A Frequency and Timing Period Acquisition Technique for OFDM Systems

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Abstract This paper introduces a frequency and timing period acquisition technique for orthogonal frequency division multiplexing (OFDM) systems. The proposed technique estimates both the frequency and timing period offsets at the time by using only one pilot symbol with its suitable frequency assignment. Pseudo noise (PN) sequences are introduced to assign these frequencies of the pilot symbol so that the acquisition range can be widened. Numerical examples are given to show the estimate variances for both additive white Gaussian noise (AWGN) and multipath fading channels.

1 Introduction

Recently, OFDM[1] has been receiving a lot of attention in the field of broadcasting because of its robustness against the effects of multipath propagation which are main obstructions for mobile reception and realization of single frequency network (SFN). OFDM is already decided as the modulation method for the digital audio broadcasting in Europe[2], and is continuously recommended for the digital terrestrial TV systems by many proponents.

A frequency offset due to the frequency difference of local oscillators between the transmitter and receiver disturbs orthogonality of the received carriers, so that a demodulated signal of one carrier is distributed into other carriers as interference. This interference is called intercarrier interference (ICI) and causes degradation of the bit error rate of the demodulated signals.

The amount of ICI has been studied by Moose[3], and also by Classen and Meyr [4]. According to the studies, for example, in an OFDM system using 1000 carriers a frequency offset of several percent of the intercarrier spacing results in interference of -20dB to a signal. Also they have derived frequency offset estimation techniques in which repeated pilot symbols are transmitted and maximum likelihood estimation techniques are employed.

Since a timing period offset also perturbs orthogonality of received carriers, the timing period synchronization is another important factor to obtain responsible demodulated signals.

In this paper we derive a frequency and timing period acquisition scheme which estimates both frequency and timing period offsets at the same time by using only one pilot symbol. We also elaborate our acquisition technique to cope with large frequency offsets by introduction of PN sequences for frequency assignment to the pilot symbol.

PN sequences have been used for frequency synchronization algorithms in [4, 5]; however, they are used for symbol assignment to the differential encoding of adjacent carriers.

The remainder of the paper is organized as follows. Section 2 describes the system model. The frequency and timing period acquisition scheme is derived in Section 3. Numerical examples for AWGN and multipath fading channels are shown in Section 4. Finally, some concluding remarks are provided in Section 5.

2 System Model

An OFDM transmission system is depicted in Fig.1, where the block shown by the broken line is a proposed frequency and timing period offset estimator, whose operation will be described later.

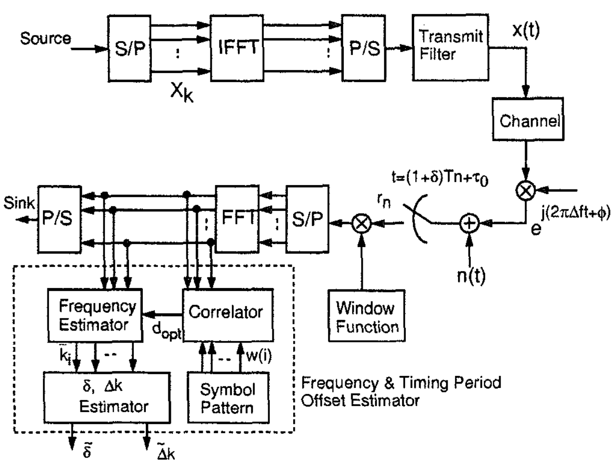

Figure 1: OFDM system block diagram

An OFDM transmitted baseband symbol using N car-

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riers is given by

$$x(t) = \frac{1}{N} \sum_{k=-\frac{N}{2}+N_r}^{\frac{N}{2}-1-N_r} X_k e^{j2\pi f_k t} g(t) \qquad (1)$$

where X_k is a complex symbol for the carrier of $f_k = k/(NT)$, T denotes the basic transmission timing period, and N_r is the number of the carriers (for each side) which are not transmitted and prevent the received signal from aliasing. The pulse shape $g(t)$ is rectangular and given by

$$g(t) = \begin{cases} 1 & -T_g \leq t < T_s \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

where $T_g = N_g T$ is called the guard interval, $T_s = NT$ the useful or effective period, and $T_s + T_g$ is the transmitted symbol period.

