NBER WORKING PAPER SERIES

A FRICTIONLESS VIEW OF U.S. INFLATION

John H. Cochrane

Working Paper 6646
http://www.nber.org/papers/w6646

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 1998

I thank Henning Bohn, Eric Leeper, Julio Rotemberg, and Mike Woodford for many helpful comments, and I thank Andrea Eisfeldt for research assistance. My research is supported by the CRSP, Graduate School of Business and by a grant from the National Science Foundation administered by the NBER. Data and updated copies of this paper can be found at http://www-gsb.uchicago.edu/fac/john.cochrane/ Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

A Frictionless View of U.S. Inflation
John H. Cochrane
NBER Working Paper No. 6646
July 1998
JEL No. E4, E5, E6

ABSTRACT

Financial innovation challenges the foundations of monetary theory, and standard monetary theory has not been very successful at describing the history of U.S. inflation. Motivated by these observations, I ask: Can we understand the history of U.S. inflation using a framework that ignores monetary frictions?

The fiscal theory of the price level allows us to think about price level determination with no monetary frictions. The price level adjusts to equilibrate the real value of nominal government debt with the present value of surpluses. I describe the theory, and I argue that it is a return to pre-quantity theoretic ideas in which money is valued via a commodity standard or because the government accepts it to pay taxes. Both sources of value are immune to financial innovation and the presence or absence of monetary frictions.

I then interpret the history of U.S. inflation with a fiscal-theory, frictionless view. I show how the fiscal theory can accommodate the stylized fact that deficits and inflation seem to be negatively, not positively correlated. I verify its prediction that open market operations do not affect inflation. I show how debt policy has already smoothed inflation a great deal.

John H. Cochrane
Graduate School of Business
University of Chicago
1101 E. 58th Street
Chicago, IL 60637
john.cochrane@gsb.uchicago.edu

Contents

1 Introduction

The standard, quantity-theoretic approach to the price level is based on a transactions demand for money. Financial innovation challenges the foundations of this monetary theory: More and more transactions are handled electronically or via credit and debit cards, while ATM's, sweep accounts and banking by computer have a deep impact on cash management. Meanwhile, a wide array of privately provided, liquid, interest-paying and often nonreservable assets have been created, leaving the supply of transaction-facilitating assets beyond the Fed's control. The quantity theory has also not had much success in describing the history of postwar U.S. inflation: Inflation seems to have very little to do with the history of monetary aggregates or interest rates. Money demand relations are dominated by "velocity shocks," unrelated to changes in financial structure. Recent inflation has been remarkably stable despite continuing financial innovation.

Motivated by these observations, I ask: Can we understand the history of U.S. inflation using a framework that *ignores* monetary frictions? Until recently, there was no coherent way to think about this question: some friction seemed necessary to determine any value for unbacked fiat money. Recently, however, a series of authors including Leeper (1991), Sims (1994, 1997), and Woodford (1995, 1996, 1997) have advocated a *fiscal* theory of the price level. The analytical content of the fiscal theory is just the government's intertemporal budget constraint, versions of

$$\frac{\text{nominal debt}}{\text{price level}} = \text{present value of real surpluses.} \tag{1}$$

In a fiscal analysis, this equation determines the price level in much the same way that $Mv = py$ determines the price level in the quantity theory. However, since total government debt rather than the supply of transactions-facilitating assets appears on the left, fiscal price level determination is immune to financial innovation, including elastically provided private media of exchange, and even a cashless or frictionless economy. More generally, as I will show below, the budget constraint provides an implicit backing or commodity standard for even apparently unbacked fiat money; these sources of value are transparently independent of financial structure or any special exchange or liquidity properties of money.

Since we see money and frictions, why abstract from them in studying the price level? First, monetary frictions have at best second-order effects on the price level in fiscal models, so why not start with the simple model. Second, a frictionless economy with lots of inside, privately provided media of exchange is, at the level of ingredients, a much more plausible abstraction for the U.S. economy than an economy with rigorously separate liquid "money" used for transactions and illiquid "bonds" used for saving. Now that we *can* determine the price level in a frictionless model, it seems sensible to do it. Third, though economic theorists have a great deal of experience with analytically convenient devices with which to introduce monetary frictions — cash-in-advance, money in the utility function, overlapping generations — none of these devices provides an *empirically* successful description of money demand or inflation. If we had a realistic and empirically successful monetary theory — a stable, exploitable and well-understood money demand function, a well-defined and agreed-on monetary aggregate, and an empirically successful account of U.S. inflation — most of our interest in the fiscal theory would vanish. The fiscal theory would be a small dusty corner in which theorists battle over "foundations" of a successful empirical framework.

In this paper, I first exposit a frictionless economy with fiscal price level determination. Though the formal theory is well worked out by the above-cited authors, the interpretation, applicability and plausibility of the fiscal theory are still disputed. I show how the fiscal theory describes a backed commodity standard and a tax-based theory of value. I clarify the vexing red herring of "Ricardian" and "non-Ricardian" regimes, and budget constraints that do or don't hold at off-equilibrium prices. I review the extension of the fiscal theory to long-term debt, which tells us when a shortfall in future surpluses can be met by a decline in long-term bond prices rather than an increase in the price level, and I show how explicit monetary frictions make small changes to the fiscal-theoretic description of the price level.

I then interpret the history of U.S. inflation with a fiscal theory, frictionless view. This is potentially a tough assignment. The history of postwar U.S. inflation does not have obvious fiscal roots, nor does it offer the kind of clean exogenous movements in debt or surpluses that one hopes for in a test. Also, the fiscal theory (with short-term debt) relates the price level to the *present value* of future surpluses. In contrast, the quantity theory relates the price level to the *flow* of transactions or income. Present values are notoriously hard to measure. Most importantly, the correlations in the data seem wrong: The 1970's were a decade of low deficits and high inflation, while the 1980's saw a dramatic increase in government debt with low inflation. Large deficits also occur in the depths of recessions with low, not high inflation, and with rising, not declining, values of the debt. The centerpiece of the empirical work is to show how one can plausibly understand these correlations.

On the other hand, it is potentially a much too easy assignment. One's first impulse is to *test* the fiscal theory; perhaps to run some VAR to see whether surplus shocks rather than monetary shocks affect the price level. However, I show that the fiscal theory *per se* has no testable implications for the joint time series of prices, debt, and surpluses. Briefly, the identity (1) holds, in equilibrium, whether fiscal or monetary considerations determine the price level. Therefore, one can always rationalize the price level by reference to debt and subsequent surpluses. Additional identifying assumptions are not easy to find in U.S. experience. For this reason, the main focus of the empirical work is to construct a plausible story for the time series rather than pursue a test. The fiscal theory does predict that open market operations should have no effect on the price level, and this is fairly easy to see in the data.

I construct a detailed dataset on total outstanding Federal debt, broken down by maturity on a zero-coupon basis. I infer the surplus from debt transactions, rather than use accounting data. I start by documenting the patterns of surpluses, debt, and inflation in the U.S. since 1960 (when useful data start). I find some surprises. For example, the biggest primary deficit occurs in 1975, along with the onset of serious inflation. The primary "Reagan deficits" are surprisingly small, and even those are largely accounted for by the dramatic recessions of 1980-82. I also find that fluctuations in the rate of return of government bonds are as large as fluctuations in surpluses, so the rate at which future surpluses are discounted may be as important to the present value of the surplus as are changing surplus expectations. I find interesting variations in maturity structure, correlated with inflation: maturities were very short in the 1970s, but have lengthened since long-term bond sales were reemphasized in 1975. Longer maturities have led to wider fluctuations in the rate of return on government debt, and they allow debt sales to immediately affect the price level.

The central issue is understanding fluctuations in the real value of the debt. The fiscal theory requires a forward-looking story: the value of the debt is determined by the present value of future surpluses. The standard story is backward-looking: the value of the debt is determined by the accumulation of past deficits and a money-determined price level. To tell the forward-looking story, I pursue models with exogenous surpluses that replicate important correlations in the data. The important ingredient of the models is that extra nominal debt sales in recessions must come with implicit promises to increase subsequent surpluses. Finally, I consider whether expected return variation and maturity structure are important elements of the story.

I ask what policies could have avoided postwar inflation. I find that fiscal policy already does a lot of price-level smoothing, and that variation in inflation comes from comparatively small failures to smooth. Even larger fluctuations in nominal debt would have been required to stabilize inflation; a k percent rule would have resulted in disastrously fluctuating inflation.

2 The fiscal theory of the price level

2.1 A simple frictionless economy

Start with a simple frictionless economy with one-period government debt. At the beginning of each period t, nominal bonds $B_{t-1}(t)$ are left outstanding from period $t-1$ and will mature at t. Bondholders can use the maturing bonds to pay net real taxes (net of government spending and transfers) s_t or to acquire new bonds at price

$Q_t(t+1)$. (I use capital letters for nominal quantities and lowercase letters for real quantities.) Accounting for the flow of bonds, then, we have

$$B_{t-1}(t) - Q_t(t+1)B_t(t+1) = p_t s_t. \tag{2}$$

Fiscal price determination is easiest to see in a terminal period, or a period in which the government sells no new debt. Then, the budget constraint simplifies to

$$\frac{B_{t-1}(t)}{p_t} = s_t. \tag{3}$$

Nominal debt $B_{t-1}(t)$ is predetermined, so the price level must adjust to equate the real value of the debt to the real value of surpluses that will retire the debt.

To extend the analysis to infinite-period economies, define the ex-post real return on government bonds

$$r_{t+1}^b \equiv \frac{1}{Q_t(t+1)} \frac{p_t}{p_{t+1}}. \tag{4}$$

Then we can write the accounting identity (2) as

$$\frac{B_{t-1}(t)}{p_t} - \frac{1}{r_{t+1}^b} \frac{B_t(t+1)}{p_{t+1}} = s_t. \tag{5}$$

Iterating forward and imposing the usual transversality condition or taking the limit of finitely lived economies with a terminal period described by (3), we obtain

$$\frac{B_{t-1}(t)}{p_t} = \sum_{j=0}^{\infty} \prod_{k=1}^{j} \frac{1}{r_{t+k}^b} s_{t+j}. \tag{6}$$

These accounting identities hold ex-post for each realization, so they also hold ex-ante, after taking time-t conditional expectations. We can write

$$\frac{B_{t-1}(t)}{p_t} = E_t \sum_{j=0}^{\infty} \prod_{k=1}^{j} \frac{1}{r_{t+k}^b} s_{t+j}. \tag{7}$$

Equation (7) is the multiperiod analogue of (3). The price level adjusts to equate the real value of nominal debt to the *present value* of the surpluses that will retire it.

It is often a convenient simplification to assume a constant expected real return on government bonds r. With this assumption, we can take expectations of (4) and find that the price of new debt is

$$Q_t(t+1) = \frac{1}{r} E_t \left(\frac{1}{p_{t+1}} \right) p_t,$$

the flow budget constraint is

$$\frac{B_{t-1}(t)}{p_t} - \frac{1}{r}E_t\left(\frac{1}{p_{t+1}}\right)B_t(t+1) = s_t. \tag{8}$$

and the present value budget constraint is

$$\frac{B_{t-1}(t)}{p_t} = E_t\sum_{j=0}^{\infty}\frac{1}{r^j}s_{t+j}. \tag{9}$$

Equations (8) or (9) determine the *sequence* $\{p_t\}$ given an exogenous sequence of surpluses $\{s_t\}$ and of nominal debt $\{B_{t-1}(t)\}$. I emphasize the determination of the *sequence* $\{p_t\}$ from the *sequences* $\{B_{t-1}(t), s_t\}$ to avoid an analysis that distinguishes between "date zero" events and subsequent history.

A reader may be uncomfortable with the absence of the rest of the economy – where are preferences, technology, and shocks? The answer is that a wide specification of models includes equations such as (8)-(9); they will determine the price level no matter what the rest of the economy looks like, so we don't have to spell it out.

"Budget constraint" is a poor terminology for equations (8) or (9). The whole point of the theory is that these equations are *not* "constraints" on the government's actions, instead they describe price level determination. However, the form of these equations is so familiar with the name "budget constraint" that I will continue to use this phrase to describe them.

The budget constraints become more complicated as one includes money, potentially held overnight despite an interest rate penalty, long-term debt, and other realistic complications. In general, we add real or indexed assets and liabilities such as social security on the right-hand side, and other nominal claims including money and long-term debt to the left-hand side. Policy rules with feedback can be included, for example by writing $s_t(p_t, p_{t-1}, ...)$. Then one solves for the price level sequence that solves the budget constraint at each date. In these more general situations, this *solution* for the price level sequence will not be the same as the present value budget constraint equation (9).

2.2 Interpretations

The claim that fiscal considerations can determine the price level, even in a completely frictionless economy, is so strange at first that it merits closer examination. The fiscal mechanisms and equations apply to a wide variety of different institutional arrangements. Spelling out some of those arrangements makes fiscal price determination much more plausible and understandable, and makes it easier to apply the fiscal theory in practice.

2.2.1 Money in frictionless economies

A frictionless economy need not be a cashless economy. The budget constraints and hence the price level are completely unaffected if the government redeems some maturing bonds for cash during the period, and if this cash rather than maturing bonds is used for transactions, tax payments and the purchase of new bonds. The split between cash and maturing bonds at any moment in time – a form of open market operation – similarly has no effect at all on the budget constraint and hence on the price level.

Furthermore, the government can provide cash elastically with no effect on the price level. If the government prints a dollar and issues it as an interest-free intraday loan, that dollar is used for transactions, and then the loan is repaid by the end of the day, the budget constraints are again unaffected. Since Fedwire transactions are netted at the end of the day, this is in fact close to the current institutional arrangement. Unlimited inside moneys – private claims to reserves, cash, or maturing government bonds – can also be created and used to make transactions, with no effect on the budget constraint and hence on the price level.

The above timing and budget constraints are the same as those in a cash-in-advance economy in which the security market is always open. One can add a cash-in-advance constraint that bonds must be exchanged for cash to make purchases with no effect on the price level. "Frictionless" means the security market is always open; transactions may still require cash. The friction in typical cash-in-advance models is that the security market is only open part of the day, requiring people to hold some cash overnight to make transactions.

2.2.2 Commodity standards

Credible commodity standards or exchange rate pegs are intuitively transparent instances of the fiscal theory of the price level. The fiscal theory looks past the promised price level or exchange rate and past any official backing such as gold stocks to the overall real resources that in the end back the promises.

Suppose the government stands ready to exchange each dollar for a bushel of wheat, and that it maintains a warehouse with enough wheat to do so. The classical (100% backed) gold standard embodies this idea. Currency boards that peg exchange rates are more recently popular implementations.

This regime would seem to nail the price level at \$1/bushel by arbitrage. Furthermore, the price level under a commodity standard is transparently immune to financial innovation. Any amount of privately issued, interest-paying, liquid assets, or private banknotes can be created with no effect on the price level. Private banknotes are valued by their own fiscal theory, and may trade at a discount due to default risk. In a cashless economy, electronic claims to "dollars" are valued as claims to "one bushel wheat." Even monetary frictions are at most important for determining interest rate spreads and quantities of liquid assets, but they have no effect on the price level.

A commodity standard is an instance of the fiscal theory. Credibility is the crucial issue with a commodity standard or a peg. 100% backing regimes – warehouses full of wheat, a Ft. Knox full of gold, or a currency board holding foreign securities – are thought to provide such credibility, since the last dollar can be extinguished just as the backing vanishes. Such backing is an asset on the right-hand side of the budget identity, put there to guarantee that the budget constraint can always hold at the promised price level.

On closer inspection, however, the overall government budget constraint really matters, not the backing, reserves, or the promised rate. For this reason, we write the fiscal theory with overall real resources on the right-hand side, not just whatever resources are explicitly devoted to backing. If a government is in financial trouble, it will try to appropriate the real assets or currency board backing that is "uselessly" sitting in a warehouse, or (equivalently) it will devalue. On the other hand, a government with healthy finances can peg an exchange rate or commodity standard with no reserves, buying reserves on the spot market as needed, raising taxes, selling real assets or borrowing against future surpluses to do so. Exchange rate pegs do not fall to speculative attack when the government "runs out of reserves," they fall apart when the government becomes unable or unwilling to *buy* reserves.

Furthermore, the government must back the entire stock of nominal debt, not just whatever currency is currently outstanding. For this reason, we write the fiscal theory with all government debt on the left-hand side, not just currency or the monetary base. If the currency outstanding is 100% backed, but there is a large stock of maturing nominal debt relative to real assets and current and future real surpluses, everyone can see that the backing promise or peg must soon be broken.

