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A fuzzy methodology to solve the groundwater flow problem in the presence of uncertain hydrogeological data

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Abstract

Several technical and regulatory uncertainties affect the evaluation of the future performance of a High-Level radioactive Waste (HLW) repository. For example, quantitative scientific data obtained during periods of site characterization, repository operation and performance confirmation is often incomplete and imprecise. However, such data can be complemented with a significant amount of qualitative data. To date, there is no proven technique that is flexible enough to incorporate uncertain or qualitative data within the constitutive equations for some of the processes responsible for the long term evolution of the repository system. This prompted the development of an innovative methodology based on *fuzzy logic* to solve the groundwater flow equation in the presence of qualitative data. In this paper, we describe a fuzzy logic based flow model, followed by some verification results.

1 Introduction

There exist several proven techniques to handle uncertainties in geologic media (Freeze et al., 1990). However, none is readily compatible or sufficiently flexible enough to incorporate uncertain or qualitative data. As a result, conventional methods are, typically, based on hard or quantitative data which are generally assumed to be error free. A technique introduced by Journel (1986), called *soft kriging*, is capable of dealing with data of varying quality but is limited by several constraining assumptions. This, besides the relatively excessive computational burden, leads to solution of the groundwater flow problem through the use of elaborate mathematical techniques without a compatible level of parameter knowledge. It is this observation that has provided the impetus to conduct the research described in this paper.

For the purpose of this paper, we denote *hard* data as those quantitative data that are collected through instruments during tests, and *soft* data as those that are qualitative, and are imprecisely expressed in non-arithmetic linguistic form (e.g., formation has *low* permeability). In this paper, we describe an innovative methodology that is based on *fuzzy logic* to solve the groundwater flow equation in the presence of soft data. In the following sections, we describe the fuzzy logic based flow model, followed by verification experiments of the developed fuzzy flow model. Some previous applications of fuzzy logic in risk assessment include, for example, studies of uncertainty in prediction of radionuclide migration (Shaw, 1990).

2 Fuzzy Logic Based Flow Model

An appropriate set of *fuzzy rules* is crucial to the success of any fuzzy logic based system. These rules can either be derived from mathematical models or by using heuristic and common sense observations based on experience and the physics of the underlying problem. This results in a fuzzy logic based methodology that is conceptually consistent with the process of incorporating soft/qualitative data during performance assessment efforts. At present, probabilistic/stochastic models do not readily have the capability of processing this type of data because of the inherent incompatibility between the *fuzzy* nature of soft data and the *crisp* nature of these techniques. In the following, we introduce an initial set of fuzzy rules that was developed from a simple heuristic description of the physical behavior of groundwater flow.

Consider two adjacent cells with some hydraulic conductivity at their interface. Since the flow of water from one cell to the other is determined by their hydraulic head gradient and interfacial hydraulic conductivity (Darcy's law), the following fuzzy rules can be postulated:

IF the absolute value of the hydraulic head gradient between two adjacent cells is *large* **AND** their interfacial hydraulic conductivity is *high* **THEN** flow of water into the cell with the *lower* hydraulic head is *large*.

IF the absolute value of the hydraulic head gradient between two adjacent cells is *medium* **AND** their interfacial hydraulic conductivity is *very low* **THEN** flow of water into the cell with the *lower* hydraulic head is *small*.

In the above rules, the adjectives large, high, medium, etc. are referred to as *fuzzy quantifiers*. If the absolute hydraulic head gradient (∇h) between two adjacent cells, the conductivity (k) at the interface of these cells, and the flow of water (Q) into the cell with the lower head are expressed by the following 7 fuzzy sets: VVL (Very Very Low), VL (Very Low), LOW, MED (Medium), HIGH, VH (Very High) and VVH (Very Very High), then variations of the above two fuzzy rules can be used to build a fuzzy rule base which is described by Table 1.

Table 1. Fuzzy Rule Base for Q (Influx or Efflux)

	VVL	VL	LOW	MED	HIGH	VH	VVH
VVL	VVL	VVL	VVL	VL	VL	VL	VL
VL	VVL	VL	VL	VL	LOW	LOW	LOW
LOW	VVL	VL	LOW	LOW	MED	MED	HIGH
MED	VVL	VL	LOW	MED	MED	HIGH	HIGH
HIGH	VVL	VL	LOW	MED	HIGH	VH	VH
VH	VVL	LOW	MED	MED	HIGH	VH	VVH
VVH	VL	LOW	MED	HIGH	VH	VH	VVH

Each entry in Table 1 represents a fuzzy rule. For example, the fuzzy rule contained in the highlighted cell is:

IF the absolute head gradient between two adjacent cells is *LOW* **AND** the interfacial hydraulic conductivity between these cells is *MEDium* **THEN** the flow into the cell with the lower head is *LOW*.

Furthermore, a no flow boundary condition can be represented as:

IF the interfacial hydraulic conductivity is *zero* **THEN** the flow across the interface is *zero*.