A frame structure shown in Fig.2 is assumed to transmit the OFDM symbols. The first interval in the frame is a so-called null symbol ($X_k = 0, \forall k$), and the second is assigned for the pilot symbol whose main roles involve frequency and timing synchronization. By observing the null symbol it becomes easy to find a beginning time of the pilot symbol. The remainder of the frame is used for data transmission.

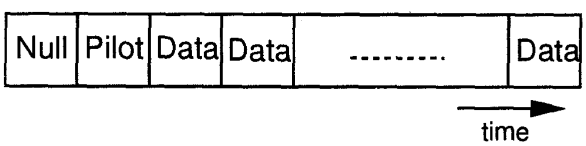

Figure 2: Transmission frame structure

The impulse response of the channel, including the physical channel and the receiver filter, is assumed to be in the form of

$$h(\tau; t) = \sum_{m=0}^{M-1} h_m(t) \delta(\tau - T_m) \qquad (3)$$

where M is the number of the multipaths in the channel and $h_m(t)$ represents the equivalent lowpass impulse response for the m-th propagation path. It is also assumed that the guard interval is set to be longer than the maximum length of the delay T_m ($0 \leq T_m < T_g$) so that no intersymbol interference (ISI) occurs.

The output signal $y(t)$ from the channel is written as

$$\begin{aligned} y(t) &= \int_0^{T_g} h(\tau; t) x(t - \tau) d\tau \\ &= \sum_{m=0}^{M-1} h_m(t) \frac{1}{N} \sum_k X_k e^{j2\pi f_k (t - T_m)} g(t - T_m). \end{aligned}$$

Before the frequency synchronization, the received signal is affected by the frequency offset Δf and the phase offset ϕ. Hence $y(t)$ is modified as

$$z(t) = y(t) e^{j(2\pi \Delta f t + \phi)}. \qquad (4)$$

Let z_n denote the discrete signal of $z(t)$ sampled at $t = n(1 + \delta)T + \tau_0$, where δ is the timing period offset and τ_0 is the timing phase offset at $n = 0$. By defining $\Delta k = \Delta f \cdot NT$, it is easy to see that

$$\begin{aligned} z_n &= \frac{1}{N} e^{j(\phi + 2\pi \frac{\Delta k}{NT} \tau_0)} \sum_{m=0}^{M-1} h_m(n(1+\delta)T + \tau_0) \\ &\quad \cdot \sum_k X_k e^{j \frac{2\pi k}{NT} (\tau_0 - T_m)} e^{j2\pi \frac{n(1+\delta)(k + \Delta k)}{N}} \\ &\quad \cdot g(n(1+\delta)T + \tau_0 - T_m). \end{aligned} \qquad (5)$$

For easy derivation, it is assumed that the impulse response $h(\tau; t)$ can be regarded as a constant at least one symbol period, i.e., $h_m(n(1+\delta)T + \tau_0) \equiv h_m$, $n = 0, ..., N-1$. Additionally, it is assumed that the pilot symbol period has an adequate length, namely, longer than the other data symbol period so that no adjacent symbol is sampled even in the case of the supposed largest timing phase shift. Under these assumptions, z_n can be written as

$$z_n = \frac{1}{N} e^{j(2\pi \frac{\Delta k s}{N} + \phi)} \sum_k H_k e^{j2\pi \frac{ks}{N}} X_k e^{j2\pi \frac{(1+\delta)(k+\Delta k)n}{N}} \qquad (6)$$

where $\tau_0 = sT$, and $H_k = \sum_{m=0}^{M-1} h_m e^{-j2\pi \frac{kT_m}{NT}}$ represents the response at the frequency [1] of k.

Since the input signal to FFT, denoted by r_n, is corrupted by AWGN, it follows that

$$r_n = z_n + \eta_n \qquad (7)$$

where η_n is the sample of the equivalent lowpass noise of $n(t)$ and is a zero-mean complex Gaussian random variable with variance $\sigma_\eta^2 = \frac{1}{2} E[|\eta_n|^2]$.