By pointing out that the overall budget matters and that all nominal debt must be backed, I do not mean to deny that 100% backing schemes, run by suitably independent agents with explicit rules and public accounting, are useful precommitment devices for government finance. These considerations do suggest however that their operation is really a matter of political economy or game theory, not, as is often argued, simple accounting.

2.2.3 Tax and demand interpretations

The fiscal theory of the price level also formalizes an old view that fiat money is valued because the government requires its use for tax payments. Since the U.S. no longer follows an explicit commodity standard, a tax-based theory of value is a more plausible description of current institutions.

Consider a terminal period, or any period in which the government does not sell new debt. To tell the simplest story, suppose that the government redeems all the outstanding debt $B_{t-1}(t)$ for cash M_t at the beginning of the period, and that the government has no assets or explicit backing left. Now, the budget constraint simplifies to

$$\frac{M_t}{p_t} = s_t. \tag{10}$$

The government's surplus s_t is the private sector's net real tax liability, which must be paid with cash.

Suppose the price level is too high. There is too little cash around to pay taxes, so taxpayers try to sell goods for cash, which lowers the price level. Conversely, suppose the price level is too low. Taxpayers have more cash than they need to pay taxes, so they try to buy more goods, driving up the price level. The same story applies to the multiperiod model, strung out through time.

These stories have a familiar feel: Inflation results from too much money chasing too few goods; money is a hot potato that individuals can try to get rid of, but in aggregate such actions only change the price level. Intuition and observations that inflation occurs in periods of high "aggregate demand" for goods and services are perfectly consistent with a fiscal theory. The crucial change, and one that would be easy to miss in analyzing the data, is that an excess of cash is measured relative to *tax liabilities* that soak it up, not relative to a transactions-based demand.

A tax-based theory of value is also transparently immune to financial innovation: the economy may operate cashlessly, and/or any amount of inside liquid assets may be created, with no change in the price level. To operate cashlessly, the government can simply accept maturing government bonds directly for tax liabilities, or may electronically convert them to dollars for a nanosecond before accepting them. (This is what happens now if you pay your tax bill with a check on a money market mutual fund.) Tax liabilities define dollars as a unit of account, and taxes give meaning to a bond's promise to pay 100 "dollars" at maturity in a cashless economy. Inside moneys do not matter, for in the end taxes must be paid with government-issued nominal claims. Like the commodity standard, the tax-based story suggests that even monetary frictions will have at best second-order effects on the price level: The basic valuation story is not much affected if people obtain money to pay taxes a day or two in advance, suffering an interest penalty.

2.2.4 A stylized history and dramatic implications

We started with a commodity standard. Then we realized that the overall real resources that back nominal debt matter in the end, not the promised redemption rate. Finally, when the resources are in place the promise can vanish as well, as in the tax story.

This progression of ideas is also a useful stylized history. Early economies used raw metal or coins for transactions, and the price level was understood primarily in terms of the commodity content. Then, banknotes, checks and government-issued paper claims to gold or silver were created. At first, people worried that these devices would not maintain their value relative to the commodity unit of account. However, 100% backed notes that explicitly promised redemption in commodity terms did in fact have stable values. Next, it was found that most of the backing was gathering dust in a warehouse. Why insist on 100% reserves? Despite reservations about price level stability that remain to this day, it was gradually found that lower and lower reserve ratios could be used and still maintain the value of the notes, *if* the issuing entity was in sound enough financial condition so that it could always purchase enough backing if required (and, often dramatically, not conversely). Eventually, the explicit promises, in the form of the gold standard, also disappeared. Observers again worried (and still do) that removing the promise would lead to price level explosion. But nothing worse than the slow postwar inflation has occurred in the U.S.

The quantity theory offers an explanation for the last experience. In the quantity theory, an unbacked fiat money has value if and only if there is an inventory demand for it due to a special use in transactions, and if it and competing special assets are limited in supply. The price level did not explode when the gold standard was dropped, because its quantity-theoretic liquidity value already accounted for its value under the gold standard.

The fiscal theory offers an alternative explanation. In a fiscal theory, the backing is all that matters to a commodity standard in the first place. The price level did not explode when the gold standard was dropped, because it was already at its *fiscal* equilibrium level. A tax-based determination of value has the additional advantage over the gold standard that the implicit "commodity" is the full basket of government purchases, so changes in the relative price of gold do not disturb the price level.

As described above, we are in the midst of a new round of financial innovation. Quantity theorists are once again worried that this new set of financial innovations will destabilize the price level. This worry has even led to proposals to limit financial innovation in order to maintain price level control. However, the price level has remained remarkably stable given the level of financial innovation. At best, a quantity theorist explains this fact by noting that the Fed follows an interest rate policy, allowing money supply to accommodate shifts in velocity. However, the shifts in velocity are not traceable to financial innovation. A fiscal theorist is not surprised: financial innovations should have no effect at all on the price level.

This argument has dramatic implications. The fiscal theory is at heart a repudiation of (at least) 100 years of the quantity theory, and a return to backing theories of the value of money. In fact, there is no sharp distinction between commodity backing regimes and unbacked fiat money. Apparently unbacked fiat money can be valued, and apparently was all along, through the implicit backing of overall government surpluses.

While a promised rate does not matter to the theory, the theory will be easiest to apply when there is an explicit promise. Then, we just have to evaluate whether the promise is credible given current and future government revenues. When promises are implicit, as in the case of the U.S. economy, we can only look at the actual history of overall surpluses to see if the price level does indeed correspond to its fundamental backing.

2.3 Ricardian and non-Ricardian regimes

2.3.1 A simple example

In the one-period example,

$$\frac{B_{t-1}(t)}{p_t} = s_t, \tag{11}$$

we can see right away a special case in which the fiscal theory may not determine prices. If the government sets a *nominal* surplus S_t rather than a real surplus s_t, then the budget constraint is

$$B_{t-1}(t) = S_t.$$

Either the government commits to redeem the outstanding stock of nominal debt $S_t = B_{t-1}(t)$, or it does not, $S_t \neq B_{t-1}(t)$. In the former case the price level is indeterminate while in the latter case no value of the price level can eliminate the discrepancy.

In fact, the government determines *real* rather than nominal surpluses. Nominal tax liabilities are given by a rate θ times nominal income, θpy, and thus real tax liabilities θy are determined. If either the rate or output are non-neutral, so that θy declines one for one with p, we could have a Ricardian regime. But if anything, nominal brackets mean that the real tax rate θ is higher with a higher price level, and nonneutralities are usually thought to give higher output with higher price level. Perhaps more importantly, the *limits* on tax collection are also real; the top of the Laffer curve is real and government can't tax more than 100% of real GDP, no matter what the price level.

2.3.2 Ricardian regimes

The above example with a fixed nominal surplus is a *Ricardian regime.* (This is Woodford's (1995) terminology. Canzoneri, Cumby and Diba (1997) call the same thing a *money-determined* regime.) If we think of the two sides of (11) as two curves (functions of price) that determine the price level, a Ricardian regime is the special case in which the curves happen to fall right on top of each other. A Ricardian regime is the fiscal analogue to interest rate targets or accommodative money supply rules that can leave the price level indeterminate in the quantity theory. More generally, I use the following definition:

A **Ricardian regime** is any policy rule $\{B_t(t+j),\ s_t\}$ in which the sequence of government budget constraints holds for any sequence of price levels.

In an infinite-period context, when some new debt is sold every period, the present value budget constraint is

$$\frac{B_{t-1}(t)}{p_t} = E_t \sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+j}. \tag{12}$$

If the surplus $\{s_{t+j}\}$ reacts to p_t in such a way that (12) would hold for any p_t, then we have a "Ricardian regime" and (12) can no longer determine the price level. If the surplus does not react in just this way, then the price level must adjust to bring (12) into balance, and we have a fiscal regime.

2.3.3 The quantity theory as a Ricardian regime

The quantity theory is a particularly important case of a Ricardian regime. In the quantity theory, we add another equation, $Mv = py$. Fixing v and y, and with government control of M, the quantity equation now determines the price level. (I discuss more general cases with varying velocity and interest rate policies below.)

However, the budget constraint (12) is still part of the system. (Strictly speaking, one must account for the interest advantage to the government of money held overnight; I do so below and the difference is not important to the current discussion.) Since nominal debt $B_{t-1}(t)$ and p_t are now determined, the budget constraint is interpreted as a constraint on fiscal policy $\{s_{t+j}\}$. If the right-hand side of (12) is insufficient for a given real value of the debt, the government must raise future surpluses, by seignorage if explicit taxation is insufficient. Thus, the quantity theory is a Ricardian regime.

Quantity theorists have long recognized the tension between two equations, $Mv = py$ and the budget constraint, each of which seems to determine the price level, which can each alone determine the price level. Therefore, fiscal considerations have long been important in the quantity theory. Sargent (1986) interprets Friedman's k percent rules in part as a way of precommitting the monetary authority in a game of chicken with the Treasury over whether surpluses would be met by taxes or seignorage. cash-in-advance models following Lucas (1980) (See Sargent (1987) p.162) explicitly rebate seignorage revenues; they write down fiscal policies in which the government follows a Ricardian regime *by choice* so that the budget constraint will not fight with the quantity theory for price level determination.

In fact, the intellectual history of the fiscal theory comes precisely from thinking hard about the government budget constraint in the quantity theory. Leeper (1991), Sims (1994) and Woodford (1994) asked, what happens to a cash-in-advance model if the government does *not* choose to follow a fiscal policy that renders the budget constraint vacuous? As the above analysis shows, the budget constraint and a non-Ricardian regime can determine the price level in a cash-in-advance model, even if the security market is always open.

2.3.4 Equilibrium and off-equilibrium; is a fiscal regime possible?

A long and rather confusing debate pervades the fiscal theory over whether the government *must* follow a Ricardian regime. We usually derive demand curves by having the auctioneer announce a price vector, and then finding utility or profit maximizing quantities that satisfy the budget constraint at those prices. Demands satisfy budget constraints, even at off-equilibrium prices. This logic suggests that the government *must* adjust future surpluses in response to an off-equilibrium price level, so there is a fundamental mistake in using the budget constraint as we do to determine the price level given surpluses.

There must be a flaw in the reasoning: what happens if the auctioneer calls out such a low price level that the required surpluses are *impossible*, for example twice real GDP, forever? To think about this issue, return to a 100% backed commodity standard as an instance of a fiscal regime. One dollar equals one bushel wheat. The government keeps a warehouse with enough wheat to back the entire nominal debt open 24 hours a day. Institutional arrangements are strong enough that the government can never raid the warehouse. This arrangement would seem to decisively nail the price level at \$1/bushel[1]. Yet a Ricardian regime advocate would argue that it does *nothing* to determine the price level. He would argue that if the Walrasian auctioneer were to announce a price of 50¢ per bushel, the government would not have enough wheat to back the debt. It would then be forced to raise taxes to obtain more wheat, selling it at and validating the lower price.

[1] As long as there is any money outstanding. If all the money is redeemed, the price level is indeterminate as is the relative price of any nonexistent object.

In the example, it's easy to see that the argument is false. There is nothing that prevents the government from sticking to a \$1/bushel redemption rate no matter what the auctioneer or secondary market announces. If the secondary market price is 50¢, the government will buy lots of wheat; but there is no limit to the amount of nominal bonds or cash it can create in exchange for wheat. Conversely, if the secondary market price is \$2, the government will sell lots of wheat at \$1. The last ounce of wheat leaves the warehouse just as the last cent of nominal government debt is redeemed. The policy might seem foolish: the government wastes resources by selling wheat at \$1 when the secondary market price is \$2, or by buying it at \$1 when the secondary market price is \$0.50. But the argument is about constraints, not objectives; if the government wants to freely buy and sell at \$1/bu., there is no constraint that stops it from doing so.

The argument does not hinge on the commodity standard or promised redemption rate. Suppose there are 100 bushels of wheat in the warehouse, \$100 in notes outstanding, and no further taxes or assets on the last day of an economy, but the government will accept its notes for wheat at the market (auctioneer) price rather than posting a price. If the market price is \$2/bu., the government will wind up with unsold wheat at the end of the day. If the price is 50¢/bu., the government may run out of wheat before consumers have redeemed all the money. There is nothing wrong with either outcome. The government can certainly waste or consume wheat at the end of the period. Conversely, the budget constraint must allow consumers to keep some money. The fact that money is not intrinsically valuable to consumers, and wheat is not valuable to the government must come from preferences, not constraints.

The mistake, from a Walrasian view, is in insisting that government debt must be paid off because of budget constraints. The government starts with an endowment of wheat and consumers start with an endowment of money. The budget constraint says that *trades* away from the endowment points must take place at or worse than market (auctioneer) prices. This constraint is satisfied in both of the above examples.

What about previous dates at which the money is issued or bonds are sold? The same points extend to multiperiod models of course. Though money and bonds at each date are the result of previous period's trades, we always come down to an initial period with endowments outstanding, and then each period markets reopen as if the previous period's outcomes were endowments.

Similarly, there is a longstanding suspicion that one must assume something special about a government in a fiscal theory; that the government is a special agent that can announce demands that do not satisfy budget constraints or repay debts at off-equilibrium prices; that it enjoys a special 'first mover' status in some game with the private sector. As the examples make clear, there is nothing special about the government. If I give away 100 I.O.U.s saying "John Cochrane will pay the bearer \$1 on demand," I have \$100 in my wallet that I will only use for repaying I.O.U.s, and all this is perfectly credible, visible, etc., I can nail the price of my I.O.U.s at \$1 each. If for some reason they become worth more (if the auctioneer announces a different price), I can (and will!) print up I.O.U.s like mad; if they become worth less I can redeem them all. I may not *choose* to, but I *can*. I can also issue "equity claims on John Cochrane's wallet," and then repurchase them via auction. If the auctioneer announces the wrong price, I can leave the room when I.O.U.s or dollars in my wallet run out.

In the same way, private entities as well as multiple governments can create nominal claims, with or without explicit promises about redemption in dollars or real baskets of goods. The only special thing about the government is the convention or legal restriction that the rest of the economy uses its I.O.U.s as numeraire.

This is reassuring. If we relied on the government being able to violate budget constraints, nonsensical conclusions would follow. The government could announce lots of spending and zero taxes[2], the budget constraint be damned.

2.4 Testing for fiscal determination

Even though governments *can* follow "non-Ricardian" regimes in which fiscal theory determines the price level, they may not *choose* to do so. Looking forward to our task of bringing a fiscal theory to data, one's natural impulse is to "test" the fiscal theory, and the natural "test" is whether the government has chosen to follow a Ricardian vs. non-Ricardian policy regime.

Unfortunately, *the fiscal theory of the price level per se has no testable implications for the time series of debt, surplus and price level.*

The budget constraint (12) holds in equilibrium for both "fiscal" and "Ricardian" regimes. The issue is whether, in determining or adjusting towards equilibrium, the price level adjusts to expected future surpluses, or whether the path of surpluses adjusts in response to the price level. All we ever observe is an equilibrium; we do not observe "who adjusted" to bring about that equilibrium, or what off-equilibrium behavior looks like. Analogously, if one observes a market, one sees the transactions price and quantity, but not the slopes of the underlying supply and demand curves.

It is tempting to test "who adjusted" by looking at dynamic responses to shocks as in a VAR. But the (state-contingent) *sequence* of price levels, surplus and debt $\{p_t, s_t, B_{t-1}(t)\}$ is a *single* equilibrium. It is not a sequence of equilibria, or even less a tatônnement process for the formation of an equilibrium. The issue is which sequence, $\{p_t\}$ or $\{s_t\}$ adjusts to the other sequence, *not* whether shocks to p_t precede

[2] I thank Larry Christiano and Martin Eichenbaum for repeatedly stressing this point, back when I thought one *did* have to assume something special about the government.

those to s_t in an equilibrium sequence $\{p_t, s_t\}$.

Woodford's (1995) analysis argues even more strongly that a test for fiscal determination is *meaningless.* As I will review below, Woodford argues that all monetary regimes (money demand specification and monetary policy rule) that are vaguely plausible descriptions of the U.S. economy leave the price level indeterminate. Therefore, Woodford's analysis implies that if the price level is determined at all it *must* be determined by fiscal means. There is no coherent alternative.

Clear as these points are in the abstract, it is helpful to apply them to empirical approaches one might attempt and see how those approaches break down.

2.4.1 Feedback rules?

Feedback rules to generate Ricardian regimes

We often think of policy in terms of rules plus innovations. Fixing a nominal surplus is equivalent to a feedback rule $s_t(p_t) = S_t/p_t$ that increases the real surplus 1% for every 1% decrease in the price level. The budget constraint in the one-period case now reads

$$\frac{B_{t-1}(t)}{p_t} = s_t(p_t) = \frac{S_t}{p_t}$$

and the price level drops out as before. We can also think of this case as a commitment to adjust the *real* surplus to soak up the *real* value of outstanding debt, a rule $s_t = B_{t-1}(t)/p_t$.

