A FRACTAL-FRACTIONAL MODEL ON IMPACT STRESS OF CRUSHER DRUM

by

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In this paper, a fractal-fractional model of the impact stress on the crusher drum is established by using He's fractal derivative and the fluid-solid coupling vibration equation. The two-scale transform is used to obtain its solution, which can be used to improve the safety performance of beating machines.

Key word: *He's fractal derivative, the fluid solid coupling vibration equation, beating engine, two-scale transform method, impact stress*

Introduction

As an indispensable mechanical equipment in industrial and agricultural production, the beater has attracted extensive attention because of the complex and harsh working environment, wide application fields and strong production applicability [1-4]. The crushing drum used in the market beating machine is mostly immersed in a complex environment of solid-liquid mixing [5]. Therefore, the stress calculation on the crushing drum is also cumbersome.

The crushing drum working in the liquid environment is mainly affected by the repeated impact stress [6, 7]. In the study of fluid impact stress in engineering, the micro-element method is usually used to cut the fluid into an infinite number of discontinuous micro--elements, therefore, the sweet potato serous fluid can be regarded as a porous environment [8], see fig. 1.

According to the momentum theorem, we can calculate the impact stress caused by each serous fluid unit and the fluid solid coupling vibration equation can be used to obtain the impact stress caused by the whole serous fluid [9-12]. The momentum model of the serous fluid unit is shown in fig. 2.

Figure 1. Porous structure model of sweet potato serous fluid

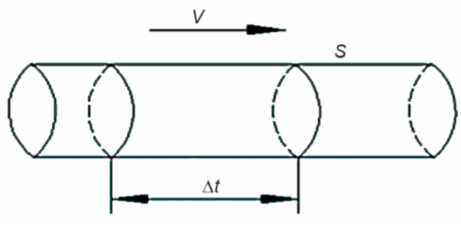

Figure 2. Fluid unit model of the sweet potato serous fluid

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In order to better complete the relevant calculation and improve the accuracy of the calculation model, this paper uses the definition of fractal derivative proposed by He [13-20] to establish a mathematical model between discontinuous fluid units.

The 2-D fractal derivatives are defined [20-22]:

$$\frac{{}^{H}\partial\Psi}{\partial t^{\alpha}}(t_0,x)=\Gamma(1+\alpha)\lim_{\substack{t-t_0=\Delta t\\ \Delta t\neq 0}}\frac{\Psi(t,x)-\Psi(t_0,x)}{(t-t_0)^{\alpha}} \tag{1}$$

$$\frac{{}^{H}\partial\Psi}{\partial x^{\beta}}(t,x_0)=\Gamma(1+\beta)\lim_{\substack{x-x_0=\Delta x\\ \Delta x\neq 0}}\frac{\Psi(t,x)-\Psi(t,x_0)}{(x-x_0)^{\beta}} \tag{2}$$

The previous fractal derivatives follow the chain rule [23]

$$\frac{{}^{H}\partial\Omega}{\partial t^{\alpha}\partial x^{\beta}}=\frac{{}^{H}\partial}{\partial t^{\alpha}}\left(\frac{{}^{H}\partial\Omega}{\partial x^{\beta}}\right) \tag{3}$$

$$\frac{{}^{H}\partial\Omega}{\partial t^{3\beta}}=\frac{{}^{H}\partial}{\partial t^{\beta}}\left[\frac{{}^{H}\partial}{\partial t^{\beta}}\left(\frac{{}^{H}\partial\Omega}{\partial t^{\beta}}\right)\right] \tag{4}$$

$$\frac{{}^{H}\partial\Omega}{\partial t^{3\alpha}\partial x^{\beta}}=\frac{{}^{H}\partial}{\partial t^{\alpha}}\left\{\frac{{}^{H}\partial}{\partial t^{\alpha}}\left[\frac{{}^{H}\partial}{\partial t^{\alpha}}\left(\frac{{}^{H}\partial\Omega}{\partial x^{\beta}}\right)\right]\right\} \tag{5}$$

