A Fuzzy Analytic Hierarchy Process Model For Supplier Selection And A Case Study

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Abstract - In today's competitive manufacturing and service industries decision making is a critical process. Supply chain management is a network of businesses and in this network there are several critical decision making problems. One of them is supplier selection decision. Supplier selection is a multi-criteria decision making problem and a fuzzy decision making model is proposed to this problem area in supply chain management. The extent analysis method and integral value calculation is used in the study for computing the priority weights of criteria and alternatives. In addition, a case study is added to the study.

Keywords: **Supplier Selection, Fuzzy Analytic Hierarchy Process**

I.INTRODUCTION

Supply chains are value-adding chains of sub-suppliers, suppliers, manufacturers, distributors and end customers, where the success of each participant is to certain degrees dependent on the supply chain as a whole.

The management of supply chains is concerned with building the most optimal supply chains [1]. One of the most important problem of the SCM is the selection of suppliers. Supplier selection is a multi-criteria decision making problem [2]. Optimal supply chain is the one that is optimal in all aspects of SCM. In order to reach optimal supply chains or during the journey of reaching optimality several techniques have been used for several subjects of SCM. In this study, a fuzzy AHP model will be designed for the problem of supplier selection in SCM. The reason why fuzzy AHP is used in supplier selection decision is that the decision making environment in such a system is so complex and with the help of fuzzy logic approach the model will represent the real life case more accurately.

II. LITERATURE SURVEY ON SUPPLIER SELECTION PROCESS

Supplier selection is a multi-criteria decision-making problem which consists of both qualitative and quantitative metrics. A lot of investigations have been published in the supplier selection area and it has been notified that in the majority of these publications supplier selection and evaluation and development have the same meaning.

However, one needs integrated models to cover all of these stages. In addition, most of the proposed models focused on manufacturing environments and a few papers have been allocated for service industries [3].

There is also a list of criteria for supplier selection that can be seen in Table 1 given by Guneri et al. 2009 [4].

There is also a detailed literature study on multi-criteria decision making approaches for supplier evaluation and selection. The interested readers are referred to the study given by Ho et al. [5].

There are several solution approaches to the supplier selection problem in the literature some of which are Analytic Hierarchy Process, Fuzzy Analytic Hierarchy Process, Data Envelopment Analysis, Mixed Integer Programming, TOPSIS, Fuzzy TOPSIS, QFD, Fuzzy QFD, Analytic Network Process and Expert Systems.

The extent analysis method of Chang [6] is the technique that is widely used in the literature for fuzzy AHP problems. However, Wang et al. [7] state that they found that the extent analysis method cannot estimate the true final weights from a fuzzy comparison matrix and has led to quite a number of misapplications in the literature. Therefore in this study, after calculating the synthetic values with extent analysis method of Chang, the final weights are calculated from weighted index values of integral values of Liou and Wang [8]. In the next sections the steps of fuzzy AHP approach is given and these steps are explained in a case study of a shoe manufacturing firm with using the mostly used criteria of supplier selection in the literature.

III. A FUZZY AHP MODEL FOR SUPPLIER SELECTION PROCESS

In classical AHP directly the numerical values of linguistic variables are used for evaluation of criteria. If the environment where the decision making process takes place is fuzzy, then fuzzy numbers are used for evaluation concerning some deviations of decision makers. Nowadays, especially in complex economic conditions, many of the decisions are made in such an environment. Thus, fuzzy version of AHP or similar method should be used in spite of its complexity during the calculations [9].

