A Fundamental Plane of black hole activity

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ABSTRACT

We examine the disc-jet connection in stellar mass and supermassive black holes by investigating the properties of their compact emission in the X-ray and radio bands. We compile a sample of ~ 100 active galactic nuclei with measured masses, 5-GHz core emission, and 2–10 keV luminosities, together with eight galactic black holes with a total of \sim 50 simultaneous observations in the radio and X-ray bands. Using this sample, we study the correlations between the radio (L_R) and the X-ray (L_X) luminosity and the black hole mass (M). We find that the radio luminosity is correlated with both M and L_X , at a highly significant level. In particular, we show that the sources define a 'Fundamental Plane' in the three-dimensional (log $L_{\rm R}$, log $L_{\rm X}$, log M) space, given by log $L_{\rm R} = (0.60^{+0.11}_{-0.11})$ log $L_{\rm X} + (0.78^{+0.11}_{-0.09})$ log $M + 7.33^{+4.05}_{-4.07}$, with a substantial scatter of $\sigma_{\rm R} = 0.88$. We compare our results to the theoretical relations between radio flux, black hole mass, and accretion rate derived by Heinz & Sunyaev. Such relations depend only on the assumed accretion model and on the observed radio spectral index. Therefore, we are able to show that the X-ray emission from black holes accreting at less than a few per cent of the Eddington rate is unlikely to be produced by radiatively efficient accretion, and is marginally consistent with optically thin synchrotron emission from the jet. On the other hand, models for radiatively inefficient accretion flows seem to agree well with the data.

Key words: accretion, accretion discs – black hole physics – galaxies: active – radio continuum: general – X-rays: binaries – X-rays: general.

1 INTRODUCTION

The ultimate observational evidence of a celestial body being a black hole comes from dynamical studies, by measuring the gravitational influence of the central object on neighboring stars and gas, both in binary systems and in the nuclei of galaxies. However, there are a number of distinctive signatures of black hole-powered activity that are usually regarded as proxy of black hole existence. Relativistic jets emitting synchrotron radiation in the radio band are one such signature, the second most common being the presence of strong, compact power-law X-ray emission commonly associated with the inner part of an accretion flow. Indeed, as a general property, accretion onto compact objects and the launch of relativistic outflows/jets seem to be correlated (or *symbiotic*, Falcke & Biermann 1995) phenomena (Begelman, Blandford & Rees 1984; Rawlings & Saunders 1991). This implies that, at some level, a correlation between jet and disc flux is unavoidable.

Observationally, jet morphologies and spectral properties of both radio and X-ray cores are remarkably similar in the case of black holes of stellar mass (galactic black holes, hereafter GBH) and of their supermassive counterparts in the nuclei of galaxies (hereafter SMBH). If iets are launched in the innermost parts of the accretion flows, as commonly assumed, then these similarities suggest that it should be possible to understand the physics of both black hole accretion and jet production by studying all those systems as a single class. To this end, radiation emerging at radio and (hard) X-ray frequencies is the most direct probe of the immediate vicinity of the black hole: effects of extinction are unimportant in the radio band, while in the X-rays, where a large fraction of the accretion energy is radiated, such effects can be accounted for with good enough spectral capabilities. Thus, radio and X-ray observations, together with the information on the central black hole mass, should allow us to study the relationship between the fundamental parameters characterizing black hole activity such as the central black hole mass, $M_{\rm BH}$, and the accretion rate, \dot{M} , as well as the disc-jet coupling.

Until less than 10 yr ago, quasars and active galactic nuclei, and the supermassive black holes believed to power them, were regarded as exceptional (and extreme) objects. However, beginning with the work of Kormendy & Richstone (1995) and, in particular, Magorrian et al. (1998), the idea that SMBH reside in the nuclei

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¹ For the case of stellar mass objects, these signatures are common to most neutron stars, too. In this cases, only dynamical studies can provide a proof of the black hole nature of the source.

of virtually every galaxy in the nearby Universe has become almost commonplace. A sharp picture of the demographics of local SMBH comes from the work of Ho, Filippenko & Sargent (1997b), who have carried out a detailed optical spectroscopic survey of a large number of nearby galaxies. From this work, it was found that between a third and a half of the sample has active galactic nuclei (AGN) like spectra, albeit of low luminosity, (LLAGN; either of the LINER, Seyfert or Transition type; see Ho, Filippenko & Sargent 1997a), thus confirming that SMBH are not only present in galaxies, but that they are also active (at least as some level). As a matter of fact, follow-up radio surveys of optically selected LLAGN (Ho & Ulvestad 2001; Nagar et al. 2002a) have yielded extremely high detection rates, with the radio emission having predominantly a compact core morphology, occasionally accompanied by jet-like features. X-ray studies at arcsec (or sub-arcsec) resolution with the Chandra X-ray Observatory have also allowed us to firmly pin down the emission properties of the nearby dim galactic nuclei through systematic surveys of LLAGN (Ho et al. 2001; Terashima et al. 2002; Terashima & Wilson 2003). These pieces of information on local, low-luminosity objects nicely complement those on more luminous (and more distant) 'classical' AGN [i.e. Seyfert galaxies, quasi-stellar objects (QSOs), radio galaxies] and allow us to investigate the dependences of observable properties on black hole mass and accretion rate.

As a matter of fact, alongside the work on black hole demographics, there have been various claims for the existence of a correlation between radio luminosity or radio-loudness (a measure of the ratio of radio to bolometric luminosity) and SMBH masses (Franceschini, Vercellone & Fabian 1998; McLure & Dunlop 2001; Lacy et al. 2001; Nagar et al. 2002a; Laor 2000). Comparisons of radio emission from GBH and SMBH has also led to the suggestion that there is a systematic difference in radio-loudness between the two classes, SMBH being on average more radio-loud (Falcke & Biermann 1996). Furthermore, among X-ray binaries, black holes tend to be more radio-loud than neutron stars (Fender & Kuulkers 2001), possibly also indicating a mass dependence of the radio-loudness parameter. However, the difficulty of separating the dependence of the radio power output on the accretion rate (due to the lack, or the neglect, of an independent observational indicator of it for SMBH) unavoidably makes the evidence of any such correlation rather weak. Indeed, other authors, using different samples, have recently found no evidence for such correlations (Ho 2002; Woo & Urry 2002; Oshlack, Webster & Whitting 2002).

A link between the disc accretion rate and the generation of relativistic radio jets has been suggested by Willott et al. (1999) for radio galaxies and by Ho & Peng (2001) for Seyfert 1 nuclei, on the basis of the strong observed correlations between radio and optical powers. Moreover, by studying a sample of galactic nuclei with measured black hole masses, Ho (2002) found clear evidence of radio-loudness being anticorrelated with some estimator of the dimensionless accretion rate. Interestingly, Brinkmann et al. (2000) have also correlated the *ROSAT* All-Sky Survey and the VLA FIRST catalogue, and have shown that there is some degree of correlation between the monochromatic X-ray luminosity at 2 keV and the radio (5 GHz) luminosity for bright AGN and quasars (for both radio-loud and radio-quiet sources), while Sambruna, Eracleous & Mushotzky (1999) have found a weak correlation between lobe radio power and 2–10 keV luminosity in a sample of radio-loud AGN.

The situation is rather different for GBH where the measured black hole masses span a very narrow range (around 10 solar masses). In this case, the observed, rather large, changes in luminosity can only be explained if they are somehow linked to variations in the accretion rate. On this note, recent work has shown that GBH in the low/hard state (where their spectra are dominated by a powerlaw X-ray emission with typical bolometric luminosities L_{bol}/L_{Edd} $\leq 10^{-2}$) always possess compact radio cores (Fender 2001), the radio luminosity of which is tightly correlated with their X-ray luminosity over more than three orders of magnitude, with $L_{\rm R} \propto L_{\rm x}^{0.7}$ (Gallo, Fender & Pooley 2002, 2003; Corbel et al. 2003). The evidence of such a correlation emphasizes the effects of the dependence of the radio luminosity on the accretion rate and hence the connection between accretion and jet processes. However, an alternative model has been put forward recently, based on the broad-band spectral energy distributions of some GBH in the low/hard state, in which the X-ray emission from those objects is produced by optically thin synchrotron radiation emerging from the jet itself (Markoff, Falcke & Fender 2001). The tightness of the observed correlation between radio and X-ray luminosity, and its slope, seem to support such an idea (Markoff et al. 2003; Corbel et al. 2003).

Finally, we should also note that GBH with bolometric luminosity close to their Eddington limit display a more complicated behaviour when observed in the radio band. There is evidence that in the high/soft state (when the spectral energy distribution is dominated by a quasi-thermal component with $kT \sim 1 \text{ keV}$) continuous jet production is inhibited, while at still higher luminosities (in the so-called very high state) powerful, episodic super-luminal ejection events have been observed (in particular in the prototypical microquasar GRS 1915+105, Mirabel & Rodriguez 1994). Thus, high-luminosity black hole X-ray binaries seem to display a (temporal) dichotomy between radio-loud and radio-quiet states, reminiscent of that of powerful quasars.

This rapid (albeit somewhat disorderly) progress in the observational studies of black holes of all classes seems to offer the possibility for understanding fundamental scalings of black hole properties with $M_{\rm BH}$ and \dot{M} . Accretion disc theory can provide us with the relationships between the observed radiative output (hence X-ray luminosity) and the black hole masses and accretion rates. Unfortunately, theoretical modelling of the relation between jet properties (related to the observed radio luminosity) and the physical parameters of the system $(M_{\rm BH}, \dot{M})$ has been hampered by the lack of a clear understanding of the mechanism by which jets are accelerated and collimated. Relating the X-ray and radio properties of a black hole system to only $M_{\rm BH}$ and/or \dot{M} has not been straightforward. However, in a recent paper, Heinz & Sunyaev (2003) have demonstrated that, under the general assumption that the jet formation process is not qualitatively different among SMBH of different mass or between SMBH and GBH, it is in fact possible to derive a universal scaling between the jet (radio) luminosity at a given frequency, L_{ν} , and both mass and accretion rate. The derived relation is independent of the jet model and has scaling indices that depend only on the (observable) spectral slope of the synchrotron emission in the radio band, and on the accretion model (see Section 5 below).

Motivated by these findings, the aim of this work is twofold. First, we want to examine (or re-examine) the significance of the correlations between radio and X-ray luminosities with $M_{\rm BH}$ and with each other in a large sample of black holes with measured masses that includes *both* GBH and SMBH. This will be dealt with in the first part of the paper: in Section 2, we describe the selection criteria and the properties of our sample, while in Sections 3 and 4 we present the results of the correlation analysis and we describe the observational properties of the Fundamental Plane of black hole activity. Then, in the second part of the paper (Section 5) we will make use of the model of Heinz & Sunyaev (2003) to relate observed correlation coefficients that define the Fundamental Plane to

theoretically predicted ones. This will allow us to place constraints on the physical properties of the accretion flow from which the jet is launched, and in particular on its radiative efficiency. Section 6 is devoted to a discussion of our findings and of their general implications for our understanding of black hole activity in different regimes. Finally, we summarize our results in Section 7

2 THE SAMPLE

2.1 Supermassive black holes in galactic nuclei

We have selected from the existing literature a sample of black hole powered systems with measured masses, the nuclei of which have been observed both at 5 GHz (mostly with arcsec resolution with the VLA) and in the 2–10 keV band. For future reference we define the dimensionless black hole mass $M = M_{\rm BH}/M_{\odot}$ and accretion rate $\dot{m} \equiv (L_{\rm bol}/\eta)/L_{\rm Edd} = \dot{M}c^2/L_{\rm Edd} \propto \dot{M}/M$, where η is the accretion efficiency.

We first considered the full sample of ~40 nearby inactive, or weakly active galaxies with existing nuclear black hole mass measurements from observations of spatially resolved kinematics. To these we have added a comparable number of bright AGNs (and QSOs) with nuclear black hole mass measured from reverberation mapping of their broad line region (a similar sample was compiled by Ho et al. 2000). From this sample we selected all objects which have been observed in both the radio and X-ray bands.

In order to obtain a more statistically representative sample, we also searched the existing literature for both nearby low-luminosity galactic nuclei (Ho et al. 2001; Terashima et al. 2002; Nagar et al. 2002a; Terashima & Wilsom 2003) and for relatively bright Seyfert nuclei (either type 1, type 2 or narrow line Seyfert 1) and radio galaxies with available radio and X-ray flux measurements. We assign black hole masses to these systems using the observed correlation between black hole masses and stellar velocity dispersion (Gebhardt et al. 2000; Ferrarese & Merritt 2000), mostly using the values of the velocity dispersion provided by the HYPERLEDA catalogue (Simien & Prugniel 2002). We note here that it is beyond the scope of this paper to discuss the different methods (and their qualities) for estimating black hole masses in the nuclei of galaxies, and refer the reader to the comprehensive literature on the subject (see e.g. De Zeeuw 2003, and references therein).

We do not include in our sample distant quasars, for which the $M-\sigma$ relation cannot be used because of the lack velocity dispersion measurements. In practice, we want to avoid any indirect method for black hole mass estimation that relies on e.g. the continuum optical/UV luminosity (as done by Woo & Urry 2002, for example).

