A Fuzzy Description Logic for Multimedia Knowledge Representation

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Abstract. Today quite a lot of multimedia systems and applications use knowledge representation formalisms to encode and reason with knowledge that exists within the multimedia documents. The goal of this direction is to narrow the semantic gab between the content of a multimedia object, as perceived by a human being, and as "viewed" by an information system. Ontologies are quite often used to capture such a knowledge. Ontology languages are based on Description Logics, which though they are expressive enough, they lack the ability to encode and reason with imprecise knowledge. To this end we extend the DL language $\mathcal{SI}$ with fuzzy set theory and provide sound and complete reasoning algorithms for the extended language.

1 Introduction

Multimedia processing, like image processing or multimedia information retrieval, is an inherently difficult task. This is due to the well known problem of the semantic gap [ZG02] between human perception of entities, that exists within a multimedia document, and the computer perception of meaningless values of pixels. To bridge the semantic gap numerous approaches have been proposed which use knowledge based systems and logical formalisms in order to encode human knowledge within information systems [ABS03,BSC00]. This knowledge can be used later by the system to produce "intelligent" answers, regarding the retrieval of multimedia documents, or assist during the analysis of images or video sequences.

Many of these approaches use the concept of an ontology, in order to encode and reason with the knowledge that exists within multimedia objects. Today, in the semantic web context, quite a lot of ontology languages exist, like the OWL [BvHH⁺04] and DAML+OIL [HPS01]languages. Both these languages use Description Logics (DLs) as their underlying formalism for the representation of knowledge as well as for performing reasoning tasks. More precisely they are almost equivalent to the $\mathcal{SHIF}(\mathbf{D}^+)$ and $\mathcal{SHOIN}(\mathbf{D}^+)$ DLs [HPS03]. Though DLs are very expressive formalisms they feature expressive limitations, regarding their ability to capture and reason about vague and imprecise information. Such types of uncertainties are apparent when dealing with multimedia applications, like retrieval and processing. Even more, several approaches that enhance image processing algorithms by dealing with imprecision have been employed in the past [KK92].

In the current paper we extend the DL language $\mathcal{SI}$ [HS99] with fuzzy set theory [KY95]. $\mathcal{SI}$ extends the well known DL $\mathcal{ALC}$ [SSS91] with transitive and inverse roles. As pointed in literature [HS99], such types of roles is crucial to represent aggregated objects and part-whole relations. Such a logic can assist the detection of composite objects in multimedia processing applications. Though $\mathcal{SI}$ is less expressive than $\mathcal{SHIF}(\mathbf{D}^+)$ and $\mathcal{SHOIN}(\mathbf{D}^+)$ it is the first difficult step towards extending such complex DLs, mainly due to the effects of transitivity [HS99].

2 Syntax and Semantics of f-$\mathcal{SI}$

In fuzzy $\mathcal{SI}$, f-$\mathcal{SI}$ for short [3], we are dealing with transitive and inverse roles. The set of transitive roles $\mathbf{R}_+$ is a subset of the set of roles $\mathbf{R}$. In addition, for any role $R \in \mathbf{R}$, the role R^- is interpreted as the inverse of R. Similarly to [HS99] we introduce two functions. The first one is the function Inv which given a role R it returns its inverse, R^-, and given an inverse role, R^-, it returns the role R. At last, for transitive roles $R \in \mathbf{R}_+$ we define the function $\mathsf{Trans}(R)$ which returns *true* iff $R \in \mathbf{R}_+$ or $\mathsf{Inv}(R) \in \mathbf{R}_+$. Complex f-$\mathcal{SI}$ concepts are defined by the following syntax rule:

$$C, D \longrightarrow \top | \bot | A | \neg C | C \sqcup D | C \sqcap D | \exists R.C | \forall R.C$$

A *terminology*, or *TBox*, is defined by a finite set of *fuzzy concept inclusion* axioms of the form $A \sqsubseteq C$ and *fuzzy concept equalities* of the form $A \equiv C$. Observe that C represents an arbitrary concept, while A an atomic one. This is because dealing with *general terminologies* [HS99] still remains an open problem in fuzzy concept languages.