Another similar rule base can be obtained by the following observation made from the mass balance equation:

IF *more* water flows in than out of the cell **THEN** the hydraulic head within this cell will *increase*.

Note that the rule base of Table 1 can be applied to obtain both flow of water into a cell (influx) and out of it (efflux). Similarly, if the influx (Q_{in}), the efflux (Q_{out}) and the hydraulic head change (Δh) within a cell are fuzzified into the following 7 fuzzy sets: LN (Large Negative), MN (Medium Negative), SN (Small Negative), Z (Zero), SP (Small Positive), MP (Medium Positive), LP (Large Positive) then variations of the aforementioned fuzzy rule will result in the following fuzzy rule base, shown in Table 2.

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Table 2. Fuzzy Rule Base for Change in Head

	LN	MN	SN	Z	SP	MP	LP
LN	Z	SP	MP	LP	LP	LP	LP
MN	SN	Z	SP	MP	LP	LP	LP
SN	MN	SN	Z	SP	MP	LP	LP
Z	LN	MN	SN	Z	SP	MP	LP
SP	LN	LN	MN	SN	Z	SP	MP
MP	LN	LN	LN	MN	SN	Z	SP
LP	LN	LN	LN	LN	MN	SN	Z

3 Testing and Verification of the Fuzzy Logic Flow Model

The fuzzy rules described in the previous section were integrated to form two fuzzy logic modules using the PC based fuzzy TIL™ shell. The two modules together defined the proposed Fuzzy **LO**gic **FLO**w **SI**mulation **M**odel (FLO^{2}SIM). The first fuzzy logic module determined influx (flow of water into a cell) and efflux (flow of water out of a cell) based on hydraulic head gradient and interfacial hydraulic conductivity information, while the second module evaluated change in hydraulic head based on the influx and efflux of each individual cell.

FLO^{2}SIM was initially tested on a single row of sequentially connected cells with the following boundary conditions: Left edge was fixed at a hydraulic head of 50 m, while a zero hydraulic head was imposed on the right edge. No flow boundary conditions were imposed on both the top and bottom of the cells. All cell interfaces were assigned the same hydraulic conductivity. This physically corresponds to a 1D saturated flow problem, analogous to a heat conduction problem. The results of the above 1D experiment were in good agreement with the hydraulic head distribution obtained from a numerical model. Figure 1 shows the hydraulic head distribution obtained by FLO^{2}SIM for the above described problem over a series of iterations. The steady-state linear distribution of the hydraulic head determined by FLO^{2}SIM corresponds to the expected solution for the above described 1D saturated flow problem. FLO^{2}SIM was also tested on 1D heterogeneous fields. There was a good agreement between the steady state distribution of hydraulic heads obtained by FLO^{2}SIM and a conventional numerical solver. These results are not presented for reasons of brevity.

FLO^{2}SIM was further tested to determine its validity in a 2D heterogeneous medium. Figure 2 shows the schematic for this 2D test problem. The region of low hydraulic conductivity was set to one-tenth that of the high hydraulic conductivity zone.

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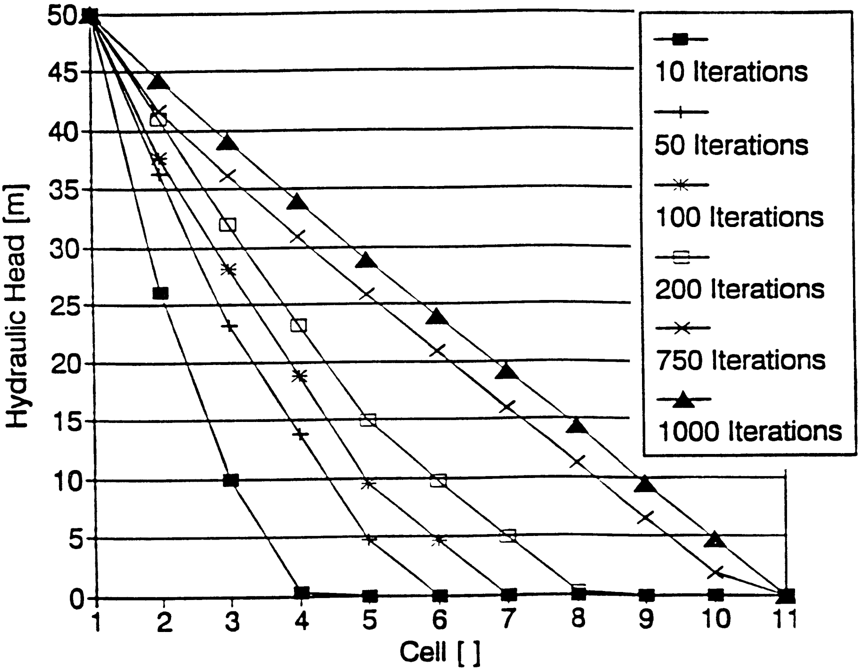

Figure 1 Hydraulic head distribution for a 1D homogeneous flow problem at 10, 50, 100, 200, 750, and 1000 iterations