Relativistically beamed sources (i.e. those whose jet axis points towards our line of sight) are dominated by the boosted jet emission, and cannot be used to test the disc–jet coupling. We therefore excluded from our sample BL Lac objects. Among the quasars in our sample, only 3C 273, which has an extremely high radio-loudness and a blazar-like spectrum, is likely to suffer from strong Doppler boosting of the radio jet. On the other hand, according to the unification scheme, Seyfert 2 nuclei should not be preferentially viewed pole on. For all the other sources (mainly low-luminosity AGN and

Seyfert 1), for which the nature of the (relatively faint) radio emission is not well established, we have assumed that the orientation of their jets with respect to line of sight is randomly distributed. A more detailed discussion of the possible incidence of relativistic beaming as a selection effect will be presented in Section 6.2.

2.2 Galactic black hole sources

The Galactic X-ray binaries included in our sample have been selected to have (i) simultaneous X-ray and radio observations, or *RXTE* All-Sky-Monitor (ASM) X-ray data in conjunction with radio fluxes available from the literature, and (ii) publicly available *RXTE*-ASM X-ray and Green-Bank Interferometer (GBI) radio light curves (from which we estimated the 5-GHz fluxes by interpolating between the 2.25- and the 8.3-GHz channels).

We treated the GBI and ASM data of each object in the following way: we first re-gridded the ASM X-ray light curve to the radio light curve, and then sorted the data into bins of ascending X-ray flux. This procedure assumes that individual bins are not correlated and can thus be re-ordered and rebinned to improve statistics. Rebinning the data in time intervals does not change the results significantly, but reduces the dynamic range in X-ray flux. In this sense, X-ray-flux-ordered binning produces a better representation of the scatter in this variable. Because the GBI becomes noise dominated below about 15 mJy, we conservatively chose to consider data at or below this value as upper limits and split the sample into detection and upper limits before binning. ASM fluxes were converted to 2–10 keV luminosities using the a conversion factor of 3.2×10^{-10} [erg s⁻¹ cm⁻²]⁻¹ [ct s⁻¹] (Grimm, Gilfanov & Sunyaev 2002), which assumes a crab-like source spectrum.

Black hole masses for GBH are derived from stellar kinematics and we use the value provided in the literature. For the microquasar LS 5039, no mass estimate is available, and we assumed $M_{\rm BH}=10~{\rm M}_{\odot}$.

2.3 Global properties of the sample

Our final sample (Table 1) comprises 149 entries, of which 99 are individual SMBH systems. The remaining 50 correspond to the observations of eight different GBH at various luminosity levels. Out of these eight sources, five are so-called X-ray transients (Cyg X-3, GRO J1655-40, GRS 1915+105, XTE J1118+480 and XTE J1859+226) and three are persistent sources (Cyg X-1, GX 339-4 and LS 5039). The SMBH sample includes 14 quasars; 19 Seyfert galaxies of type 1 (in this class we include all the Seyfert spectrally classified as types 1 to 1.9); 32 Seyfert 2 galaxies; seven narrow line Seyfert 1 galaxies (NS1); 11 low-ionization nuclear emission line regions (LINERS) of type 1.9; 13 LINERS of type 2 and two LINER/H II transition nuclei. Separating mass estimators into direct ones (stellar kinematics, maser emission, gas kinematics and, to a lesser extent, reverberation mapping) and indirect ones (all those that infer a measure of the central black hole mass from the observed $M-\sigma$ relation), we can assess possible biases introduced in our sample by the different mass measurement techniques. 55 masses are measured indirectly (four Sy1, four NS1, 30 Sy2, 15 LINERs and the two transition objects) and 44 directly (all the 14 QSOs, 15 Sy1, two Sy2, three NS1, eight LINERs plus Sgr A* and M 32).

For the radio and X-ray luminosities taken from the literature, we have assumed $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$, correcting the quoted values when necessary. When available, we have included information on the radio spectral properties of the sources. In Table 1 we have marked all the sources with flat radio spectrum ($\alpha_R < 0.4$, where

² We use here the relationship $M = 1.3 \times 10^8 (\sigma/200 \text{ km s}^{-1})^{4.58}$, as derived in Ferrarese (2002). See however Tremaine et al. (2002) for a thorough statistical discussion of the different scalings that have been claimed for the $M-\sigma$ relation.

³ Available on-line at http://www-obs.univ-lyon1.fr/hypercat/

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Table 1. Radio and X-ray properties of black holes with measured masses.

Ark 564 99 Cyg A 224 Fairall 9 199 IC 1459 29 IC 4296 (PKS 1333-33) 188 IC 4329A 65 Mrk 3 52	S2/L2 .8 S1 .2 L2 .1.9 .5 S1 .0 S2 .5 S2	<37.68 39.71 39.68 38.94	α_R U F U F S	Ref 1 11 4	Log L _X 43.74 44.22 44.14	2 59	6.5 9.4	Method $I_{\text{[O III]}}$ G	Ref 3
Cyg A 224 Fairall 9 199 IC 1459 29 IC 4296 (PKS 1333-33) 188 IC 4329A 65 Mrk 3 52	S2/L2 .8 S1 .2 L2 .1.9 .5 S1 .0 S2 .5 S2	41.43 <37.68 39.71 39.68 38.94	F U F	11 4	44.22	59			
Fairall 9 199 IC 1459 29 IC 4296 (PKS 1333-33) 188 IC 4329A 65 Mrk 3 52	.8 S1 .2 L2 L1.9 .5 S1 .0 S2 S2	<37.68 39.71 39.68 38.94	U F	4			9.4		
IC 1459 29 IC 4296 (PKS 1333-33) 188 IC 4329A 65 Mrk 3 52	.2 L2 L1.9 .5 S1 .0 S2 S2	39.71 39.68 38.94	F		44 14			G	58
IC 4296 (PKS 1333-33) 188 IC 4329A 65 Mrk 3 52	L1.9 .5 S1 .0 S2 S2	39.68 38.94		4 52		5, 6	7.91	R	21
IC 4329A 65 Mrk 3 52	.5 S1 .0 S2 S2	38.94	S	4, 53	40.66^{a}	9	9	S	10
Mrk 3 52	.0 S2 S2			11	41.20	12	9.1	I_{σ}	13
	S2		U	4	43.72	5	6.69	R	21
		39.86 39.86	S	14, 51	42.7 <44	14, 15	8.81	I_{σ}	16
Mrk 78 149 Mrk 279 127		39.86	S S	14, 53 4, 49	<44 43.72	14 5	7.92 7.62	I_{σ} R	16 21
Mrk 335 107		38.27	s F	4, 49 4, 49	43.72	2, 17	6.79	R R	21
Mrk 348 60		39.70	F	14, 52	42.85	14	7.17	I_{σ}	16
Mrk 478 316		38.75	U	18	43.87	2	7.30	$I_{\mathrm{[OIII]}}$	3
Mrk 507 216		38.78	Ü	1	42.97	2	7.10	$I_{\rm [OIII]}$	3
Mrk 509 144		38.30^{c}	S	19, 56	44.00	20	7.86	R	4
Mrk 573 69		38.22	S	14, 49	<43.00	14	7.25	I_{σ}	16
Mrk 590 109		38.70	S	4, 49	43.58	57	7.23	R	21
Mrk 766 51		38.32	S	1, 22	42.90	2, 6	6.64	$I_{ m [OIII]}$	3
Mrk 1066 48		38.68	S	14, 53	<41.85	14	6.93	I_{σ}	16
` /	.810 –	<33.3	U	67	35.97	67	6.40	S	68
NGC 315 66		40.41	F	23	41.68	24	9.10	I_{σ}	16
NGC 1052 19		39.86	S	25	41.57	24	8.29	I_{σ}	16
NGC 1068 14		39.12	S	4, 27	41.00	15	7.20	M	4
NGC 1275 70 NGC 1365 21		41.74 38.80	F S	14, 27 53	43.40 40.60	32, 34 24	8.64 7.66	I_{σ}	16 26
NGC 1386 21		36.70	S U	53 53	40.64	24	7.00	I_{σ} I_{σ}	26 16
NGC 1580 11 NGC 1667 61		37.34	S	14, 27	40.04	15	7.20	I_{σ}	16
NGC 2110 31		38.99	F	14, 53	42.60	14, 15	8.41	I_{σ}	16
NGC 2273 28		37.83	S	27	41.40	15, 24	7.27	I_{σ}	16
	.5 L1.9	37.22	F	4, 29	38.40	24, 28	7.59	Ğ	10
NGC 2841 12	.0 L2	36.00	F	29	38.26	28	8.42	I_{σ}	30
NGC 2992 30	.8 S2	38.64	U	14	42.10	14, 15	7.75	I_{σ}	16
	.9 S1.5	36.82	F	4, 29	39.90	24, 28	7.79	S	4
NGC 3079 20		38.39	F	27, 50	40.20	15	7.65	I_{σ}	26
NGC 3147 40		38.01	F	27, 31	41.61	15, 31	8.79	I_{σ}	26
NGC 3169 16		37.19	F	31, 54	41.35	31	7.91	I_{σ}	33
NGC 3226 15		37.20	F	31, 54	40.74	31	8.23	I_{σ}	34
NGC 3227 20 NGC 3362 111		37.94 38.47	S S	4, 27 14, 49	41.99 <43.6	5 14	7.59 6.68	R I_{σ}	21 16
NGC 3516 38		37.98	S	4, 27	43.20	5, 6	7.36	r_{σ} R	4
	.6 S2	35.45	F	29	<37.6	28	7.26	I_{σ}	30
NGC 3675 12		<35.99	U	29	<37.99	28	7.11	I_{σ}	30
NGC 3998 14		37.98	S	4, 53	41.66	24	8.75	S	4
NGC 4051 17		37.30	S	4, 27	41.50	5, 6	6.11	R	21
NGC 4117 124	S2	35.70	S	14	39.4^{b}	45	6.74	I_{σ}	16
NGC 4143 17		37.16	F	29, 31	40.03	31	8.32	I_{σ}	34
NGC 4151 20		38.49	S	4, 27	42.83	5, 6	7.17	R	21
NGC 4203 14		36.79	F	29, 31	40.23	28, 31	7.90	I_{σ}	30
	.3 S1.9	35.94	F	4, 29	40.52	35	7.60	M	4
NGC 4261 (3C 270) 29		39.21	U	11	41.17	24	8.72	G	4
	.7 L1.9	37.91	F	29, 31	39.96	31	9.20	S	60
NGC 4321 16 NGC 4374 (M84) 18		<36.17 38.81	U F	29 4, 29	<38.59 40.34	28 24	6.80 9.20	I_{σ} G	30 4
NGC 4374 (M64) 16 NGC 4388 16		36.95	S	4, 29 27	40.34	15	6.80		26
	.6 S1.5	35.56	S	4, 27	39.50	36	< 5.04	I_{σ} S	4
NGC 4450 16		36.53	F	29	40.34	24	7.30	I_{σ}	26
NGC 4457 17		<35.70	U	29	39.97	24	6.86	I_{σ}	26
NGC 4472 18		36.69	S	27, 29	<38.80	37	8.80	I_{σ}	38
NGC 4486 (M87) 16		39.78	F	4, 29	40.55	39	9.48	Ğ	4
	.7 L2	<35.65	U	29	38.86	28	7.65	I_{σ}	30
NGC 4501 16	.8 S2	36.20	S	27	40.28	14	7.90	I_{σ}	26
NGC 4548 16		36.30	F	29	39.79	31	7.40	I_{σ}	26
	.7 S1.9	36.15	F	29, 31	39.56	31	7.70	I_{σ}	26
NGC 4579 16	.8 S1.9	37.65	F	29, 31	41.14	28, 31	7.85	I_{σ}	26

