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Notes and Comment

A further test of the superposition model for the redundant-signals effect in bimodal detection

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The superposition model proposed by Schwarz (1989) to account for the redundant-signals effect in a bimodal detection task with visual and auditory signals is tested further. It is shown that the model does not fit the observed standard deviations reported by Miller (1986) if the residual (motor) component is assumed to be independent of the waiting time.

The *redundant-signals effect* in a bimodal detection task with visual and auditory signals refers to the observation that a response is made more quickly when it is indicated by two simultaneously presented signals, one on each channel, than when a single signal is presented on either channel alone (see, e.g., Diederich & Colonius, 1987; Miller, 1982, 1986; Raab, 1962). Raab proposed a *probability summation* explanation for the speedup of responses: the two signals are processed simultaneously within different channels, and each channel produces a separate activation. A response is initiated as soon as a certain activation level is exceeded in either channel. Responses to redundant signals are especially fast because the expected processing time of the channel "winning the race" is smaller than the expected processing time of either channel in the single-signal condition.

Subsequent empirical and theoretical studies (Colonius, 1986, 1988, 1990; Gielen, Schmidt, & van den Heuvel, 1983; Miller, 1982, 1986; Ulrich & Giray, 1986) suggest that the probability summation explanation of the effect is not sufficient to account for the entire amount of the reaction time (RT) reduction. In an alternative approach, Miller (1982) advanced the idea of a *coactivation* mechanism whereby the evidence coming from different signals is pooled by a central decision maker to produce faster responses on redundant-signal than on single-signal trials (see also Fournier & Eriksen, 1990). Recently, Schwarz (1989) proposed a specific formalization of the coactivation idea, the *superposition model*, and presented a successful fit of the model to the mean RTs of Miller's (1986) study.

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The purpose of this note is to further explore and test the superposition model. In particular, it will be shown that the model, in its present form, is unable to account for the observed standard deviations (*SD*s) in Miller's data.

The Superposition Model

It is assumed that the presentation of a stimulus induces a stream of neural events that can be represented as a renewal counting process, $\{N(t), \geq 0\}$ (see, e.g., Smith, 1988). Thus, for any point in time t, $t \geq 0$, $N(t)$ is a discrete random variable counting the number of events up to time t. Moreover, let T_i denote the waiting time for the ith event to occur. In an ordinary renewal process, the interarrival times $Z_1 \equiv T_1$, and $Z_i \equiv T_i - T_{i+1}$, $i > 1$, are independently and identically distributed random variables. It is assumed that as soon as a critical number of events, c, has been registered at some central decision mechanism, a response execution process is started. The main postulate of the superposition model is that in the redundant-signals condition the renewal processes generated by either signal are superposed, thereby reducing the waiting time for the critical count. Specifically, if $N_A(t)$ and $N_V(t)$ denote the renewal counting processes for the auditory and the visual signals, respectively, the counting process for the double stimulus is $N_R(t) = N_V(t) + N_A(t)$. If the visual stimulus is presented τ msec ($\tau > 0$) before the auditory stimulus, we consider

$$N_R(t) = N_V(t) + N_A(t-\tau), \tag{1}$$

where $N_A(t-\tau) = 0$ for $t < \tau$. The subscripts V and A

Table 1
Fit of Superposition Model to Miller's (1986) Data Using Schwarz's (1989) Parameter Estimates

	Subject B.D.				Subject K.Y.			
SOA	Mean RT		*SD*		Mean RT		*SD*	
$-\infty$	231	(231)	56	(38)	211	(215)	60	(21)
−167	234	(231)	58	(38)	216	(215)	74	(21)
−133	230	(230)	40	(37)	217	(215)	76	(21)
−100	227	(230)	40	(35)	214	(215)	78	(21)
−67	228	(227)	32	(32)	218	(215)	76	(20)
−33	221	(222)	28	(29)	215	(213)	66	(18)
0	217	(214)	28	(29)	208	(208)	64	(16)
33	238	(238)	28	(28)	237	(232)	62	(18)
67	263	(261)	26	(30)	249	(251)	58	(27)
100	277	(282)	30	(36)	256	(263)	46	(37)
133	298	(299)	32	(45)	273	(271)	54	(46)
167	316	(313)	34	(55)	278	(276)	62	(54)
∞	348	(348)	92	(106)	282	(283)	62	(69)