3 Frequency and Timing Period Acquisition Technique

3.1 Δk and δ estimation

Several papers have studied frequency offset estimation techniques for OFDM systems [3-6]. In [3, 4], for example, a maximum likelihood estimation technique is employed for the phase estimation of the repeated transmitted pilot symbols. In these studies, however, the correct timing has been assumed.

Here we propose a technique which estimates frequency and timing period offsets at the same time by using only one pilot symbol. In our proposal, the pilot symbol uses only $P(< N)$ carrier frequencies of $k = k_i$, $i = 0, ..., P-1$ with symbols $X_{k_i} (\neq 0)$, and no other frequencies are transmitted. An example of frequency assignment is illustrated in Fig.3 for the carrier spacing of $L_P = 64$ and the number of carriers $P = 13$.

To explain the basic idea for the offset estimation, suppose that two carriers of $k = k_1$ and k_2 are transmitted. Noting that the sampled signals at the receiver are given

[1] For the sake of convenience, k is simply called frequency for f_k.

by (6), the corresponding frequency to the transmitted frequency of $k = k_i$ is given by

$$k_i' = (1+\delta)(k_i + \Delta k), \quad i = 1,\ 2. \tag{8}$$

The estimated offset values

$$\tilde{\Delta} k = \frac{k_2\tilde{k}_1 - k_1\tilde{k}_2}{\tilde{k}_2 - \tilde{k}_1}, \quad \tilde{\delta} = \frac{\tilde{k}_2 - \tilde{k}_1}{k_2 - k_1} - 1 \tag{9}$$

can be calculated if we find the estimate frequencies $\tilde{k}_1,\ \tilde{k}_2$ for $k_1',\ k_2'$ from the received signal sequence $\{r_n\}$.

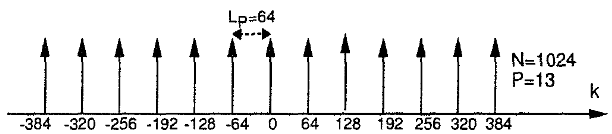

Figure 3: Frequency assignment of the pilot symbol with equal spacing

Since the received signal is corrupted by AWGN, $\tilde{k}_1$ and $\tilde{k}_2$ have to have uncertainties. To mitigate these uncertainties, more than two carriers are transmitted as shown in Fig.3 and averages are taken. In the receiver the i-th estimated offsets

$$\begin{aligned} \tilde{\Delta} k_i &= \frac{k_{i+1}\tilde{k}_i - k_i\tilde{k}_{i+1}}{\tilde{k}_{i+1} - \tilde{k}_i}, & (10) \\ \tilde{\delta}_i &= \frac{\tilde{k}_{i+1} - \tilde{k}_i}{k_{i+1} - k_i} - 1 & (11) \end{aligned}$$

are calculated for each pair of frequencies $(k_i,\ k_{i+1})$, $i = 0, ..., P-2$, and then the estimated offset values of Δk and δ are given by

$$\begin{aligned} \tilde{\Delta} k &= E[\tilde{\Delta} k_i], & (12) \\ \tilde{\delta} &= E[\tilde{\delta}_i] & (13) \end{aligned}$$

where $E[\cdot]$ denotes the ensemble average over all i.

Under multipath fading channels or lower signal to noise ratios (SNR), some $\tilde{k}_i$'s might not be found. If $\tilde{k}_{i+1}$ is not found, a different pair of frequencies (k_i, k_{i+2}), or more generally (k_i, k_{i+j}) for some j, can be used instead of (k_i, k_{i+1}) for calculation of $\tilde{\Delta} k_i$ and $\tilde{\delta}_i$ in (10) and (11).

3.2 Frequency Estimation by using Hanning Window Function

In order to find $\tilde{k}_i$, we use a well known frequency estimation method which employs FFT with the Hanning window function. The frequency is estimated from the FFT output of the signal sequence $\{b_n r_n\}_{n=0}^{N-1}$ where b_n is the Hanning window function defined by

$$b_n = 1 - \cos(\frac{2\pi n}{N}), \quad n = 0, ..., N-1. \tag{14}$$

This method has two points: one is that it attains high precision for estimation, and the other is this method can estimate plural frequencies at the same time since the Hanning window strongly suppresses side-lobes. However, the interspace of transmitted carrier frequencies has to be larger than 4 because the Hanning window broadens the spectrum.

