x}{\rho' V_{\text{cr}}} \right)_3 \frac{A_3}{A_4 \cos \beta_4}}{\frac{p''_4}{p''_{4, \text{id}}} \left(\frac{T''}{T'} \right)_3^{(\gamma+1)/2(\gamma-1)} \left(\frac{T''_4}{T''_3} \right)^{(\gamma+1)/2(\gamma-1)}} \quad (\text{B17a})$$

$$\frac{p''_4}{p''_{4, \text{id}}} = 1 - \frac{\left\{ \frac{\gamma-1}{\gamma+1} \left[\text{mk} \left(\frac{W}{W_{\text{cr}}} \right)_4^2 + \frac{\left(\frac{W}{W_{\text{cr}}} \right)_3^2}{\frac{T''_4}{T''_3}} \left[\text{mk} + \sin^2 (\beta_3 - \varphi) \right] \right\}^{\gamma/(\gamma-1)}}{1 - \frac{\gamma-1}{\gamma+1} \left(\frac{W}{W_{\text{cr}}} \right)_4^2} \quad (\text{B17b})$$

$$\left(\frac{\rho W}{\rho'' W_{\text{cr}}} \right)_4 = \left(\frac{W}{W_{\text{cr}}} \right)_4 \left[1 - \frac{\gamma-1}{\gamma+1} \left(\frac{W}{W_{\text{cr}}} \right)_4^2 \right]^{1/(\gamma-1)} \quad (\text{B17c})$$

$$W_{\text{cr}, 3} = \frac{W_3}{\left(\frac{W}{W_{\text{cr}}}\right)_3} \quad (\text{B18a})$$

$$W_{\text{cr}, 4} = W_{\text{cr}, 3} \left(\frac{T''_4}{T''_3}\right)^{1/2} \quad (\text{B18b})$$

$$W_4 = W_{\text{cr}, 4} \left(\frac{W}{W_{\text{cr}}}\right)_4 \quad (\text{B18c})$$

$$\left(\frac{T'}{T''}\right)_4 = 1 + \frac{\gamma - 1}{\gamma + 1} \left(\frac{U_4^2 + 2U_4 W_{u, 4}}{W_{\text{cr}, 4}^2}\right) \quad (\text{B18d})$$

$$W_{u, 4} = W_4 \sin \beta_4 \quad (\beta_4 \text{ will be negative}) \quad (\text{B18e})$$

$$\frac{p'_4}{p'_0} = \frac{p'_2}{p'_1} \frac{p'_1}{p'_0} \frac{p''_4}{p''_{4, \text{id}}} \left[\left(\frac{T'''}{T'''}\right)_3 \left(\frac{T''_4}{T''_3}\right) \left(\frac{T'}{T''}\right)_4 \right]^{\gamma/(\gamma-1)} \quad (\text{B19a})$$

$$V_{\text{cr}, 4} = W_{\text{cr}, 4} \left(\frac{T'}{T''}\right)_4^{1/2} \quad (\text{B19b})$$

$$W_4 = W_{\text{cr}, 4} \left(\frac{W}{W_{\text{cr}}}\right)_4 \quad (\text{B19c})$$

$$V_4 = \left[W_4^2 \cos^2 \beta_4 + (U_4 + W_4 \sin \beta_4)^2 \right]^{1/2} \quad (\text{B19d})$$

$$\frac{p_4}{p'_4} = \left[1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{V}{V_{\text{cr}}}\right)_4^2 \right]^{\gamma/(\gamma-1)} \quad (\text{B20a})$$

$$\frac{p_4}{p_0} = \frac{p_4'}{p_0'} \frac{p_4}{p_4'} \quad (\text{B20b})$$

$$V_{u,4} = U_4 + W_4 \sin \beta_4 \quad (\text{B21a})$$

$$T_4' = \frac{T_4'}{T_4''} \frac{T_4''}{T_3'} T_3' \quad (\text{B21b})$$

$$\Delta h' = \frac{V_{u,3} U_3 - V_{u,4} U_4}{gJ} \quad (\text{B22})$$

$$\Delta h'_{id,t} = c_p T_0' \left[1 - \left(\frac{p_4'}{p_0'} \right)^{(\gamma-1)/\gamma} \right] \quad (\text{B23a})$$

$$\Delta h'_{id,s} = c_p T_0' \left[1 - \left(\frac{p_4}{p_0'} \right)^{(\gamma-1)/\gamma} \right] \quad (\text{B23b})$$