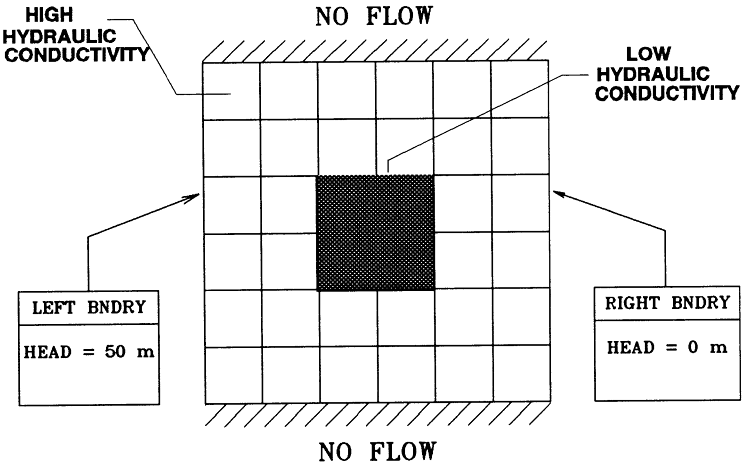

Figure 2 Schematic for the 2D test problem (heterogeneous case)

Figure 3 depicts a comparison between FLO^{2}SIM and CMVSFS, a conventional direct simulation code (Bagtzoglou et al., 1992) for a flow problem with a zone (a square of 10 x 10 cells) of a tenfold contrast in hydraulic conductivity being located in the middle of the 2D domain (a square of 25 x 25 cells) shown in Figure 2. The left and right boundary conditions as well as the no flow boundaries for this test problem are described in Figure 2. The solid lines in Figure 3 correspond to the numerical solution obtained with CMVSFS, the dashed lines correspond to the head distribution as predicted by FLO^{2}SIM, and the dotted lines corresponds to the standard deviation of the head distribution computed by FLO^{2}SIM. In general, a good agreement is observed between the outputs of FLO^{2}SIM and CMVSFS.

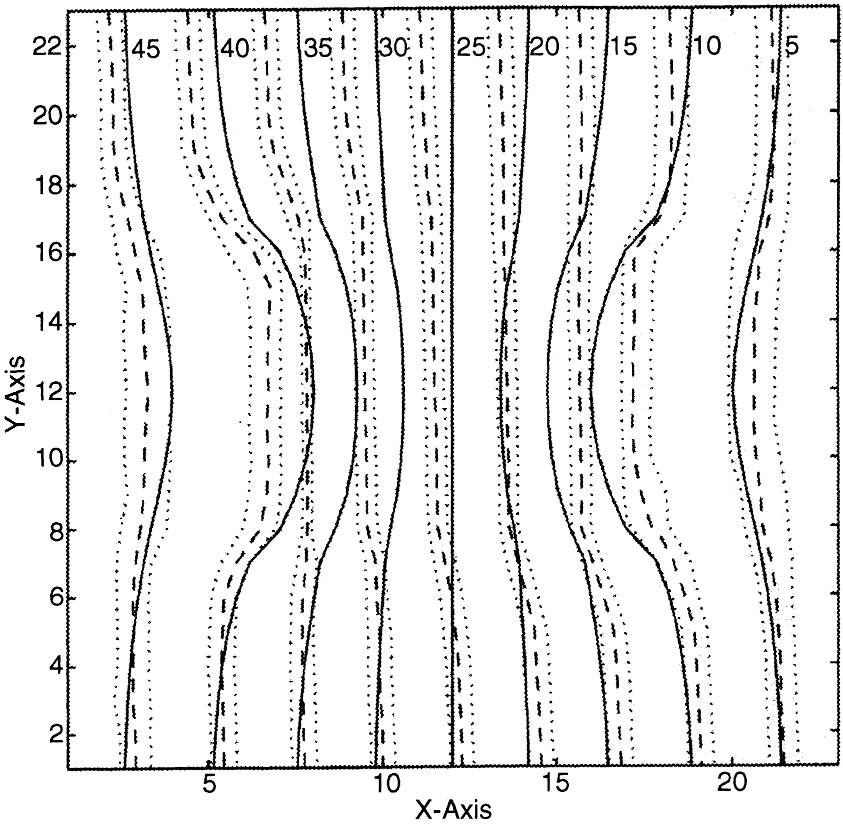

Figure 3 Comparison of hydraulic head distribution between FLO^{2}SIM (dashed) and CMVSFS (solid)

4 Conclusions

In this paper, we have introduced a flow simulation model based on fuzzy logic to solve the classical groundwater flow problem. The advantages of such a model are twofold: (i) the ability to handle imprecise or vague data, and (ii) the fuzzy flow model relies on simple fuzzy rules instead of numerically solving complex flow equations to determine hydraulic head distributions in a given domain. We have presented some computer simulation results to test the described fuzzy flow model in 1D and 2D homogeneous and heterogeneous domains. Although, FLO^{2}SIM does not exactly match the results of a conventional simulation code, it does produce a very good first approximation of the head distribution using a fraction of the CPU resources that conventional numerical codes require.

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