Let G_l denote the FFT output of $g_n = b_n r_n = b_n(z_n + \eta_n)$, then

$$\begin{aligned} G_l \;=\; & -\frac{1}{N} e^{j(2\pi\frac{\Delta k s}{N}+\phi)} \\ & \cdot \sum_k H_k e^{j2\pi\frac{ks}{N}} X_k e^{j\pi\{(1+\delta)(k+\Delta k)-l\}} \\ & \cdot \frac{\sin\pi\{(1+\delta)(k+\Delta k)-l\}}{\sin\frac{\pi}{N}\{(1+\delta)(k+\Delta k)-l\}} \\ & \cdot \frac{\sin^2(\frac{\pi}{N})}{\sin\frac{\pi}{N}\{(1+\delta)(k+\Delta k)-l+1\}} \\ & \cdot \frac{\cos\frac{\pi}{N}\{(1+\delta)(k+\Delta k)-l\}}{\sin\frac{\pi}{N}\{(1+\delta)(k+\Delta k)-l-1\}} \\ & + (\text{noise component}). \end{aligned} \tag{15}$$

The sequence $\{|G_l|\}$ has local peaks at the frequencies of $l = l_i$ which are on the integer values near $(1+\delta)(k_i+\Delta k)$ for $i = 0, ..., P-1$. By using $|G_l|$ at $l = l_i$ and l_{i-1}; or $|G_l|$ at $l = l_i$ and l_{i+1}, the frequencies are estimated as

$$\tilde{k}_i = l_i + \frac{1-2\gamma_i}{1+\gamma_i}, \text{ where } \gamma_i = \frac{|G_{l_i-1}|}{|G_{l_i}|}, \tag{16}$$

or

$$\tilde{k}_i = l_i - \frac{1-2\beta_i}{1+\beta_i}, \text{ where } \beta_i = \frac{|G_{l_i+1}|}{|G_{l_i}|}. \tag{17}$$

3.3 Correspondence Detection by Correlation

The basic estimation scheme for Δk and δ was derived. In ordinary cases, it is safe to assume $|\delta| << 1$ because the timing period offset is rather small, e.g., less than several 100 ppm. However, the frequency offset Δk may be larger than 1 in terms of the carrier spacing (i.e., $\Delta f > 1/(NT)$), and some cases, may be more than several tens. Taking into account this fact, the transmitted frequency of $k = k_i$ does not always appear at or near the frequency of $l = k_i$ after the FFT in the receiver. Therefore, the first thing to recognize is the correspondence of the frequencies of l_i detected in the receiver to the transmitted frequencies of k_i.

This can be realized by taking correlation between the transmitted frequency pattern and the absolute values of the FFT output over the frequency domain. To represent the transmitted frequency pattern, a function $w(k)$ is defined by

$$w(k) = \begin{cases} 1 & \text{if } X_k \neq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

The receiver searches the optimal frequency shift, denoted by d_{opt}, which gives the maximum correlation of $|G_k|$ and

$w(k-d)$ in the frequency shift range $-d_{max} \leq d \leq d_{max}$; in words

$$d_{opt} = d; \text{ such that } \max_{d} \sum_{k} w(k-d)|G_k|. \quad (19)$$

The frequency shift range can be regarded as the frequency acquisition range.

In case of $|\delta| << 1$, d_{opt} almost gives the nearest integer to Δk and hence a rough estimation of Δk. Thus the correspondence of the detected frequencies of l_i to the transmitted frequencies of k_i is now recognized by $l_i = k_i + d_{opt}$. Therefore (10) and (11) can be calculated under the correct relationship.

Here it should be noted that correlation in (19) should be calculated only for the integer values of d in the frequency shift range. Minute evaluation, which is necessary for the acquisition process shown in [4], is not required.

The acquisition algorithm is summarized below:

Step 1) Find the rough frequency offset d_{opt} by (19).

Step 2) Estimate $\tilde{k}_i$ which locates near $l_i = k_i + d_{opt}$ by (16) or (17) for each i.