We retain fiscal price level determination if the government follows a policy rule with some feedback, so long as the feedback is not exactly one-for-one. If the government responds linearly to real debt,

$$s_t = \bar{s} + \alpha \frac{B_{t-1}(t)}{p_t}, \tag{13}$$

only the case $\bar{s} = 0$, $\alpha = 1$ implies that the budget constraint is vacuous.

In an infinite-period context, the constraint is

$$\frac{B_{t-1}(t)}{p_t} = E_t \sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+j}.$$

The constraint holds for any price level and the regime is Ricardian if $\left\{\sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+j}\right\}$ reacts to p_t in a one-for-one manner.

It is more common to think about feedback rules for the one-period surplus than rules for the discounted value of future surpluses. Generally speaking, policies in which surpluses adjust to the price level in such a way that real debt does not grow faster than the real interest rate generate a Ricardian regime. The constraint

$$\frac{B_{t-1}(t)}{p_t} = \sum_{j=0}^{k} \prod_{l=1}^{j} \frac{1}{r^b_{t+l}} s_{t+j} + \prod_{l=1}^{k} \frac{1}{r^b_{t+l}} \frac{B_{t+k}(t+k+1)}{p_{t+k}}$$

holds as an accounting identity, and if the last term or its expectation converges to zero for any p_t, the budget constraint holds for any p_t. This statement is the natural infinite-period counterpart to the one-period example in which real surpluses adjust to soak up the real value of the debt.

As a specific example, consider linear feedback rules that raise the surplus in response to increases in real debt

$$s_t = \alpha \left[\frac{B_{t-1}(t)}{p_t}\right] + \varepsilon_t. \tag{14}$$

The one-period identity is then

$$\frac{B_t(t+1)}{p_{t+1}} = r^b_{t+1}\left(\frac{B_{t-1}(t)}{p_t} - s_t\right) = r^b_{t+1}(1-\alpha)\left(\frac{B_{t-1}(t)}{p_t}\right) + r^b_{t+1}\varepsilon_t$$

Thus, any $\alpha > 0$ implies real debt that grows slower than the interest rate and so a Ricardian regime.

One may wish to be a bit more restrictive, if one wants to consider only infinite-period results that are the limits of finitely-lived economies. If $0 < \alpha < r - 1$, real debt still explodes, though slower than the interest rate. Feedback $\alpha \geq r - 1$ is necessary to keep real debt bounded, and $\alpha \geq r-$ GDP growth is necessary to keep the debt/GDP ratio bounded. (Canzoneri, Cumby, and Diba (1997) derive these results as special cases of considerably more general feedback rules.)

Testing feedback rules?

Given these feedback restrictions, it is natural to test for a Ricardian regime by running regressions of surpluses on real debt to see if surpluses do adjust enough to real debt. The trouble is, *this is always true* in the data. In the one-period context, the constraint

$$s_t = \frac{B_{t-1}(t)}{p_t}$$

does hold, in equilibrium. We cannot tell which variable – p or s – adjusted to the other in order to produce the equilibrium. We could run the regression with s on the left and interpret the results as an estimation of (13), giving the Ricardian result, or we could put p on the left and interpret the result as confirmation of fiscal price determination from B and s.

Similarly, it is tempting in an infinite-period context to run a regression of (14), and test whether surpluses adjust to the value of debt, $\alpha > 0$ (or $\alpha > r - 1$). Alas, this coefficient again tells us nothing about the regime. For example, suppose the surplus is completely exogenous, $s_t = \rho s_{t-1} + \varepsilon_t$ and B is constant. In a "fiscal" regime, prices are then

$$p_t = \frac{B_{t-1}(t)}{E_t \sum \frac{1}{r^j} s_{t+j}} = \frac{B}{s_t}(1 - \rho/r).$$

In this example, a low surplus leads to a low real value of the debt,

$$B/p_t = \frac{1}{1-\rho/r} s_t.$$

But one could easily put s on the left and, mistakenly, find a surplus feedback rule that generates a Ricardian regime. I give an explicit example below in which an exogenous surplus process generates a VAR in which debt forecasts future surpluses.

Feedback on equilibrium vs. off-equilibrium price levels

Even more fundamentally, the government *can* distinguish the nominal quantity of debt from the price level. Therefore, it can follow a policy which systematically responds to the real value of debt *for the equilibrium price level* while refusing to validate *out-of-equilibrium* price levels.

To give a precise example, suppose the government wants to attain a price level p^*. It may follow a one-for-one feedback rule, promising to change taxes so as to soak up the real value of any debt $B_{t-1}(t)$ that happens to be outstanding due to stochastic variation in debt or surplus along the way – it may follow the feedback rule $s_t = B_{t-1}(t)/p^*$. However it does *not* promise to validate an out-of-equilibrium price $p_t \neq p^*$; it will *not* change taxes to $s_t = B_{t-1}(t)/p_t$ for $p_t \neq p^*$. We observe a one-for-one feedback rule, over time and across states of nature, but the price level *is* determinate at p^*.

2.4.2 A VAR?

With words like "exogeneity" and "causality" around, it is tempting to examine "who adjusts" in the context of a VAR, watching the response of variables to innovations. But again, since we are watching the evolution over time of one equilibrium, a "fiscal regime" poses no restrictions on such VARs.

A suggestion based on the surplus→debt response function

Canzoneri, Cumby and Diba (1997) propose the following test for a fiscal regime. If a surplus shock leads to higher surpluses but lower real value of the debt, they find a Ricardian or "money-determined" regime. If it leads to higher real value of the debt, they find a non-Ricardian or "fiscal-determined" regime. Not surprisingly, they find that positive shocks to surpluses reduce the real value of debt, and hence a "money-determined" regime.

These restrictions flow from the central idea that the value of the debt is *forward-looking* in the fiscal theory – debt is the present value of *future* surpluses–and *backward-looking* in a Ricardian or Monetary regime – debt is the accumulation of past surpluses. Denote the real value of debt $v_{t+1} = B_t(t+1)/p_{t+1}$. Then, the identity

$$v_{t+1} = r^b_{t+1}(v_t - s_t) \quad (15)$$

motivates the idea that a positive surplus shock should lower next period's real debt. The time $t+1$ present value constraint

$$v_{t+1} = E_{t+1} \sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+1+j}.$$

motivates the idea that a positively autocorrelated surplus shock should raise the real value of the debt.

One should of course be suspicious, since both equations hold in both regimes. In fact, the response function sign prediction requires a different surplus driving process, not a difference in regime. The time t present value constraint is

$$v_t = \frac{B_t(t)}{p_t} = E_t \sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+j}.$$

$B_t(t)$ is predetermined. In a money-determined regime, the price level is set by $p_t = M_t v/y$ and therefore does not change. If s_t rises and $\{E_t s_{t+j}\}$ do not decline, *the budget constraint no longer holds.* The only way to salvage the budget constraint is if the Fed agrees to monetize – if M moves with the innovation in s – producing exactly the price level rise that would be predicted by the fiscal theory. Then, the ex-post real interest rate in (15) adjusts so that the real value of the debt can rise next period. If the Fed does not monetize the deficit, future surpluses *must* decline.

2.4.3 Budget explosions?

Hamilton and Flavin (1986) pursue interesting tests for present value budget balance. Essentially, they test whether the debt or debt/GDP ratio are explosive. This seems like a natural test for a non-Ricardian regime. But the non-Ricardian regime only specifies explosive paths for real debt (the infinite-period counterpart to resources or money left at the end of single period economies) in response to never-observed, *off-equilibrium* prices. In equilibrium, the budget constraint holds, and we do not observe explosive debt.

2.4.4 Identification and non-testability

It may seem that I have made too much of the lack of testable restrictions. The pure quantity theory does not have testable implications either: $Mv = py$ is also an accounting identity – a definition of velocity. It too requires additional assumptions; that velocity is not affected by some variable or shock; that some monetary or income shocks are exogenous, etc. Every economic theory requires some extra assumptions, why pick on the fiscal theory?[3]

With identifying assumptions, the fiscal theory does make predictions. Again, look at the budget constraint,

$$\frac{B_{t-1}(t)}{p_t} = E_t \sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+j} \tag{16}$$

If we could find a shock to nominal debt that leaves future surpluses unchanged, the fiscal theory does predict that the price level should rise. If we find a shock to surpluses with no effect on nominal debt, the price level should decline. Shocks to the composition of nominal debt that leave its value the same, such as open market operations, should have no effect on the price level if they are not associated with changes in future surpluses.

All of these are valid predictions of the theory. The trouble is that the constraint (16) holds under all of the alternatives as well. For example, the quantity theory includes $Mv = py$ and the constraint (16). A shock to nominal debt with no change in future surpluses must come with just enough increase in M to give the same price level prediction as the fiscal theory; if not the shock *must* lead to a change in future surpluses. If one showed that neither happened, then one could reject the $Mv = py$ part of the quantity theory. But there is no way to reject the *fiscal* part of the theory.

2.5 Uncertainty and longer-maturity debt

So far, I have simplified the analysis by assuming one-period debt. In addressing the data, it is important to consider longer-maturity debt as well. In perfect foresight models, the addition of long-term debt makes no difference, but with uncertainty, long-term debt changes the fiscal theory in some crucial ways. For example, news of future deficits can be met by a decline in long-term bond prices rather than by a rise in the price level.

[3] I thank Benjamin Friedman for raising this point emphatically at the conference.

2.5.1 Statement of the budget constraints

Suppose a full spectrum of bonds is outstanding at the beginning of period t, and let

$$B_{t-1}(t+j) = \text{ bonds outstanding at the beginning of } t \text{ that mature at } t+j,$$

$$Q_t(t+j) = \text{ price at } t \text{ of bonds that mature at } t+j.$$

Again, we start from the accounting identity that just-maturing bonds plus net revenue from the purchase or sale of long-term bonds must add up to the nominal net of interest surplus,

$$B_{t-1}(t) - \sum_{j=1}^{\infty} Q_t(t+j)\left[B_t(t+j) - B_{t-1}(t+j)\right] = p_t s_t.$$

We can express the ex-post real rate of return on government bonds equivalently as value at tomorrow's prices/value today or as a weighted sum of individual bond returns. The generalization of (4) is thus

$$\begin{aligned} r^b_{t+1} &\equiv \frac{p_t}{p_{t+1}} \frac{\sum_{j=1}^{\infty} Q_{t+1}(t+j)B_t(t+j)}{\sum_{j=1}^{\infty} Q_t(t+j)B_t(t+j)} \\ &= \frac{p_t}{p_{t+1}} \sum_{j=1}^{\infty} \frac{Q_{t+1}(t+j)}{Q_t(t+j)} \times \frac{Q_t(t+j)B_t(t+j)}{\sum_{k=1}^{\infty} Q_t(t+k)B_t(t+k)}. \end{aligned}$$

If we write the real value of the debt

$$v_t = \frac{\sum_{j=0}^{\infty} Q_t(t+j)B_{t-1}(t+j)}{p_t}$$

then our earlier identities (5) and (6) still hold;

$$v_t = \frac{1}{r^b_{t+1}} v_{t+1} + s_t \tag{17}$$

$$v_t = \sum_{j=0}^{\infty} \left(\prod_{k=1}^{j} \frac{1}{r^b_{t+k}} \right) s_{t+j}. \tag{18}$$

It is again often convenient to assume that the real rate of interest is a constant r both across time and bonds. Then, again using $Q_t(t+j) = 1/r^j \times E_t\left[p_t/p_{t+j}\right]$, the identities simplify to, first, the value of maturing bonds - revenue from new bond sales = surplus,

$$\frac{B_{t-1}(t)}{p_t} - \sum_{j=1}^{\infty} \frac{1}{r^j} E_t\left(\frac{1}{p_{t+j}}\right)\left[B_t(t+j) - B_{t-1}(t+j)\right] = s_t \tag{19}$$

and second, the value of outstanding debt = present value of surpluses

$$v_t = \frac{B_{t-1}(t)}{p_t} + \sum_{j=1}^{\infty} \frac{1}{r^j} E_t\left(\frac{1}{p_{t+j}}\right) B_{t-1}(t+j) = E_t \sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+j}. \tag{20}$$

2.5.2 Solving for prices in terms of debt policy and surplus

With only one-period debt, equation (20) expressed the price level at t in terms of the present value of future surpluses. It was also a *solution* for the equilibrium price level sequence $\{p_t\}$ in terms of exogenous debt and surplus paths $\{B_t(t+j),\ s_t\}$. With long-term debt, future p_{t+j} enter into the equation on the left-hand side, not just p_t. Therefore, we have to work more to get an expression with p_t on the left and other variables on the right.

Cochrane (1998) derives the general solution for the price level sequence $\{p_t\}$ given any surplus and debt path. The general formula is rather cumbersome. To get a flavor of some of the possibilities, we can directly solve three special cases, generated by three special types of debt policy:

1. one-period debt. If the government follows a policy of always rolling over one-period debt, $B_t(t+j) = 0$ for $j > 1$, then equation (20) reduces to

$$\frac{B_{t-1}(t)}{p_t} = E_t \sum_{j=0}^{\infty} \frac{1}{r^j} s_{t+j}. \tag{21}$$

2. No new debt. Suppose the government does not issue new debt, but pays off existing debt (perhaps a perpetuity) as it matures. Then, we have $B_t(t+j) - B_{t-1}(t+j) = 0$ and equation (19) becomes

$$\frac{B_{t-1}(t)}{p_t} = s_t. \tag{22}$$

   The price level is now set only by debt that comes due each day and that day's surplus.

3. Geometric maturity structure. Suppose the government commits to a maturity structure

$$B_{t-1}(t+j) = \phi^j B_{t+j-1}(t+j).$$

   Then, (19) or (20) imply

$$\frac{B_{t-1}(t)}{p_t} = s_t + (1-\phi) \sum_{j=1}^{\infty} \frac{1}{r^j} E_t\left(s_{t+j}\right)$$

   which nicely nests (21) and (22).

We have spent so much time looking at equations like (21) that these examples are worth examining closely: *It is not generally true that the each period's price level is determined by the value of all debt relative to the present value of surpluses.* The latter conclusion is a very special case that comes entirely from the restriction that short-term debt is continuously rolled over.

2.5.3 Reaction to surplus news

To understand the effects of long-term debt, let us ask how the price level reacts if there is bad news about future surpluses. With one-period debt, the answer is simple: the price level must rise today.

$$\frac{B_{t-1}(t)}{p_t} = E_t \sum_{j=1}^{\infty} \frac{1}{r^j} s_{t+j}, \tag{23}$$

so if $E_t s_{t+j}$ declines, p_t must rise.

With long-term debt, the equation value of debt = present value of surpluses is instead from (20),

$$\frac{B_{t-1}(t)}{p_t} + \sum_{j=1}^{\infty} \frac{1}{r^j} E_t \left(\frac{1}{p_{t+j}}\right) B_{t-1}(t+j) = \sum_{j=0}^{\infty} \frac{1}{r^j} E_t s_{t+j}. \tag{24}$$

Now when expected future surpluses decline, expected *future* prices p_{t+j} can rise to reestablish the budget constraint. Equivalently, the real or relative price of long-term debt $q_t(t+j) = 1/r^j E_t\left(1/p_{t+j}\right)$ can decline.

Which of these options happens? The answer depends on debt policy, $\{B_{t-1}(t+j)\}$. For example, in the extreme case that the government sells no new debt, equation (22) shows that a decline in surpluses at date $t+j$ has *no* effect on the price level at time t. It only affects the price level at $t+j$, and hence affects only the price of outstanding $t+j$ bonds at time t. The geometric case gives an intermediate result.

2.5.4 Reaction to debt sales

What happens if at period t, the government issues more debt, with no change in surpluses? If no long-term debt is outstanding, again, the price level at t is set by

$$\frac{B_{t-1}(t)}{p_t} = E_t \sum_{j=1}^{\infty} \frac{1}{r^j} s_{t+j} \tag{25}$$

Hence, a change in $B_t(t+1)$ has no effect on the price level at period t. It does affect the price level at $t+1$:

$$\frac{B_t(t+1)}{p_{t+1}} = E_{t+1} \sum_{j=1}^{\infty} \frac{1}{r^j} s_{t+1+j} \tag{26}$$

so every 1% increase in debt sold at t translates into a 1% increase in the price level at $t+1$.

