Comments

A Further Exploration in the Theory of Exchange Rate Regimes

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I. Introduction

In an interesting paper in this *Journal,* Elhanan Helpman (1981) develops a simple two-country model to compare the welfare levels that are achieved under different exchange rate regimes. In a world characterized by perfect foresight, his basic finding is that equilibrium consumption allocations, and hence welfare levels, are identical under a floating exchange rate system and a (one-sided peg) fixed exchange rate regime. This result is due to Helpman's assumption that output is determined exogenously, which allows a separation of the real and monetary sides of the economy. The purpose of this paper is to extend Helpman's model by making two alterations in his setup: (1) to allow production to be endogenously determined as a function of labor force participation and (2) to allow welfare levels to be dependent on labor service as well as on consumption. The main conclusion then obtained is that a country may achieve a higher welfare level under a flexible exchange rate regime than under a fixed exchange rate regime, since it is able to choose an optimal rate of inflation (or currency depreciation).

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II. The Model

Consider the following model of a "small" open economy inhabited by an immortal representative agent who is blessed with perfect foresight. The agent's goal is to maximize his lifetime utility, $\underline{U}(\cdot)$, as given by

$$\underline{U} = \sum_{t=1}^{\infty} \rho^{t-1} U(c_{Ht} + c_{Ft}, l_t), \quad \rho \in (0, 1),$$

where ρ is the individual's (constant) subjective discount factor, $U(\cdot)$ is his momentary utility function, c_{Ht} and c_{Ft} are his period t consumption of the home country's output and the foreign country's output, respectively, and l_t is the quantity of labor services he supplies in period t. The momentary utility function, $U(\cdot)$, is assumed to be strictly quasi-concave and twice differentiable with consumption being a normal good and labor service being an inferior one.

The representative agent has two sources of income. His primary source of income is through the owner operation of a firm. The firm produces the consumption good using labor, l, supplied by the individual. The firm's output of this good, y, is described by the production process

$$y_t = f(l_t),$$

where the function $f(\cdot)$ is taken to be twice differentiable and to exhibit diminishing returns to scale. Also, in each period t the individual receives a nominal transfer payment, X_{Ht}, from the domestic government (this transfer payment may be negative).

There is an international bond market in which the individual can freely participate. In this market he can issue (or buy) 1-period real bonds, b, denominated in terms of the foreign output. These bonds pay the internationally determined real rate of return, r. If, for instance, the representative agent were to issue bonds worth b units of the foreign-produced good he would have to repay the equivalent of $(1 + r)b$ units of the foreign output next period to the buyer of these bonds.

In the model, the individual must use currency to purchase goods. For example, if in period t the individual purchases c_{Ht} units of the home-produced good, this must be bought using currency from the individual's current holdings of domestic money, M_{Ht}. Likewise, in this period his purchases of the foreign-produced good must be bought using currency from his holdings of foreign money, M_{Ft}.

A time profile of the individual's life in period t will now be given so as to highlight the circulation of money in the model. The representative agent enters this period with a certain amount of domestic and foreign money to spend left over from the previous period, $t - 1$. Now, at the beginning of period t the individual receives in domestic currency the income his firm accrued from sales during the previous period. This amounts to $P_{Ht-1}f(l_{t-1})$, where P_{Ht-1} represents the domestic nominal price of the home-produced good in period $t - 1$. At this time the agent also receives a nominal transfer payment from the government in the amount X_t.

The individual then takes his cash to the international bond cum foreign exchange market and liquidates the debts he incurred during the previous period. These debts now amount in domestic currency terms to $(1 + r_{t-1})e_tP_{Ft}b_{Ft-1}$, where P_{Ft} is the foreign nominal price of their output in period t and e_t is the price in domestic currency of a unit of foreign currency in period t. After doing this, the individual issues new foreign-denominated real bonds worth in domestic currency terms $e_tP_{Ft}b_{Ft}$. His resulting new holdings of cash are then allocated between holding domestic and foreign currency in the magnitudes M_{Ht} and M_{Ft}.