Step 3) Calculate $\tilde{\Delta} k_i$ and $\tilde{\delta}_i$ for each i by (10) and (11). If $\tilde{k}_{i+1}$ is not detected near $k_i + d_{opt}$, then use k_{i+2} and $\tilde{k}_{i+2}$ instead of k_{i+1} and $\tilde{k}_{i+1}$, and so on. Estimate $\tilde{\Delta} k$ and $\tilde{\delta}$ by (12) and (13).

Step 4) Control the local oscillators in the receiver so that the Δk and δ will become zero. After the initial acquisition process, a tracking operation is performed if necessary [2]. Compensation for the timing phase shift τ_0 can be achieved for example by estimation of the channel impulse response[2].

3.4 Frequency Assignment by PN Sequences

As long as $d_{max} < \frac{L_P}{2}$, only one main peak exists in the frequency acquisition range in which the correlations are taken, and d_{opt} can be found correctly if Δk is within the frequency acquisition range.

To cope with a larger frequency offset, the frequency range has to be widened and hence d_{max} might be larger than $\frac{L_P}{2}$. In this case, more than one dominant correlation peak appears with the period of L_P along the frequency shift d when the frequencies of the pilot symbol are assigned with equal spacing of L_P. Consequently, a wrong d_{opt} could be chosen for lower SNRs or deep fades under frequency selective channels.

We introduce PN sequences for frequency assignment to the pilot symbol for the correct acquisition of d_{opt} over wider frequency ranges.

Let $V(i)$ denote a PN sequence with length of P, and define L_0 as a constant integer satisfying $0 < L_0 < L_P$. Using $V(i)$, the frequencies of the pilot symbol are defined by

$$k_i = i \cdot L_P + L_0 \cdot V(i) + const., \quad i = 0, ..., P-1 \quad (20)$$

[2] For example, this can be done by a decision directed method.

An example of the frequency assignment is shown in Fig.4. In this example $V(i)$ is selected as a PN sequence of length $P = 13$ and given by $V(i) = \{0101001100000\}$[8]. The remainder conditions are set as $L_P = 64, L_0 = 32$, and $const. = -384$.

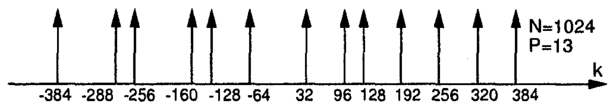

Figure 4: Frequency assignment of the pilot symbol with PN sequence

An alternative is that only the frequencies of $k_i = i \cdot L_P + const.$ for i such that $V(i) = 1$ are sent as the pilot symbol.

Applying these techniques to the frequency assignment of the pilot symbol, the search of d_{opt} becomes more correct in wider frequency shift ranges ($d_{max} > \frac{L_P}{2}$), and hence wider acquisition ranges can be attained.

4 Numerical Examples

Computer simulation has been conducted for evaluation of the characteristics of the acquisition process for both AWGN and multipath fading channels. In both cases, $N = 1024$, $N_r = 96$, and the initial offsets are set as $\Delta k = 10.2$ and $\delta = 10^{-4}$.

4.1 AWGN Channels

In the first example a system under AWGN channels is considered. In this example, the frequencies of the pilot symbol are assigned by equal spacings.

Fig.5 shows the simulated results of the frequency estimate variance $\sigma^2_{\tilde{\Delta} k} = E[(\tilde{\Delta} k - E[\tilde{\Delta} k])^2]$ against the received SNRs for several L_Ps from 8 to 64. For all L_Ps, $d_{max} = 96$. Here the received SNR implies the total signal power of the carriers in the pilot symbol to the noise power ratio. For $L_P = 64$ and $P = 13$, the $\sigma^2_{\tilde{\Delta} k}$ for the SNRs over 15dB attain 10^{-4} or less.

The results of the timing period estimate variance $\sigma^2_{\tilde{\delta}} = E[(\tilde{\delta} - E[\tilde{\delta}])^2]$ are given in Fig.6. For $L_P = 64$, the $\sigma^2_{\tilde{\delta}}$ for the SNRs of 20dB or more are less than 10^{-10}.