Table 1 - continued

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Object	D (Mpc)	SC	$Log L_R$	$\alpha_{\rm R}$	(6) Ref	$\log L_{ m X}$	(8) Ref	$\log M$	Method	Ref
NGC 4594 (M104)	9.8	L2	37.84	F	4, 53	40.70	15	9.04	S	4
NGC 4636	17.0	L1.9	36.40	U	29	<38.43	37	7.90	I_{σ}	38
NGC 4725	12.4	S2	<35.90	U	29	39.16	28	7.49	I_{σ}	30
NGC 4736	4.3	L2	34.80	F	29	39.62	24	7.30	I_{σ}	26
NGC 5033	18.7	S1.5	36.79	S	27	41.00	24, 28	7.30	I_{σ}	30
NGC 5194	7.7	S2	35.50	S	27	39.80	15	6.90	I_{σ}	16
NGC 5252	92.3	S2	38.96	S	14, 49	42.97	14	8.12	I_{σ}	16
NGC 5273	21.3	S2	36.22	F	14, 24	<42.0	14	6.37	I_{σ}	16
NGC 5347	31.2	S2	37.10	F	14, 55	40.0^{b}	45	6.70	I_{σ}	16
NGC 5548	70.2	S1.5	38.58	S	4, 27	43.55	5	8.03	R	21
NGC 5929	33.2	S2	38.30	S	14, 49	40.7^{b}	45	7.22	I_{σ}	16
NGC 6166	116	S2	39.95	S	40	40.56	41	9.19	I_{σ}	26
NGC 6251	94.8	S2	40.93	S	4, 53	42.15	15	8.73	G	4
NGC 6500	40.0	L2	38.90	F	31, 54	40.11	31	8.28	I_{σ}	30
NGC 7469	66.6	S1	38.38	S	4, 49	43.31	42	6.81	R	21
NGC 7672	53.2	S2	37.25	S	14, 55	43.37	14	6.80	I_{σ}	16
NGC 7682	68.0	S2	38.88	S	14, 49	<43.2	14	7.25	I_{σ}	16
NGC 7743	24.4	S2	36.99	S	27	39.71	24	6.47	I_{σ}	16
PG 0026+129	627	Q	40.17	U	4	44.44	43	7.73	R	21
PG 0052+251	690	Q	39.42	U	4	44.66	44	8.34	R	21
PG 0804+761	430	Q	39.41	U	4	44.26	43	8.28	R	21
PG 0844+349	268	Q	38.12	U	4	43.29	43	7.34	R	21
PG 0953+414	1118	Q	40.14	U	4	44.50	43	8.26	R	21
PG 1211+143	362	Q	41.08	U	4	43.61	43	7.61	R	21
PG 1226+023 (3C 273)	705	Q	44.03	F	4, 56	45.70	44	8.74	R	21
PG 1229+204	268	Q	38.45	U	4	44.29	44	7.88	R	21
PG 1307+085	690	Q	38.98	U	4	44.51	44	8.44	R	21
PG 1411+442	380	Q	38.71	U	4	43.53	43	7.90	R	21
PG 1426+015	366	Q	38.98	U	4	43.89	44	8.67	R	21
PG 1613+658	565	Q	39.75	F	4, 56	44.34	44	8.38	R	21
PG 1700+518	1406	Q	40.92	S	4, 53	<43.20	43	7.78	R	21
PG 2130+099	255	Q	38.89	S	4, 56	43.55	44	8.16	R	21
3C120	138	S1	41.55	F	11, 53	43.95	11	7.36	R	21
3C 390.3	241	S1	41.09	S	11	44.00	11	8.53	R	21
UGC 6100	116	S2	38.50	U	14	<43.6	14	7.72	I_{σ}	16
Sgr A*	0.008	_	32.50	F	62	33.34	63	6.41	S	64
Cyg X-1	0.0021	GBH	<29.44	F	61	36.44	61	1.06	S	69
Cyg X-1	0.0021	GBH	<29.45	F	61	36.57	61	1.06	S	69
Cyg X-1	0.0021	GBH	29.63	F	61	36.48	61	1.06	S	69
Cyg X-1	0.0021	GBH	29.65	F	61	36.57	61	1.06	S	69
Cyg X-1	0.0021	GBH	29.66	F	61	36.64	61	1.06	S	69
Cyg X-1	0.0021	GBH	29.24	F	72	36.58	61	1.06	S	69
Cyg X-1	0.0021	GBH	29.30	F	72	36.65	61	1.06	S	69
Cyg X-1	0.0021	GBH	29.18	F	72	36.66	61	1.06	S	69
Cyg X-3	0.009	GBH	<30.64	F	61	36.94	61	1.00	S	70
Cyg X-3	0.009	GBH	31.62	F	61	37.19	61	1.00	S	70
Cyg X-3	0.009	GBH	31.85	F	61	37.41	61	1.00	S	70
Cyg X-3	0.009	GBH	32.17	F	61	37.60	61	1.00	S	70
Cyg X-3	0.009	GBH	32.43	U	61	37.88	61	1.0	S	70
GRO J1655-40	0.0032	GBH	<29.76	F	61	36.64	61	0.85	S	65
GRO J1655-40	0.0032	GBH	<29.69	F	61	37.55	61	0.85	S	65
GRO J1655-40	0.0032	GBH	29.94	F	61	35.27	61	0.85	S	65
GRS 1915+105	0.0125	GBH	<30.89	F	61	38.17	61	1.20	S	65
GRS 1915+105	0.0125	GBH	<30.89	F	61	38.47	61	1.20	S	65
GRS 1915+105	0.0125	GBH	<30.89	F	61	38.73	61	1.20	S	66
GRS 1915+105	0.0125	GBH	31.60	F	61	38.25	61	1.20	S	66
GRS 1915+105	0.0125	GBH	31.76	F	61	38.40	61	1.20	S	66
GRS 1915+105	0.0125	GBH	31.76	F	61	38.65	61	1.20	S	66
GX 339-4	0.004	GBH	29.91	F	7	36.48	7	1.00	S	8
GX 339-4	0.004	GBH	29.87	F	7	36.42	7	1.00	S	8
GX 339-4	0.004	GBH	29.89	F	7	36.40	7	1.00	S	8
GX 339-4	0.004	GBH	29.62	F	7	36.12	7	1.00	S	8
GX 339-4	0.004	GBH	29.71	F	7	36.12	7	1.00	S	8
	0.001	0211		-	,	20.12	•	1.00	5	O

Table 1 - continued

(1) Object	(2) D (Mpc)	(3) SC	(4) Log L _R	(5) α _R	(6) Ref	(7) Log <i>L</i> _X	(8) Ref	(9) Log <i>M</i>	(10) Method	(11) Ref
-					7					
GX 339-4	0.004	GBH	29.66	F	7	36.14	7	1.00	S	8
GX 339-4	0.004	GBH	29.45	F	7	35.81	7	1.00	S	8
GX 339-4	0.004	GBH	29.11	F	7	35.32	7	1.00	S	8
GX 339-4	0.004	GBH	28.34	F	7	34.21	7	1.00	S	8
GX 339-4	0.004	GBH	28.02	F	7	33.91	7	1.00	S	8
GX 339-4	0.004	GBH	28.38	F	7	33.84	7	1.00	S	8
GX 339-4	0.004	GBH	28.49	F	7	34.01	7	1.00	S	8
GX 339-4	0.004	GBH	<27.51	F	7	<33.34	7	1.00	S	8
GX 339-4	0.004	GBH	<27.25	F	7	33.19	7	1.00	S	8
LS 5039	0.003	GBH	29.80	S	61	35.62	61	1.00	_	_
LS 5039	0.003	GBH	30.09	S	61	35.45	61	1.00	_	_
LS 5039	0.003	GBH	30.07	S	61	35.67	61	1.00	_	_
LS 5039	0.003	GBH	30.09	S	61	35.81	61	1.00	_	_
LS 5039	0.003	GBH	30.08	S	61	35.96	61	1.00	_	_
XTE J1118+480	0.0018	GBH	<28.98	F	46	35.43	47	1.00	S	48
XTE J1118+480	0.0018	GBH	28.92	F	71	35.46	61	1.00	S	48
XTE J1118+480	0.0018	GBH	28.92	F	71	35.57	61	1.00	S	48
XTE J1118+480	0.0018	GBH	28.92	F	71	35.56	61	1.00	S	48
XTE J1118+480	0.0018	GBH	28.92	F	71	35.47	61	1.00	S	48
XTE J1118+480	0.0018	GBH	28.92	F	71	35.45	61	1.00	S	48
XTE J1859+226	0.011	GBH	<29.24	F	61	36.58	61	>0.88	S	71
XTE J1859+226	0.011	GBH	<29.30	F	61	36.65	61	>0.88	S	71
XTE J1859+226	0.011	GBH	29.18	F	61	36.66	61	>0.88	S	71

Notes. aL_X calculated from the known luminosity in the 0.3–8 keV band and the observed spectral index; bL_X calculated from the GIS count rate, assuming $\Gamma=2$ and $\text{Log}(N_{\text{H}})=22.5$; ${}^cL_{\text{5Ghz}}$ extrapolated from observations at 8 GHz.

Column (1): name of the object. Column (2): distance in Mpc (for $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$). Column (3): spectral class; GBH: galactic black hole; L: LINER; S: Seyfert; NS1: narrow line Seyfert 1; T: transition object (LINER/H II); Q: quasar. Column (4): logarithm of nuclear luminosity at 5 GHz. Column (5): radio spectral index α_R (where $F_{\nu} = \nu^{-\alpha_R}$); F: flat spectrum ($\alpha_R < 0.4$); S: steep spectrum ($\alpha_R > 0.4$); U: undetermined. Column (7) logarithm of the intrinsic rest-frame luminosity in the 2–10 keV band. Column (9): logarithm of the black hole mass. Column (10) mass measurement method; S: stellar kinematics; G: gas kinematics; M: maser kinematics; R: reverberation mapping; $I_{[O \text{III}]}$: inferred from the mass-[O III] line-width correlation; I_{σ} : inferred from the mass-velocity dispersion correlation.

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 $\alpha_{\rm R}$ is the radio spectral index $\alpha \equiv -\partial \ln L_{\nu}/\partial \ln \nu$ evaluated at the frequency $\nu = 5$ GHz) with flag (F). All the sources with $\alpha_{\rm R} > 0.4$ are instead classified as steep spectrum (S), while those for which the radio spectral index couldn't be determined are marked with a (U). Overall, we have 77 flat spectrum sources (32 SMBH and 45 GBH, respectively); 43 steep spectrum sources (38 and five) and 29 for which the spectral slope is undetermined (all SMBH).

In Fig. 1 we show the distributions of black hole masses, radio and X-ray luminosities and that of the ratio of their X-ray to the Eddington luminosity, $L_{\rm X}/L_{\rm Edd} = L_{\rm 2-10\,keV}/1.3\times10^{38}M$. It is worth emphasizing that the shape of those distributions reflects more the nature of our selection procedure rather than the global intrinsic properties of accreting black holes. For example, a precise measurement of the central black hole mass is a prerequisite for a given source to be included in the sample. As recently discussed by De

Zeeuw (2003), not all mass ranges are equally well probed by the different methods, and this should introduce a strong selection effect in our sample. Similarly, because BH masses are more easily measured in the nuclei of nearby galaxies, our sample is biased against the most luminous quasars. This should be taken into account when examining, for example, the $L_{\rm X}/L_{\rm Edd}$ distribution of the black holes in our sample.

In Fig. 2 we show the radio luminosity versus the black hole mass for objects of different spectral classes. Panel (a) shows the whole sample, while panel (b) concentrates on the SMBH only. Overplotted are some of the linear regression fits discussed in the recent literature (see Section 3 for details).

Finally, Fig. 3 shows the core radio luminosity versus the nuclear X-ray luminosity in the 2–10 keV band (left-hand panel) and versus the ratio of the X-ray nuclear luminosity to the Eddington luminosity

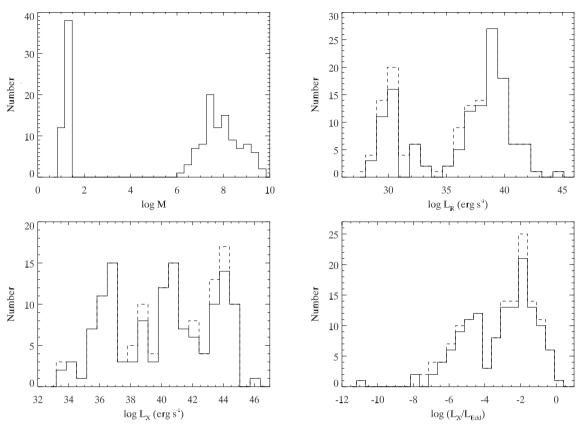


Figure 1. Distribution of black hole masses (top left-hand panel), 5 GHz radio luminosity (L_R , top right-hand panel), 2–10 keV X-ray luminosity (L_X , bottom right-hand panel) and of the ratio $L_X/L_{\rm Edd}$. Solid histograms denote detected sources, dashed ones are for upper limits.

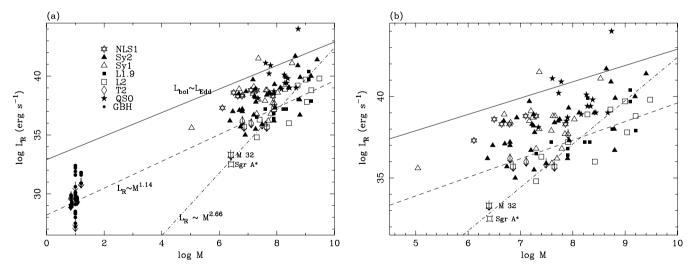


Figure 2. Radio core luminosity at 5 GHz versus black hole mass. Upper limits are marked with arrows. Panel (a) shows the whole sample, including GBH (black filled circles) and SMBH, with different symbols indicating objects belonging to different spectral classes. The right-hand panel (b) is a blow-up of the SMBH sample. The dot-dashed line gives the regression fit proposed by Franceschini et al. (1998), the dashed line that proposed by Nagar et al. (2002a), both obtained using different samples of SMBH only. The thick solid upper line gives the maximum core radio power as calculated by Ho (2002) for sources accreting at the Eddington rate. Although a correlation analysis of our sample would give results similar to those obtained by Nagar et al. (2002a), as we discuss in the text, *none* of the above relations reflects the real physical scaling of radio power and black hole mass.

(right-hand panel). We represent objects in different mass bins with different colours to highlight a possible segregation of different mass bins in the L_R - L_X plane.

In the next section we present a more quantitative statistical analysis of the correlations among L_R , L_X and M.

3 CORRELATION ANALYSIS

Luminosity-luminosity correlations have to be tested for possible spurious effects introduced by their common dependence on the distance. Such tests can be done by performing a partial correlation

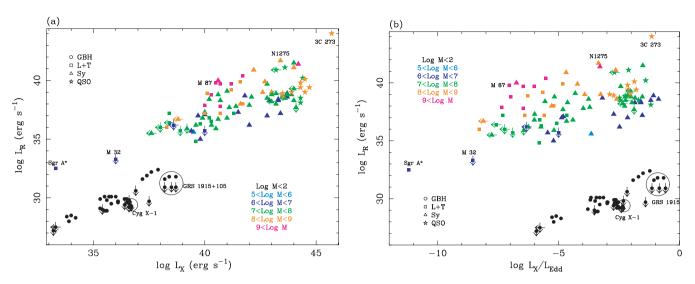


Figure 3. Radio core luminosity at 5 GHz versus X-ray luminosity in the 2–10 keV band (a) for the whole sample. Different colours correspond to different mass bins, while different symbols correspond to different classes of objects. In panel (b) we plot the same radio core luminosity at 5 GHz versus the ratio $L_{\rm X}/L_{\rm Edd}$ of X-ray to Eddington luminosity. The colour-coding of the different mass bins makes the mass segregation more evident.