We can describe this result as a unit-elastic demand curve for nominal debt at each date: extra debt sales simply drive down the price of debt, and real revenue raised by bond sales is independent of the number sold. To see this, write the real value of debt at the end of time t as

$$q_t(t+1)B_t(t+1) = \frac{1}{r}E_t\left(\frac{1}{p_{t+1}}\right)B_t(t+1).$$

From (26), the quantity $B_t(t+1)/p_{t+1}$ is the same no matter what the level of debt sales $B_t(t+1)$.

With long-term debt outstanding, unexpected long-term bond sales with constant surpluses can raise revenue and thus lower the price level. Unexpected debt sales dilute the claims of existing long-term debt to the real resources that will be available to redeem debt on the maturity date. This is an attractive story for 1980, or for the fact that inflation often moderates in recessions when long-term debt sales are particularly high.

2.6 Monetary frictions

2.6.1 Money demand in the fiscal theory

The essence of the fiscal theory does not involve money or monetary frictions. Since we have spent so much time thinking about money, however, it is important to verify that adding money back in to a fiscal regime does not alter the basic story of price level determination.

We have already considered how *money* can be added to a fiscal and frictionless regime. Here, we consider how monetary *frictions* affect a fiscal regime. Above, people could use money for transactions during the day, but nobody held non-interest paying money overnight. Now they may; the security market may not always be open, so money for the next day's purchases must be held overnight, or real (overnight) money balances may enter the utility function.

The summary is simple: If monetary considerations can determine the price level, they do. If $Mv = py$ holds, with constant v and well-controlled M, and if the Treasury adapts fiscal policy to the money-determined price level, the fiscal theory has little to say. However, in most monetary models that describe anything like modern institutions, monetary considerations alone *do not* determine the price level. In this case, the fiscal theory determines the price level, leaving monetary frictions at best to determine interest-rate spreads for liquid assets. As monetary considerations get weaker, fiscal considerations get stronger. (This is the central point of Woodford (1995), and much of this section is a simplified version of Woodford's analysis.)

Introduce money, and let M_t denote money balances held overnight from time $t-1$ to time t. For simplicity, revert to one-period debt. The flow budget constraint now reads

$$B_{t-1}(t) - B_t(t+1)/R_t + M_t - M_{t+1} = p_t s_t \tag{27}$$

where $R_t = 1/Q_t(t+1)$ denotes the nominal interest rate. A useful form of the present value budget constraint is

$$\frac{B_{t-1}(t)}{p_t} = E_t \sum_{j=0}^{\infty} \frac{1}{r^j}\left[s_{t+j} + \frac{M_{t+j+1} - M_{t+j}}{p_{t+j}}\right]. \tag{28}$$

In this form we see how money can introduce potential seignorage revenues. Consider a simple money demand function,

$$M_t v(R_t) = p_t y_t. \tag{29}$$

Now, why doesn't control of the money supply, plus the money demand equation (29) determine the price level?

Constant velocity

In the most simplified quantity-theoretic tradition, it does. If the money supply is controlled, and if velocity is independent of interest rates, $Mv = py$ determines the price level p. If this price level agrees with the government budget constraint, fine. If it does not, one of the two determinants of the price level must give. As we have seen, most users of this model specify that the government chooses a Ricardian fiscal policy in which the budget adapts.

Varying velocity.

Truly constant velocity, in the face of arbitrarily large interest differentials, is an extreme and unlikely assumption. With an interest-elastic demand and fixed supply, money demand can still determine the expected rate of inflation or expected price level, but it does not determine the (ex-post) price level. The government budget constraint then determines the price level.

For example, write money demand

$$\ln M_t = \ln p_t + \ln y - b(\ln r + E_t \ln p_{t+1} - \ln p_t). \tag{30}$$

Assume constant output and real interest rate, and fixed money supply. Now, money demand = money supply gives a log-linear difference equation for the price level, and hence pins down the rate of inflation at each date,

$$E_t \ln p_{t+1} = \frac{1+b}{b} \ln p_t + \frac{1}{b}\left[\ln y - \ln M_t - \ln r.\right. \tag{31}$$

Depending on the initial price level p_t, there are an infinite number of paths that satisfy this difference equation. It is conventional to pin down the price level as the non-explosive solution,

$$\ln p_t = \sum_{j=0}^{\infty} \left(\frac{b}{1+b} \right)^j \frac{1}{1+b} \left[E_t \ln M_{t+j} - \ln y - b \ln r \right]. \tag{32}$$

But this choice is an extra condition, not derived from money demand, optimization or any other principle. We may be able to rule out *real* quantities that grow faster than the real rate of interest, but nominal quantities may happily explode.

With a non-Ricardian fiscal regime, the budget constraint pins down the (ex-post) price *level* at each date. If that choice implies an explosive expected future price level despite constant money supply, so be it. As Woodford (1995) notes, if we do not observe exploding price levels with constant money supply, that just means that governments do not follow constant money supply policies in the face of exploding price levels.

Varying velocity, money pays interest.

Money demand = money supply really determines the interest *differential* between monetary and non-monetary assets. In the above example, the convention that money pays no interest then means that money demand = money supply must determine the inflation rate. If monetary assets pay interest R_t^m, however, as more and more monetary assets do, then the money demand equation becomes

$$M_t v(R_t - R_t^m) = p_t y_t.$$

Now the price level at each date can be entirely determined by the fiscal condition (28). Money demand = money supply *only* affects the interest differential between money and other assets.

Interest rate targets

The money supply regime matters as well as money demand. For example, even with $Mv = py$, monetary considerations alone do not determine the price level if money supply is sufficiently accommodative. Then, the government budget constraint can do so. A nominal interest rate target is the classic example. If the government provides whatever quantity of money is necessary for the nominal interest rate to equal R, then $Mv(R) = py$ determines M/p but not the level of either M or p.

2.6.2 Why abandon frictions for studying inflation?

I use the fiscal theory to avoid monetary frictions altogether in the study of the price level, rather than follow the above style of analysis and the bulk of the fiscal theory literature including Woodford (1995, 1996, 1997), Leeper (1991), Sims (1994), Dupour (1997), Schmitt-Grohé and Uribe (1997), by including standard theoretical devices for generating monetary frictions. I argued in the introduction that these frictions have at best second-order implications for the price level, as we have seen, and that they are not a realistic or empirically successful description of the U.S. economy. Here, I document the latter claims. Of course, monetary and other frictions will be important if one wants to study interest rate spreads among liquid assets. Similarly, one will have to specify some frictions in order to be precise about why we *care* about inflation.

No explanation for inflation

Figure 1 presents the history of CPI inflation together with growth in the popular monetary aggregates. The history of business-cycle and even decade-long variation in inflation has essentially nothing to do with the history of monetary aggregates. The swings of inflation in the 1970s and especially the dramatic end of inflation in the early 1980s occurred without any obvious corresponding changes in monetary growth. If anything, M1 and M2 growth are *negatively* correlated with inflation in the 1970's, requiring artfully-specified long and variable lags if one is going to insist that money growth caused the inflation. Base and M1 growth were much more volatile in the 1980-1990s with stable inflation than they were when inflation was more volatile in the 1970s.

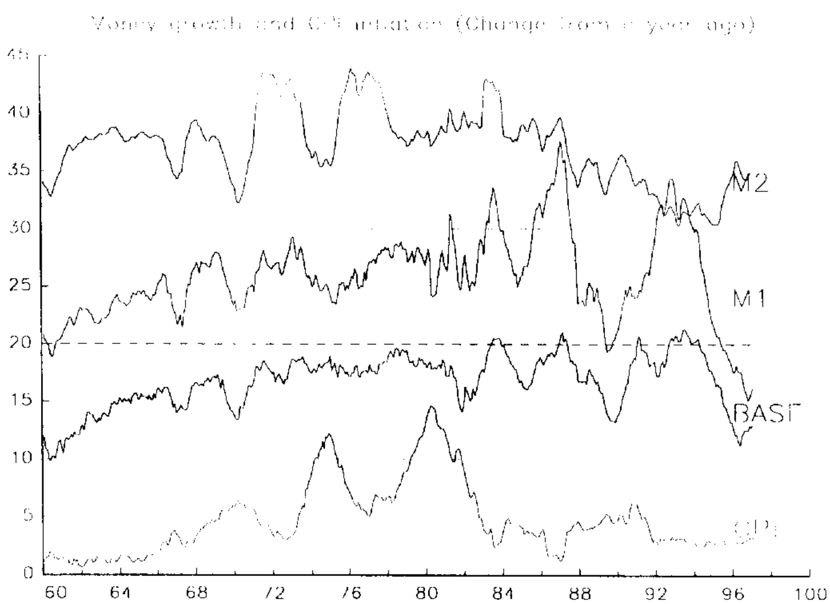

Figure 1: Inflation and growth in monetary aggregates. All series are monthly observations of annual growth rates. Base, M1, M2 are shifted up for clarity.

Monetary VARs provide a more formal accounting of variation. Despite a wide range of monetary policy indicators, identification schemes and specifications, such VARs regularly assign trivial fractions of price level variance to monetary shocks, and almost all to "price shocks." Cochrane (1994b) surveys this literature. As a specific example, Table 1 is an abridged version of Tables 3 and 4 from Christiano, Eichenbaum and Evans' (1998) survey. Note in particular that federal funds rate shocks in the top left quadrant explain essentially no price variation. Therefore, the VARs refute the standard analysis that price variation is results from unusually "tight" or "loose" interest rate policy.

Policy Shock:	FF				NBR				NBR / TR			
k	2	4	8	12	2	4	8	12	2	4	8	12
Y	0.4	21	44	38	0	7	10	8	0	17	30	22
P	0.5	0.3	0.4	2.5	0	0	1	1	0	0	0	1

Policy Shock:	MB				$M1$				$M2$			
k	2	4	8	12	2	4	8	12	2	4	8	12
Y	0	5	5	3	0	0	3	6	2	14	29	24
P	0	3	4	2	0	0	0	0	0	1	6	15

Table 1. Per cent of k quarter ahead forecast error variance due to policy shock. Source: Christiano, Eichenbaum, Evans (1998).

Since the total quantity of nominal debt appears on the left-hand side of the budget constraint, the irrelevance of open market operations to the price level is an obvious fiscal proposition. Of course, exact irrelevance is a special case. Open market operations do slightly alter the maturity structure of the debt, which can affect the timing of inflation; open market operations can change the rate of return on government debt; and open market operations may forecast changes in surpluses and so inherit a non-causal association with inflation.

To check whether monetary shocks forecast surpluses, Table 2 presents regressions of the surplus data described below on Christiano Eichenbaum and Evans' federal funds shock. With negative $\bar{R}$ and most t statistics below one, there is not a shred of statistical evidence that federal funds shocks forecast surpluses. The point estimates in the first three rows are large: a 1 percentage point federal funds shock results in roughly 1 percentage point rise in the surplus/consumption ratio for as much as 2 years. Since the real debt/consumption ratio is about 0.5, such an estimate implies as much as a 4% decline in the price level, roughly consistent with the VAR point estimates. However, as one expects from the t values and $\bar{R}^2$, the coefficients are driven by two outliers (1969,1975) in an otherwise symmetrical scatter-plot. The negative point estimates in the last row add up to an economically as well as statistically insignificant effect.

	ε_t^{ff}	ε_{t-1}^{ff}	ε_{t-2}^{ff}	ε_{t-3}^{ff}	ε_{t-4}^{ff}	ε_{t-5}^{ff}	$\bar{R}^2$	p-value
coef.		1.11					-0.00	0.40
t-stat.		(0.86)						
coef.	1.52	0.59					-0.006	0.41
t-stat.	(1.06)	(0.42)						
coef.	1.56	0.58	-0.33				-0.07	0.69
t-stat.	(1.01)	(1.55)	(-0.23)					
coef.	1.23	0.14	-1.52	0.09	-0.10	0.85	-0.26	0.96
t-stat.	(0.69)	(0.08)	(-0.82)	(0.05)	(-0.06)	(0.05)		

Table 2. Regressions of annual surplus/consumption ratio on annual averages of Christiano, Eichenbaum Evans (1998) federal funds shocks.

Money demand and velocity shocks.

Money demand functions explain some of the fluctuations in money growth rates documented in Figure 1 via income and interest elasticities (endogenous velocity), but not much. To quote a recent review by Baba, Hendry, and Starr (1992), "Estimated U.S. M1 demand functions appear unstable, regularly breaking down...(e.g., missing money, great velocity decline, M1-explosion)." Even the staunchest defenders of empirical money demand relations such as Lucas (1988) at best point to a stable income and interest elasticity over very long time scales, 50 years or more.

Suspiciously, "velocity shocks" are not traceable to changes in financial structure, and changes in financial structure do not seem to lead to velocity shocks. Velocity shocks also do not appear exogenous: Times such as 1980-82 when the Fed pushes hard on the monetary lever are precisely the times when velocity becomes least predictable.

The following conceptual experiment offers one interpretation of elusive elasticities and velocity shocks. It is not surprising that the short run interest elasticity of money demand is low, since changes in interest costs are trivially small. If you keep \$1,000 in cash and a noninterest bearing checking account, a change from 5% to 6% annual interest rates increases monthly interest costs by 83¢. And since holding extra cash has benefits that at the margin are equal to marginal costs, the utility cost is another order of magnitude smaller than 83¢, say 8¢ per month. Consumers can be forgiven if they don't immediately change their cash management habits for 8¢ monthly utility gains!

But suppose instead that the government moved \$100 from each person's savings account or mutual fund to their checking account overnight. A small interest elasticity implies that interest rates must jump dramatically in response to this change. For now, instead of looking at

$$\ln M = \ln P + a \ln y - br$$

and noticing b is small, we are looking at

$$r = \frac{1}{b}\left[-\ln M + \ln P + a \ln y\right]$$

and noticing that $1/b$ is very large!

Intuition suggests the opposite reaction however: people would just hold the extra $100, because, again, fine details of cash management don't matter that much. At a 5% interest spread, an extra $100 implies 42¢ per month interest cost and an order of magnitude smaller utility cost, say 4¢ per month. If consumers do not adjust immediately to gain the extra 4¢, we see an endogenous velocity shock, associated with the open market operation.

Obviously, if the elasticity depends on which variable is pushed, one would not want to impose a rigid money demand curve on any model. (This analysis owes a strong debt to Akerlof (1979) and Akerlof and Milbourne (1980). They show that $s - S$ money demand policies have similar mushy implications since people must change the $s - S$ bounds before interest rates have any effect.)

Theoretical objections

A generation of theorists have argued that the quantity theory is an increasingly implausible description of modern economies with competitive banking systems, including Black (1970), Fama (1980, 1983, 1985), Hall (1983), King (1983), White (1984), and Cowen and Kroszner (1994). Considering the vast number of liquid, non-reservable inside assets, as well as trade credit, credit cards, debit cards, and other means of financing transactions, and considering the flexibility and competitiveness of financial institutions, it is difficult to believe that an artificial scarcity of one liquid asset can have any systematic effect.

As these authors recognized, some sort of perfectly competitive, frictionless model is a more sensible first-order approximation to the U.S. financial system than is a rigid separation of assets into liquid "money" or "transactions-facilitating assets" and illiquid "investment assets." However, these authors could not get around the view that the price level had to be determined by an explicit commodity-based unit of account, or a special transactions demand for the monetary base together with a limited supply. The fiscal theory gives us a structure that can determine the price level within the natural perfectly competitive or frictionless approximation, while preserving the fact of apparently unbacked fiat money.

One can of course study an infinite-velocity limit of the quantity theory, as advocated by Woodford (1997) and the "currency ghost" view of Cowen and Kroszner (1994). However, it does not seem productive to hinge the price level on whether U.S. transactions can be accomplished with fast-moving claims to one dollar bill, or whether two will be required. At some point, and especially as the interest costs of holding the remaining money become vanishingly small, velocity must become endogenous rather than rigidly linking money to transactions via an ever-longer lever.

What about open market operations and money demand?

What about the common view and empirical evidence that monetary policy affects output? For example, Table 1 shows that federal funds rate shocks explain up to 40% of the variance of output, while explaining none of the variance of prices.

Choosing a fiscal and even frictionless description of the *price level* does not require that open market operations must have *output*-neutral effects. Open market operations can still affect the interest rate spreads of monetary assets, and interest rate spreads can affect output. For example, Bernanke's (1983) non-monetary view of the great depression stresses the disruption of credit arrangements following open market operations. These output effects can occur while the fiscal constraint alone determines the price level. As Goodfriend (1988) reminds us, central banks pursued active interest rate policy, with visible output effects, even under the classical gold standard. One *can* add sticky prices to a fiscal model with monetary frictions, as in Woodford (1997), to generate output variation related to inflation, but one *need not* tie output to inflation (counterfactually, I might add) in order to explain output effects of open market operations.