Let $\mathbf{I} = \{a, b, c, ...\}$ be a set of individual names. A *fuzzy assertion* [Str01] is of the form $\langle a : C \bowtie n \rangle$ or $\langle (a, b) : R \bowtie n \rangle$, where $\bowtie$ stands for $\geq, >, \leq$ and $<$. We call assertions defined by $\geq, >$ *positive* assertions, while those defined by $\leq, <$ *negative* assertions. A finite set of fuzzy assertions defines a fuzzy *ABox* $\mathcal{A}$. In [Str01] the concept of conjugated pairs of fuzzy assertions has been introduced, in order to represent pairs of assertions that form a contradiction. The possible conjugated pairs are defined in table 1, where ϕ represents a concept expression.

A fuzzy set $C \subseteq X$ is defined by its *membership function* (μ_C), which given an object of the universal set X it returns the membership degree of that object

[3] In a previous approach to fuzzy DLs the prefix μ is used, but this letter is reserved by DLs with *fixed point constructors* [BMNPS02]. In some other approaches the naming $\mathcal{ALC}_F$ is used but this can easily be confused with $\mathcal{ALCF}$ ($\mathcal{ALC}$ plus *functional restrictions*[HS99]), when pronounced.

	$\langle \phi < m \rangle$	$\langle \phi \leq m \rangle$
$\langle \phi \geq n \rangle$	$n \geq m$	$n > m$
$\langle \phi > n \rangle$	$n \geq m$	$n \geq m$

Table 1. Conjugated pairs of fuzzy assertions

$$
\begin{array}{rcl}
\top^{\mathcal{I}}(a) & = & 1 \\
\bot^{\mathcal{I}}(a) & = & 0 \\
(\neg C)^{\mathcal{I}}(a) & = & 1 - C^{\mathcal{I}}(a) \\
(C \sqcup D)^{\mathcal{I}}(a) & = & max(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a)) \\
(C \sqcap D)^{\mathcal{I}}(a) & = & min(C^{\mathcal{I}}(a), D^{\mathcal{I}}(a)) \\
(\forall R.C)^{\mathcal{I}}(a) & = & \inf_{b \in \Delta^{\mathcal{I}}}\{max(1 - R^{\mathcal{I}}(a,b), C^{\mathcal{I}}(b))\} \\
(\exists R.C)^{\mathcal{I}}(a) & = & \sup_{b \in \Delta^{\mathcal{I}}}\{min(R^{\mathcal{I}}(a,b), C^{\mathcal{I}}(b))\} \\
(R^{-})^{\mathcal{I}}(b,a) & = & R^{\mathcal{I}}(a,b) \\
\text{For } R \in \mathbf{R}_{+} & & R^{\mathcal{I}}(a,c) \geq sup_{b \in \Delta^{\mathcal{I}}}\{min(R^{\mathcal{I}}(a,b), R^{\mathcal{I}}(b,c))\}
\end{array}
$$

Table 2. Semantics of $\mathcal{SI}$-concepts

to the fuzzy set. By using membership functions we can extend the notion of an *interpretation function*[BMNPS02] to that of a *fuzzy interpretation*. More formally a fuzzy interpretation $\mathcal{I}$ consists of a pair $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is the domain of interpretation, as in the classical case, and $\cdot^{\mathcal{I}}$ is an interpretation function which maps concepts (roles) to a membership function $\Delta^{\mathcal{I}} \longrightarrow [0,1]$ $(\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \longrightarrow [0,1])$, which defines the fuzzy subset $C^{\mathcal{I}}$ $(R^{\mathcal{I}})$. For example if $a \in \Delta^{\mathcal{I}}$ then $A^{\mathcal{I}}(a)$ gives the degree that the object a belongs to the fuzzy concept A, e.g. $A^{\mathcal{I}}(a) = 0.8$.

In order to extend a fuzzy interpretation to cover arbitrary concepts, created by the syntax rule, we have to interpret the concept forming operators $(\neg, \sqcup, \sqcap, \exists, \forall)$. As in previous approaches to fuzzy DLs [Str01,HKS02,ST04] we use the standard operations to interpret the above concept constructors. These are the Lucasiewicz negation $(c(a) = 1-a)$, the Gödel t-norm $(t(a,b) = min(a,b))$, the Gödel t-conorm $(u(a,b) = max(a,b))$, the Kleen-Dienes fuzzy implication $(\mathcal{J}(a,b) = max(1-a,b))$ and the supremum and infimum for the existential and universal quantifiers. We refer to the language created by the above operations as f$_{KD}$-$\mathcal{SI}$ after the initials of the name of the fuzzy implication. The semantics of f$_{KD}$-$\mathcal{SI}$ are depicted in table 2.