Two-scale transformation

The two-scale transformation method proposed by He in 2019 [24] is a main mathematical tool, which is widely used in computing fractal derivatives [25-28]. Using this method we can easily solve the fractal-fractional models. In order to elucidate the two-scale transformation method, we consider the following fractal-fractional differential equation:

$$\frac{{}^{H}\partial}{\partial x^{\beta}}\left[\frac{{}^{H}\partial\Psi}{\mathrm{d}t^{\alpha}}(t,x)\right]+\frac{{}^{H}\partial\Omega}{\partial x^{\beta}}(t,x)+p(t,x)=0 \tag{6}$$

In this regard, we make the following assumptions

$$T=t^{\alpha} \tag{7}$$

$$X=x^{\beta} \tag{8}$$

where t and x are considered from the microscopic point of view, T and X are considered from a macro perspective, α and β are the fractal dimensions [27, 29, 30].

Based on this formula, we can convert eq. (6) to the following form:

$$\frac{\partial}{\partial X}\left[\frac{\partial\Psi}{\partial T}(T,X)\right]+\frac{\partial\Omega(T,X)}{\partial X}+p(T,X)=0 \tag{9}$$

In the end, the fractal fractional differential equation is transformed into the form of traditional differential equation, which can be solved by Taylor series method [31, 32], the finite integration method [33], the exp-function method [34], and the homotopy perturbation method [35].

Based on the mentioned methods, this paper will solve the impact stress model by using the two-scale transformation formula after we establish the impact stress fractal model.

The fractal model of impact stress

In the actual working environment, the crushing drum will be repeatedly impacted by the serous fluid, at the same time, the kinetic energy of the serous fluid will change under the joint action of the impact stress of the serous fluid on the cylinder wall and the reaction force of the cylinder wall on the serous fluid due to rotation. Considering the material of the drum wall and the physical properties of the impact fluid, the fluid structure coupling analysis of the impact circuit is carried out in this paper.

As shown in fig. 3, it is a 3-D model of the crushing drum. According to the working principle of the beater, the periodic movement of the crushing drum will cause the volume in the drum to change, which will result the flow pulsation [36]. The flow pulsation will cause the velocity of serous fluid to change periodically with time, and the pulsating fluid velocity can be expressed:

$$v = v_0 \left[1 + \sum_{i=1}^{n} \Lambda \sin(\omega_i t) \right] \tag{10}$$

where v_0 is the average velocity of the fluid, Λ – the velocity fluctuation amplitude, and ω – the velocity fluctuation frequency.

Since there is only one inlet and outlet of the crushing drum, we can see that $n = 1$. Therefore, eq. (10) can be simplified:

$$v = v_0 [1 + \Lambda \sin(\omega t)] \tag{11}$$

Letting $v_0 = 3$ m/s, $\Lambda = 0.86$, we can plot the function between speed and time when $\omega = 0.5\pi, \pi, 1.5\pi, 2\pi$, fig. 4.

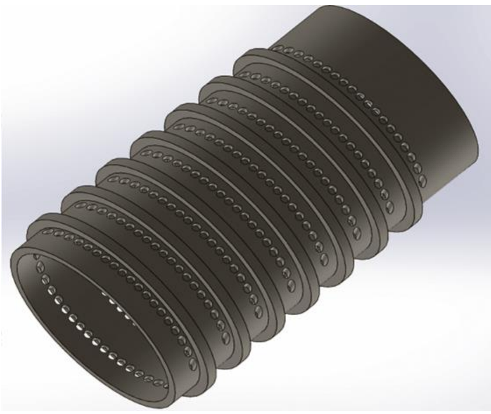

Figure 3. The 3-D model of the crushing drum

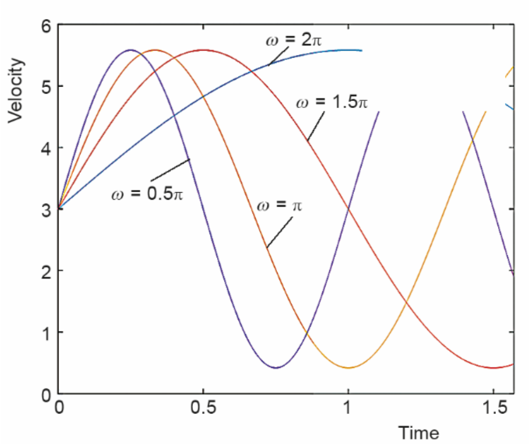

Figure 4. The velocity fluctuation with different frequencies (*ω* = 0.5*π*, *π*, 1.5*π*, 2*π*)