During the remainder of the period, the individual uses his holdings of domestic and foreign currency to purchase his consumption quantities of the home-produced good, c_{Ht}, and the foreign-produced good, c_{Ft}. Note that if in this typical period t equilibrium is to prevail in both the domestic and foreign countries' goods markets, then the law of one price,

$$P_{Ht} = e_tP_{Ft} \quad \forall\, t = 1, 2, \ldots, \infty, \tag{1}$$

must hold.[1] Also, during the remainder of this period the agent supplies labor input to his firm. He then enters period $t + 1$ with $M_{Ht} - P_{Ht}c_{Ht}$ units of domestic currency and $M_{Ft} - P_{Ft}c_{Ft}$ units of foreign currency and the process begins again. It has of course been assumed in the discussion above that in period t the individual made his decisions about how much money to hold, how many bonds to issue, how many goods to consume, and how much labor to supply in an optimal fashion. Attention will now be directed to this matter.

Formally, the recursive equation of the agent's dynamic programming problem is shown by (2) with the choice variables of the individual being c_{Ht}, c_{Ft}, l_t, M_{Ht}/P_{Ht}, M_{Ft}/P_{Ft}, and b_t:[2]

$$V(a_t; P_t) = \max\,[U(c_{Ht} + c_{Ft}, l_t) + \rho V(a_{t+1}; P_{t+1})], \tag{2}$$

[1] This proposition is demonstrated formally in Aschauer and Greenwood (1983). It is perhaps worth emphasizing that the law is a consequence, and not an assumption, of the setup being employed here. Different types of cash-in-advance setups can alter the traditional form of the law of one price as is shown by Helpman and Razin (1981).

[2] It is being assumed that the rest of the world is facing an analogous maximization problem. The following solvency condition should also be imposed on the agent's dynamic programming problem:

$$\lim_{t\to\infty} \prod_{j=1}^{t} \left(\frac{1}{1 + r_{j-1}}\right) b_t \leq 0.$$

subject to

$$\frac{M_{Ht}}{P_{Ht}} + \frac{M_{Ft}}{P_{Ft}} = \frac{P_{Ht-1}}{P_{Ht}} f(l_{t-1}) + \frac{P_{Ht-1}}{P_{Ht}}\left(\frac{M_{Ht-1}}{P_{Ht-1}} - c_{Ht-1}\right)$$
$$+ \frac{P_{Ft-1}}{P_{Ft}}\left(\frac{M_{Ft-1}}{P_{Ft-1}} - c_{Ft-1}\right) \tag{3}$$
$$+ b_t - (1 + r_{t-1})b_{t-1} + \frac{X_{Ht}}{P_{Ht}} \quad \forall\, t = 1, 2, \ldots, \infty,$$

$$c_{Ht} \leq \frac{M_{Ht}}{P_{Ht}} \quad \forall\, t = 1, 2, \ldots, \infty, \tag{4}$$

$$c_{Ft} \leq \frac{M_{Ft}}{P_{Ft}} \quad \forall\, t = 1, 2, \ldots, \infty, \tag{5}$$

and where

$$a_t \equiv \left[f(l_{t-1}), \left(\frac{M_{Ht-1}}{P_{Ht-1}} - c_{Ht-1}\right), \left(\frac{M_{Ft-1}}{P_{Ft-1}} - c_{Ft-1}\right), b_{t-1}, \frac{X_{Ht}}{P_{Ht}}\right]$$

and

$$P_t \equiv \left(\frac{P_{Ht-1}}{P_{Ht}}, \frac{P_{Ft-1}}{P_{Ft}}, r_{t-1}\right).$$