A fuzzy concept C is *satisfiable* iff there exists some fuzzy interpretation $\mathcal{I}$ for which there is some $a \in \Delta^{\mathcal{I}}$ such that $C^{\mathcal{I}}(a) = n$, and $n \in (0,1]$. A fuzzy interpretation $\mathcal{I}$ satisfies a *TBox* $\mathcal{T}$ iff $\forall a \in \Delta^{\mathcal{I}} A^{\mathcal{I}}(a) \leq D^{\mathcal{I}}(a)$, for each $A \sqsubseteq C$, and $\forall a \in \Delta^{\mathcal{I}} A^{\mathcal{I}}(a) = D^{\mathcal{I}}(a)$, for each $A \equiv C$.

Fuzzy interpretations are also extended to interpret individuals and assertions that appear in an *ABox*. For a fuzzy *ABox*, an interpretation maps, additionally, each individual $a \in \mathbf{I}$ to some element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. An interpretation $\mathcal{I}$ satisfies a fuzzy assertion

$$\langle a : C \geq n \rangle \quad \text{iff} \quad C^{\mathcal{I}}(a^{\mathcal{I}}) \geq n,$$
$$\langle (a, b) : R \geq n \rangle \quad \text{iff} \quad R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \geq n$$

The satisfiability of fuzzy assertions with $\leq, >$ and $<$ is defined analogously.

A fuzzy *ABox* $\mathcal{A}$ is *consistent* iff there exists an interpretation $\mathcal{I}$ that satisfies each fuzzy assertion in the fuzzy *ABox*. We then say that $\mathcal{I}$ is a *model* of $\mathcal{A}$. The *entailment* and *subsumption* problems can be reduced to *ABox* consistency as shown in [Str01].

3 A Fuzzy Tableau for $\mathbf{f}_{KD}$-$\mathcal{SI}$

Consistency of an *ABox* $\mathcal{A}$ can be checked with tableaux algorithms that try to prove the satisfiability of an assertion by constructing a model for it [HST00]. This is accomplished by providing a set of decomposition rules which unfold the possibly complex concept expressions appearing in $\mathcal{A}$. The model is represented by a so-called *completion-forest*, a collection of *completion-trees* some of whose nodes correspond to individuals in the model, each node being labelled with a set of triples of the form $\langle D, \bowtie, n \rangle$ which denote the type, the concept and the membership degree that the individual of the node has been asserted to belong to concept D. As for fuzzy assertions, also when we are dealing with triples of a single node, the concepts of conjugated, positive and negative triples can be defined in the obvious way. Since expansion rules decompose the complex concepts, the concepts that appear in triples are subconcepts of the initial concept. Subconcepts of a concept D are denoted by $sub(D)$. Hence, the set of all subconcepts that appear within an *ABox* is denoted by $sub(\mathcal{A})$.

The operators that we used for the semantics of the language satisfy the De'Morgan laws. Thus, the *negation normal form* (NNF) [BMNPS02] of a concept can be produced. Hence, we assume that all concepts are in their NNF form.

In the present paper we will extend the notions of a *tableau* for an *ABox* $\mathcal{A}$ [HST00], to a *fuzzy tableau.* In the following we use the symbols $\rhd$ and $\lhd$ as a placeholder for the inequalities $\geq, >$ and $\leq, <$ and the symbol $\bowtie$ as a placeholder for all types of inequations. Furthermore we use the symbols $\bowtie^-$, $\rhd^-$ and $\lhd^-$ to denote their *reflections*. For example the reflection of $\leq$ is $\geq$ and that of $>$ is $<$.