Similarly, a fiscal theory of the price level is not inconsistent with the observation that money and nominal income often move together. Money *is* useful for transactions, and governments typically provide it elastically as needed, for example by following interest-rate policies. It does not follow from this observation that *if* the government exchanged bonds for money, there would be any effect on the price level.

2.7 Episodes

Dramatic episodes of hyperinflation, stabilization, currency collapse and so forth are perhaps the most natural place to start evaluating the fiscal theory. I focus instead on U.S. data below, in part because theories that are only good for extreme events in the unstable monetary arrangements of faraway (to admittedly parochial U.S. observers) lands will not in the end impact monetary analysis of the U.S. economy. However, a quick look at such episodes helps to illustrate the fiscal theory and make it plausible.

There is no tight relation between debt and the price level in many historical episodes. Wars offer the most dramatic example. Nominal debt increases substantially, often with relatively little change in the price level. Of course, such nominal debt increases also come with the explicit or implicit promise that future taxes will be raised to retire the debt after the war. Hence both sides of the budget identity change, and the price level need not be affected. In fact, recall that (short-term) debt sales with no change in future surpluses produce no extra revenue. The whole point of selling extra nominal debt in a war is to raise revenue. If they raise revenue, such sales *must* have come with an implicit promise to raise future taxes.

The same lesson applies in peacetime: If the government raises revenue by selling additional (short-term) bonds, the debt sale must have come with an explicit or implicit promise to raise future surpluses. Both sides of the identity move at the same time, so we should not expect a tight relation between total nominal debt and the price level.

Hyperinflations are classic pieces of evidence for the quantity theory, since money and the price level both grow very quickly. However, hyperinflating countries issue little nominal debt other than money, so money and nominal debt are the same thing and the fiscal theory predicts the same hyperinflation as the quantity theory.

Hyperinflations are of course linked to government finances. In the standard quantity-theoretic analysis, an intractable budget shortfall forces the government to print money to pay its bills, and the money causes inflation. If a country tried to finance an intractable budget shortfall by rolling over explosive quantities of one-week interest-paying debt, while rigidly controlling the money stock, this would provide a nice experiment: the quantity theory predicts no inflation while the fiscal theory predicts hyperinflation. Alas, it hasn't happened. A country that had explosive *inside* money growth with no government budget problems would provide another nice experiment: The fiscal theory predicts no inflation and the quantity theory predicts hyperinflation. Unfortunately, all the recorded hyperinflations resulted from explosive growth in nominal government debt.

Sargent's (1986) classic study of the ends of hyperinflations again points to a fiscal link, though the analysis is quantity-theoretic and Ricardian. The budget problem is solved; seignorage stops, so inflation stops. Again, the fiscal theory makes the identical prediction that inflation will stop once the budget problem is solved, though directly rather than via its inducements to seignorage. In fact, the fiscal theory nicely accommodates a troubling fact: money growth usually does not stop at the time of the fiscal announcement that ends the hyperinflation. This fact usually has to be explained by an increase in money demand at lower nominal interest rates.

Sargent's analysis and a fiscal theory can differ substantially over the effects of news about *future* surpluses or deficits. In a quantity-theoretic analysis, news about future deficits that will result in future seignorage primarily affects only future inflation. Cagan-style hyperinflation dynamics are the only way that future seignorage can affect today's price level, but such effects are weak since future money growth is discounted at the interest elasticity of money demand (see equation (32)), which is on the order of 0.15. In a fiscal model with short-term debt, future deficits are discounted at the much higher gross interest rate, producing discount factors on the order of 0.95. Thus, the fiscal theory can predict a much stronger reaction of current prices to news of far-off deficits. This prediction depends on details in both cases: if news of future deficits causes the government to start printing money now, the quantity theory can also predict current inflation, and if there is a lot of long-term debt the fiscal theory can predict no current inflation but instead a fall in long-term bond prices reflecting expected future inflation.

In this line, the Asian currency plunges of late 1997 cry for a fiscal analysis. It seems much more plausible that the currencies plunged on bad fiscal news, induced by a wave of bank insolvencies, than on news that open market purchases or seignorage would soon double the money supply. Bad fiscal news also lowers the price – raises the interest rate – of longer-term debt, and high interest rates are characteristic of these crises. The fiscal story also makes sense of the fact, surprising to a standard analysis, that many governments had ample foreign exchange reserves (Burnside, Eichenbaum and Rebelo (1998)).

Similarly, Argentia suffered great stress on its currency board during the Peso crisis, including very high interest rates. Reserves were high in this case as well– the currency was 100% backed. Again, this was a time of great fiscal stress for the government. The temptation to abrogate the board and devalue might well have turned into necessity. The high interest rates make sense again as high nominal rates that include this probability.

Brash (1996) unwittingly offers an essentially fiscal view of New Zealand's celebrated monetary reforms. Though he is the governor of the Reserve Bank of New Zealand, his description of that country's disinflation spends 40 pages on microeconomic reforms, tax reforms, and large and successful fiscal policy reforms before even talking about monetary policy. Then he describes only the political economy of a inflation contract, without once mentioning monetary restriction or open market operations by which this contract is supposed to be implemented. A fiscal theorist sees direct causality from dramatically good fiscal news to the price level in which the actions of the central bank were largely irrelevant.

As Woodford (1996) emphasizes, a fiscal analysis makes sense of the otherwise pointless deficit targets for entry into the European Monetary Union. If (say) Italian debt is to trade at par with (say) German debt, then either Italian surpluses must be sufficient to value that debt, or Germany must implicitly or explicitly stand ready to bail out the Italian budget. Of course, Italian debt may instead trade at a discount, reflecting a possibility of explicit default. Therefore, one must read the deficit targets as an attempt to avoid explicit default as well as subsidy.

3 U.S. debt and inflation

3.1 Data

Easily available U.S. government deficit and debt data are potentially poor approximations to the economic concepts one wants. Above, all, it is important to create a surplus series that corresponds to the revenue from debt operations. (Auerbach Gokhale and Kotlikoff (1994) stress the larger difficulties of deficit measurement.)

To produce more accurate data, I created annual data on privately held U.S. government debt from the CRSP government bond files, which in turn record data from the *Treasury Bulletin*. From these data, I created an annual series on the total real value of the debt, v_t^a as the sum of December 31 price times quantity of all bonds outstanding, divided by the December CPI. I estimated the annual rate of return on the government bond portfolio r_{t+1}^a from the Fama-Bliss (1987) zero-coupon bond return series multiplied by December 31 (t) portfolio weights. Then, I estimated the annual surplus from the identity $s_{t+1}^a = v_t^a r_{t+1}^a - v_{t+1}^a$. (It is more natural in annual data to date December 31, year t debt as v_t^a rather than v_{t+1}^a, so this identity has slightly different timing than the discrete time identities studied above in which v_t denotes beginning of period debt.) I also created a zero-coupon equivalent maturity structure $B_t(t+j)$ by adding up the principal and all coupons of all bonds outstanding at December 31, year t that come due in year $t+j$. I count the monetary base as zero-maturity debt. The data is described in detain in an Appendix available on the author's website.

3.2 Surplus, debt, and returns

Before addressing the fiscal theory directly, I characterize the new data on surplus, debt and returns. I also develop the central patterns that we must try to match.

3.2.1 Surplus

Figure 2 presents the primary surplus inferred from bond data as above together with the conventionally measured surplus or deficit. The two series correlate well through the 1970s, when interest payments on the debt were small. In the 1980s, however, the primary surplus does become positive, while the deficit remains large, reflecting large interest payments on the outstanding debt.

One's first reaction to a view that the price level is set by the interaction of nominal debt and real surpluses might be to ask: what surpluses? Has not the Federal government been in deficit continuously for the last 30 years? Of course, the theory refers to the *primary* surplus, correctly measured; the graph offers hope for the view that debt is eventually repaid with primary surpluses.

The surplus shows a clear cyclical pattern, dipping in the recessions of 1975, 1982 and 1990. Interestingly, the primary surplus suffers its biggest negative shock in 1975, with the onset of severe inflation, not during the Reagan deficits of the 1980s.

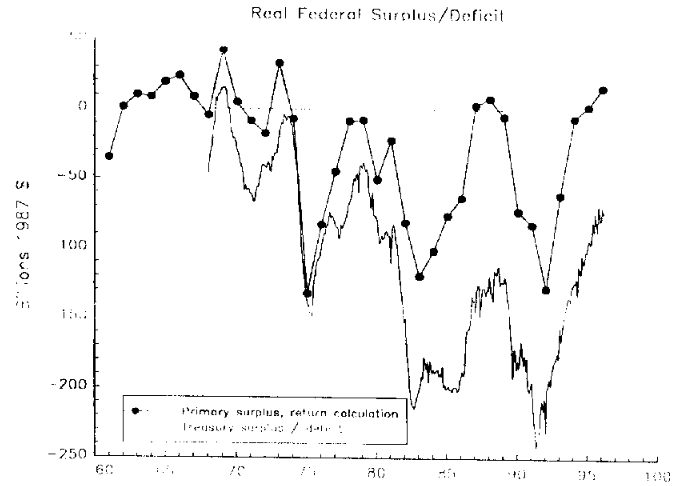

Figure 2: Real primary surplus inferred from bond data and Federal surplus/deficit. All series deflated by the consumer price index. The Federal Surplus or Deficit is monthly observations of annual averages.

Figure 3 contrasts three measures of the primary surplus. The preferred "return calculation" infers the surplus from growth in total debt and the estimated rate of return on government bonds, while the "revenue calculation" sums up revenue from bond transactions during the year, as described in the Appendix. The net of interest surplus is reported by the Treasury, and consists of the total surplus or deficit less interest payments, but not gains and losses incurred from bond sales or purchases.

The three series correlate well, but not perfectly. Unusually active debt policy in 1990-1991 and 1995 drive a wedge between the revenue and return calculations. Both measures somewhat more pessimistic than the net of interest surplus series. The difference is substantial most recently: rather than a $80 billion primary surplus, the bond data show almost no primary surplus or deficit.

Figure 4 presents the components of the revenue-based real surplus series. One can see that the cyclical variation in surplus (the negative of revenue) is driven by variation in new bond and bill sales. However, the need to pay coupons and redeem maturing bonds soon catches up with new sales. From 1983 to 1987 for example, new

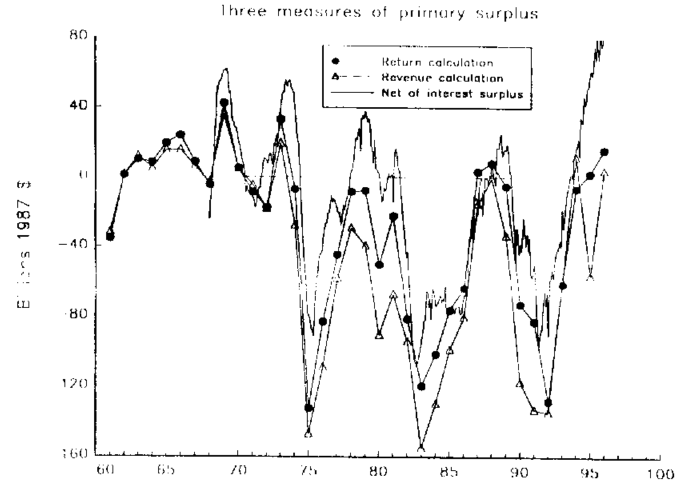

Figure 3: Real surplus inferred from bond data and net of interest surplus reported by the Treasury. The "return calculation" starts by estimating the rate of return on government bonds, and then imputes the surplus from the rate of return and growth in total value. The "revenue calculation" is based on a direct estimate of revenue from debt transactions.

sales continue to rise but revenue declines. Bond sales are spread over maturities, and thus the maturing bonds are much smoother than the bond sales. For example, with only one-period debt, maturing bonds would equal the previous period's sales. Lately, the Treasury has started to sell more of existing issues. Seignorage – change in the monetary base – is an insignificant contributor to government revenue.

3.2.2 Surplus and output

In order to focus on the cyclical properties of the surplus, Figure 5 contrasts the surplus/consumption ratio with the output/consumption ratio. Dividing by consumption allows us to scale variables with growth, producing plausibly stationary series. I divide the surplus by consumption rather than output to avoid avoids putting business-cycle output variation in the surplus measure. The output/consumption ratio exploits the relative stability of consumption (permanent income) to produce a business cycle indicator. (See Cochrane (1994a).)

The graph emphasizes that most variation in the surplus is the predictable result of output variation. The dramatic deficit of 1975 is associated with a severe output drop. The initial Reagan deficits also line up nicely with output. Only 1984 and 1985 are

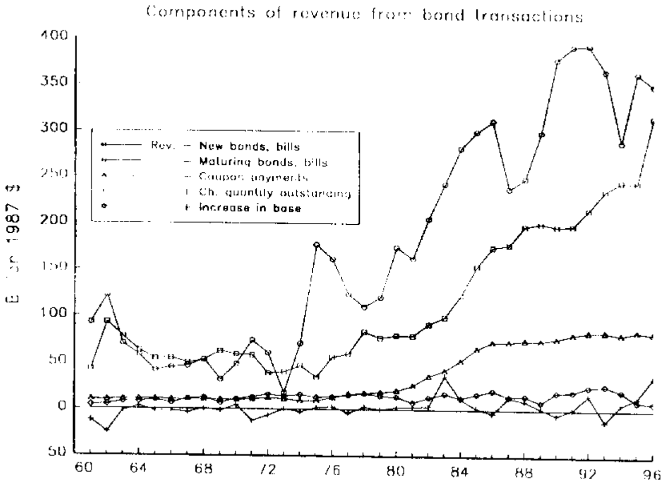

Figure 4: Components of real revenue from bond sales. Revenue (negative surplus) is, by definition, equal to new bond sales + change in quantity outstanding + change in base - maturing bonds -coupon payments.

years with somewhat larger surpluses than would be expected. The "Reagan deficits" resulted from large interest costs on a stock of debt built up over several recessions, *not*, as is often claimed, from an unusually loose primary fiscal policy. The graph also points to a secular relation between surpluses and output. The output slowdown that started in about 1973, even relative to consumption, is associated with a similar secular decline in the surplus.

3.2.3 Surplus and inflation

Figure 6 presents the surplus/consumption ratio together with annual CPI inflation. Since inflation and the surplus are both procyclical, it is little surprise that business cycle movements in the surplus are *positively* correlated with business cycle movements in inflation through the 1970s. On the other hand, the longer-term variation in the surplus and inflation are negatively correlated, as shown by the moving averages.

3.2.4 Bond returns and debt growth

Figure 7 presents the real rate of return on the government bond portfolio, together with the three-month and five-year real rates of return which, along with other returns, are used to construct it.

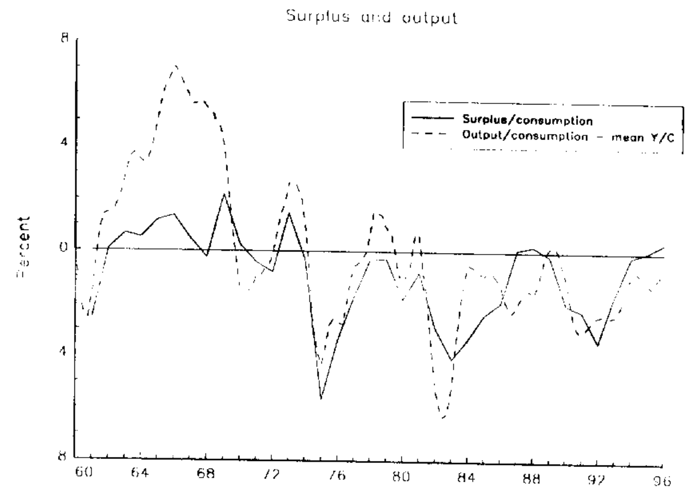

Figure 5: Surplus/consumption and output/consumption ratio. The output/consumption line graphs $y/c - E(y/c)$ so that the graphs fit on the same scale. Consumption is nondurable plus services consumption, output is GDP.

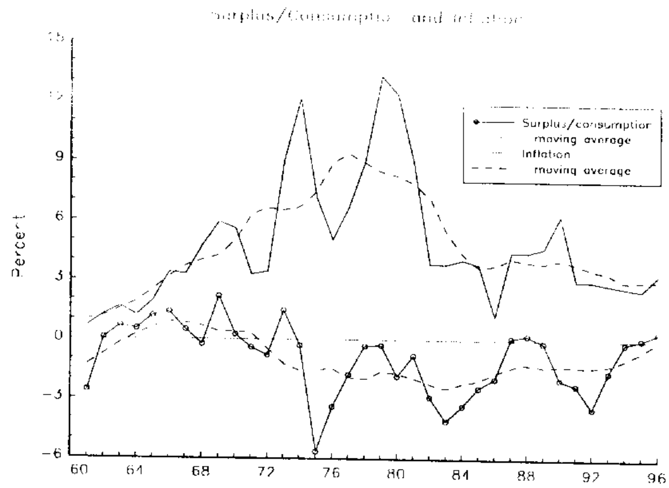

Figure 6: Real primary surplus divided by ND&S consumption, and annual CPI inflation. Moving averages use 3 years' data on each side.