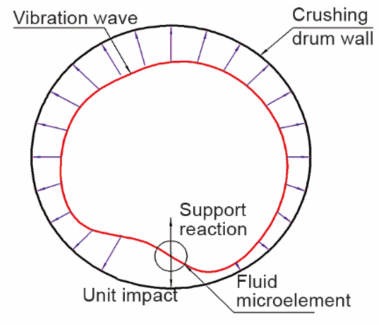

Figure 5. Schematic picture of impact stress on crushing drum

As shown in fig. 5, according to the coupled vibration formula, the momentum equation under fluid impact had been obtained [36, 37], on this basis, we can establish its fractal-fractional form:

$$m_\kappa \frac{{}^H\partial}{\partial t^\alpha}\left(\frac{{}^H\partial\Omega}{\partial t^\alpha}\right)+\frac{{}^H\partial(pA_\kappa)}{\partial x^\beta}+$$
$$+2m_\kappa v_0\frac{{}^H\partial}{\partial t^\alpha}\left(\frac{{}^H\partial\Omega}{\partial x^\beta}\right)+\frac{1}{2}m_\kappa v_0^2\Lambda^2\frac{{}^H\partial}{\partial x^\beta}\left(\frac{{}^H\partial\Omega}{\partial x^\beta}\right)+$$
$$+m_\kappa g\sin\theta+f_t=$$
$$=-m_\kappa v_0\Lambda\left(1+\frac{{}^H\partial\Omega}{\partial x^\beta}\right)\cos(\omega t)-2m_\kappa v_0\Lambda\cdot$$
$$\cdot\left[\frac{{}^H\partial}{\partial t^\alpha}\left(\frac{{}^H\partial\Omega}{\partial x^\beta}\right)+v_0\frac{{}^H\partial}{\partial x^\beta}\left(\frac{{}^H\partial\Omega}{\partial x^\beta}\right)\right]\sin(\omega t)+\frac{1}{2}m_\kappa v_0^2\Lambda^2\frac{{}^H\partial}{\partial x^\beta}\left(\frac{{}^H\partial\Omega}{\partial x^\beta}\right)\cos(2\omega t) \tag{12}$$

where m_κ is the unit mass of fluid, Ω – the axial displacement, A_κ – the cross sectional area of fluid, θ – the angle between work plane and horizontal plane, p – the unit pressure, f_t – the axial internal force of pipeline, and ${}^H\partial\Omega/\partial t^\alpha$ and ${}^H\partial\Omega/\partial t^\beta$ – the fractal derivative.

According to the axial and radial vibration equations of the cylinder wall under fluid impact, the change rate of fluid cross-section area A_κ to time can be obtained:

$$\frac{{}^H\partial A_\kappa}{\partial t^\alpha}-\frac{2RA_\kappa}{E\delta}\frac{{}^H\partial p}{\partial t^\alpha}+\frac{\mu A_p R}{\delta}\left[\frac{{}^H\partial}{\partial t^\alpha}\left(\frac{{}^H\partial\Omega}{\partial x^\beta}\right)+\frac{{}^H\partial\psi}{\partial x^\beta}\frac{{}^H\partial}{\partial x^\beta}\left(\frac{{}^H\partial\Omega}{\partial x^\beta}\right)\right]+$$
$$+\left\{\frac{{}^H\partial A_\kappa}{\partial x^\beta}-\frac{2RA_f}{E^*}\frac{{}^H\partial p}{\partial x^\beta}+\frac{\mu A_p R}{\delta}\left[\frac{{}^H\partial}{\partial x^\beta}\left(\frac{{}^H\partial\Omega}{\partial x^\beta}\right)+\frac{{}^H\partial\psi}{\partial x^\beta}\frac{{}^H\partial}{\partial x^\beta}\left(\frac{{}^H\partial\Omega}{\partial x^\beta}\right)\right]\right\}v_0=$$
$$=\left\{\frac{2RA_\kappa}{E^*\delta}\frac{{}^H\partial p}{\partial x^\beta}-\frac{{}^H\partial A_f}{\partial x^\beta}-\frac{\mu A_\kappa R}{\delta}\left[\frac{{}^H\partial}{\partial t^\alpha}\left(\frac{{}^H\partial\Omega}{\partial x^\beta}\right)+\frac{{}^H\partial\psi}{\partial x^\beta}\frac{{}^H\partial}{\partial x^\beta}\left(\frac{{}^H\partial\Omega}{\partial x^\beta}\right)\right]\right\}v_0\Lambda\sin(\omega t) \tag{13}$$