Definition 1. *If $\mathcal{A}$ is an f_{KD}-$\mathcal{SI}$ ABox, $\boldsymbol{R}_A$ is the set of roles occurring in $\mathcal{A}$ together with their inverses, $\mathbf{I}_A$ is the set of individuals in $\mathcal{A}$ and $\mathcal{X}$ is the set $\{\geq, >, \leq, <\}$. A fuzzy tableau T for $\mathcal{A}$ is defined to be a quadruple ($\boldsymbol{S}$, $\mathcal{L}$, $\mathcal{E}$, $\mathcal{V}$) such that: $\boldsymbol{S}$ is a set of individuals, $\mathcal{L} : \boldsymbol{S} \to 2^{sub(A)} \times \mathcal{X} \times [0, 1]$ maps each individual to a set of triples which denote the membership degree and the type of assertion of each individual to a concept that is a subset of $sub(A)$, $\mathcal{E} : \boldsymbol{R}_A \to 2^{\boldsymbol{S} \times \boldsymbol{S}} \times \mathcal{X} \times [0, 1]$ maps each role to a set of triples which denote the membership degree and the type of assertion of a pair of individuals to the role in $\boldsymbol{R}_A$, and $\mathcal{V} : \mathbf{I}_{\mathcal{A}} \to \boldsymbol{S}$ maps individuals occurring in $\mathcal{A}$ to elements in $\boldsymbol{S}$. For all $s, t \in \boldsymbol{S}$, $C, E \in sub(\mathcal{A})$, and $R \in \boldsymbol{R}_A$, T satisfies:*

1. *If $\langle \neg C, \bowtie, n \rangle \in \mathcal{L}(s)$, then $\langle C, \bowtie^-, 1 - n \rangle \in \mathcal{L}(s)$,*

2. *If* $\langle C \sqcap E, \geq, n\rangle \in \mathcal{L}(s)$ *then* $\langle C, \geq, n\rangle \in \mathcal{L}(s)$ *and* $\langle E, \geq, n\rangle \in \mathcal{L}(s)$*,*
3. *If* $\langle C \sqcup E, \leq, n\rangle \in \mathcal{L}(s)$ *then* $\langle C, \leq, n\rangle \in \mathcal{L}(s)$ *and* $\langle E, \leq, n\rangle \in \mathcal{L}(s)$*,*
4. *If* $\langle C \sqcup E, \geq, n\rangle \in \mathcal{L}(s)$ *then* $\langle C, \geq, n\rangle \in \mathcal{L}(s)$ *or* $\langle E, \geq, n\rangle \in \mathcal{L}(s)$*,*
5. *If* $\langle C \sqcap E, \leq, n\rangle \in \mathcal{L}(s)$ *then* $\langle C, \leq, n\rangle \in \mathcal{L}(s)$ *or* $\langle E, \leq, n\rangle \in \mathcal{L}(s)$*,*
6. *If* $\langle \forall R.C, \geq, n\rangle \in \mathcal{L}(s)$ *and there exists a triple* $\langle\langle s,t\rangle, \rhd, n_1\rangle \in \mathcal{E}(R)$ *which is conjugated with* $\langle\langle s,t\rangle, \leq, 1-n\rangle$ *then,* $\langle C, \geq, n\rangle \in \mathcal{L}(t)$*,*
7. *If* $\langle \exists R.C, \leq, n\rangle \in \mathcal{L}(s)$ *and there exists a triple* $\langle\langle s,t\rangle, \rhd, n_1\rangle \in \mathcal{E}(R)$ *which is conjugated with* $\langle\langle s,t\rangle, \leq, n\rangle$ *then,* $\langle C, \leq, n\rangle \in \mathcal{L}(t)$*,*
8. *If* $\langle \exists R.C, \geq, n\rangle \in \mathcal{L}(s)$*, then there exists* $t \in \boldsymbol{S}$ *such that* $\langle\langle s,t\rangle, \geq, n\rangle \in \mathcal{E}(R)$ *and* $\langle C, \geq, n\rangle \in \mathcal{L}(t)$*,*
9. *If* $\langle \forall R.C, \leq, n\rangle \in \mathcal{L}(s)$*, then there exists* $t \in \boldsymbol{S}$ *such that* $\langle\langle s,t\rangle, \geq, 1-n\rangle \in \mathcal{E}(R)$ *and* $\langle C, \leq, n\rangle \in \mathcal{L}(t)$*,*
10. *If* $\langle \exists R.C, \leq, n\rangle \in \mathcal{L}(s)$*,* $\mathsf{Trans}(R)$ *and there exists a triple* $\langle\langle s,t\rangle, \rhd, n_1\rangle \in \mathcal{E}(R)$ *which is conjugated with* $\langle\langle s,t\rangle, \leq, n\rangle$ *then,* $\langle \exists R.C, \leq, n\rangle \in \mathcal{L}(t)$*,*
11. *If* $\langle \forall R.C, \geq, n\rangle \in \mathcal{L}(s)$*,* $\mathsf{Trans}(R)$ *and there exists a triple* $\langle\langle s,t\rangle, \rhd, n_1\rangle \in \mathcal{E}(R)$ *which is conjugated with* $\langle\langle s,t\rangle, \leq, 1-n\rangle$ *then,* $\langle \forall R.C, \geq, n\rangle \in \mathcal{L}(t)$*,*
12. $\langle\langle s,t\rangle, \bowtie, n\rangle \in \mathcal{E}(R)$ *iff* $\langle\langle t,s\rangle, \bowtie, n\rangle \in \mathcal{E}(\mathsf{Inv}(R))$*,*
13. *There do not exist two conjugated triples in any set of triples for any individual* $x \in \boldsymbol{S}$*,*
14. *If* $\langle a : C \bowtie n\rangle \in \mathcal{A}$*, then* $\langle C, \bowtie, n\rangle \in \mathcal{L}(\mathcal{V}(a))$*,*
15. *If* $\langle (a,b) : R \bowtie n\rangle \in \mathcal{A}$*, then* $\langle\langle \mathcal{V}(a), \mathcal{V}(b)\rangle, \bowtie, n\rangle \in \mathcal{E}(R)$