Before about 1980, the average maturity structure is still quite short. Therefore, the government bond portfolio return tracks the three month rate pretty well. In fact, since the monetary base, which pays no interest, is such a large fraction of government debt in this period, the rate of return on government bonds is typically a few points *less* than the 3 month rate. In the 1980s and 1990s, the maturity structure lengthens. Now the government bond average behaves much more like a long-term rate, subject to large swings as long-term bond prices move around.

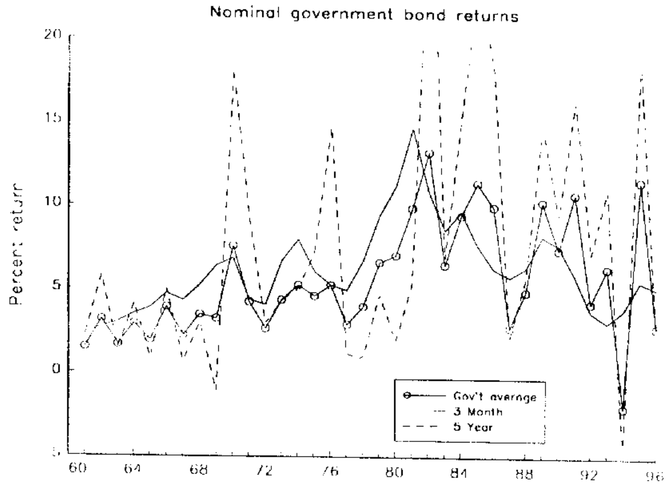

Figure 7: Real rate of return on government bond portfolio, and three-month and five-year zero coupon rates of return. The government bond portfolio return is estimated as the average of all zero-coupon returns weighted by the zero-coupon maturity structure at the beginning of the year.

Figure 8 presents the real rate of return on government bonds and the real percent increase in the value of the debt. The surplus (as a fraction of value) is the difference between the growth in total debt and the return on the government bond portfolio,

$$\frac{s_{t+1}^a}{v_t^a} = r_{t+1}^a - \frac{v_{t+1}^a}{v_t^a},$$

so this graph documents the sources of surplus variation.

In 1975 there was a large (25%) increase in the total value of government debt. However, the rate of return on government bonds was not large at all, so we estimate a large primary deficit, as shown above. Debt also grew very quickly in the early 1980s, but around half of that growth was due to very high real returns on outstanding debt. This is why the surplus measure above did not find extreme primary deficits, as one might have expected.

The graph also reminds us that the real rate of return on government debt increased dramatically in the early 1980s and has stayed high and variable since. This is in part due to higher real returns on government bonds, and partially due to the smaller proportion of monetary base in the debt.

Finally, the graph documents an important and interesting correlation pattern. The surplus is very well negatively correlated with debt growth; returns are positively correlated with debt growth, and the surplus is negatively, though weakly correlated with returns.

	r^a_{t+1}	v^a_{t+1}/v^a_t	s^a_{t+1}/v^a_t
r^a_{t+1}	1	0.70	-0.16
v^a_{t+1}/v^a_t		1	-0.82
s^a_{t+1}/v^a_t			1

Table 3. Correlation matrix of government bond return, real debt growth and surplus/debt ratio.

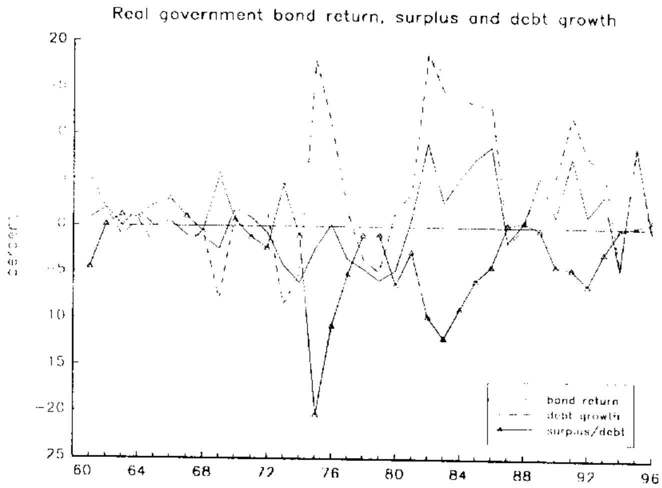

Figure 8: Real return on government bond portfolio, real debt growth, and surplus/debt ratio. The real bond return is estimated from the returns on zero-coupon bonds weighted by the beginning of year maturity structure. Real debt growth is the growth in total market value of the debt. The three series are related by the accounting identity $v_t/v_{t-1} = r^b_t - s_t/v_{t-1}$.

3.2.5 Surplus, value and inflation

Figure 9 presents the real value of the debt value and the surplus, each scaled by consumption, and inflation. The surplus is positively associated with inflation, and negatively associated with changes in value; we will work hard to understand these correlations.

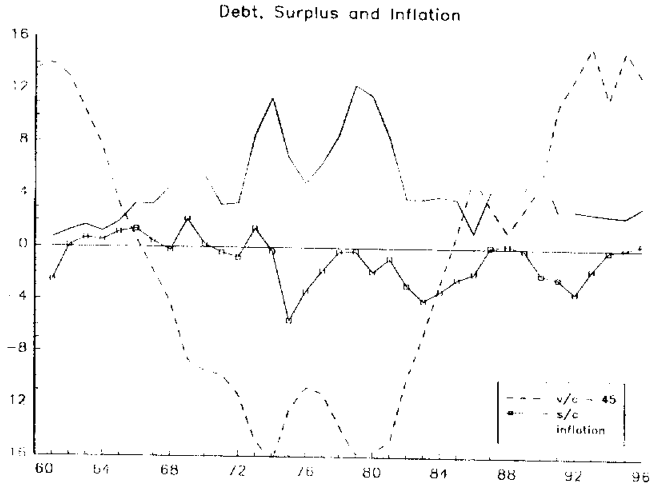

Figure 9: Value/consumption, surplus/consumption and inflation. Value/consumption is shifted down by 45 percentage points to fit on the graph with the other two series.

3.3 Explaining the correlations

Our task is now to understand the pattern of correlations documented above. I start by viewing the surplus and nominal debt as policy choices. The price level is then determined by the government budget constraint. The central issue in matching the data this way is understanding the real value of the debt. If the government controls the nominal value, then the price level is trivially the ratio of nominal to real value of the debt.

A Ricardian or monetary story is *backward-looking:* the nominal value of the debt is determined by the accumulation of past deficits, the price level is determined by $Mv = py$, and these two determine the real value of the debt. The present value of future surpluses must then adjust to pay off this debt. A fiscal story is *forward-looking*. The real value of the debt is determined by the present value of expected future surpluses, and the price level (and/or long-term bond prices) adjusts to equate that real value to the nominal value. Equivalently, the monetary and fiscal analyses tell different stories about *growth* in value. In either case, the accounting identity

$$\frac{v_{t+1}^a}{v_t^a} = r_{t+1}^a - \frac{s_{t+1}^a}{v_t^a}$$

holds. In a monetary story, the ex-post real rate of return on government bonds is the predetermined nominal rate deflated by the monetary-determined price level. Therefore, this identity determines the new real value of the debt v^a_{t+1} from current and past information. In a fiscal story, the value of the debt v^a_{t+1} is determined first; this identity then determines the ex-post rate of return on government bonds r^a_{t+1}; the price level (or the prices of long-term bonds) adjust $r^a_{t+1} = rp_t/p_{t+1}$ so that the identity holds.

From this perspective, the central puzzles are that the level of the real value of the debt seems to have very little to do with surpluses, and, worse, high surpluses are associated with declines in the value of the debt. This pattern of correlations is what a backward looking-view with relatively stable money and hence prices might expect: high deficits mean growing real debt, and the current value of the debt is just cumulated past surpluses.

To see the puzzle from the perspective of a forward-looking view, consider an AR(1) model for the surplus, which the graphs suggest is reasonable. If we write $s^a_t = \rho s^a_{t-1} + \varepsilon_t$ with constant expected returns we obtain a perfect *positive* correlation between surpluses and debt.

$$v^a_t = E_t \sum \beta^j s^a_{t+j} = \frac{\beta\rho}{1-\beta\rho} s_t$$

Currently high surpluses indicate high surpluses in the future, and thus should indicate a *high* real value of the debt. But this positive correlation between surpluses and the value of the debt is completely counterfactual; the value of the debt declines when surpluses are high. (This is the basic idea of Canzoneri, Cumby and Diba's (1997) rejection.)

Of course, an AR(1) surplus process is obvious but perhaps too simple. If we model the surplus as an AR(2) or higher process, low current surpluses can come with news of higher future surpluses, so that the value of the debt rises. While appeal to such a model may seem contrived at first, on second thought it is in fact the most plausible view. Deficits go up – surpluses decline – when taxes decrease and spending increases in a recession. In this situation, the government sells more nominal debt precisely to raise revenue. As we have seen, the only way extra nominal debt sales can raise revenue is if they come with a promise to raise surpluses in the future. If a low surplus did not come with promises of increased surpluses in the future, the government would not raise any extra revenue with extra nominal debt sales.

To understand the issue, it is worth thinking about alternative policies that the government might follow. The extra revenue to cover the declining surplus must come from somewhere. If current surpluses decline in a recession and the government holds future surpluses constant, the price level must increase. This implies a low or negative (net) real return on government bonds; the "extra revenue" comes by inflating away the real value of outstanding debt. This policy – financing cyclical deficits by inflating away outstanding debt – would obviously lead to much more volatile and countercyclical inflation. Thus, the fact that the government follows the current policy, selling more debt in recessions while promising to raise surpluses in the following booms, smooths inflation and the value of government bonds, at least to some extent.

3.3.1 An exogenous-surplus model with short-term debt

To tell a quantitative version of this story, I specify an exogenous path for the surplus and debt. I find the real value of the debt as the present value of the surplus, and the price level is the ratio of real to nominal debt. I specify the processes to deliver the correlations in the data; surpluses are correlated with declining values of the debt, the debt moves much more slowly than the surplus and its level is poorly correlated with the surplus; real and nominal debt growth track closely, and the surplus is negatively correlated with inflation.

Modeling surplus and value.

To match the model with stationary time series, I examine the real value/ consumption and surplus/consumption ratios and the inflation rate (rather than price level). As an accounting identity, the value/consumption and surplus/consumption ratios obey

$$\frac{v_t^a}{c_t} = \frac{1}{r_{t+1}^a}\frac{c_{t+1}}{c_t}\left(\frac{s_{t+1}^a}{c_{t+1}} + \frac{v_{t+1}^a}{c_{t+1}}\right). \tag{33}$$

Define $\beta = E\left[c_{t+1}/(c_t r_{t+1}^a)\right]$. I start by assuming this discount factor is constant over time. Then, we can iterate (33) forward, take expectations and write the value/consumption ratio as the present value of the surplus/consumption ratio. Denote $vc_t \equiv v_t^a/c_t - E\left(v_t^a/c_t\right)$, $sc_t \equiv s_t^a/c_t - E\left(s_t^a/c_t\right)$, and then the value/consumption ratio also obeys the familiar identity,

$$vc_t = E_t \sum_{j=1}^{\infty} \beta^j sc_{t+j}$$

I model the surplus as the sum of a "business cycle" component a_t and a "long run" component z_t,

$$\begin{aligned} sc_t &= z_t + a_t \\ z_t &= \eta_z z_{t-1} + \varepsilon_{zt} \\ a_t &= \eta_a a_{t-1} + \varepsilon_{at} \end{aligned} \tag{34}$$

We saw above how the surplus is highly correlated with output. Therefore, think of the business cycle component a_t as driven by varying output at constant tax and spending policies, and not controlled by the government. Think of the long-term component z_t as reflecting tax rates, spending policies and so forth, which the government does control. However, for optimal-taxation reasons, the government does not want to vary z_t period by period to offset a_t, for example increasing tax rates in recessions in order offset the loss of tax revenue. Thus, as in much of the tax-smoothing literature write z_t as a very persistent process, almost if not exactly a random walk, and the government chooses ε_{zt} each period.

Given this surplus process, the real value of the debt is

$$vc_t = E_t \sum_{j=1}^{\infty} \beta^j sc_{t+j} = \frac{\beta\eta_z}{1-\beta\eta_z} z_t + \frac{\beta\eta_a}{1-\beta\eta_a} a_t \tag{35}$$

Putting together (34) and (35), we can write the observable series s, v in terms of the unobservable surplus components z, a as

$$\begin{bmatrix} sc_t \\ vc_t \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{\beta\eta_z}{1-\beta\eta_z} & \frac{\beta\eta_a}{1-\beta\eta_a} \end{bmatrix} \begin{bmatrix} z_t \\ a_t \end{bmatrix} = B \begin{bmatrix} z_t \\ a_t \end{bmatrix} \tag{36}$$

Parameters

I pick parameters so that sc, vc follow

$$\begin{aligned} \begin{bmatrix} sc_t \\ vc_t \end{bmatrix} &= \begin{bmatrix} 0.55 & 0.06 \\ -0.55 & 0.96 \end{bmatrix} \begin{bmatrix} sc_{t-1} \\ vc_{t-1} \end{bmatrix} + \delta_t; \\ \begin{bmatrix} \sigma(\delta_s) & \rho_{sv} \\ & \sigma(\delta_v) \end{bmatrix} &= \begin{bmatrix} .013 & -0.55 \\ & .035 \end{bmatrix}. \end{aligned} \tag{37}$$

This is the OLS estimate, except for the lower left -0.55 coefficient of vc_t on sc_{t-1}. The OLS estimate is -0.75 ($s.e.$ = 0.26); I use -0.55 instead in order to satisfy the constraint that this coefficient equal the negative of the coefficient of sc_t on vc_{t-1} implied by the structural model, or more generally by the fact that vc is the present value of sc. The corresponding "structural" parameters are

$$\begin{aligned} \begin{bmatrix} a_t \\ z_t \end{bmatrix} &= \begin{bmatrix} 0.64 & 0 \\ 0 & 0.87 \end{bmatrix} \begin{bmatrix} a_{t-1} \\ z_{t-1} \end{bmatrix} + \varepsilon_t; \\ \begin{bmatrix} \sigma(\varepsilon_z) & \rho_{az} \\ & \sigma(\varepsilon_a) \end{bmatrix} &= \begin{bmatrix} .023 & -0.95 \\ & .011 \end{bmatrix}; \ \beta = 0.988. \end{aligned} \tag{38}$$

The difference between the "structural" representation (38) and the surplus, value VAR (37) is very important. The surplus $s = a + z$ is exogenous; it does not respond to the real value of the debt or to prices. Yet the surplus seems to respond to the debt in the VAR representation, what Bohn (1998) interprets as "corrective action." In this case, the value of the debt reveals changing surplus forecasts.

As expected, the "structural" representation has one business cycle component, $\eta_a = 0.64$, and one slow moving component with $\eta_z = 0.87$.

The negative correlation between surplus and value innovations in the data, -0.55, induces an even stronger negative correlation between business cycle and long-run surplus innovations in the "structural" model, -0.95. As above, this is a central part of the story: when there is a negative business-cycle surplus shock, the government wants to raise revenue by nominal debt sales; to do so it must increase the present value of future surpluses by increasing the long-run component of the deficit.

Inflation and nominal debt.

I consider only one-period debt, whose nominal value is V_t^a. Real and nominal debt are of course related by

$$\frac{V_t^a}{p_t} = v_t^a \tag{39}$$

In this simple model, inflation is directly controlled by the government via the decision of how much nominal debt to issue for a given real value of the surplus. Therefore, we can model inflation or nominal debt and find the value of the other. I model inflation, and then calculate the supporting nominal debt policy later. (In discrete time, the government can only affect next period's price level by changing nominal debt. It is best to think of the model operating at higher than annual frequency, however, so the government can control this year's price level with this year's debt.)

The government chooses debt so that inflation is a function of the two state variables z, a :

$$dp_t \equiv \Delta \ln p_t - \Delta \ln p = \left[\begin{array}{cc} \alpha_z & \alpha_a \end{array} \right] \left[\begin{array}{c} z_t \\ a_t \end{array} \right].$$

where $\Delta \ln p = E(\Delta \ln p_t)$ is the steady state. I chose the parameters α so that

$$dp_t = \left[\begin{array}{cc} 1 & -0.21 \end{array} \right] \left[\begin{array}{c} sc_t \\ vc_t \end{array} \right] \tag{40}$$

One can recover the underlying nominal debt policy by differencing (39).