$$\eta_t = \frac{\Delta h'}{\Delta h'_{id,t}} \quad (\text{B24a})$$

$$\eta_s = \frac{\Delta h'}{\Delta h'_{id,s}} \quad (\text{B24b})$$

The foregoing equations have given the flow conditions through the turbine and the efficiencies. The following equations give quantities which are useful for special purposes, such as identifying sources of loss in the turbine:

$$L_R = mk \frac{W_3^2 + W_4^2}{2gJ} \quad (\text{rotor loss}) \quad (\text{B25})$$

$$V_0 = V_{cr, 0} \left(\frac{V}{V_{cr}} \right)_0 \quad (\text{B26a})$$

$$V_2 = V_{cr, 0} \left(\frac{V}{V_{cr}} \right)_2 \quad (\text{B26b})$$

$$L_S = k \frac{V_0^2 + V_2^2}{2gJ} \quad (\text{stator loss}) \quad (\text{B26c})$$

$$\Delta \eta_R = \frac{L_R}{\Delta h'_{id, s}} \quad (\text{fractional rotor loss}) \quad (\text{B27})$$

$$\Delta \eta_S = \frac{L_S}{\Delta h'_{id, s}} \quad (\text{fractional stator loss}) \quad (\text{B28})$$

$$L_E = \frac{V_4^2}{2gJ} \quad (\text{exit loss}) \quad (\text{B29a})$$

$$\Delta \eta_E = \frac{L_E}{\Delta h'_{id, s}} \quad (\text{fractional exit loss}) \quad (\text{B29b})$$

$$L_I = \frac{W_3^2 \sin^2(\beta_3 - \varphi)}{2gJ} \quad (\text{incidence loss}) \quad (\text{B30a})$$

$$\Delta \eta_I = \frac{L_I}{\Delta h'_{id, s}} \quad (\text{fractional incidence loss}) \quad (\text{B30b})$$

$$\left. \begin{aligned} \eta_E &= \eta_S + \Delta\eta_E \\ \eta_R &= \eta_E + \Delta\eta_R \\ \eta_I &= \eta_R + \Delta\eta_I \end{aligned} \right\} \quad (\text{B31})$$

$$\theta_{\text{cr}} = \left(\frac{V_{\text{cr}, 0}}{1019.46} \right)^2 \quad (\text{B32a})$$

$$\epsilon = \frac{1.40}{\gamma} \left[\frac{\left(\frac{\gamma + 1}{2} \right)^{\gamma/(\gamma-1)}}{1.8929} \right] \quad (\text{B32b})$$

$$\delta = \frac{p'_0}{2116.22} \quad (\text{B32c})$$

$$w^{\text{eq}} = w \frac{\sqrt{\theta_{\text{cr}}}}{\delta} \epsilon \quad (\text{equivalent weight flow}) \quad (\text{B33})$$

$$\frac{wN}{\delta} \epsilon \quad (\text{weight flow-speed parameter}) \quad (\text{B34})$$

$$N_{\text{eq}} = \frac{N}{\sqrt{\theta_{\text{cr}}}} \quad (\text{equivalent speed}) \quad (\text{B35})$$

$$\nu = \frac{U_3}{\sqrt{2gJ \Delta h'_{\text{id}, s}}} \quad (\text{blade-jet speed ratio}) \quad (\text{B36})$$

$$\Delta h'_{\text{eq}} = \frac{\Delta h'}{\theta_{\text{cr}}} \quad (\text{equivalent specific work}) \quad (\text{B37})$$

$$\text{NOT} = \frac{N}{\sqrt{T_0}} \quad (\text{B38})$$

$$W_{TOP} = \frac{w \sqrt{T'_0}}{p'_{in}} \quad (B39)$$

where $p'_{in} = p'_0/144$

$$TOP = \frac{\Gamma}{p'_{in}} \quad (B40)$$

where $\Gamma = 89\ 155\ w\ \Delta h'/N$ (in. -lb)

$$NSP = \frac{N\sqrt{Q}}{H^{3/4}} \quad (\text{specific speed}) \quad (B41)$$

where $Q = w/\rho_4$ and $H = J\ \Delta h'_{id,t}$.

For calculation of ρ_4 ,

$$p_4 = \frac{p_4}{p'_0} p'_0$$

$$T_4 = T'_4 \left[1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{V}{V_{cr4}} \right)^2 \right]$$

$$\rho_4 = \frac{p_4}{RT_4}$$

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