Table 2. Results of correlation analysis.

	Variables			Sub	sample				Correlati	on
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
X	Y	Z	Objects	N	$N_{ m ul}^{ m X}$	$N_{ m ul}^{ m Y}$	$N_{ m ul}^{ m Z}$	τ	σ	$P_{ m null}$
$Log L_R$	$\text{Log } L_{ ext{X}}$	Log D	All Objects	149	20	14	0	0.255	0.0414	7.3×10^{-10}
$\text{Log } L_{\text{R}}$	$\text{Log } L_{\text{X}}$	Log D	SMBH only	99	7	13	0	0.200	0.0518	1.12×10^{-4}
$\text{Log } L_{\text{R}}$	$Log L_X$	M	All Objects	149	20	14	1	0.448	0.0495	$<1 \times 10^{-10}$
$\text{Log } L_{\text{R}}$	$\text{Log } L_{\text{X}}$	M	SMBH only	99	7	13	1	0.450	0.0523	$<1 \times 10^{-10}$
$\text{Log } L_{\text{R}}$	M	$Log L_X$	All Objects	149	20	1	14	0.432	0.0469	$<1 \times 10^{-10}$
$Log L_R$	M	$\text{Log } L_{\text{X}}$	SMBH only	99	7	1	13	0.310	0.0547	1.45×10^{-8}
$\text{Log } L_{\text{X}}$	M	$Log L_R$	All Objects	149	14	1	20	0.184	0.0452	4.68×10^{-5}
$\text{Log } L_{\text{X}}$	M	$Log L_R$	SMBH only	99	13	1	7	-0.022	0.052	0.672

Notes. Column (1): variable X. Column (2): variable Y. Column (3): variable Z. Correlation between variables X and Y is studied, taking into account the mutual correlation of X and Y with Z. Column (4): subsample. Column (5): number of objects in the subsample. Columns (6)–(8): number of upper limits in X, Y and Z. Columns (9)–(11): results of partial correlation analysis, giving the partial Kendall's τ correlation coefficient, the square root of the calculated variance σ , and the associated probability P_{null} for accepting the null hypothesis that there is no correlation between X and Y.

analysis, taking distance as the third variable. Here we choose the so-called partial Kendall's τ correlation test, proposed by Akritas & Siebert (1996) in the case of censored data sets. Applying this test to our data, we find that the L_R - L_X correlation is strong: the probability of the null hypothesis (i.e. that there is no correlation) is less that 10^{-10} for the whole sample and $\sim 1.12 \times 10^{-4}$ for the SMBH only (see Table 2).

However, it is apparent from Fig. 3 that, when the data points are grouped into mass bins, objects in different bins tend to lie on parallel tracks. Such a behaviour can be seen both in the $L_{\rm R}-L_{\rm X}$ plane (panel a), and, perhaps even more clearly, in the $L_{\rm R}-L_{\rm X}/M$ plane (panel b). The presence of a mass segregation suggests that the radio luminosity of an object likely depends both on its X-ray luminosity and on its mass. In order to assess this hypothesis wo once again use partial correlation analysis, now taking $L_{\rm R}$ (or $L_{\rm X}$) as the dependent variable and testing its partial correlation with $L_{\rm X}$ ($L_{\rm R}$) where we take M as the third variable. In addition, we also look for a partial correlation between $L_{\rm R}$ ($L_{\rm X}$) with mass itself, by taking $L_{\rm X}$ ($L_{\rm R}$) as the third variable. In so doing we can effectively discriminate between intrinsic and spurious correlations among the three quantities.

The results of such tests (see Table 2) show that the radio luminosity is strongly correlated with both black hole mass and X-ray luminosity ($P_{\rm null} < 1 \times 10^{-10}$ for the whole data set), while in turn, the X-ray luminosity correlates with both mass and radio luminosity only if we include both GBH and SMBH ($P_{\rm null} \simeq 4.68 \times 10^{-5}$). These results imply that any regression fit used to find correlations between any two variables that does not account for the dependence on the third one (as, for example trying to find the dependence of $L_{\rm R}$ on M without accounting for the dependence on $L_{\rm X}$), inevitably leads to an incorrect estimate of the correlation coefficients.

If, in any case, we compute the correlation coefficients between any two of these variables 4 to allow a comparison with earlier works, we find that $\log L_{\rm R}^{\rm SMBH} = (29.54 \pm 1.60) + (1.23 \pm 0.20) \log M$, with a very large scatter (standard deviation $\sigma_{\rm RM}^{\rm SMBH} = 1.65$). Including GBH, we find $\log L_{\rm R}^{\rm all} = (28.75 \pm 0.18) + (1.20 \pm 0.04) \log M$ (standard deviation $\sigma_{\rm RM}^{\rm all} = 1.51$), which is indeed very similar to

⁴ We use a the linear regression method by parametric EM algorithm (that deals with censored data) as implemented in the ASURV package (Isobe, Feigelson & Nelson 1986).

what found by Nagar et al. (2002a), but very different from what was proposed by Franceschini et al. (1998), based on a much smaller sample (as shown by the different linear regression fits plotted in Fig. 2).

We emphasize, however, that the partial correlation analysis presented so far and summarized in Table 2, implies that a far better representation of the mutual dependencies of L_R , L_X and M (and the one that minimizes the scatter) should be searched for with multivariate linear regression tests, that allow simultaneous fitting over the three-dimensional space defined by the three variables.

3.1 Finding the multivariate correlation coefficients

The standard multivariate regression formalism does not treat dependent and independent variables symmetrically (see e.g. Fasano & Vio 1988). In other words, if a linear regression analysis yields a linear regression coefficient b for the dependence of y on x, it does not necessarily yield a coefficient b^{-1} for the dependence of x on y. Closely related to this is the fact that standard χ^2 statistics, for which errors are only associated with the dependent variable, artificially reduce the estimate of the correlation coefficient in the presence of intrinsic scatter in the independent variable. The net result of these shortcomings is that χ^2 distributions over the fitting parameter space are not fair representations of the confidence in the resulting best fit.

In order to alleviate these shortcomings, we extend the statistical approach used by the 'Nukers' group (Gebhardt et al. 2000; Tremaine et al. 2002) to obtain an unbiased estimator of the best fit regression coefficients in multivariate problems. This approach uses a modified chi-square estimator, called the merit function (Fasano & Vio 1988; Press et al. 1992; Tremaine et al. 2002), defined by

$$\hat{\chi}^2 = \sum_i \frac{\left(y_i - a - \sum_j b_j x_{ij}\right)^2}{\sigma_{y_i}^2 + \sum_j (b_j \sigma_{x_{ij}})^2} \tag{1}$$

where y_i is the dependent variable, x_{ij} are the independent variables and σ_{y_i} and $\sigma_{x_{ij}}$ the associated error estimates, a is the zero intercept, and \boldsymbol{b} are the linear regression coefficients, which are to be found. The argument of the sum in equation (1) is a measure of the projected distance of the data point $P_i = (x_{ij}, y_i)$ to the regression hyperplane given by the equation $y = a + \sum_j b_j x_j$, measured in the χ^2 space around point P_i . In other words, the argument of the sum in equation (1) counts the number of χ^2 intervals between P_i and the regression hyperplane. Because equation (1) is nonlinear in b_j , it is no longer possible to minimize $\hat{\chi}^2$ analytically. However, for a set of parameters \boldsymbol{b} , we can still find the optimal value $a_{\min}(\boldsymbol{b})$ for which $\hat{\chi}^2$ is minimized as

$$a_{\min}(\boldsymbol{b}) = \frac{\sum_{i} \left[\left(y - \sum_{j} b_{j} x_{ij} \right) / \sigma_{y_{i}}^{2} + \sum_{j} \left(b_{j} \sigma_{x_{ij}} \right)^{2} \right]}{\sum_{i} \left[\sigma_{y_{i}}^{2} + \sum_{j} \left(b_{j} \sigma_{x_{ij}} \right)^{2} \right]^{-1}}$$
(2)

Finding the best-fitting parameters is then a simple minimization problem which can be treated with standard numerical minimization routines. Because our analysis is restricted to a bivariate problem in this paper, it is possible to use a graphical solution in the form of 2D $\hat{\chi}^2$ contour plots.

Unfortunately, such a technique cannot handle censored data. However, it is easy to verify that the incidence of the censored data points in our sample on the estimate of the linear regression coefficient is small compared to the intrinsic scatter in the data. In order to do so, we have performed standard linear regression fit using specific techniques that are able to handle censored data, as

implemented in the ASURV package (Isobe et al. 1986), on the whole data sample both with and without the upper limits. The differences in the resulting multivariate linear correlation coefficients in the two cases are much smaller than the estimated errors.

In the limit of $\sigma_{y_i} \gg \sigma_{x_{ij}}$, the merit function in equation (1) recovers the traditional χ^2 estimator. This implies that the usefulness of equation (1) is limited to cases where the estimated errors are an accurate representation of thintrinsic uncertainty, including intrinsic scatter. Artificially large errors in one variable will introduce an asymmetry similar to that intrinsic to the traditional regression formalism. At face value, the errors in our sample are dominated by the uncertainties in the mass measurements. However, because we are using a sample of predominantly low-redshift, low-luminosity AGN, error in the distance measurements can be large due to peculiar velocities, which leads to large errors in the luminosity measurements as well. For Galactic sources, distance estimates typically carry even larger uncertainties. Worse yet, the resulting errors in $L_{\rm X}$ and L_R will be correlated. As a first step, we therefore decided to follow the Nuker approach and assume isotropic uncertainties σ_{M_i} = $\sigma_{L_{\rm X},i}=\sigma_{L_{\rm R},i}$ in all three variables and re-normalize these errors to produce a minimum reduced $\hat{\chi}_{\text{red}}^2 = \hat{\chi}^2/n_{\text{dof}}$ of unity.

4 RESULTS

We fit the data with the function

$$\log L_{\rm R} = \xi_{\rm RX} \log L_{\rm X} + \xi_{\rm RM} \log M + b_{\rm R},\tag{3}$$

or, alternatively with its reciprocal

$$\log L_{\rm X} = \xi_{\rm XR} \log L_{\rm R} + \xi_{\rm XM} \log M + b_{\rm X},\tag{4}$$

where we have used the merit function to estimate the best fit linear regression coefficients. Because the regression technique we adopt is symmetric,⁵ equations (3) and (4) contain the same amount of information.

Our final results of the multivariate regression analysis are shown in the form of χ^2 contour plots (using equation (1) as a χ^2 estimator) in the (ξ_{RM}, ξ_{RX}) parameter space in Fig. 4. The top panel shows the results of the multivariate fit performed on the entire data set. We obtain $\xi_{RX}=0.60\pm0.11, \xi_{RM}=0.78^{+0.11}_{-0.09}$, and $b_R=7.33^{+4.05}_{-4.07}$ (all the errors are one sigma confidence), with a dispersion $\sigma_R=0.88$, or, cast in the form of equation (3):

$$\log L_{\rm R} = \left(0.60^{+0.11}_{-0.11}\right) \log L_{\rm X} + \left(0.78^{+0.11}_{-0.09}\right) \log M + 7.33^{+4.05}_{-4.07} \tag{5}$$

The value we obtain for the ξ_{RX} correlation coefficient is consistent, within the errors, with that found in GBH ($\xi_{RX} \approx 0.7$) by Gallo et al. (2003). This also means that individual GBH sources for which the correlation between radio and X-ray luminosities is well established (GX 339-4 and V404 Cyg) do indeed follow the same global trend defined by black holes of all masses included in our sample.

Our data set provides fairly tight constraints on the values of the correlation coefficients ξ_{ij} (i, j = R, X, M). This in turn implies that in the three-dimensional space ($\log L_R$, $\log L_X$, $\log M$) the sources are distributed preferentially on a plane, that we call hereafter the 'Fundamental Plane' describing black hole activity. Fig. 5 shows an edge-on view of the plane.

⁵ We verified that the standard regression technique is instead not symmetric: for the same fitting function (3), the best-fitting parameters are $\xi_{RX}=0.57$ and $\xi_{RM}=0.77$, while by fitting equation (4) we obtain $\xi_{XR}=1.05$ and $\xi_{XM}=-0.49$, instead of the expected $\xi_{XR}=1/\xi_{RX}=1.75$ and $\xi_{XM}=-\xi_{RM}/\xi_{RX}=-1.35$.