Note that as long as $\pi_{Ht} \equiv (P_{Ht+1} - P_{Ht})/P_{Ht}$ and $\pi_{Ft} \equiv (P_{Ft+1} - P_{Ft})/P_{Ft}$ are both greater than $-r_t/(1 + r_t)$, the constraints (4) and (5) will hold as strict equalities, since in this situation bonds will dominate money as an abode of purchasing power.[3] Assuming that this is the case, it follows from the maximization problem above that the time profile $\{c_t \equiv c_{Ht} + c_{Ft}, l_t\}_1^\infty$ describing the agent's consumption-labor choice can be characterized by the following two Euler equations, (6) and (7), and the intertemporal budget constraint (8):[4]

$$\frac{\delta U(\cdot_t)}{\delta c_t} = \rho(1 + r_t)\frac{\delta U(\cdot_{t+1})}{\delta c_{t+1}} \quad \forall\, t = 1, 2, \ldots, \infty, \tag{6}$$

$$-\frac{\delta U(\cdot_t)}{\delta l_t} = \frac{f'(l_t)}{(1 + \pi_{Ht})}\frac{\rho\delta U(\cdot_{t+1})}{\delta c_{t+1}} \quad \forall\, t = 1, 2, \ldots, \infty, \tag{7}$$

[3] See Helpman (1981) for a more complete discussion of this point.

[4] For simplicity, it will be assumed that the world starts at the beginning of period 1. In other words, for the domestic economy let $c_{H0} = c_{F0} = l_0 = f(l_0) = b_0 = M_{H0} = M_{F0} = 0$. A similar set of initial conditions will hold for the rest of the world. Finally, make the following definitions: $r_0 \equiv 0$, $P_{H0} \equiv 1$, $P_{F0} \equiv 1$, and $e_0 \equiv 1$. In deriving the representative agent's intertemporal budget constraint (8), the solvency condition mentioned in n. 2 was used in addition to eq. (3), eqq. (4) and (5) holding as strict equalities.

$$\sum_{t=1}^{\infty}\left[\prod_{j=1}^{t}\left(\frac{1}{1+r_{j-1}}\right)\right]c_t = \sum_{t=1}^{\infty}\left[\prod_{j=1}^{t}\left(\frac{1}{1+r_j}\right)\right]\left[\frac{f(l_t)}{1+\pi_{Ht}}\right] + \sum_{t=1}^{\infty}\left(\frac{1}{1+r_{j-1}}\right)\frac{X_{Ht}}{P_{Ht}}. \tag{8}$$

In the model's general equilibrium, the goods and money markets clear each period, implying that $\Sigma_{j=1}^{t} X_{Hj} = P_{Ht}f(l_t)$ for all t.[5] When this result is used, the agent's intertemporal budget constraint (8) can be replaced in the model's general equilibrium by the simpler one (9) shown below, so that the agent's equilibrium consumption-labor time profile, $\{c_t, l_t\}_1^{\infty}$, can be described completely by (6), (7), and (9):

$$\sum_{t=1}^{\infty}\left[\prod_{j=1}^{t}\left(\frac{1}{1+r_{j-1}}\right)\right]c_t = \sum_{t=1}^{\infty}\left[\prod_{j=1}^{t}\left(\frac{1}{1+r_{j-1}}\right)\right]f(l_t). \tag{9}$$

It can be seen quite easily from (6), (7), and (9) that the domestic rate of inflation, π_H, affects the agent's consumption-labor choice. If (6) and (7) are combined, it follows that for all t

$$\frac{-\delta U(\cdot_t)/\delta l_t}{\delta U(\cdot_t)/\delta c_t} = \frac{f'(l_t)}{(1+r_t)(1+\pi_{Ht})}. \tag{10}$$

The left-hand side of the equation above represents the marginal rate of substitution between working and consuming in t. The right-hand side represents the marginal rate of transformation between work effort and consumption during the same period. Specifically, by working an extra unit in t, the individual could increase his consumption in this period by $f'(l_t)/[(1 + r_t)(1 + \pi_{Ht})]$. This can be explained intuitively as follows. An extra unit of work in period t leads to an increase in the nominal value of the firm's output in this period of $P_{Ht}f'(l_t)$. However, the individual does not receive the firm's earnings until period $t + 1$, at which time the real value of these earnings will be $(P_{Ht}/P_{Ht+1})f'(l_t) = (1 + \pi_{Ht})^{-1}f'(l_t)$. The amount the individual could borrow in period t against this increased future income, so as to increase his period t consumption, would be the right-hand side of (10).[6]