Analogous properties apply if we substitute $\geq$ by $>$ and $\leq$ by $<$.

Lemma 1. *A fuzzy* $\mathcal{SI}$*-ABox* $\mathcal{A}$ *is consistent iff there exists a fuzzy tableau for* $\mathcal{A}$*.*

3.1 An algorithm for constructing an f_{KD}-$\mathcal{SI}$ Fuzzy Tableau

As it is obvious in order to decide *ABox* consistency a procedure that constructs a fuzzy tableau for an f_{KD}-$\mathcal{SI}$ *ABox* has to be determined. In the current section we will provide the technical details for constructing a correct tableaux algorithm.

As pointed in [HST00] algorithms that decide consistency of an *ABox* work on *completion-forests* rather than on *completion-trees*. This is because an *ABox* might contain several individuals with arbitrary roles connecting them. Such a forest is a collection of trees that correspond to the individuals in the *ABox*.

The nodes of the forest correspond to the individuals that have been generated in order to satisfy positive and negative existential and value restrictions, respectively, and the edges between two nodes, to the relations that connect two individuals. Nodes are labelled with a set of triples $\mathcal{L}(x)$ (*node triples*), which contain concepts that are subsets of $sub(\mathcal{A})$, augmented with the membership degree and the type of assertion that the node belongs to the specific concept. More precisely we define $\mathcal{L}(x) = \{\langle C_i, \bowtie, n_i\rangle\}$, where $C \in subA$, $\bowtie \in \{\geq, >, \leq, <\}$ and $n_i \in [0,1]$. Furthermore, edges $\langle x,y\rangle$ are labelled with a set $\mathcal{L}(\langle x,y\rangle)$ (*edge triples*) defined as, $\mathcal{L}(\langle x,y\rangle) = \{\langle R, \bowtie, n\rangle\}$, where $R \in \mathbf{R}_A$. The algorithm expands each tree either by expanding the set $\mathcal{L}(x)$, of a node x with new triples, or by adding new leaf nodes.