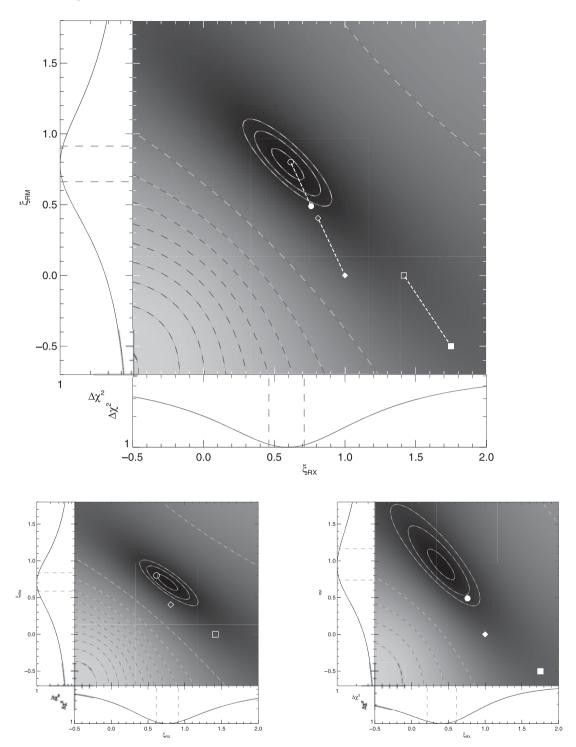


Figure 4. Shaded areas show the χ^2 density distribution, and dashed lines the χ^2 contours, for the observed correlation coefficients ξ_{RM} and ξ_{RX} . The inner three contours show the formal 1, 2 and 3σ confidence levels, the remaining contours further out show levels of $\Delta\chi^2_{red}=10$. Shown are the results for: the entire data set (upper panel), the flat spectrum sources (lower left-hand panel) and the steep spectrum sources (lower right-hand panel). Overplotted on each panel are the theoretically predicted values of the correlation coefficients where circles, diamonds and squares represent the ADAF, jet and standard disc models respectively. Empty symbols show the values for $\alpha_R=0$ and filled ones for $\alpha_R=0.5$. The lines connecting the points represent the tracks of ξ_{RX} and ξ_{RM} traced out by variation of α_R .

The other two panels of Fig. 4 show the constraints on the two correlation coefficients obtained with the regression analysis on different subsamples assembled according to the spectral properties in the radio band. For flat spectrum radio sources we obtain $\xi_{\rm RX,f}=0.76\pm0.13$, $\xi_{\rm RM,f}=0.71\pm0.10$ and $b_{\rm R,f}=1.31^{+4.85}_{-5.07}$, with a dis-

persion $\sigma_{R,f}=0.81$, smaller than that of the whole data set. The difference in the coefficient for the L_R-L_X correlation with respect to the whole sample result may be due to a larger incidence of GBH in the flat spectrum subsample, which seem to have a slightly larger measured value of ξ_{RX} (Gallo et al. 2002); the constraints on

the other coefficient, ξ_{RM} , are almost as good as for the entire data set, and in very good agreement with it.

On the other hand, in our steep spectrum sources subsample, only one GBH is included (LS 5039), and this results in a much larger uncertainty, in particular on the radio luminosity–mass correlation coefficient: ($\xi_{\rm RX,s} = 0.41^{+0.16}_{-0.18}, \xi_{\rm RM,s} = 0.94^{+0.21}_{-0.19}, b_{\rm R,s} = 14.08^{+5.92}_{-6.31}$ with dispersion $\sigma_{\rm R,s} = 0.87$).

5 PHYSICAL INTERPRETATION OF THE FUNDAMENTAL PLANE RELATION

As stated in the introduction, a correlation between X-ray and radio emission is expected if there is a fundamental connection between accretion flows and jet activity. Thus, at a qualitative level, the existence of the Fundamental Plane found in Section 4 is not surprising. However, on a quantitative level, the presence of such a plane and the measurement of the correlation coefficients associated with it provide powerful probes of accretion physics and of the inner jet.

In the following section we will therefore lay out an avenue of how to use the Fundamental Plane relation to constrain accretion and jet physics. In order to do this, we shall first turn to a discussion of the theory of the radio and of the X-ray emission by the jet–disc system. We will first make use of the scale invariant assumption about the disc–jet coupling (Heinz & Sunyaev 2003) to derive the expected scalings of the radio luminosity with black hole mass and accretion rates. Then, we will show how these theoretically predicted correlation coefficients translate into observable ones between black hole mass and X-ray luminosity ($\xi_{\rm RM}$, $\xi_{\rm RX}$) for different models of the X-ray emission. By comparing these theoretical predictions to the observed values from equation (5) we will then discuss the constraints we can put on the models themselves.

5.1 Synchrotron emission from scale invariant jets

It was recently shown by Heinz & Sunyaev (2003) that the dependence of radio luminosity $L_{\rm R}$ on black hole mass M and dimensionless accretion rate \dot{m} can be cast into a model independent form if the underlying jet physics is scale invariant. In this case, all the model dependent uncertainties can be absorbed into the observable spectral index α . The relationships between $L_{\rm R}$ and M and between $L_{\rm R}$ and \dot{m} depend only on the boundary conditions at the base of the jet, set by the conditions in the accretion flow feeding the jet. We will briefly review their argument (the interested reader is encouraged to consult Heinz & Sunyaev 2003, for more details) before comparing their predictions to the correlations derived in the previous section.

The nature and conditions in the inner disc are most likely governed by only a few parameters (M, m, a) and possibly the black hole spin a). The fundamental scale imposed on the problem is the gravitational radius of the black hole, $R_g = GM_{\rm BH}/c^2$. Jet formation occurs in the innermost regions of accretion discs, thus it is natural that jet formation, too, is primarily governed by these three parameters only.⁶

Then, we make the Ansatz that jet structure and dynamics are (at least approximately) invariant under changes of $R_{\rm g} \propto M$ and \dot{m} . In other words, taking a jet produced by a black hole of mass M_1 , scaling its dimensions by a factor of M_2/M_1 we should obtain a jet around a black hole of mass M_2 .

This proposed invariance can be cast into a simple mathematical form. Any quantity f needed to calculate the synchrotron emission from jets can be decomposed into a structure function $\psi_f(R/R_g, a)$, which describes the spatial variation of f along the jet, and which depends on M only through R/R_g , and a normalization $\phi_f(M, \dot{m}, a)$, which is set by the boundary conditions at the base of the jet (thus, by accretion disc physics):

$$f(\mathbf{R}, M, \dot{m}, a) = \phi_f(M, \dot{m}, a)\psi_f(\mathbf{R}/R_\sigma, a)$$
(6)

The quantities necessary for calculating the jet synchrotron emission are the magnetic field strength B, the jet diameter $D_{\rm jet}$, and the normalization C of the electron power-law distribution ${\rm d}n_{\rm e}/{\rm d}\gamma=C\gamma^{-p}$ (typically, the spectral index of the particle distribution is $p\sim 2$ –3). For example, according to the prescription (6), the magnetic field should follow $B=\phi_B(M,\dot{m},a)\psi_B(R/R_{\rm g},a)$, where $\phi_B=B_0=B(R_0)$ is the value of the field at the base of the jet R_0 . As a geometric quantity, the jet diameter D should be directly proportional to the characteristic scale $R_{\rm g}$ such that $\phi_D=D_0=D(R_0)\propto R_{\rm g}\propto M$. Scale invariance also implies that dynamical time-scales are proportional to $R_{\rm g}/c\propto M$, and thus that characteristic velocities are scale invariant.

Using the standard formulae for synchrotron emission (Rybicki & Lightman 1979), Heinz & Sunyaev (2003) showed that the synchrotron luminosity L_{ν} at a given frequency ν emitted by the jet must then depend non-linearly on M and \dot{m} , following

$$\frac{\partial \ln(L_{\nu})}{\partial \ln(M)} = \frac{2p+13+2\alpha}{p+4} + \frac{\partial \ln(\phi_B)}{\partial \ln(M)} \left(\frac{2p+3+\alpha p+2\alpha}{p+4}\right) + \frac{\partial \ln(\phi_C)}{\partial \ln(M)} \left(\frac{5+2\alpha}{p+4}\right) \equiv \xi_M \tag{7}$$

and

$$\frac{\partial \ln(L_{\nu})}{\partial \ln(\dot{m})} = \frac{\partial \ln(\phi_B)}{\partial \ln(\dot{m})} \left(\frac{2p+3+\alpha(p+2)}{p+4} \right) + \frac{\partial \ln(\phi_C)}{\partial \ln(\dot{m})} \left(\frac{5+2\alpha}{p+4} \right) \equiv \xi_{\dot{m}} \tag{8}$$

where α is the spectral index at frequency ν .

Note that the model-dependent structure functions $\psi_f(R/R_g,a)$ scale out from these expressions. Only the spectral indices $(\alpha$ and p) and the boundary conditions ϕ_B and ϕ_C for the magnetic field B and the electron power-law distribution normalization C, respectively, remain. α and p are observables: the electron spectral index p can be deduced from the optically thin synchrotron spectral index at high frequencies. The functions ϕ_B and ϕ_C , on the other hand, need to be provided by accretion disc theory. It is reasonable to assume that the relativistic particle pressure at the injection radius is a fixed fraction (i.e. independent of M and m) of the total pressure at injection, ϕ_C α ϕ_B^2 . This leaves ϕ_B as the only model-dependent parameter of the theory. Given a prescription for ϕ_B , we can predict how the synchrotron luminosity of jets should scale with M and m.

Because ξ_M and $\xi_{\bar{m}}$ are constants, we have in general (with $L_{\nu} = L_{\rm 5GHz} = L_{\rm R}$)

$$\log L_{\mathcal{R}} = \xi_M \log M + \xi_{\dot{m}} \log \dot{m} + K_1, \tag{9}$$

where K_1 is a normalization constant.⁷ Variations in other parameters, such as the viewing angle or the black hole spin, will only

⁶ The influence of a and the associated second scale of the problem, the light cylinder radius, on jet formation is unclear at this point. We will henceforth keep a fixed and assume that variations in a will only introduce a scatter in any relation derived below which is independent of M and \dot{m} .

⁷ We note here that for flat spectrum jets with $\alpha_{\rm R} \sim 0$, the canonical value of $p \sim 2$, and $\phi_B^2 \propto M^{-1}\dot{m}$, the dependence of $L_{\rm R}$ on M and \dot{m} follows $L_{\rm R} \propto (M\dot{m})^{17/12} = \dot{M}^{17/12}$, as had been found by Falcke & Biermann (1996) for the specific case of the 'canonical conical' (Blandford & Königl 1979) jet model.

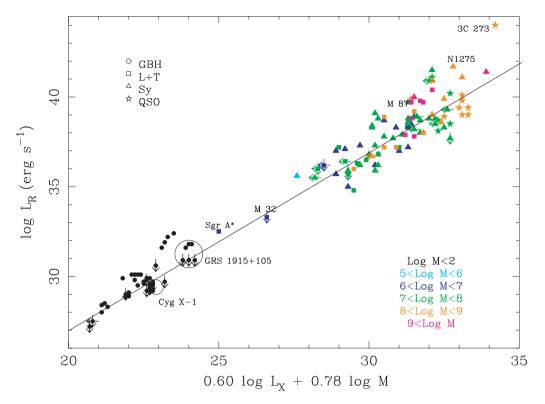


Figure 5. The edge-on view of the 'Fundamental Plane of black hole activity'. The solid line shows the best fitting function (5).

introduce a scatter in this relationship that is *independent* of M and \dot{m} .

Because the expressions for ξ_M and ξ_m do not depend on the shape functions ψ_f , they are independent of the model details. Any scale-invariant jet model that reproduces the observed radio spectral index α_R *must* satisfy equations (7) and (8). This implies that (i) measurements of ξ_M and ξ_m cannot be used to constrain the functions ψ_f , i.e. they cannot be used to distinguish between different jet models but that (ii) measurements of ξ_M and ξ_m can be used to place constraints on the boundary conditions at the base of the jet ϕ_f , or in other words on the accretion disc model.

While the black hole mass M is observable (see above), the accretion rate \dot{m} can only be inferred through radiation in other bands, where the emission is dominated by the accretion disc. Thus, in addition to a prescription of the boundary conditions ϕ_f through accretion disc theory, we must also provide a relation between the accretion disc luminosity (or $L_{\rm X}$) and \dot{m} in order to model the observed $L_{\rm R}-\dot{m}$ correlation. We must therefore discuss the different possible sources of the X-ray emission. We will start with those models that describe X-ray emission as produced by an accretion flow of some kind.

5.2 Accretion flow origin of X-ray radiation

For the accretion-powered X-ray luminosity we can write, in analogy to equation (9):

$$\log L_{\rm X} = \log M + q \log \dot{m} + K_2,\tag{10}$$

where K_2 is a normalization constant. The efficiency coefficient q need not be constant, though the linear correlation analysis from Section 3 is limited to this case.

Using equations (7)–(9) together with equation (10), we obtain the following general expression for the *observable* correlation coefficients as defined in equations (3) and (4):

$$\xi_{\text{RM}} = \frac{2p + 13 + 2\alpha_{\text{R}}}{p + 4} + \frac{\partial \ln \phi_{B}}{\partial \ln M} \left(\frac{2p + 13 + \alpha_{\text{R}} p + 6\alpha_{\text{R}}}{p + 4} \right) - \frac{\partial \ln \phi_{B}}{\partial \ln m} \left[\frac{2p + 13 + \alpha_{\text{R}} p + 6\alpha_{\text{R}}}{q(p + 4)} \right] \\ \xi_{\text{RX}} = \frac{\partial \ln \phi_{B}}{\partial \ln m} \left[\frac{2p + 13 + \alpha_{\text{R}} p + 6\alpha_{\text{R}}}{q(p + 4)} \right].$$
(11)

Different accretion models imply different values of q and different scalings of the magnetic energy density at the base of the jet ϕ_B with M and \dot{m} , resulting in different values of $\partial \ln \phi_B / \partial \ln M$ and $\partial \ln \phi_B / \partial \ln \dot{m}$. In the next two sections we calculate the expected scalings for radiatively efficient and inefficient flows (the results of the calculations are summarized in Table 3). For the case of radiatively efficient accretion, we will consider the X-ray emission to be produced by a standard, geometrically thin and optically thick disc coupled with a hot corona. In the case of radiatively inefficient accretion flows, for the sake of clarity we will describe a detailed spectral modelling in the framework of a pure advective disc (advection dominated accretion flows -ADAF). Currently this is the simplest existing dynamical model of a radiatively inefficient flow from which we can derive detailed spectral energy distributions. However, one should keep in mind that radiatively inefficient flows may be significantly modified by convection or outflows (see below). We do not, however, expect the arguments laid out below to be significantly modified in such cases.