[5] This condition follows from the fact that in each country the cash-in-advance constraints hold as strict equalities, implying that $P_{Ht}c_{Ht} = M_{Ht}$ and $P_{Ht}c_{Ht}^{*} = M_{Ht}^{*}$ for all t, where c_{Ht}^{*} and M_{Ht}^{*} represent the foreign demands for domestic output and currency in period t. Equilibrium in the domestic goods and money markets in each period t requires that $c_{Ht} + c_{Ht}^{*} = f(l_t)$ and $M_{Ht} + M_{Ht}^{*} = \Sigma_{j=1}^{t} X_{Hj}$ for all t. Thus, $\Sigma_{j=1}^{t} X_{Hj} = P_{Ht}f(l_t)$ for all t.

[6] Note that the foreign rate of inflation does not enter into the system of eqq. (6), (7), and (9), characterizing the agent's equilibrium consumption-labor choice through time. This is because domestic residents—consumers and firms—can avoid holding foreign currency over adjacent time periods.

III. The Optimal Rate of Inflation and the Choice of Exchange Rate Regime

It is useful to compare the results just obtained with those that would arise in a pure barter economy with zero transactions costs of exchange. In this costless barter environment, the agent's optimization problem would be to maximize his lifetime utility (2) subject to his intertemporal budget constraint (9). This corresponds to the agent's solving the following dynamic programming problem (11) with his decision variables being c_{Ht}, c_{Ft}, l_t, and b_t:

$$V(b_{t-1}; r_{t-1}) = \max \left[U(c_{Ht} + c_{Ft}, l_t) + \rho V(b_t; r_t)\right] \tag{11}$$

subject to

$$c_{Ht} + c_{Ft} = f(l_t) + b_t - (1 + r_{t-1})b_{t-1} \quad \forall\, t = 1, 2, \ldots, \infty. \tag{12}$$

Here, the time profile describing the agent's equilibrium consumption-labor choice, $\{c_t, l_t\}_1^\infty$, may be characterized by the following set of conditions:

$$\frac{\delta U(\cdot_t)}{\delta c_t} = (1 + r_t)\frac{\delta U(\cdot_{t+1})}{\delta c_{t+1}} \quad \forall\, t = 1, 2, \ldots, \infty, \tag{13}$$

$$-\frac{\delta U(\cdot_t)}{\delta l_t} = f'(l_t)\frac{\delta U(\cdot_t)}{\delta c_t} \quad \forall\, t = 1, 2, \ldots, \infty, \tag{14}$$

$$\sum_{t=1}^{\infty}\left[\prod_{j=1}^{t}\left(\frac{1}{1 + r_{j-1}}\right)\right]c_t = \sum_{t=1}^{\infty}\left[\prod_{j=1}^{t}\left(\frac{1}{1 + r_{j-1}}\right)\right]f(l_t). \tag{15}$$

The individual's marginal rate of substitution between working and consuming in period t follows directly from equation (14). This results in equation (16):

$$\frac{-\delta U(\cdot_t)/\delta l_t}{\delta U(\cdot_t)/\delta c_t} = f'(l_t) \quad \forall\, t = 1, 2, \ldots, \infty. \tag{16}$$

A comparison of the equation above with its analogue (10) arising in the cash-in-advance environment points out how the necessity of using money distorts the agent's consumption-leisure choice. As can be seen from (16), in the costless barter economy the individual's marginal rate of substitution between working and consuming in period t is equated to the marginal product of labor in this period. In the cash-in-advance economy, a wedge of the amount $1/(1 + i_{Ht})$ is driven between these two quantities, where i_{Ht} is defined as the domestic nominal rate of interest in period t, so that $i_{Ht} \equiv \pi_{Ht} + r_t + \pi_{Ht}r_t$. Thus in equation (10) the marginal product of labor is being discounted by a factor of one plus the domestic nominal interest rate. This wedge, $1/(1 + i_{Ht}) = 1 - [i_{Ht}/(1 + i_{Ht})]$, occurs because the private opportunity cost of holding domestic money in period t, $i_{Ht}/(1 + i_{Ht})$, is not equal to the social cost of holding money in this period. The latter cost is zero because money is costless to produce.