where A_p is the cross-sectional area of crushing drum, which is easy to know in practical engineering, ψ – the radial displacement, R – the radius of curvature, μ – the Poisson ratio, δ – the wall thickness of crushing drum, and ${}^H\partial A_\kappa/\partial t^\alpha$, ${}^H\partial p/\partial t^\alpha$, and ${}^H\partial A_f/\partial t^\beta$ – the fractal derivative.

Next, we use the method of two-scale transformation.

Assume:

$$T=t^\alpha \tag{14}$$

$$X=x^\beta \tag{15}$$

Equation (12) can be converted into:

$$m_\kappa \frac{\partial^2 \Omega}{\partial T^2} + \frac{\partial (pA_\kappa)}{\partial X} + 2m_\kappa v_0 \frac{\partial}{\partial T}\left(\frac{\partial \Omega}{\partial X}\right) + \frac{1}{2} m_\kappa v_0^2 \Lambda^2 \frac{\partial^2 \Omega}{\partial X^2} + m_\kappa g \sin\theta + f_t =$$

$$= -m_\kappa v_0 \Lambda \left(1 + \frac{\partial \Omega}{\partial X}\right)\cos(\omega T) - 2m_\kappa v_0 \Lambda \left[\frac{\partial}{\partial T}\left(\frac{\partial \Omega}{\partial X}\right) + v_0 \frac{\partial^2 \Omega}{\partial X^2}\right]\sin(\omega T) +$$

$$+\frac{1}{2} m_\kappa v_0^2 \Lambda \frac{\partial^2 \Omega}{\partial X^2}\cos(2\omega T) \tag{16}$$

In the same way, double scale change is performed for eq. (13):

$$\frac{\partial A_\kappa}{\partial T} - \frac{2RA_\kappa}{E^*\delta}\frac{\partial p}{\partial T} + \frac{\mu A_p R}{\delta}\left[\frac{\partial}{\partial T}\left(\frac{\partial \Omega}{\partial X}\right) + \frac{\partial \psi}{\partial X}\frac{\partial}{\partial X}\left(\frac{\partial \Omega}{\partial X}\right)\right] +$$

$$+\left\{\frac{\partial A_\kappa}{\partial X} - \frac{2RA_\kappa}{E^*}\frac{\partial p}{\partial X} + \frac{\mu A_p R}{\delta}\left[\frac{\partial}{\partial X}\left(\frac{\partial \Omega}{\partial X}\right) + \frac{\partial \psi}{\partial X}\frac{\partial}{\partial X}\left(\frac{\partial \Omega}{\partial X}\right)\right]\right\} v_0 =$$

$$= \left\{\frac{2RA_\kappa}{E^*\delta}\frac{\partial p}{\partial X} - \frac{\partial A_\kappa}{\partial X} - \frac{\mu A_p R}{\delta}\left[\frac{\partial}{\partial T}\left(\frac{\partial \Omega}{\partial X}\right) + \frac{\partial \psi}{\partial X}\frac{\partial}{\partial X}\left(\frac{\partial \Omega}{\partial X}\right)\right]\right\} v_0 \Lambda \sin(\omega t) \tag{17}$$

When $\alpha = \beta = 1$, we find that eq. (12) equals eq. (16), and eq. (13) equals eq. (17).

Conclusion

In this paper, the derivative of the coupled vibration equation is modelled by the fractal-fractional differential model using He's fractal derivative, and the impact stress model of slurry on the wall of the crushing drum is obtained. Obviously, when α and β are equal to 1, the fractal-fractional model is converted to the traditional one.

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