Both in Fig.5 and 6, the estimate variances $\sigma^2_{\tilde{\Delta} k}$ and $\sigma^2_{\tilde{\delta}}$ become smaller for the smaller P at the same SNR since the signal power for a sub-carrier becomes larger. The rate of increase for the estimate variances for the larger P becomes larger because the amount of ICIs increases with the number of P.

4.2 Multipath Fading Channels

The second example evaluates the acquisition characteristics for a system under multipath Rayleigh fading

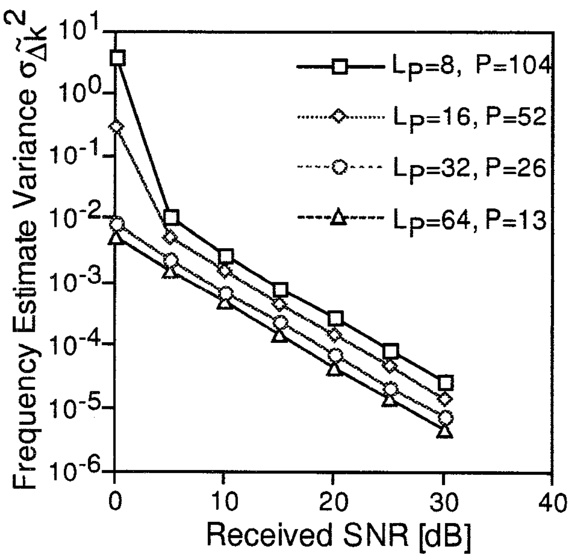

Figure 5: Frequency estimate variances for AWGN channel

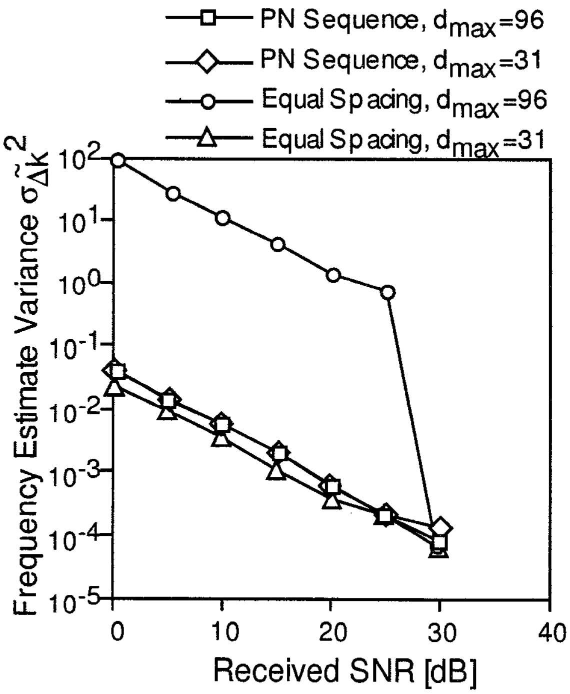

Figure 7: Frequency estimate variances for multipath fading channel

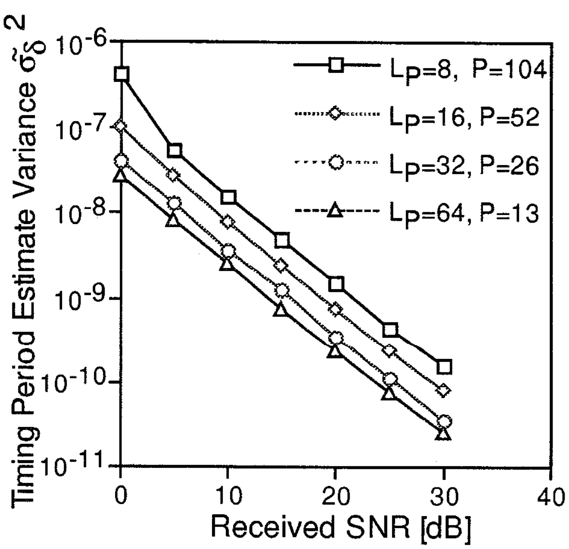

Figure 6: Timing period estimate variances for AWGN channel

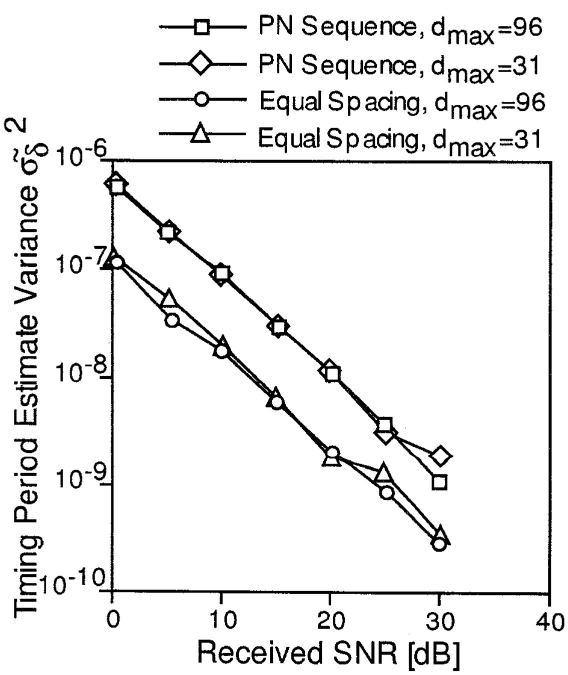

Figure 8: Timing period estimate variances for multipath fading channel

channels. In this example it is assumed that the channel has tow paths ($M = 2$) at $T_0 = 0$ and $T_1 = 10T$, and the tap coefficients h_0 and h_1 are independent zero-mean complex valued Gaussian random processes [3] with the variances $E[|h_0|^2] = E[|h_1|^2] = 0.5$. The adequate guard interval $N_g = 128$ is assumed and hence no ISIs occur. In the computer simulation, the tap coefficients are generated by Jake's model[9]. The maximum Doppler frequency of 25Hz and $1/T = 6.77$MHz are assumed.

Fig.7 compares the frequency estimate variances $\sigma^2_{\tilde{\Delta}k}$ between the frequency assignment of the equal spacing ($L_P = 64$) and the PN sequence shown in Fig.4 for the wide ($d_{max} = 96$) and narrow ($d_{max} = 31$) frequency acquisition ranges. In both cases the same number of the carriers ($P = 13$) are transmitted.

For $d_{max} = 96(> \frac{L_P}{2})$, the results of $\sigma^2_{\tilde{\Delta}k}$ for the equal spacing assignment are quite large over the lower to moderate SNRs, since the rough estimation of the frequency shift does not work correctly, as described in Section 3.4. However, the estimate variance drops around the SNR of 30dB since the estimation of d_{opt} becomes correct around that SNR.