TABLE I
LIST OF CRITERIA USED IN SUPPLIER SELECTION PROBLEM

Criteria	**Dickson**	**Lehmann et.al**	**Abratt**	**Weber et. al**	**Min et. al**	**Stravropolous**	**Ghodsypour et al.**	**Chan et. al**	**Chen et.al**	**Lin et. al**
Price	X	X	X	X	X	X	X	X		
Quality	X			X	X		X	X	X	
Delivery	X	X		X	X		X	X		
Warranties and Claims	X	X								
After Sales Service	X	X	X	X				X		
Technical Support		X	X					X		
Training Aids	X	X		X						
Attitude	X		X	X						
Performance History	X			X				X		
Financial Position	X	X		X				X	X	
Geographical Location	X			X				X		
Management and Organization	X			X						
Labor Relations	X			X						
Communication System	X			X				X		X
Response to Customer Request		X						X		
E-commerce Capability					X	X				
JIT Capability										
Technical Capability	X			X					X	
Production Facilities and Capacity	X			X			X	X		
Packaging Ability	X			X						
Operational Controls	X			X						
Ease-of-Use		X	X							
Maintainability		X	X							
Amount of Past Business	X	X		X						
Reputation and position in industry	X	X	X	X				X		X
Reciprocal Arrangements	X			X						
Impression	X	X	X	X						
Environmentally Friendly Products					X					
Product Appearance						X				
Catalog Technology						X				
Relationship Closeness									X	X
Conflict Resolution									X	X
Political Stability								X		
Economy								X		
Terrorism								X		

Within the scope of this study, a fuzzy AHP model will be designed for supplier selection in Supply Chain Management. The solution procedure of the fuzzy AHP approach involves six essential steps as follows:

Step 1. Define the problem and state clearly the objectives and results
Step 2. Decompose the complex problem into a hierarchical structure with decision elements (criteria and alternatives)
Step 3. Employ pair-wise comparisons among decision elements and form comparison matrices with fuzzy numbers
Step 4. Use the extent analysis method to estimate the relative weights of the decision elements (The defuzzification process is carried out here.)

The steps of Chang's [6] extent analysis can be given as in the following (with triangular fuzzy numbers)**:**
*1***:** The value of fuzzy synthetic extent with respect to the i[th] object is defined as:

$$S_i = \sum_{j=1}^{m} M_{gi}^{j} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} \qquad (1)$$

To obtain $\sum_{j=1}^{m} M_{gi}^{j}$, perform the fuzzy addition operation of m extent analysis values for a particular matrix such that:

$$\sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right)$$

and to obtain $\left[\sum_{j=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1}$, perform the fuzzy addition operation of M^j_{gi} (j = 1, 2, ...,m) values such that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i \right) \qquad (3)$$

and then compute the inverse of the vector above, such that:

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right) \qquad (4)$$

*2***:** As $\widetilde{M}_1 = (l_1, m_1, u_1)$ and $\widetilde{M}_2 = (l_2, m_2, u_2)$ are two triangular fuzzy numbers, the degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ defined as:

$$V\left(\widetilde{M}_2 \geq \widetilde{M}_1\right) = \sup_{y \geq x} \left[\min\left(\mu_{\widetilde{M}_1}(x), \mu_{\widetilde{M}_2}(y) \right) \right] \qquad (5)$$

and can be equivalently expressed as follows:

$$V\left(\widetilde{M}_2 \geq \widetilde{M}_1\right) = hgt(\widetilde{M}_1 \cap \widetilde{M}_2) = \mu_{M_2}(d)$$

$$= \begin{cases} 1, & if \quad m_2 \geq m_1 \\ 0, & if \quad l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & otherwise \end{cases} \qquad (6)$$

According to fuzzy sets theory [10], the representation of a fuzzy triangular number is given in Figure 1.In Figure 1, d is the ordinate of the highest intersection point D between μ_{M_1} and μ_{M_2} to compare M1 and M2, we need both values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$.

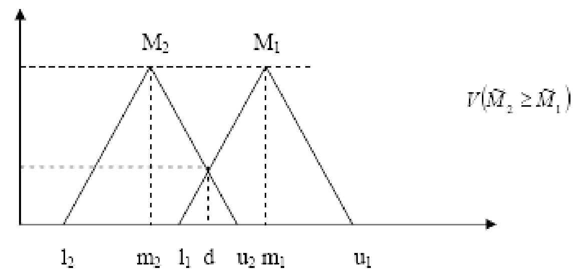

Fig.1. The intersection between M_1 and M_2

*3***:** The degree possibility for a convex fuzzy number to be greater than k convex fuzzy M_i (i=1, 2, k) numbers can be defined by

$$V(M \geq M_1, M_2, \ldots. M_k) = V[(M \geq M_1) \ and \ (M \geq M_2) \ and \ldots and \ (M \geq M_k)]$$
$$= \min \ V(M \geq M_i), \ i = 1,2,3,\ldots.,k \qquad (7)$$