The fact that the necessity to use money in a cash-in-advance constraint environment can distort real allocations has been noted before by others. Wilson (1979) and Aschauer (1980) discuss how the rate of inflation affects individuals' consumption-leisure choice in cash-in-advance economies.[7] Similarly, Helpman and Razin (1981) and Stockman (1981) analyze how the inflation rate influences agents' savings-investment decisions.

To maximize domestic welfare in the cash-in-advance economy, the wedge that arises in each period between the agent's marginal product of labor and his marginal rate of substitution between working and consuming must be removed. This can be done by adopting a domestic monetary policy and a foreign exchange rate regime that allows the domestic nominal rate of interest to be set equal to zero, thereby equating the private and social opportunity costs of holding money. The domestic rate of inflation required for this result is

$$\hat{\pi}_{Ht} = \frac{-r_t}{1 + r_t} \quad \forall\, t = 1, 2, \ldots, \infty, \tag{17}$$

which is the familiar optimum quantity of money result as discussed in Friedman (1969).

Finally, what implication does the inflation rate requirement (17) have for the choice of exchange rate regimes? Assuming an exogenous foreign inflation rate in period t of the amount π_{Ft}, the adoption of the rule (17) and the law of one price (1) necessitate that the domestic exchange rate appreciate in period t at the rate minus $\hat{d}_t$, where[8]

$$-\hat{d}_t = \frac{i_{Ft}}{1 + i_{Ft}}, \quad i_{Ft} \equiv \pi_{Ft} + r_t + \pi_{Ft} r_t, \tag{18}$$

and $d_t \equiv (e_{t+1} - e_t)/e_t$ is the rate of depreciation of the exchange rate. In other words, the rate of appreciation of the domestic exchange rate in period t will be equal to the opportunity cost of holding foreign money, $i_{Ft}/(1 + i_{Ft})$.

Thus, unless the foreign country is adhering to the rule for the

[7] It can be shown that the domestic economy's steady-state levels of consumption, labor force participation, output, and welfare are inversely related to its steady-state inflation rate. This has the implication of a positively sloped long-run Phillips curve. For further details see Aschauer and Greenwood (1983).

[8] The law of one price (1) implies that $\pi_{Ht} = \pi_{Ft} + d_t + \pi_{Ft} d_t$ for all t, so that $d_t = (\pi_{Ht} - \pi_{Ft})/(1 + \pi_{Ft})$ for all t. Plugging eq. (17) into this expression for d_t yields (18) in the text.

optimum quantity of money (i.e., unless $i_{Ft} = 0$ for all t), the domestic country should adopt a flexible exchange rate system and follow the rule itself. This allows the domestic country to maximize its own welfare rather than adopt a fixed exchange rate system that constrains it to accept the foreign suboptimal rate of inflation and the associated inferior level of welfare.[9]

IV. Conclusion

This paper has extended Helpman's (1981) model by allowing production to be endogenous and welfare levels to be dependent on labor service as well as consumption. In this situation, it was shown that there exists an optimal rate of inflation. A case for flexible exchange rates, so as to allow the domestic country to achieve this optimal inflation rate, may then be made unless the foreign country is maximizing its own welfare by choosing the optimal rate of inflation. Then the choice between adopting a system of fixed or flexible exchange rates would be a matter of indifference.

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[9] By definition, a fixed exchange rate system implies that $d_t = 0$ for all t, so that as a consequence of the law of one price $\pi_{Ht} = \pi_{Ft}$ for all t, a fact which is readily deduced from n. 8.