On the other hand, in case of the PN sequence assignment, the $\sigma^2_{\tilde{\Delta}k}$ for both $d_{max} = 96$ and 31 are almost same values. This shows that the PN sequence assignment can effectively realize wider acquisition ranges.

Fig.8 shows the comparison of the simulation results for the timing period estimate variance $\sigma^2_{\tilde{\delta}}$. From Fig.8 the estimate variances $\sigma^2_{\tilde{\delta}}$ are independent of d_{max} for either case of the equal spacing or PN sequence assignments. The reason is that $\tilde{\delta}$ depends on only the relative difference of the estimated frequencies as shown in (11) , and hence even an incorrect estimation of d_{opt} does not affect $\sigma^2_{\tilde{\delta}}$.

Fig.8 also shows that the equal spacing assignment attains smaller variances than the PN sequence assignment.

Comparing Fig.7 and 8 to Fig.5 and 6, the fading channels deteriorate the estimate variances. This can be regarded as the result from the additional ICIs due to the channel variations during one symbol duration.

5 Concluding Remarks

In this paper a frequency and timing period acquisition scheme has been derived. The technique estimates both the frequency and timing period offsets at the same time by using only one pilot symbol. To realize wider frequency acquisition ranges, PN sequences have been introduced to assign the frequencies of the pilot symbol. Numerical examples have been shown for both AWGN and multipath fading channels, and the results have shown that the frequency assignment technique using PN sequences can effectively make the acquisition range wider.

[3] In this example, the tap coefficients are not constant even in one symbol period.

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