Note—Predicted values are in parentheses; negative SOA values refer to the condition in which the auditory stimulus was presented first. Parameters: $m = 165$, $c = 3$, $\alpha_A = 0.455$, $\alpha_V = 0.0164$ for B.D. and $m = 185$, $c = 2$, $\alpha_A = 0.0667$, $\alpha_V = 0.0204$ for K.Y.

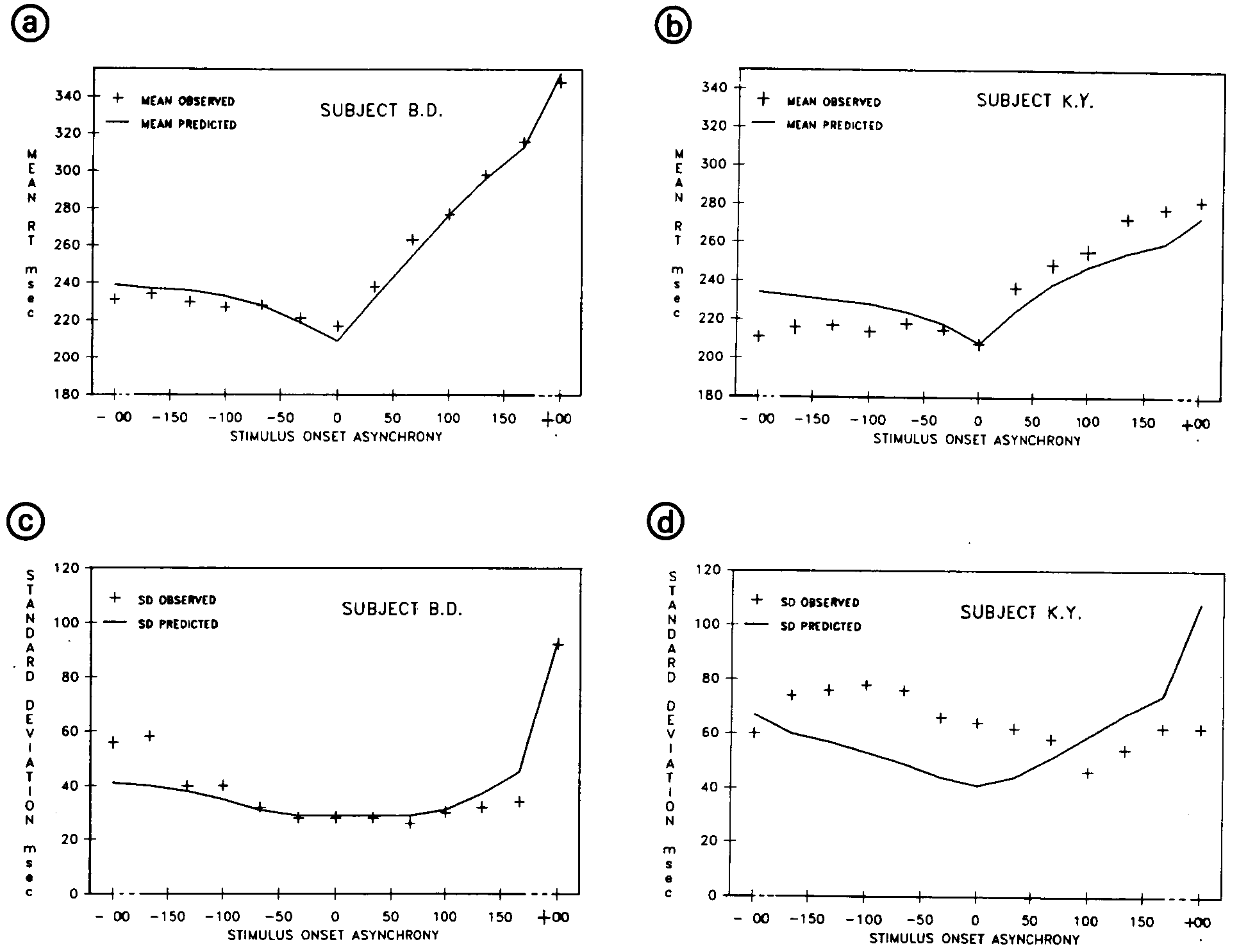

Figure 1. Observed mean RT and standard deviation versus mean RT and standard deviation predicted by the superposition model for Subject B.D. (a, c) and Subject K.Y. (b, d) over all SOAs.