Clearly, this model will only capture the parts of inflation that are correlated with surplus and value. This is in some sense the interesting part: we want to understand the puzzling positive correlation of surplus and inflation. To fully capture the inflation time series, we can add an additional inflation or nominal debt shock.

Artificial time series

Figure 10 presents artificial time series from this system. The shocks are generated by a random number generator. Comparing to actual data on the same series in

Figure 9, we see that the system replicates the initially puzzling features of the data: The value of the debt is poorly correlated with the surplus and moves more slowly than the surplus; the short run correlations between value and surplus are negative – the surplus is highly negatively correlated with *growth* in the value of the debt, and business cycle movements in inflation are positively correlated with the surplus.

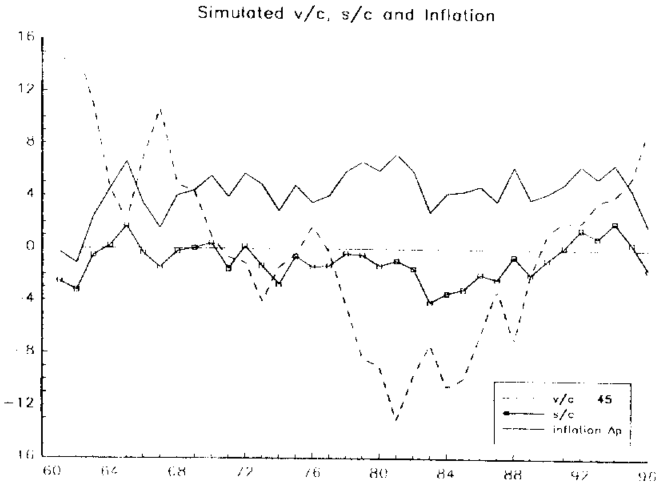

Figure 10: Artificial data on surplus/consumption, value/consumption and inflation. Shocks are drawn from a random number generator.

If we use the sample residuals from the sc, vc VAR representation rather than draw residuals from a random number generator, the sample and artificial sc, vc series match by construction. However, since we do not have an inflation shock, inflation does not match exactly. Figure 11 shows actual and simulated inflation, using parameters (40). The figure shows that the model does a good job of matching both the secular and cyclical fluctuations in inflation.

The parameters relating inflation to sc, vc are ad-hoc. I tried picking parameters to replicate the OLS regression $dp_t = 0.08 \times sc_t - 0.21 \times vc_t + \varepsilon_t$. This simulation tracks the level of inflation better but misses the cyclical fluctuations. Since we cannot statistically fit a three series model with two shocks, one cannot argue between the two parameterizations on statistical grounds; I stick with the former parameterization since it produces a subjectively more convincing story at the business cycle frequencies.

Debt policy and inflation smoothing

We think of the government as picking the nominal debt V_t along with the long-run surplus shock ε_{zt} each period. For convenience, I have characterized this policy by its

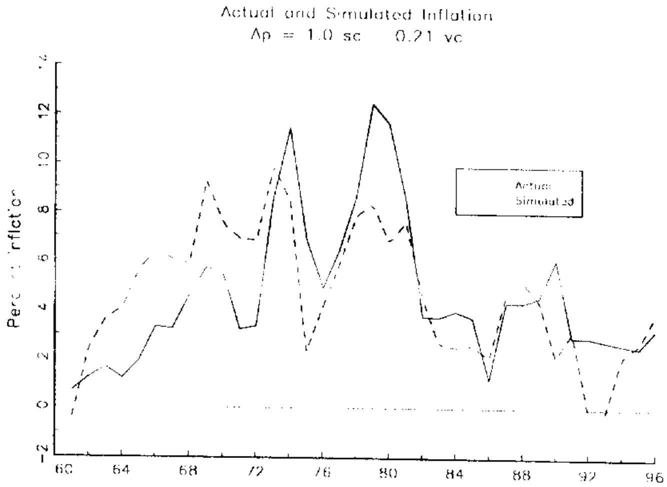

Figure 11: Actual and simulated inflation.

inflation outcome, but we should look at the actual nominal debt policy. The actual and simulated *real* debt growth (vc_{t+1}/vc_t) are exactly the same. Hence, I calculate the nominal debt growth that generates simulated price level p^{sim} by $V_t = p_t^{sim} v_t^a$. Actual nominal debt growth similarly generates actual inflation. Figure 12 presents real and nominal debt growth; inflation is of course the difference between real and nominal debt growth.

Figure 12 emphasizes that *fluctuations in debt growth are far larger than fluctuations of inflation.* Furthermore, *fluctuations in nominal debt growth closely mirror fluctuations in read debt growth.*

What policy for nominal debt growth would have resulted in *zero* inflation? The answer is one way of getting at the question, what *caused* inflation? One answer in this case is easy: if nominal debt growth had been the same as real debt growth, inflation would have been zero. However, the *character* of such a policy is quite surprising, since real debt growth and inflation are so strongly negatively correlated. Nominal debt growth should have been *more* volatile. It should have declined even more sharply that it already did in 1973 and 1979, for example.

We are used to the monetarist claim that bad inflation outcomes have come from excessively volatile monetary policy; that stable (k-percent) money growth rules would have led to stable inflation. The exact opposite is the case here. Growth in the *real* value of the debt is so volatile, that steady (k-percent) nominal debt growth would have resulted in wildly fluctuating inflation. Wild swings in nominal debt growth in fact did a great deal to stabilize inflation.

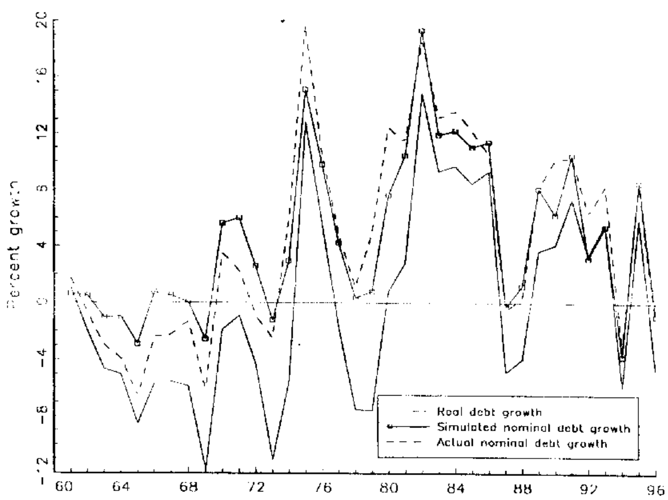

Figure 12: Real and nominal debt growth, and simulated nominal debt growth = real debt growth × simulated inflation.

Real debt growth could also have been different. Of course, constant surplus along with constant nominal debt would have given a constant price level, but it is more convincing to think about alternative surplus policies within the constraint that the cyclical component a_t is beyond the government's control, and that the persistence of the long run component z_t must be respected, leaving the government only the choice of its innovation ε_{zt}. To have a constant price level in this way, the government must choose positive long run shocks to exactly offset bad cyclical shocks. From the identity (33), with $\Delta \equiv E_t - E_{t-1}$, no innovation in the rate of return implies

$$-\Delta sc_t = \Delta vc_t = \Delta \sum \beta^j sc_{t+j}$$

The model implies

$$\Delta \sum \beta^j sc_{t+j} = \frac{1}{1-\beta\eta_z}\varepsilon_{zt} + \frac{1}{1-\beta\eta_a}\varepsilon_{at}$$

$$\Delta sc_t = \varepsilon_{zt} + \varepsilon_{at}$$

Solving for ε_{zt}

$$\varepsilon_{zt} = -\frac{\left(1+\frac{1}{1-\eta_a\beta}\right)}{\left(1+\frac{1}{1-\eta_z\beta}\right)}\varepsilon_{at}.$$

Only this choice for ε_{zt} will leave no innovation in the price level.

Evaluating the terms in parentheses using the above persistence parameters η_a, η_z, the government could have eliminated inflation with a z process that had -1.00 correlation with the a process, rather than the actual -0.95 correlation, and with a standard deviation of 0.105 rather than 0.11. Again, we see how much inflation smoothing is already in debt policy. The orthogonal component of the surplus process that "caused" inflation is quite small.

This example also shows quantitatively how the initially puzzling features of the data flow naturally from a government that is trying to smooth inflation despite large cyclical surplus shocks. In order to smooth inflation, long-term surpluses *must* rise when short-term surpluses decline, and low surpluses must be associated with declining value of the debt.

This logic and Figure 12 also suggest how we could have missed a fiscal determination of inflation all along. If we had lived in an economy with stable nominal debt, fluctuating real values and correspondingly fluctuating inflation, or if we had lived in an economy with stable real values of the debt but fluctuating nominal values causing inflation, we would have noticed. In fact, we lived in an economy with wildly fluctuating real values of the debt, and with nominal values that almost–but not quite–smoothed inflation. Equivalently, there is so little independent variation in real and nominal values of the debt, that we never see time series corresponding to classic experiments, in which sone of surplus or debt is varied and the other is held constant.

The idea that the government can separately determine nominal debt and surpluses is strained, however. In fact, nominal debt sales are the most likely signal of future surpluses. If the government simply sold less debt in the late 70's, consumers may have misread this to mean that future surpluses were also going to be lower, so revenue from bond sales would have been less. The next step in this kind of modeling therefore should be to recognize a *regime*. Consumers rationally infer expected future surpluses from nominal debt sales. Then, the government can really only choose one quantity at each date. To change inflation, the government must change regimes to one in which nominal debt sales do and are understood to carry larger changes in future real surpluses.

Over the long run of decades, the nominal debt growth and inflation are *positively* rather than negatively correlated. Hence, at very low frequencies we can understand inflation as the consequence of excessive nominal debt growth with relatively constant surpluses rather than require changing expectations of future surpluses to do the work.

3.4 Bond returns

So far, I have assumed that the expected government bond return is constant. Variation in the expected rate of return at which future surpluses are discounted may account for substantial variation in the real value of the debt. Suggestively, the government bond return varies by about as much as the surplus/consumption ratio, so variation in bond returns is at least a plausibly important source of variation in the real value of the debt. Also, fluctuations in government bond returns are clearly associated with cyclical movements in inflation, and the disinflation of the early 1980s was associated with a large increase in bond returns.

In this section, I give a preliminary assessment of whether expected return variation is an important part of the story. I apply the methodology used by Campbell and Shiller (1988) and Cochrane (1992) to decompose the variation in stock market prices into expected dividend growth, expected return and bubble components. They find that almost all variation in stock market values is due to varying expected returns and almost none due to variation in expected future dividend growth, so perhaps the same is true here.

Linearizing the present value relation

To separate the present value identity into additive return and surplus components, I Taylor approximate the one-period identity (33) around its steady state and iterate forward, following Campbell and Shiller (1988). Table 4 summarizes steady state values and deviations from steady state

Variable	Notation: Steady state	Notation: Deviation from steady state	Sample Mean[4]	s.s. value in calculation
v_t^a/c_t	vc	$vc_t = \ln(v_t^a/c_t) - vc$	43%	43%
s_t^a/c_t	sc	$sc_t = s_t^a/c_t - sc$	-0.9%	+0.4%
c_{t+1}/c_t	Δc	$dc_t = \ln(c_{t+1}/c_t) - \Delta c$	3.2%	
r_{t+1}^a	r^a	$\tilde{r}_{t+1}^a = \ln(r_{t+1}^a) - \ln(r^a)$	0.64%	
$\frac{1}{r_{t+1}^a}\frac{c_{t+1}}{c_t}$	ρ		1.025	0.99

Table 4. Variable definitions and values for approximate present value identity.

[4] Geometric mean where appropriate, e.g.

$$vc = \exp\left[E(\ln(v_t/c_t))\right].$$

The linearized version of the iterated identity is

$$vc_t \approx -\sum_{j=1}^{k} \rho^{j-1} \tilde{r}^a_{t+j} + \sum_{j=1}^{k} \rho^{j-1} dc_{t+j} + \frac{1}{vc} \sum_{j=1}^{k} \rho^j sc_{t+j} + \rho^k vc_{t+k} \tag{41}$$

s^a_t/c_t can be negative so I do not approximate it in logs. I verified the accuracy of the approximation by graphing vc_t constructed back from the last date in the sample according to (41) against the actual value, and it is quite accurate. Letting the horizon go to infinity and taking expectations yields a linearized version of the present value formula

$$vc_t \approx -E_t \sum_{j=1}^{\infty} \rho^{j-1} \tilde{r}^a_{t+j} + E_t \sum_{j=1}^{\infty} \rho^{j-1} dc_{t+j} + \frac{1}{vc} E_t \sum_{j=1}^{\infty} \rho^j sc_{t+j} \tag{42}$$

Real debt/consumption is high if the effective discount rate is low, or if future surpluses are high. This identity holds for any information set that includes v^a_t/c_t.

The steady state is defined by $(1-\rho)vc = \rho \times sc$ and $\rho \equiv dc/r^a$. We may rightly worry that the steady state and the fiscal theory in general requires consistently positive and high surpluses. However, since ρ is quite near one – the average real bond return is close to the average consumption growth rate – a very small steady state surplus can service a large real debt.

The sample mean surplus/consumption ratio is in fact negative, and the sample mean real bond return in Table 4 is less than the sample mean consumption growth, implying $\rho > 1$ which means all the sums explode! However, as Figure 8 shows, the low returns are driven by the 1970s and the low average surplus is driven by the 1980s. One might reasonably regard these experiences as unusual. For this reason I impose $\rho = 0.99$ rather than use sample means. The implied steady state surplus/consumption ratio is 0.4%, which is positive but not unbelievably high given the sample experience that has ranged from +2% to −6% (see Figure 5).

Perfect foresight

The next question is, how much information should we consider when evaluating the identity? First, let us assume perfect foresight. Suppose people knew exactly what the path of future real government bond returns and future surpluses was going to be. Does variation in surpluses or variation in returns account for variation in the real value of the debt?

To answer this question, I calculate at each date t the terms in (41), iterated to the end of the sample. For example at date t, the "component due to future returns" is

$$\text{return component}_t = -\sum_{j=t+1}^{T} \rho^{j-t-1} \tilde{r}^a_j$$

Figure 13 presents the results. The sloping dashed line is the mean or steady state plus the present value of the endpoint, $\rho^{T-t} v^a_T/c_T$. The solid and dashed variable lines add the return component and the return plus surplus components to the endpoint component. As we move back through time, these variable components explain variation of the v^a/c ratio through time. The difference between "end + return + surplus" and the actual v^a/c ratio is due to the consumption component and approximation error, both of which are small.

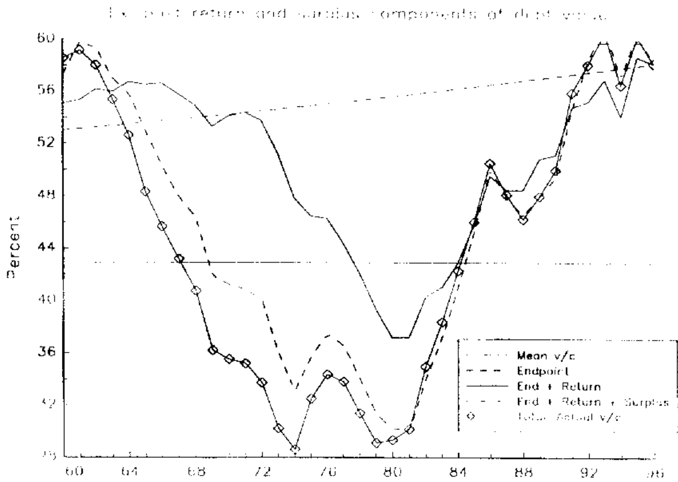

Figure 13: Components of value/consumption ratio.

Starting in the mid 60s, the value of the debt falls dramatically. Since the "end+return line" is fairly steady, but the "end+return+surplus" line tracks the decline; the decline is attributed to changing surpluses. The large deficits of 1975 and the early 80s were starting to appear on the horizon, driving down the value of the debt. Starting in 1972, however, the "end+return+surplus" line is fairly steady, while the "end+return" line drops rapidly. Now the high returns of the 1980s are starting to matter. By 1980, future budgets moved back into surplus, but the value of the debt is still low because interest (discount) rates are so high. As we move to the present, good surpluses are discounted with fewer and fewer high interest rates, raising the value of the debt.

With perfect foresight, then, both surplus and return variation are important in understanding the real value of the debt.