Rule	Description
$(\neg)$	if 1. $\langle \neg C, \bowtie, n\rangle \in \mathcal{L}(x)$ 2. and $\langle C, \bowtie^-, 1-n\rangle \notin \mathcal{L}(x)$ then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{\langle C, \bowtie^-, 1-n\rangle\}$
$(\sqcap_\rhd)$	if 1. $\langle C_1 \sqcap C_2, \rhd, n\rangle \mathcal{L}(x)$, x is not indirectly blocked, and 2. $\{\langle C_1, \rhd, n\rangle, \langle C_2, \rhd, n\rangle\} \not\subseteq \mathcal{L}(x)$ then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{\langle C_1, \rhd, n\rangle, \langle C_2, \rhd, n\rangle\}$
$(\sqcup_\lhd)$	if 1. $\langle C_1 \sqcup C_2, \lhd, n\rangle \in \mathcal{L}(x)$, x is not indirectly blocked, and 2. $\{\langle C_1, \lhd, n\rangle, \langle C_2, \lhd, n\rangle\} \not\subseteq \mathcal{L}(x)$ then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{\langle C_1, \lhd, n\rangle, \langle C_2, \lhd, n\rangle\}$
$(\sqcup_\rhd)$	if 1. $\langle C_1 \sqcup C_2, \rhd, n\rangle \in \mathcal{L}(x)$, x is not indirectly blocked, and 2. $\{\langle C_1, \rhd, n\rangle, \langle C_2, \rhd, n\rangle\} \cap \mathcal{L}(x) = \emptyset$ then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C\}$ for some $C \in \{\langle C_1, \rhd, n\rangle, \langle C_2, \rhd, n\rangle\}$
$(\sqcap_\lhd)$	if 1. $\langle C_1 \sqcap C_2, \lhd, n\rangle \mathcal{L}(x)$, x is not indirectly blocked, and 2. $\{\langle C_1, \lhd, n\rangle, \langle C_2, \lhd, n\rangle\} \cap \mathcal{L}(x) = \emptyset$ then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C\}$ for some $C \in \{\langle C_1, \lhd, n\rangle, \langle C_2, \lhd, n\rangle\}$
$(\exists_\rhd)$	if 1. $\langle \exists R.C, \rhd, n\rangle \in \mathcal{L}(x)$, x is not blocked, 2. x has no R-neighbour y connected with a triple $\langle R^*, \rhd, n\rangle$ and $\langle C, \rhd, n\rangle \in \mathcal{L}(y)$ then create a new node y with $\mathcal{L}(\langle x, y\rangle) = \{\langle R, \rhd, n\rangle\}$, $\mathcal{L}(y) = \{\langle C, \rhd, n\rangle\}$,
$(\forall_\lhd)$	if 1. $\langle \forall R.C, \lhd, n\rangle \in \mathcal{L}(x)$, x is not blocked, 2. x has no R-neighbour y connected with a triple $\langle R^*, \lhd^-, 1-n\rangle$ and $\langle C, \lhd, n\rangle \in \mathcal{L}(y)$ then create a new node y with $\mathcal{L}(\langle x, y\rangle) = \{\langle R, \lhd^-, 1-n\rangle\}$, $\mathcal{L}(y) = \{\langle C, \lhd, n\rangle\}$,
$(\forall_\rhd)$	if 1. $\langle \forall R.C, \rhd, n\rangle \in \mathcal{L}(x)$, x is not indirectly blocked, and 2. x has an R-neighbour y with $\langle C, \rhd, n\rangle \notin \mathcal{L}(y)$ and 3. $\langle R^*, \rhd^-, 1-n\rangle$ is conjugated with a positive triple that connects x and y then $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{\langle C, \rhd, n\rangle\}$,
$(\exists_\lhd)$	if 1. $\langle \exists R.C, \lhd, n\rangle \in \mathcal{L}(x)$, x is not indirectly blocked and 2. x has an R-neighbour y with $\langle C, \lhd, n\rangle \notin \mathcal{L}(y)$ and 3. $\langle R^*, \lhd, n\rangle$ is conjugated with a positive triple that connects x and y then $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{\langle C, \lhd, n\rangle\}$,
$(\forall_+)$	if 1. $\langle \forall R.C, \rhd, n\rangle \in \mathcal{L}(x)$, Trans (R), x is not indirectly blocked, and 2. x has an R-neighbour y with $\langle \forall R.C, \rhd, n\rangle \notin \mathcal{L}(y)$ and 3. $\langle R^*, \rhd^-, 1-n\rangle$ is conjugated with a positive triple that connects x and y then $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{\langle \forall R.C, \rhd, n\rangle\}$,
$(\exists_+)$	if 1. $\langle \exists R.C, \lhd, n\rangle \in \mathcal{L}(x)$, Trans (R), x is not indirectly blocked and 2. x has an R-neighbour y with $\langle \exists R.C, \lhd, n\rangle \notin \mathcal{L}(y)$ and 3. $\langle R^*, \lhd, n\rangle$ is conjugated with a positive triple that connects x and y then $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{\langle \exists R.C, \lhd, n\rangle\}$,

Table 3. Tableaux expansion rules

If nodes x and y are connected by an edge $\langle x, y\rangle$, then y is called a *successor* of x and x is called a *predecessor* of y, *ancestor* is the transitive closure of *predecessor*. A node x is called an $R-$*neighbour* of a node x if either y is a successor of x and $\mathcal{L}(\langle x, y\rangle) = \langle R, \bowtie, n\rangle$ or y is a predecessor of x and $\mathcal{L}(\langle y, x\rangle) = \langle \mathsf{Inv}(R), \bowtie, n\rangle$. We then say that the edge triple *connects* x and y to a degree of n. If we replace $\bowtie$ with $\rhd$ we get the notion of a *positive* R-neighbour and if by $\lhd$ we get that of a *negative* R-neighbour.