in the above equation must be interchanged if the auditory stimulus is presented τ msec before the visual stimulus.

To test the superposition model against empirical data, the interarrival time distribution must be specified. The most workable case, adopted by Schwarz (1989), is to assume exponentially distributed interarrival times yielding a homogeneous Poisson counting process (see Appendix). For $\tau = 0$, the waiting time for the cth event, T_c, then follows a gamma distribution with mean c/a and variance c/a^2, where a ($a > 0$) is the intensity parameter governing the Poisson process. If the model is to be applied to data with nonzero stimulus onset asynchronies (SOAs), that is, $\tau > 0$, the waiting time for the cth event corresponding to the counting process given in Equation 1 must be considered. Unfortunately, this waiting time does no longer have a simple gamma distribution. Schwarz (1989) presented the distribution of T_c together with its mean. To allow a more stringent test of the superposition model, here we will also derive the expression for the variance of T_c (for the formal presentation, see Appendix).

Finally, the observable reaction time is assumed to be additively composed of the waiting time T_c plus all processes following (or preceding) it. The durations of these additional processes, which may include motor preparation and response execution components, are represented by a random variable, M, say. In general, T_c and M need not be stochastically independent, and for a test of the model at the level of the entire distribution functions, the bivariate distribution for (T_c, M) has to be specified. However, at the level of the expectations, we have

$$E_\tau[\mathrm{RT}] = E_\tau[T_c + M] = E_\tau[T_c] + E_\tau[M],$$

where the subscript τ refers to the distributions of T_c and M with an SOA of τ msec. If T_c and M are stochastically independent, the variance of the observable RT at an SOA of τ msec will be

$$\mathrm{Var}_\tau[\mathrm{RT}] = \mathrm{Var}_\tau[T_c] + \mathrm{Var}_\tau[M]. \tag{2}$$

For the sake of parsimony, in the application below we assume M to be a constant, that is, $M = m$. Equations 1 and 2 then simplify respectively to

$$E_\tau[\mathrm{RT}] = E_\tau[T_c] + m \tag{3a}$$

and

$$\mathrm{Var}_\tau[\mathrm{RT}] = \mathrm{Var}_\tau[T_c]. \tag{3b}$$

This leaves four parameters to be estimated from the data: the residual constant m, the critical count number c, and the intensity parameters α_A and α_V for the auditory and visual stimulus, respectively. With a sufficient number of different SOAs (more than four values of τ, say) the model maintains its predictive power even if only means and standard deviations are considered for testing.

Application to Miller's (1986) Data

Miller (1986) collected RTs in a bimodal (auditory/visual) go/no-go detection task for 2 subjects (for experimental details, we refer to his study). Schwarz (1989) presented a close fit of the expected waiting times of the superposition model (plus a constant for the residual time) to the mean observed RTs of Miller's study. Using Schwarz's parameter estimates, Equation A1 in the appendix allows a test of the model's prediction for the standard deviations as well. Table 1 presents the observed and the predicted *SD*s for both subjects over all SOAs. For convenience, the observed and the predicted means are reproduced as well.

Although the *SD*s predicted by the superposition model roughly reflect their U-shaped dependence on the SOAs, it is obvious that the fit is not satisfactory. In particular, for Subject B.D., the predicted *SD*s are too large for SOAs above 100 msec and too small for SOAs below −100 msec. For Subject K.Y., the predicted *SD*s are far too small over the entire SOA range. One would expect to improve the fit by estimating the model parameters so as to optimize the fit to both the means and the *SD*s simultaneously. The results of an iterative fitting procedure[1] are presented in Figure 1.