A Simple VAR

Perfect foresight is of course an extreme assumption. What story can we tell based on documentable patterns of forecastability in the variables? To address this question, I form the expectations in (42) with simple VARs. Denoting the VAR

$$x_t = Ax_{t-1} + \varepsilon_t$$

we measure terms in (42) by, for example,

$$E\left[\sum_{j=1}^{\infty} \rho^{j-1} \Delta \ln r^a_{t+j} \middle| x_t\right] = e_r' \left[A(I-\rho A)^{-1} x_t\right]$$

where e_r is a vector of zeros and ones that picks off the return element of the VAR.

I form the VAR with a single lag of the debt/consumption ratio, real bond return, surplus/consumption ratio, and consumption growth. Table 5 presents the estimated VAR equations. The important points: The real debt/consumption ratio is very persistent (0.96), as we expect. Higher surpluses forecast lower debt (-0.73). Again, we must think that high surpluses forecast low future surpluses to account for this fact. The surplus is also persistent (0.46). Higher debt forecasts slightly higher (0.067) surpluses. This is a key coefficient, and its statistical significance is encouraging. Higher debt also forecasts higher bond returns (0.046), though this is less significant. Overall, bond returns and consumption growth seem nearly unforecastable.

Left-hand variable	Right-hand variable $\ln \frac{v}{c}_{t-1}$	$\frac{1}{vc}\frac{s}{c}_{t-1}$	$\ln r^b_{t-1}$	$\ln \frac{c_{t-1}}{c_{t-2}}$	R^2
$\ln \frac{v}{c}_t$	0.96	-0.73	0.18	-2.31	0.95
(t-stat.)	(21)	(-3.1)	(0.6)	(-3.7)	
$\frac{1}{vc}\frac{s}{c}_t$	0.067	0.46	-0.19	1.23	0.58
(t-stat.)	(2.9)	(3.9)	(-1.4)	(4.0)	
$\ln r^b_t$	0.046	-0.23	0.13	-0.46	0.20
(t-stat.)	(1.3)	(-1.3)	(0.7)	(-1.0)	
$\ln \frac{c_t}{c_{t-1}}$	0.00	0.001	0.06	0.37	0.05
(t-stat.)	(0.00)	(0.02)	(0.8)	(2.2)	

Table 5. OLS regressions 1960-1996. T-statistics in parentheses

Figure 14 presents the contributions of return, surplus, and consumption growth terms to explaining the debt/consumption ratio, along with the actual ratio. For example, the line marked return graphs the time series $e_r' A(I-\rho A)^{-1} x_t$.

In contrast to the ex-post decomposition, *the forecastable components of the surplus now almost exactly account for all variation in the value of the debt.* The central facts behind this result is that higher value forecasts higher future surpluses, as seen in the coefficient of surplus on value in the VAR, and surpluses are persistent. Though a higher value also forecasts a lower bond return, bond returns are much less persistent.

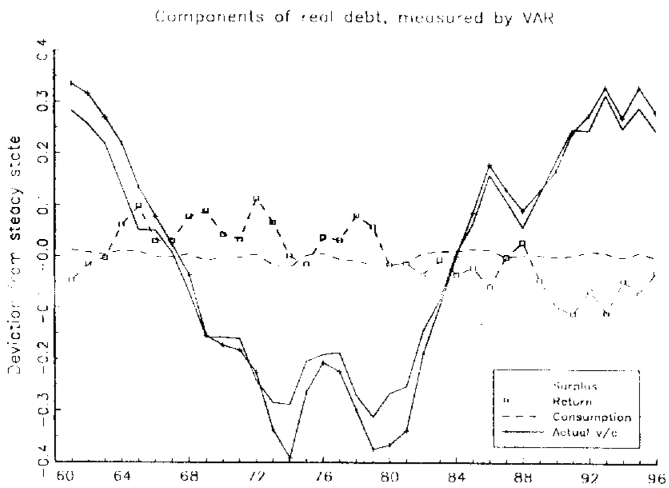

Figure 14: Components of value, as measured by VAR.

We can make the same point quantitatively with a variance decomposition. From (42),

$$\begin{aligned} var\left[vc_t\right] &\approx -\sum_{j=1}^{\infty}\rho^{j-1}cov\left(vc_t, r^n_{t+j}\right) + \sum_{j=1}^{\infty}\rho^{j-1}cov(vc_t, dc_{t+j}) \\ &\quad + \frac{1}{vc}\sum_{j=1}^{\infty}\rho^{j}cov(vc_t, sc_{t+j}) \end{aligned}$$

The debt/consumption ratio can *only* vary if it forecasts returns, surpluses or consumption growth. The question is, which components account for fluctuations in value? I estimate the above moments from the VAR representation. For example,

$$\sum_{j=1}^{\infty}\rho^{j}cov(vc_t, sc_{t+j}) = e_v' E(xx')\rho A(I-\rho A)^{-1}e_s; \quad E(xx') = \sum_{j=0}^{\infty}A^{j}\Sigma A^{j'}$$

Table 6 presents the fractions of debt value variance so explained. Again, we see that forecasts of future surpluses account for the vast majority of fluctuations in debt value. Furthermore, almost all variation is accounted for; we do not have to rely on bubble terms.

Surplus	Return	Consumption
84.7%	13.6%	2.6%

Table 6. Fractions of debt value/consumption explained by VAR forecasts of surplus/consumption, government bond returns and consumption

growth. Each term is $100 \times cov(vc_t, \sum_j \rho^j x_{t+j})/var(vc_t)$ as estimated from the VAR representation.

We are left with an unsettling picture. Based on perfect foresight, expected return variation is an important determinant of the value of the debt. Based on a simple VAR, expected return variation is unimportant. The latter result depends on the VAR: Variables such as yield spreads that forecast more long-run interest rate variation could raise the contribution of bond returns.

3.5 Maturity

The maturity structure is also potentially significant in our attempt to make sense of fiscal price determination. Bad fiscal news might be met by declines in long-term bond prices rather than a rise in the price level. Long-term debt sales can raise revenue with no change in future surpluses, by diluting the claims of existing long-term bonds. The inflation of the 70's came down sharply along with large sales of long-term debt in the early 80s, and inflation comes down with large debt sales in recessions. This mechanism may provide part of the explanation. Here I present some facts about the maturity structure that help us to see whether this is an important route to follow.

Figure 15 presents measures of the maturity structure, on a zero coupon equivalent basis as always. Overall, the maturity structure is surprisingly short: 40 to 70% of the debt has maturity one year or less, and is rolled over every year. These are *face* values; the market values of long-term debt are even smaller. As is well known from simpler measures, the maturity structure was quite short until about 1975. Then longer term debt gradually became more and more important. The build-up in long-term debt has been gradual, peaking in about 1988.

Figure 16 presents the maturity structure of debt with more than one-year maturity. (One-year and less maturity is such a large fraction of the debt that one cannot see the rest if it is included.) Starting in 1960, a few very long-term, low coupon bonds are outstanding. The spikes in the maturity structure are the principal amounts of these bonds. As the bonds age, the spikes move in towards the zero-maturity point. The government sold very little long-term debt, so the major feature of the term structure, and the root cause of the shortening maturity structure seen in Figure 15 is the aging of this long-term debt.

Starting about 1975, along with the first big primary deficit, we can see the effect of new, regular long-term debt sales. Initially, these sales also leave lumps in the maturity structure, but soon the coupons of the accumulated long-term bonds smooth the maturity structure. A few large sales in the mid-1980s show up as lumps

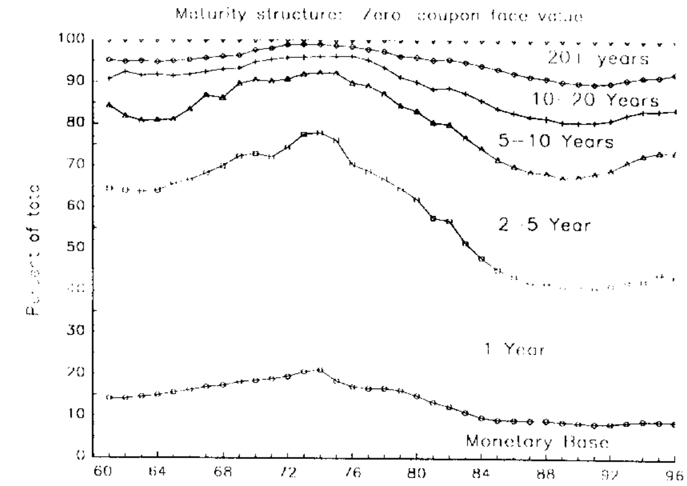

Figure 15: Maturity structure of debt, on a zero coupon equivalent basis. For example, debt between one and two years includes debt with one and two year maturity, plus all coupon payments that come due between one and two years.

that then age. One can also see increased sales of 10 year debt, and 2 year debt in the eventual height of the lines on the left-hand side.

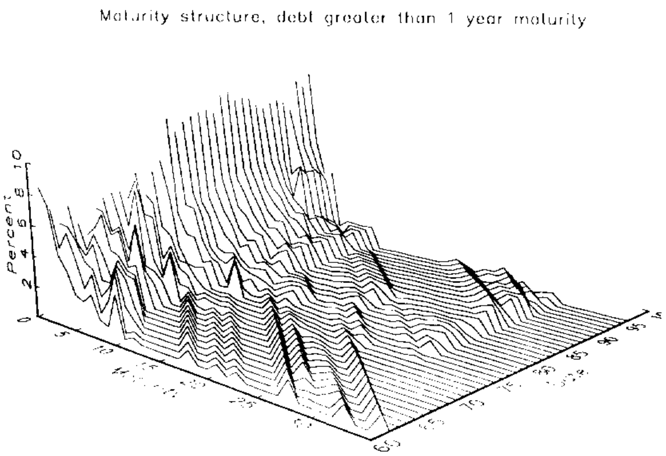

Figure 16: Maturity structure of debt, on a zero-coupon basis. Each curve is the face value of outstanding debt divided by the face value of all debt.

4 Conclusion

Two main themes recur through this article. First, one can apply the view that money is valued because it is backed to modern economies with apparently unbacked fiat money. In systems in which money is explicitly backed such as a gold standard or currency board, it turns out that the backing in terms of overall government resources is all that really matters, and this backing continues to matter when explicit backing disappears. When money is valued because it is backed, the fact that certain assets have a liquidity value in exchange has at best second-order effects on the price level, and the value of money will therefore not be affected by financial innovation. Second, in order to understand U.S. data from this fiscal perspective, we must view the primary surplus process as one in which a negative shock today induces a positive shock in the long run. While not immediately obvious, this is a natural specification. The government is faced with cyclical surplus shocks about which it can do little, yet it does not want wildly fluctuating and countercyclical inflation. Therefore, it sells extra debt in recessions, raising revenue by so doing because it implicitly promises to raise subsequent surpluses.

5 References

Akerlof, George A. 1979, "Irving Fisher on his head: The consequences of constant threshold-target monitoring of money holdings, *Quarterly Journal of Economics* 93 169-89.

Akerlof, George A. and Ross D. Milbourne 1980 "Irving Fisher on his Head II: The Consequences of the Timing of Payments for the Demand for Money" *Quarterly Journal of Economics*

Auerbach, Alan J., Jagadeesh Gokhale, and Laurence J. Kotlikoff, 1994, "Generational Accounting: A Meaningful Way to Evaluate Fiscal Policy," *Journal of Economic Perspectives,* 8, 73-94.

Baba, Yoshihisa, Hendry, David F., Starr, Ross M. 1992, "The Demand for M1 in the U.S.A., 1960-1988" *Review of Economic Studies*, 59 25-61.

Bernanke, Ben S., 1983, "Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression" *The American Economic Review,* 73, 257-276.

Black, Fisher 1970 Banking and Interest Rates in a World Without Money, *Journal of Bank Research* Autumn 9-20.

Bohn, Henning 1998, "The Behavior of U.S. Public Debt and Deficits," Working Paper, University of California at Santa Barbara.

Brash, Donald T. 1996, "New Zealand's Remarkable Reforms," Fifth IEA Annual Hayek Memorial Lecture, Occasional Paper 100, Institute for Economic Affairs

Burnside, Craig, Martin Eichenbaum and Sergio Rebelo, 1998, "Prospective Deficits and the Asian Currency Crisis," Working paper, Northwestern University

Canzoneri, Matthew B., Robert E. Cumby, Behzad T. Diba, 1997 "Is the price level determined by the needs of fiscal solvency?" Working paper, Georgetown University

Campbell, John Y. and Robert J. Shiller, 1988, "Stock Prices, Earnings, and Expected Dividends" *Journal of Finance* 43, 661-76.

Christiano, Lawrence J., Martin Eichenbaum, Charles L. Evans, 1998, "Monetary Policy Shocks: What Have We Learned and to What End?" Manuscript, Northwestern University and Federal Reserve Bank of Chicago.

Cochrane, John H. 1992, "Explaining the variance of price-dividend ratios" *Review of Financial Studies,* 5, 243-80.

Cochrane, John H. 1994a "Permanent and Transitory Components of GNP and Stock Prices," *Quarterly Journal of Economics* CIX, 241-66.

Cochrane, John H., 1994b, "Shocks" *Carnegie-Rochester Conference Series on Public Policy* 41, 295-364.

Cochrane, John H. 1998, "Maturity matters: Long-term debt in the fiscal theory of the price level" Manuscript, University of Chicago.

Cowen, Tyler and Randall Kroszner 1994 *Explorations in the New Monetary Economics* Cambridge MA: Blackwell

Dupour, Bill 1997 "Exchange Rates and Bank Notes: The Fiscal Theory of the Price Level" Manuscript, University of Pennsylvania.

Fama, Eugene F. 1980 "Banking in a theory of finance" *Journal of Monetary Economics* 6, 39-57.

Fama, Eugene F. 1983 "Financial Intermediation and Price Level Control" *Journal of Monetary Economics* 12, 7-28.

Fama, Eugene F. 1985 "What's Different About Banks?" *Journal of Monetary Economics* 15, 29-39.

Fama, Eugene F. and Robert R. Bliss 1987, "The Information in Long-Maturity Forward Rates," *American Economic Review,* 77 680- 92.

Goodfriend, Marvin, 1998, "Central Banking Under the Gold Standard," *Carnegie-Rochester Conference Series on Public Policy*, 29, 85-124.

Hall, Robert E. 1983 "Optimal Fiduciary Monetary Systems" *Journal of Monetary Economics* 12, 33-50.

Hamilton, James D. and Marjorie A. Flavin, 1986, "On the Limitations of Government Borrowing: A Framework for Empirical Testing," *American Economic Review*, 76, 808-19.

King, Robert G. 1983, "On the Economics of Private Money" *Journal of Monetary Economics* 12 127-158.

Leeper, Eric 1991, "Equilibria Under 'Active' and 'Passive' Monetary Policies," *Journal of Monetary Economics* 27, 129-147.

Lucas, Robert E. Jr., 1980 "Equilibrium in a pure currency economy," *Economic Inquiry* 18, 203-220.

Lucas, Robert E., Jr. 1988, "Money Demand in the United States: A Quantitative Review," *Carnegie-Rochester Conference Series on Public Policy*, 29, 137-67.

Porter Richard D. and Ruth A. Judson 1996 "The Location of U.S. Currency: How Much is Abroad?" *Federal Reserve Bulletin* (October)

Sargent, Thomas J. 1986, *Rational Expectations and Inflation* New York: Harper & Row

Sargent, Thomas J. 1987, *Dynamic Macroeconomic Theory*, Cambridge MA: Harvard University Press.

Schmitt-Grohé, Stephanie and Martin Uribe, 1997, "Price Level Determinacy and Monetary Policy Under a Balanced-Budget Requirement" Manuscript, Federal Reserve Board.

Sims, Christopher A. 1994, "A Simple Model for the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory* 4, 381-399.

Sims, Christopher A. 1997, "Fiscal Foundations of Price Stability in Open Economies," Working Paper, Yale University.

White, Lawrence H., 1984, "Competitive Payments Systems and the Unit of Account" *American Economic Review* 74, 699-712.

Woodford, Michael 1994, "Monetary policy and price level determinacy in a cash-in-advance economy" *Economic Theory* 4, 345-389

Woodford, Michael 1995, "Price Level Determinacy Without Control of a Monetary Aggregate," *Carnegie-Rochester Conference Series on Public Policy* 43, 1-46.

Woodford, Michael 1996, "Control of the Public Debt: A Requirement for Price Stability?" *NBER Working Paper 5684.*

Woodford, Michael, 1997 "Doing Without Money: Controlling Inflation in a Post-Monetary World" *NBER Working Paper 6188.*