A node x is *blocked* if for some ancestor y, y is blocked or $\mathcal{L}(x) = \mathcal{L}(y)$. A blocked node x is *indirectly blocked* if its predecessor is blocked, otherwise it is *directly blocked*. If x is directly blocked, it has a unique ancestor y that blocks it.

The algorithm initializes a forest $\mathcal{F}_\mathcal{A}$ to contain a root node x_0^i, for each individual $a_i \in \mathbf{I}$ occurring in the *ABox* $\mathcal{A}$ and additionally $\{\langle C_i, \bowtie, n\rangle\} \cup \mathcal{L}(x_0^i)$, for each assertion of the form $\langle a_i : C_i \bowtie n\rangle$ in $\mathcal{A}$, and an edge $\langle x_0^i, x_0^j\rangle$ if $\mathcal{A}$ contains an assertion $\langle (a_i, a_j) : R_i \bowtie n\rangle$, with $\{\langle R_i, \bowtie, n\rangle\} \cup \mathcal{L}(\langle x_0^i, x_0^j\rangle)$ for each assertion of the form $\langle (a_i, a_j) : R_i \bowtie n\rangle$ in $\mathcal{A}$. $\mathcal{F}_\mathcal{A}$ is then expanded by repeatedly applying the rules from table 3. We use the notation R^* to denote either the role R or the role returned by $\mathsf{Inv}(R)$.

In description logics the notion of a *clash* is used in order to denote that a contradiction has occurred in the completion forest. In our framework a node x is said to contain a *clash* if and only if there exist two conjugated triples within a single node, or one of the following triples exists within a node:

$$\langle \bot, \geq, n\rangle,\ \langle \top, \leq, n\rangle, \text{ for } n > 0,\ n < 1 \text{ respectively}$$
$$\langle \bot, >, n\rangle,\ \langle \top, <, n\rangle$$
$$\langle C, <, 0\rangle,\ \langle C, >, 1\rangle$$

Lemma 2. *Let $\mathcal{A}$ be an f_{KD}-$\mathcal{SI}$ ABox. Then*

1. *The tableaux algorithm terminates*
2. *$\mathcal{A}$ has a tableau if and only if the expansion rules can be applied to $\mathcal{A}$ such that they yield a complete and clash-free completion forest.*

In order to demonstrate the expressive power of the extended language and its potential use in multimedia applications, like knowledge based image processing, we will consider an example.

Suppose we have the following definitions:

$\mathcal{T} = \{Vehicle \sqsubseteq \exists hasPart.Wheel\}$

$\mathcal{A} = \{\langle (C1, Axis) : hasPart \geq 0.7\rangle,$
$\langle (Axis, W1) : hasPart \geq 0.8\rangle,$
$\langle W1 : Wheel \geq 0.6\rangle\}$

with $\mathsf{Trans}(hasPart)$.

We want to find out if $KB \models \{\langle C1 : Vehicle \geq 0.6\rangle\}$. For that purpose we initialize a completion forest, as described in section 3.1, and then check for the satisfiability of the $ABox$, $\mathcal{A} \cup \{\langle C1 : \exists hasPart.Wheel < 0.6\rangle\}$, cause of the fuzzy concept inclusion axiom. The $\exists_+$-rule adds $\langle \exists hasPart.Wheel, <, 0.6\rangle$ to $\mathcal{L}(Axis)$, while subsequently the $\exists_\lhd$-rule adds $\langle Wheel, <, 0.6\rangle$ to $\mathcal{L}(W1)$. We can see that the triples $\langle Wheel, <, 0.6\rangle$ and $\langle Wheel, \geq, 0.6\rangle$ conjugate, thus our KB entails the fuzzy assertion.

4 Conclusion

In this paper we have extended the DL language $\mathcal{SI}$ with fuzzy set theory. The combination of transitive and inverse roles enable us to capture knowledge about part-whole relationships and aggregated objects. Furthermore, the incorporation of fuzziness allows us to encode and reason with vague and imprecise knowledge. Both these properties fit well into the framework of knowledge based multimedia processing where both part-whole relationships, as well as, imprecise and vague knowledge appear in applications like multimedia information retrieval and processing.

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