As a result of our calculations, we will show how the physical properties of radiatively inefficient and efficient discs do indeed

Table 3. Scaling indices of the X-ray luminosity and of the magnetic field as functions of the physical parameters \dot{m} and M for different accretion flow models.

Accretion model	q	$\partial \ln \phi_B / \partial \ln M$	$\partial \ln \phi_B / \partial \ln \dot{m}$
Disc/corona, gas	1	-1/2	1/2
Disc/corona, rad	1/2	-1/2	1/4
ADAF	2.3	-1/2	1/2
Brehms. emission only	2	-1/2	1/2

imply very different scalings for the observed X-ray luminosity with the accretion rate \dot{m} .

5.2.1 Radiatively efficient flows

In the standard Shakura–Sunyaev disc model (Shakura & Sunyaev 1973), accretion occurs via an optically thick and geometrically thin disc. The effective optical depth in the disc is very high and photons are close to thermal equilibrium with electrons. The emission results in multicolour ($T \propto \dot{m}^{1/4} M^{-1/4} r^{-3/4}$) blackbody spectrum. This component is thought to explain the 'blue bump' in AGN and the soft X-ray emission in GBH. However, the standard model does not predict the power-law X-ray emission observed in all sub-Eddington accreting black holes. In fact, the emission in the 2–10 keV band we are considering here is completely dominated by the power-law spectral component for SMBH and for GBH in the low/hard state.

It is generally accepted that this hard X-ray power-law emission is produced by inverse Compton scattering of the soft, blackbody disc photons on a population of hot electrons that surround the cooler disc in the innermost region of the accretion flow (the so-called *corona*). In these models a fraction f_c of the gravitational power is dissipated in the corona (and eventually emerges as X-ray radiation). The X-ray luminosity can then be written as $L_X \sim f_c \dot{m} L_{\rm Edd} \propto f_c \dot{m} M$, and the coronal magnetic energy density follows $B^2 \propto \dot{m} f_c/M$ (e.g. Haardt & Maraschi 1991; Di Matteo, Celotti & Fabian 1999a; Merloni & Fabian 2002). L_X should therefore scale linearly with \dot{m} , as long as the fraction of power dissipated into the corona f_c does not change with the accretion rate. Note, however, that we expect some scatter in any such a relation due to intrinsic changes in the coronal plasma optical depth and temperature that may have some dependence on the accretion rate.

Merloni & Fabian (2002) and Merloni (2003) have shown how it is possible to derive coupled equations for the accretion disccorona systems under very simple assumptions about the nature of the turbulent magnetic viscosity inside the disc. The main property of their self-consistent coupled treatment is that f_c is indeed constant when gas pressure dominates in the disc, so for accretion rates $\dot{m} \lesssim 0.016(\alpha_{\rm v} M)^{-1/8}(1-0.84\alpha_{\rm v}^2)^{-9/8}$ (with $\alpha_{\rm v}$ viscosity parameter) we should expect q=1. On the other hand, $f_c \propto \dot{m}^{-1/2}$ in the radiation pressure dominated parts of the disc, and q=1/2 (see Table 3). It is worth noting that q and $\partial \phi_B/\partial \dot{m}$ enter equation (11) in such a way that the correlation coefficients $\xi_{\rm RM}$ and $\xi_{\rm RX}$ are identical for the gas and radiation pressure dominated disc cases.

5.2.2 Radiatively inefficient flows

The standard solution we have discussed in the previous section is not a unique solution for the accretion flow equations at low accretion rates. In recent years, much work has been devoted to the detailed study of low radiative efficiency accretion and in particular to the so called ADAF solutions (Narayan & Yi 1994, 1995; Abramowicz et al. 1995; Narayan, Mahadevan & Quataert 1998). The latter usually refers to the optically thin ADAF branch which is established only for accretion rates lower than a critical value $\dot{m} < \dot{m}_{\rm crit} \sim \alpha_{\rm v}^2$ (Rees et al. 1982). However, both from the theoretical point of view (Narayan, Igumenschev & Abramowicz 2000) and from numerical simulations (Igumenschev & Abramowicz 2000; Hawley & Balbus 2002), it has been shown that that radiatively inefficient flows are prone to strong convective instabilities and/or powerful outflows which alter the nature of the solutions significantly. Despite the extensive theoretical efforts, however, the relative importance of convection and outflow for adiabatic flows is still a matter of significant debate (Balbus & Hawley 2002; Narayan et al. 2002) and no strong observational discriminant has yet been found.

From our point of view, it is important to understand how the value of the magnetic field in such flow scales with M and \dot{m} , and to have a detailed model for their radiative output, in order to predict the scaling of the X-ray emission with these parameters. From general arguments, we expect any mechanically cooled flow to obey $\phi_B^2 \propto M^{-1} \dot{m}$ (Heinz & Sunyaev 2003). On the other hand, the exact value of q can only be determined once a detailed assessment of the radiative processes that give rise to observed luminosity is made. This is what we discuss in the following.

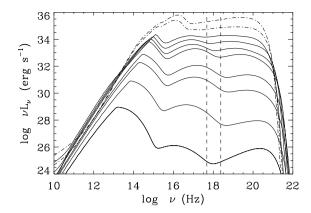
A hot accretion flow around a stellar mass (supermassive) black hole radiates mostly in the optical (radio) to X-ray bands. In the optical (radio) band, the emission results from synchrotron radiation. At higher energies, and up to the X-ray band, the emission is produced by bremsstrahlung processes for low accretion rates and inverse Compton scattering of the soft synchrotron photons or blackbody photons from the disc in the outer regions when the accretion rate approaches the critical value (Narayan, Barrett & McClintock 1997). The predicted spectrum from an ADAF depends (weakly) on the ratio of the gas to magnetic pressure β , the viscosity parameter $\alpha_{\rm v}$, and the fraction of the turbulent energy in the plasma which heats the electrons, δ . Here, we fix $\alpha_{\rm v}=0.1$, $\beta=10$, and take $\delta=0.3$.

The top panel of Fig. 6 shows the spectra of ADAFs (plus outer thin disc for $\dot{m} \sim \dot{m}_{\rm crit}$) for 10 accretion rate values starting from $\dot{m}=10^{-6}$ up to $\dot{m}\sim \dot{m}_{\rm crit}\sim 10^{-2}$. The bottom panel shows the relationship between the 2-10 keV luminosity, $L_{\rm X}$, and the accretion rate, \dot{m} (for M=10). For the chosen values of the parameters $\alpha_{\rm v}$, β and δ , the dependence of the 2–10 keV luminosity on \dot{m} is roughly given by (see also Beckert & Duschl 2002, for a similar calculation):

$$L_{X,2-10\,\text{keV}} \propto \begin{cases} \dot{m}^{3.4}, & \dot{m} \lesssim 10^{-4} \\ \dot{m}^{1.6}, & 10^{-4} \lesssim \dot{m} \lesssim 5 \times 10^{-3} \\ \dot{m}^{2}, & 5 \times 10^{-3} \lesssim \dot{m} \lesssim 2 \times 10^{-2} \end{cases}$$
(12)

The full band spectra shown in Fig. 6 with the two vertical lines indicating the 2–10 keV energy band illustrate the origin for the various dependences shown in equation (12). For $\dot{m} \lesssim 10^{-4}$ the integrated 2–10 keV emission includes both the first order Comptonized component, which drops off at around few keV, and bremsstrahlung which dominates the emission at higher energies. At higher accretion rates the first (or higher) order Compton scattering of softer photons always dominate the 2–10 keV emission (see Fig. 6, top panel). At these higher rates the optical depth increases and cooling becomes overall more efficient.

This can also been seen in bottom panel of Fig. 6, where we also plot (dashed line) the electron temperature at r = 3 as a function of \dot{m} . The temperature is nearly constant for $\dot{m} \lesssim 10^{-4}$; in this



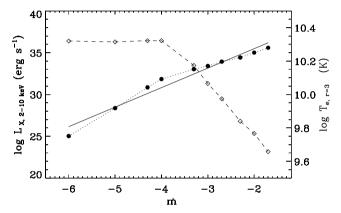


Figure 6. Predictions from ADAF models for $\dot{m}=10^{-6}, 10^{-5}, 5 \times 10^{-5}, 10^{-4}, 5 \times 10^{-4}, 10^{-3}, 2 \times 10^{-3}, 5 \times 10^{-3}, 10^{-2}$. The top panel shows the broad-band spectral energy distributions for a fixed M=10, with the vertical dashed lines enclosing the 2–10 keV spectral energy band. For the top two models (dot-dashed lines) we also plot the multicolour blackbody from an outer thin disc with an inner edge at r=40. In the bottom panel: the solid dots joined by the dotted line show the integrated 2–10 keV X-ray luminosity and the open symbols joined by dashed lines and the flow electron temperature at r=3 versus the accretion rate \dot{m} . The solid line is a linear fit to the 2–10 keV luminosity over the whole of the \dot{m} with slope 2.3.

regime the viscous heating is virtually fully balanced by advection. At $\dot{m} \gtrsim 10^{-4}$ cooling processes become more important and the temperature decreases. The overall change in the dependences in equation (12) reflects this behaviour.

However, it is important to keep in mind that the exact dependences in this narrow band will also depend on the microphysical parameters that we choose for the model. For instance, variations in the electron heating parameter, δ will result in changes in the Comptonization spectra. Therefore, in order to compare theoretical predictions with the observed data, the intrinsic scatter of which does not allow us to put tight constraints on the different emission regimes, we fit the $L_{\rm X}-\dot{m}$ relation over the whole range of \dot{m} and obtain a single power-law, $L_{\rm X} \propto \dot{m}^{2.3}$ (i.e. q=2.3, as shown by the solid line in the middle panel of Fig. 6). We also calculate the relation between $L_{2-10\,\mathrm{keV}}$, versus the black hole mass, M for the models and, as expected, the relation is very close to linear with $L_{\rm X} \propto M^{0.97}$. Overall the scalings derived from our specific computation of a specific ADAF model are consistent with the expected behaviour of the more general class of radiatively inefficient, mechanically cooled accretion flows. In fact, we note here that if the flow is modified by convection or outflows, we expect the X-ray spectrum to be dominated by bremsstrahlung emission (Quataert & Narayan 1999; Di Matteo et al. 1999b), for which we expect a=2.

5.3 Jet origin of X-ray radiation

The presence of both high-energy electrons and radio/optical radiation implies that the inner jet must emit inverse Compton X-rays, contributing to the overall X-ray spectrum at some level. However, under suitable conditions even the synchrotron component will reach X-ray energies. Especially in the presence of relativistic beaming, synchrotron emission could actually be responsible for the bulk of the X-rays. It is therefore useful to discuss the radio-X-ray-mass correlation expected for the synchrotron X-ray scenario (Markoff et al. 2001).

While at low frequencies the effects of synchrotron self-absorption lead to the flat spectrum typically observed in core dominated sources, at high frequencies the jet becomes optically thin to synchrotron self-absorption, following the well know optically thin power-law with X-ray spectral index $\alpha_X = (p-1)/2$.

Because X-rays and radio emission now originate from the same source, it is straightforward to derive the relation between the two components. The correlation coefficients ξ_M and ξ_m for the optically thick radio emission are still given by equations (7) and (8), while for the optically thin X-ray emission we can simply substitute $\alpha_X = (p-1)/2$ in those expressions. Finally, we can eliminate $\log m$ from equation (9) in favour of $\log L_X$ and arrive at the desired expressions for the correlation coefficients:

$$\tilde{\xi}_{RX} = \frac{2(2p+13+\alpha_R p+6\alpha_R)}{(p+4)(p+5)}$$

$$\tilde{\xi}_{RM} = \frac{2p+13+2\alpha_R}{p+4} - \frac{6(2p+13+\alpha_R p+6\alpha_R)}{(p+4)(p+5)}$$
(13)

where α_R is the radio spectral index and p is assumed to be the same for radio and X-ray emitting electrons, ⁸ and we have used a tilde to distinguish the synchrotron X-ray model.

5.4 Comparison with observations

In the previous sections we have shown how different theoretical models for the emission processes responsible for the observed radio and X-ray emission from black holes can be directly translated into predictions for the observable correlation coefficients ξ_{RX} and ξ_{RM} (see equation 3). The relatively tight observational constraints on these indices that we have derived in Section 3 can therefore be used to directly test these models. The theoretical coefficients for different models are shown in Fig. 4. The circles and the squares denote inefficient and efficient disc accretion respectively. Diamonds represent synchrotron X-rays from the jet. Open and closed symbols are for flat ($\alpha_R = 0$) and steep ($\alpha_R = 0.5$) spectrum sources, respectively. All model coefficients are calculated assuming p = 2 (but see Section 6.4 for a discussion of steeper electron distributions as due, for example, to the effects of cooling). To highlight the sensitivity

⁸ This implies that radiative cooling must either be negligible for the region of the jet where the X-rays are produced, leaving $p \sim 2$ over the entire spectrum, or that cooling and continuous injection have produced a universal power-law with a slope of order $p \sim 3$. This limitation, while severe, cannot be avoided because particle transport including the effects of radiative cooling cannot be formulated in a simple scale invariant fashion.

of the coefficients to variations in α_R , we also plotted the tracks in $\xi_{RX} - \xi_{RM}$ space for changes in α_R between 0 and 0.5.