Assume that $d(A_i) = \min V(S_i \geq S_k)$ for $k = 1,2,\ldots.,n; k \neq i$. Then the weight vector is given by

$$W' = (d'(A_1), d'(A_2), \ldots\ldots, d'(A_n))^T \qquad (8)$$

where $A_i = (i = 1,2,\ldots n)$ are n elements.

*4***:** Via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2), \ldots\ldots, d(A_n))^T \qquad (9)$$

where W is a non-fuzzy number.

Step 5. Check the consistency of matrices to ensure that the judgments of decision makers are consistent.

Step 6. Aggregate the relative weights of decision elements to obtain an overall rating for the alternatives.

IV. APPLICATION OF THE FUZZY AHP MODEL IN SUPPLIER SELECTION

Step 1. The problem is selecting the best supplier among 2 alternative suppliers which are A, and B, raw material suppliers of a shoe manufacturing firm. The best alternative supplier must be selected according to 4 criteria which are four most addressed criteria in the literature by Guneri et al. [4]: cost, reputation, quality and delivery time.

Step 2. The hierarchical structure of the problem is given in Figure 2:

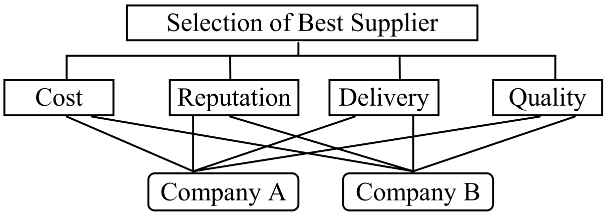

Fig.2. Hierarchical structure of the problem

Step 3. The Saaty's scale of comparisons in a multi-criteria decision making area as in this study is given in Table 2:

TABLE II
SAATY'S SCALE [11]

Intensity of Importance	Definition	Explanation
1	*Equal Importance*	*Two activities contribute equally to the objective*
3	*Moderate Importance*	*Experience and judgment slightly favor one activity over another*
5	*Strong Importance*	*Experience and judgment strongly favor one activity over another*
7	*Very Strong Importance*	*An activity is favored very strongly over another; its dominance demonstrated in practice.*
9	*Extreme Importance*	*The evidence favoring one activity over another is of the highest possible order of affirmation*
2, 4, 6, 8	*For compromise between the above values*	*Sometimes one needs to interpolate a compromise judgment numerically because there is no good word to describe it.*

In order to compare the criteria, and alternatives with each other according to criteria the fuzzy scale is given in Table 3.

TABLE III
THE FUZZY SCALE

VL	*Just Equal*	*(1, 1, 1)*
L	*Low*	*(1, 2, 3)*
ML	*Medium Low*	*(2, 3, 4)*
MH	*Medium High*	*(3, 4, 5)*
H	*High*	*(4, 5, 6)*
VH	*Very High*	*(5, 6, 7)*

The comparison matrix of criteria with fuzzy numbers is given in Table 4 below:

TABLE IV
THE COMPARISON MATRIX OF CRITERIA

	Cost	**Reputation**	**Delivery**	**Quality**
Cost	*(1,1,1)*	*(2,3,4)*	*(3,4,5)*	*(3,4,5)*
Reputation	*(0.25,0.33,0.50)*	*(1,1,1)*	*(0.20,0.25,0.33)*	*(2,3,4)*
Delivery	*(0.20,0.25,0.33)*	*(3,4,5)*	*(1,1,1)*	*(5,6,7)*
Quality	*(0.20,0.25,0.33)*	*(0.25,0.33,0.50)*	*(0.14,0.17,0.20)*	*(1,1,1)*

The comparison matrices of each alternative according to each criteria are given in Table 5, 6, 7 and 8 respectively:

TABLE V
THE COMPARISON OF ALTERNATIVES
A AND B WITH RESPECT TO COST

	A	**B**
A	*(1,1,1)*	*(0.33, 0.50, 0.33)*
B	*(1,2,3)*	*(1,1,1)*

TABLE VI
THE COMPARISON OF ALTERNATIVES
A AND B WITH RESPECT TO REPUTATION

	A	**B**
A	*(1,1,1)*	*(0.17,0.2,0.25)*
B	*(4,5,6)*	*(1,1,1)*

TABLE VII
THE COMPARISON OF ALTERNATIVES
A AND B WITH RESPECT TO DELIVERY TIME

	A	**B**
A	*(1,1,1)*	*(0.25,0.33,0.50)*
B	*(2,3,4)*	*(1,1,1)*

TABLE VIII
THE COMPARISON OF ALTERNATIVES
A AND B WITH RESPECT TO QUALITY

	A	**B**
A	*(1,1,1)*	*(5,6,7)*
B	*(0.14,0.17,0.20)*	*(1,1,1)*

Step 4. The synthetic values of each criteria according to α cut level = 0.5:
S_{cost} = (0.249, 0.406, 0.645), $S_{reputation}$ = (0.095, 0.155, 0.251), $S_{delivery}$ = (0.254, 0.380, 0.574) and $S_{quality}$ =(0.044, 0.060, 0.087)

Integral Value = I = $\frac{1}{2} * [\alpha * c + b + (1 - \alpha) * a]$ Here, a, b and c are the lower, medium and upper values of fuzzy synthetic values [8].

The integral values for each criteria:
I_{cost} = 0.427, $I_{reputation}$ = 0.164, $I_{delivery}$ = 0.397, $I_{quality}$ = 0.063

Total index value = 1.051

Normalized W = (0.427/1.051, 0.164/1.051, 0.397/1.051, 0.063/1.051)

W = (0.406, 0.156, 0.378, 0.060)

Final weights of each criteria:
$W_{cost} = 0.406$, $W_{reputation} = 0.156$, $W_{delivery} = 0.378$ and $W_{quality} = 0.060$

Final weights of alternatives A and B with respect to cost, reputation, quality and delivery:
For Table 5: $W_A = 0.701$, $W_B = 0.299$
For Table 6: $W_A = 0.166$, $W_B = 0.834$
For Table 7: $W_A = 0.750$, $W_B = 0.250$
For Table 8: $W_A = 0.858$, $W_B = 0.142$

Step 5. All the critical ratio for matrices are below 0.1, so all of them are consistent.

Step 6. Final Weight of Supplier A (FW_A) = 0.406*0.701 + 0.156*0.166 + 0.378*0.750 + 0.060*0.858 = **0.645**
Final Weight of Supplier B (FW_B) = 0.406*0.299 + 0.156*0.834 + 0.378*0.250 + 0.060*0.142 = **0.355**

Figure 3 is a graphical representation of the results. Alternative A, that is superior to B should be chosen as the best alternative.

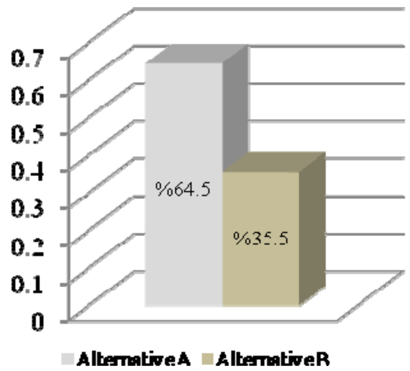

Fig.3 Comparison of alternatives

V. CONCLUSION

Supplier selection is an important problem or area in SCM. There several approaches to this problem. One of them, fuzzy AHP technique that is used in this study for this problem. By the integration of integral value calculation and the extent fuzzy approach, the ambiguities involved in the data could be effectively represented and processed to make a more effective decision.

This approach can produce beneficial results in other sectors or in other application areas. The comparison of results with an expert system model's results will be conducted as a future study. For the confidentiality of data the names of the company and the suppliers are not given in this study. In the case study, alternative A is determined as the best supplier which has the highest priority according to final weights.

VI. REFERENCES

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