Although the observed *SD*s have been accounted for somewhat better now, the fit, especially for large SOAs, is still unsatisfactory. Moreover, the quality of the fit for the means has been reduced substantially. Apparently, there is a tradeoff between fitting the central tendency and fitting the variability in the data. Although a test of the model at the level of the entire RT distribution would be feasible, this outcome of the test at the level of the means and variances obviates it.

Discussion and Conclusion

Our results imply that the version of the superposition model tested here can only partially account for the empirical data. The fit at the level of the means, and thus the account for the redundant-signals effect, was quite successful. On the other hand, the variability in the data has not been adequately described by the model. It should be noted, however, that this result does not rule out the superposition concept as such. First, it is conceivable that versions of the superposition model using other interarrival time distributions, as discussed in Schwarz (1989), would account better for the variability. Second, the assumption of a constant residual component, $M = m$, seems unrealistic in view of the fact that the magnitude of the residual component M amounts to more than 50% of the total RT. Unfortunately, the contribution of the residual processes to the variability seems to be dependent on the SOA. Consequently, simply increasing or decreasing the predicted *SD*s by a constant amount (i.e., the *SD* of M) does not improve the fit measured over all SOAs.[2] To avoid a proliferation of parameters, the functional dependence of the variance of M on the SOA needs to be specified. One way to achieve this is to drop the assumption of independence between T_c and M or, more generally, to make an assumption about the bivariate distribution of (T_c, M). We are currently exploring several alternative approaches on larger data sets.[3]

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NOTES

1. Routine "OPTMUM" of the GAUSS programming language with an unweighted sum-of-squares objective function for the means and standard deviations was used. The parameter estimates are $\alpha_V = 0.0240$, $\alpha_A = 0.0539$, $c = 5$, $m = 145$ for B.D., and $\alpha_V = 0.0093$, $\alpha_A = 0.0149$, $c = 1$, $m = 167$ for K.Y.

2. Including an additional parameter for the variance of M in the iterative fitting procedure did not yield an improved fit.

3. An interesting suggestion has recently been made by W. Schwarz (personal communication, December 21, 1990). He proposes to have the motor time depend on the relative contribution of the auditory and visual channels to the critical count c.

APPENDIX

Assuming Poisson counters, the distribution for the waiting time T_c in the double-stimulus condition with SOA $= \tau$ corresponding to Equation 1 is (cf. Schwarz, 1989):

$$P_\tau[T_c \leq t] = \begin{cases} 1-\exp(-\alpha_V t)\sum_{i=0}^{c-1}\dfrac{(\alpha_V t)^i}{i!} & \text{if} \quad 0 \leq t \leq \tau \\ 1-\exp\{-[(\alpha_V+\alpha_A)t-\alpha_A\tau]\}\sum_{i=0}^{c-1}\dfrac{[(\alpha_V+\alpha_A)t-\alpha_A\tau]^i}{i!} & \text{if} \quad \tau \leq t \end{cases}$$

The expected value for the waiting time is

$$E_\tau[T_c] = \frac{c}{\alpha_V} - \frac{\alpha_A}{\alpha_V(\alpha_V+\alpha_A)}\exp(-\alpha_V\tau)\sum_{i=0}^{c-1}\frac{(\alpha_V\tau)^i}{i!}(c-i),$$

while its variance is

$$\mathrm{Var}_\tau[T_c] = E_\tau[T_c^2] - E_\tau[T_c]^2,$$

where

$$E_\tau[T_c^2] = \frac{c(c+1)}{\alpha_V^2} - 2\exp(-\alpha_V\tau)\sum_{i=0}^{c-1}\frac{(\alpha_V\tau)^i}{i!}(c-i)\left\{\frac{1}{\alpha_V^2}\left(1+\alpha_V\tau+\frac{c-1-i}{2}\right) - \frac{1}{(\alpha_V+\alpha_A)^2}\left[1+(\alpha_V+\alpha_A)\tau+\frac{c-1-i}{2}\right]\right\}. \quad \text{(A1)}$$

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