For the models in which X-rays are produced by radiatively inefficient accretion, we have q=2.3 and $\partial \ln \phi_B/\partial \ln m=-\partial \ln \phi_B/\partial \ln m=0.5$ (see Section 5.2.2). The predicted coefficients for both flat and steep spectrum sources lie roughly within the 3σ contours of the observed sample. Thus, radiatively inefficient accretion is consistent with the observations. This conclusion is also true for the general class of models for which q=2.

For the standard disc–corona model, we have q=1 (high radiative efficiency) and $\partial \ln \phi_B/\partial \ln m=-\partial \ln \phi_B/\partial \ln M=0.5$ (see Section 5.2.1). The model predictions fall well outside the 3σ contours of the data and are therefore inconsistent with the observations.

For the models in which the X-rays are produced by optically thin synchrotron emission from the jet itself, the flat spectrum model predictions are marginally consistent with the data (the model point is close to the 3σ contour), while the steep spectrum model point lies well outside the 3σ contour.

These results suggest that, in a statistical sense, the correlations between radio luminosity, X-ray luminosity and mass of active black holes require a radiatively inefficient accretion flow coupled with a (scale-invariant) synchrotron emitting jet. It is important to stress here that this result does not suggest that the ADAF model for the accretion flow is the correct one, and convection and/or powerful ows are unimportant. In fact, the nature of the radio emission and the mere existence of the Fundamental Plane are clear indications that the full dynamical model for the accretion flow should at some level include, in a self-consistent manner, the effects of the backreaction from the outflowing gas. However, our results rather dictates that the radiative efficiency of the flow must be low. This conclusion holds over the observed range of $L_{\rm X}/L_{\rm Edd}$, and therefore for sources which are substantially sub-Eddington. In Section 6.6 we discuss what should be expected from similar studies performed on samples that include a larger number of very luminous black holes.

The jet synchrotron model for the X-ray emission with p=2 is only marginally consistent with the data. However, steeper electron distributions (p=3, for example) do predict values for the correlation coefficient that are well within our 3σ contours. An assessment of the relevance of such models should be made taking in due account the effect of cooling, as we discuss in Section 6.4.

6 DISCUSSION

The main result of our work is the discovery of a 'Fundamental Plane' of black hole activity. That is, if we define the instantaneous state of activity of a black hole of mass M (in units of solar masses), by the radio luminosity at a fixed frequency (for example at 5 GHz), and by the hard X-ray luminosity (for example in the 2–10 keV energy band) of its compact core, and represent such an object as a point in the three-dimensional space ($\log L_R$, $\log L_X$, $\log M$), all the points representing black holes (either of stellar mass or supermassive) will lie preferentially on a plane, described by equation (5).

We note that in the case the radiatively inefficient disc model (q = 2.3) the Fundamental Plane equation (5) implies that the radio luminosity satisfies:

$$L_{R,q=2.3} \propto \dot{m}^{1.38} M^{1.38} = \dot{M}^{1.38},$$
 (14)

i.e. $L_{\rm R}$ scales with the *physical accretion rate* only. This is very close to the predicted dependence of $L_{\rm R} \propto \dot{M}^{1.42}$ from 5.1 for the canonical parameter choice of $\alpha=0$, p=2, and $\phi_B \propto M^{-1/2} \dot{m}^{1/2}$, for which the kinetic jet power $W_{\rm jet}$ is directly proportional to the physical accretion rate, $W_{\rm jet} \propto \dot{M}$ (see also Falcke & Biermann

1995). Then, the total power released by the accretion/jet system may be written as $W_{\text{tot}} \simeq \dot{M}c^2 = L_{\text{bol}} + W_{\text{jet}} + W_{\text{adv,conv}}$, where the first term on the right-hand side is the total radiated luminosity and the last one include contributions from the energy advected and/or stored in the convective motions. Our results suggest that the flow must be radiatively inefficient; therefore, for small enough accretion rates we have $L_{\rm bol} \simeq \dot{M} \dot{m} c^2 \ll \dot{M} c^2 \sim W_{\rm jet} + W_{\rm adv,conv}$. On the other hand, $W_{\rm jet} \propto W_{\rm adv,conv} \propto \dot{M}c^2$. Therefore, the issue of what the relative fraction of the total accretion energy dissipated into the jet is (or, alternatively, of when a source is 'jet dominated'; Fender, Gallo & Jonker 2003; Falcke, Körding & Markoff 2003) reduces to the determination of the value of the constant $W_{\rm iet}/W_{\rm adv,conv}$. This requires the knowledge of the structure functions ψ in equation (6), i.e. the specification of a jet model or the direct measure of the total kinetic power carried by the jet, together with a dynamical model for the disc-jet coupling, which is clearly beyond the scope of this

6.1 The thickness of the Fundamental Plane

The Fundamental Plane defined in equation (5) is not, however, razor thin. The sources are substantially scattered around it, with a dispersion $\sigma = 0.88$ in log $L_{\rm R}$ (corresponding to a dispersion of σ_{\perp} = 0.62 perpendicular to the plane). Such a scatter is not at all surprising. Theoretically, it can be explained in large part by the scatter in the radio spectral index α_R . For example, if we consider the ADAF (q = 2.3) model with fixed p = 2, we expect the theoretical relationship $L_R = (0.62 + 0.29\alpha_R) \log L_X + (0.80 - 0.62\alpha_R) \log M$ + $b_{\rm R}$. For the sake of simplicity, let us assume now that the unknown radio spectral index of all the observed sources is normally distributed, with $\alpha_R = \langle \alpha_R \rangle \pm \sigma_\alpha$. In order to fit the observed data, we need $\langle \alpha_R \rangle \approx 0$ and $b_R \approx 7$. The dispersion in the radio luminosity due to the dispersion in the unknown parameter α_R is then $\sigma_{\rm R} \gtrsim \sigma_{\alpha} \sqrt{0.33^2 (\Delta \log M)^2 + 0.29^2 (\Delta \log L_{\rm X}/M)^2} \sim 2\sigma_{\alpha}$ where $2\Delta \log M$ is the range in M and $2\Delta \log L_X/M$ the range in luminosity spanned by our sample. Thus, a dispersion in the unknown radio spectral index of ≈ 0.3 (consistent with the observations) can give a large contribution to the observed intrinsic scatter.

In addition to the scatter produced by the diversity in α_R (which introduces scatter in the Fundamental Plane relation simply by the fact the *orientation* of the plane is slightly different for different α_R) the *intercept* b_R of the plane also varies as we look at different jet models and different spectral indices. It is not clear a priori how much it does so, as this depends on how the shape functions ψ_f (see Section 5.1) vary in order to produce the different α_R . While for one spectral class with uniform α_R the normalization cancels out when determining the correlation coefficients, this is not true when comparing jets from different spectral classes, i.e. with varying α_R . The fact that the scatter in the relation is moderate indicates, however, that the change in ψ_f over the range of radio spectral slopes that enter our sample is also moderate.

Ideally, we should therefore restrict the above analysis to subclasses of sources which follow a very narrow range in α_R . However, at the current stage the data do not allow such a division, both because the sample is too small and because the spectral indices are often not known to an accuracy that would allow such a treatment. The rough division of our sample into flat and steep spectrum sources nonetheless shows that such a treatment is possible and does lead to a reduction in scatter in the well defined and well sampled class of flat spectrum sources, for which we can be confident that the radio emission does indeed originate in the self-absorbed core of the jet

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(a prerequisite of the scale invariance model by Heinz & Sunyaev 2003).

A major source of scatter could be relativistic beaming, which we discuss in the following section. Also to be taken into account is the influence of the black hole spin on jet formation (Blandford & Znajek 1977; Koide et al. 2002). While we still expect the accretion disc to have significant influence on the jet power, as the magnetic field necessary to tap the black hole rotational energy must be provided and/or confined by the accretion disc (thus setting the field strength and the jet power; see Livio, Ogilvie & Pringle 1999; Meier 2001), the spin itself will enter as a sensitive parameter into the radio luminosity as well, introducing scatter in any correlation. Once again, strong future constraints on the tightness of this relation may help constrain the possible influence of black hole spin (see e.g. Fender 2003, for a discussion of the GBH case).

6.2 Relativistic beaming and selection effects

Because the jets that dominate radio emission from active black holes are known to be relativistic, Doppler boosting of synchrotron radiation (relativistic beaming) must be taken into account when studying these sources.

For the purpose of our study it is of primary importance to avoid strongly beamed sources, whose intrinsic (unbeamed) radio luminosity can be difficult to infer. This is the reason why BL Lac object have been excluded from our study. Furthermore, it is also important to take into account possible strong selection effects due to relativistic beaming, that would skew the observed correlations. Then, in a sample of randomly oriented jetted sources (but without the aligned ones), relativistic beaming should be an additional source of scatter in any correlation involving radio luminosities, the magnitude of which depends on the average relativistic speed of the jets.

For the SMBH in our sample, the first selection criterion is the availability of a mass measurement of the central black hole. Thus, the main selection biases do not come from flux limits, and no preferential selection of bright (beamed) sources should be operating, although anti-selection effect might be present.

Possible contaminations from beamed sources, however, could still be present among flat spectrum nearby LLAGN or type 1 Seyfert (we have 27 of them in the sample), given the current uncertainties on the exact nature of their radio emission. Nevertheless, there are many reasons to believe that these sources are not biased towards jet axes close to the line of sight. Flat spectrum LLAGN were originally selected in the optical band, from the Palomar spectroscopic survey (Ho et al. 1997a). Many of those type 1 low-luminosity Seyfert and LINERS do indeed possess double-peaked broad emission lines (Ho et al. 2001), clearly indicating a non-negligible inclination angle. The radio properties of a number of these sources were also studied by Nagar et al. (2002a). They found that LLAGN lie on the extrapolation of the FR I/FR II correlations in the $L_R - L_{H\alpha}$ plane, while a substantial offset would be expected if they were all strongly affected by beaming. Once again, we are led to the conclusion that nearby LLAGN and Seyfert 1 included in our sample have a random orientation angle, that should disfavour strongly beamed sources. Finally we note that even if a few of these source were indeed strongly Doppler boosted, their small number will not affect significantly the measured value of a correlation coefficient calculated in logarithmic space (logarithmic weighting).

Selection effects due to beaming in Galactic sources may be more subtle. Due to the overall scaling of radio luminosity with mass, GBH are expected to be on average more radio-quiet than SMBH (see also Section 6.6). It is indeed well known that galactic X-ray bi-

naries are difficult to detect in the radio band, and this might already determine a selection bias towards beamed sources. In particular, sources selected from GBI monitoring (see Section 2.2) might be more beamed. If indeed GBH were all affected by strong selection biases toward beamed emission, the derived correlation coefficients of the Fundamental Plane, $\xi_{\rm RM}$ in particular, might be incorrect. However, we know from observation of proper motion during radio outbursts that the jets of the classical microquasars GRO J1655-40 and GRS 1915+105 have large angle with the line of sight (see e.g. Fender 2003). Furthermore, for all the other GBH in our sample, apart from LS5039, orbital parameters have been inferred from optical spectroscopy, and in none of them the inferred inclination angle appears to be small. Therefore, unless the black hole jets are all misaligned with the orbital angular momentum (Maccarone 2002), we can exclude that strong beaming affects our sample.

A similar conclusion had already been reached by an accurate study of the observed scatter (\sim 0.7) about the L_R – L_X correlation in a larger sample of low/hard state GBHs (Gallo et al. 2003). An analogous argument could be put forward for our entire sample, with the similar conclusion (given the similar amount of scatter found here) that the maximum Doppler factor should not be too large. This conclusion would be further strengthen by noting that the observed scatter about the Fundamental Plane is not much larger than what is expected from the intrinsic scatter in the radio spectral index α_R (see Section 6.1). However, precise constraints on the Doppler factor could only be obtained by analysing more carefully selected samples with well determined (possibly fixed) radio spectral indices.

6.3 Steep spectrum sources

For the steep spectrum sources in our sample, the origin of the radio emission is not entirely clear. Steep spectra in general imply optically thin emission. This could be an indication that the emission is dominated by regions far away from the central engine, e.g. in the diffuse extended radio lobes or hot spots. This would prohibit any treatment that makes use of the scale invariant model, though scaling arguments for the large scale emission still apply (Heinz 2002). Steep spectrum sources should then be excluded from the sample completely, and the fact that they still fit into the plane would then mostly be due to selection effects. It should be noted that in this case the radio luminosity should strongly correlate with the black hole mass (which determines the kinetic jet power $W_{\rm iet}$ and thus the large scale radio luminosity), while it should not be related to the instantaneous X-ray flux, because the radio lobes reflect the mean power output by the central source, averaging out its temporal variability. We expect this to be a problem for Galactic black hole sources, where we have lower resolving power. For supermassive black holes, and for nearby LLAGN in particular, we can be more confident that the large scale emission is well resolved and thus does not contribute to the core emission used in our sample.

It is also possible that some of the steep spectrum sources can be associated to core jet emission which is optically thin at radio frequencies, i.e. their self-absorption break lies below 5 GHz. Because we would expect the self-absorption break frequency to be lower for higher M and lower accretion rates \dot{m} (Heinz & Sunyaev 2003), this effect would imply that steep spectra should predominantly be observed in SMBH which are relatively X-ray dim. While the trend with $L_{\rm X}$ is not clear in the data, the fact that the steep spectrum sources in our sample almost exclusively belong to the SMBH class does suggest that at least some of the sources in the sample might indeed be core dominated steep spectrum sources, for which the theoretical analysis of Section 5 holds.

6.4 X-ray emission from jets

Our analysis, taken at face value, does suggest that accretion models for the origin of the X-ray emission fit the data somewhat better than pure jet models. However, at X-ray energies the effects of cooling on the particle spectrum cannot be ignored. In the context of scale invariant models, taking cooling into account is not possible. Proper treatment of particle transport including the effects of cooling may lead to different results for the synchrotron X-ray model. In fact, simply using p=3 in equation (13) (as expected in a continuous particle injection model with cooling) moves the flat spectrum point close to the 1σ contour of the observed correlation in Fig. 4.

For the case of the galactic black hole GX 339-4 in the hard state, where the radio–X-rays correlation is well established, Markoff et al. (2003) were able to fit the observed slope ($\xi_{RX,339} \simeq 0.71$) with a synchrotron X-ray model assuming p=2.15 (and thus $\alpha_X=0.58$, which is also close to the average slope of the X-ray spectral index of the source in the hard state). Here we have found that, in a statistical sense, higher values of p are needed to explain the observed correlation for the entire sample in the framework of the jet synchrotron model for the X-ray emission.

Furthermore, for p=2 the X-ray synchrotron luminosity from the jet scales like $L_{\rm X,synch} \propto M^{1.25}$, while the disc X-ray luminosity follows $L_{\rm X,disc} \propto M$. Thus, if the X-ray emission in GBH jets like GX 339-4 were indeed due to synchrotron emission, we would expect an even stronger contribution from synchrotron X-rays relative to the disc emission in AGNs with the same accretion rate, where the general wisdom (mainly based on accurate analysis of detailed X-ray spectra) is still that the X-rays originate predominantly in the disc. For p=3, on the other hand, the synchrotron X-rays follow $L_{\rm X,synch} \propto M$, and the relative contribution from jet and disc will be independent of M.

Finally, the X-ray spectral variability behaviour of both GBH and Seyfert galaxies is consistent with pivoting of the spectrum at X-ray energies. If the X-ray emission is produced by synchrotron radiation and there is no cooling break at longer wavelengths, such a power-law variability would imply a radio variability which is orders of magnitude larger than observed (see discussion in Zdziarski et al. 2003).

It seems, therefore, that *if* the X-ray emission in some of the sources is dominated by synchrotron emission from the jet, the effects of cooling must play an important role in determining the observed correlation. In the context of the scale-invariant model, it is rather problematic to take cooling into account; this is why a theoretical assessment of the role of the cooling on the observed correlation is beyond the scope of this paper.

6.5 Estimating black holes masses and the nature of ultraluminous X-ray sources

Due to the intrinsic scatter, the predictive power of the $\log M - \log L_{\rm X} - \log L_{\rm R}$ relation is currently limited in scope. Nonetheless, from the Fundamental Plane equation (5), we can derive a formal relationship between the observed fluxes in the radio (at 5 GHz) and in the X-ray (2–10 keV) band (in erg s⁻¹ cm⁻²), the distance to a source D (in Mpc) and its unknown mass. We obtain

$$\log M \simeq 16.3 + \log D + 1.28(\log F_R - 0.60 \log F_X) \pm 1.06.$$
 (15)

In particular, we can predict in which region of $\log L_R - \log L_X$ space we would expect to find intermediate-mass black holes, if they exist, and specifically, where we would expect ultraluminous X-ray sources (ULX; Makishima et al. 2000) based on their X-ray

flux, should they be intermediate-mass black holes, as opposed to intrinsically beamed low-mass black holes.

A clear prediction of the Fundamental Plane equation in this sense is that, if ULX are stellar mass objects (with M < 30), and their radio emission is not strongly Doppler boosted, there is an upper limit to their radio flux at 5 GHz. For example, a ULX with an X-ray luminosity (in the 2–10 keV band) of 10^{40} erg s⁻¹ at a distance of 10 Mpc will have

$$\log(F_{\rm R}/{\rm Jy}) \lesssim -6.5 - 2\log(D/10) + 0.6(\log L_{\rm X}/40) \pm 0.9$$
 (16) so its flux will be at most few $\mu {\rm Jy}$.

As an example, we refer to the recent claim of the observation of synchrotron radio emission from the ULX 2E 1400.2–4108, in the dwarf irregular galaxy NGC 5408 (at a distance of \sim 4.8 Mpc) (Kaaret et al. 2003). The observed radio (0.26 mJy at 4.8 GHz) and X-ray fluxes (2.7 \times 10⁻¹² erg s⁻¹ cm⁻² in the 0.3–8 keV band), if indeed both associated with a compact source at the distance of NGC 5408, would imply a mass of log $M \simeq 4 \pm 1$. However, we should stress here that such an estimate only holds *if the radio emission is not beamed*. The uncomfortably high value for the estimated mass, the steep inferred radio spectral index and the radio to X-ray flux ratio, all suggest that the observed intensity of the radio emission in this source may be Doppler boosted by a relativistic jet pointing in our direction as already suggested by Kaaret et al. (2003).

6.6 Accretion mode changes

It is well accepted, both from theory and observations, that accretion can proceed in different modes (or states), with different radiative efficiencies and spectral properties (Abramowicz 1998; Done 2002; Frank, King & Raine 2002; Narayan 2002), mainly driven by variations of the dimensionless accretion rate \dot{m} . Here we have demonstrated (Section 5) that the correlation between radio and Xray luminosity in GBH and SMBH can provide valuable constraints on the emission mechanisms in these bands and on the physics of accretion. We have shown how low radiative efficiency accretion models can possibly provide the underlying scalings that most easily explain the properties of the observed Fundamental Plane. Radiatively efficient thin disc-corona systems are clearly inconsistent with the data. The only possibility for these models to be reconciled with our observations would entail a highly radiatively inefficient corona, where most of the dissipated magnetic energy is converted into bulk kinetic energy of outflowing gas (Beloborodov 1999), as described by Merloni & Fabian (2002). This would require some specific model for the magnetic dissipation processes, and for their scaling with M and \dot{m} , which are currently unknown.

However, because both such modes of accretion are expected to occur only below accretion rates \lesssim few per cent of Eddington, we would expect the $\log M - \log L_R - \log L_X$ correlation to break down at around this limit. In other words, we should expect the QSOs and the bright Seyferts in our sample, which occupy the region of high accretion rates and are independently known to have spectral characteristics inconsistent with models of low radiative efficiency, to depart from the observed correlations. Also in GBHs, it has indeed been shown that the correlation between radio and X-ray luminosity breaks down as the sources switch to their high states (Maccarone 2003; Gallo et al. 2003). In particular, high-luminosity states of GBH (high or very high states) show highly variable radio fluxes and radio to X-ray ratios (Mirabel & Rodriguez 1994; Fender et al. 1999). If this reflects a general property of the disc-jet coupling at high accretion rates, a substantial increase in the scatter should appear above the critical accretion rate.

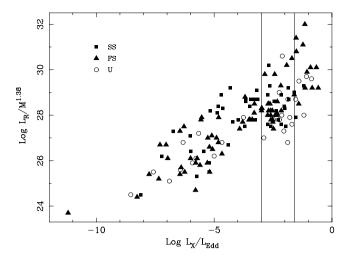


Figure 7. The radio luminosity $\log L_{\rm R}$, divided by $M^{1.38}$ as a function of the ratio $L_{\rm X}/L_{\rm Edd}$. Solid squares are for steep spectrum sources, solid triangles for flat spectrum ones and open circles for sources with undetermined radio spectral index. Two vertical lines mark the boundary of the region where we expect the critical luminosity for the mode change between radiatively inefficient and efficient accretion. The scaling for the radio luminosity with mass is obtained directly from the Fundamental Plane equation (5) by imposing that the X-ray luminosity scales linearly with black hole mass.

In order to illustrate this point further, in Fig. 7 we plot, as a function of the ratio $L_{\rm X}/L_{\rm Edd}$, the radio luminosity divided by $M^{1.38}$ [such a scaling for the radio luminosity with mass is obtained directly from the Fundamental Plane equation (5) by imposing that the X-ray luminosity scales linearly with black hole mass]. As indeed expected, by rescaling the radio luminosity in such a way all the different tracks corresponding to different mass bins in Fig. 3(b) collapse into a single one (with some residual scatter). The region between the two vertical lines corresponds to the expected values of $L_{\rm X}/L_{\rm Edd}$ above which a change of accretion mode, from radiatively inefficient to standard radiatively efficient is expected to occur. Our SMBH sample is still limited in order to test whether such a change is observed. In fact, we select out of our sample the majority of bright quasars, both radio-quiet and radio-loud, because of the lack of a reliable mass estimates for these sources. However, there is indeed a hint of an apparent increase in the scatter about the correlation, above the critical accretion rates, as seen in Fig. 7.

We may thus speculate that the famous (and still much debated, see e.g. Cirasuolo et al. 2003) radio-loud/radio-quiet dichotomy of quasars will appear only at the highest values of \dot{m} , and be caused mainly by a switch of accretion mode analogous to the high/very high transition in GBH. At low accretion rates, black holes seem to follow the more regular behaviour circumscribed by the Fundamental Plane of equation (5). Such sources not only tend to be radio-loud (Ho & Peng 2001; Ho 2002), but also their radio-loudness parameter, $R_{\rm X}$ (here defined as the ratio of radio to X-ray luminosity), obeys the following scaling: $R_{\rm X} \equiv L_{\rm R}/L_{\rm X} \propto L_{\rm X}^{-2/5} M^{4/5}$. Therefore, the smallest the X-ray luminosity, the more radio-loud these sources are (Ho 2002). In this regime, no dichotomy has to be expected, as already suggested by Nagar et al. (2002a).

7 SUMMARY

We have shown how the coupling between accretion flows and jet emission in black holes can be understood by studying large samples of *both* galactic (GBH) and supermassive (SMBH) black holes with measured mass, observed at both radio and X-ray wavelengths.

We have compiled, from the existing literature, a sample of ~ 100 supermassive black holes and \sim 50 different observations of galactic. stellar mass black holes. We required that the SMBH have a reliable measure of the central black hole mass, either (preferentially) direct, via high-resolution kinematics studies of surrounding stars and gas or reverberation mapping of the broad line region (for luminous Seyfert 1 galaxies and Quasars), or, indirectly, thanks to the empirical correlation between black hole masses and central velocity dispersion. Although such selection criteria introduce a number of biases in the final distribution of M, which are difficult to control, our final sample spans a large enough range in both mass and accretion rate to unveil major trends in the physical properties of active black holes. This is a crucial point, and represents the main novelty of our approach. Most previous studies on the subject have only considered correlations of jet (radio) luminosity with black hole mass or with luminosity in other wavebands (and hence with accretion rate) separately.

We have performed a partial correlation analysis on the sample, and concluded that the radio luminosity is strongly correlated with *both* black hole mass *and* X-ray luminosity, while in turn, the X-ray luminosity correlates with both mass and radio luminosity. In light of this, we have carried out a multivariate linear regression analysis and derived the best-fitting correlation coefficient between radio luminosity, X-ray luminosity and black hole mass.

The data from the entire sample allow us to put tight constraints on the correlation coefficients. Remarkably, we find that the sources lie preferentially on a plane ('the Fundamental Plane of black hole activity') in the three-dimensional (log $L_{\rm R}$, log $L_{\rm X}$, log M) space, described by the equation log $L_{\rm R} = (0.60^{+0.11}_{-0.11})$ log $L_{\rm X} + (0.78^{+0.11}_{-0.09})$ log $M+7.33^{+4.05}_{-4.07}$. The scatter around this plane is, however, significant ($\sigma_{\rm R}=0.88$).

From a theoretical point of view, these results clearly suggest that the Ansatz of scale invariance for the disc–jet coupling (Heinz & Sunyaev 2003) captures the main physical properties of such systems. Thus, a universal theoretical scaling between the radio flux at a given frequency and both mass and accretion rate can be derived, *independently of the jet model*, with scaling indices that depend only on the (observable) spectral slope of the synchrotron emission in the radio band, and on the accretion mode. Also, it is possible to predict the correct amount of scatter for any such relationship.

By comparing the observationally derived correlation coefficients to the theoretically predicted ones, we are able to put constraints on accretion models and on the disc–jet coupling. We demonstrate that the X-ray emission from black holes accreting at less than a few per cent of the Eddington rate cannot be produced by radiatively efficient accretion, while radiatively inefficient accretion flows agree well with the data. The optically thin X-ray jet synchrotron emission model is only marginally consistent with the observed correlation; however, cooling of the electrons should be properly taken into account before drawing a firmer conclusion on the issue.

The Fundamental Plane equation also implies that low-luminosity black holes should be more and more radio-loud as the accretion rate decreases, as observed (Ho 2002). The possible dichotomy between radio-quiet and radio-loud Quasars could instead be due to a switch of accretion mode at the highest accretion rates. Therefore, more accurately selected samples of more luminous sources, including a significant number of both radio-quiet and loud quasars with well determined radio spectral properties, should allow the observation of bends, discontinuities or even bifurcations of the Fundamental

Plane associated with accretion mode switches and with the turning on and off of the most powerful radio sources.

NOTE ADDED IN PRESS

After the submission of this manuscript another work has appeared as preprint (Falcke et al. 2003), which is similar in scope, and reaches very similar conclusions about the scale invariant properties of low-luminosity black holes.

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